

$M^8 - H$ Duality and the Two Manners to Describe Particles

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Abstract

This article is part of a longer paper “TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, and Twistors”. I found it convenient to isolate the part of paper related to the notion of particle mass to a separate article. The basic new result is that $M^8 - H$ duality allows to see particles in two manners. In M^8 picture particles are massive and correspond to a fixed $M^4 \subset M^8$: in this case symmetry group is $SO(4)$: this could correspond to low energy hadron physics. In $M^4 \times CP_2$ picture particles are massless and symmetry group is $SU(3)$: this picture would correspond to high energy hadron physics with massless quarks and gluons. It is shown that p-adic mass calculations performed $M^4 \times CP_2$ picture are consistent with the masslessness of the particles: in zero energy ontology (ZEO) it is possible to have quantum superpositions of particles with different mass and this is consistent with the description of the situation in terms of p-adic thermodynamics.

Contents

| | | |
|----------|---|----------|
| 1 | Introduction | 1 |
| 1.1 | $M^8 - H$ duality and the two manners to describe particles | 1 |
| 1.2 | Option I: fixed $M_T^4 \supset M^2$ | 2 |
| 1.3 | Option II: varying $M_L^4 \supset M^2$ | 2 |
| 2 | p-Adic particle massivation and ZEO | 2 |
| 2.1 | The problem | 2 |
| 2.2 | ZEO forces p-adic particle massivation | 4 |

1 Introduction

This article is part of a longer paper “TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, and Twistors” [L2]. I found it convenient to isolate the part of paper related to the notion of particle mass to a separate article.

1.1 $M^8 - H$ duality and the two manners to describe particles

The isometry groups for $M^4 \times CP_2$ is $P \times SU(3)$ (P for Poincare group). The isometry group for $M^8 = M^4 \times E^4$ with a fixed choice of M^4 breaks down to $P \times SO(4)$. A further breaking by selection $M^4 \subset M^8$ of preferred octonionic complex plane M^2 necessary in the algebraic approach to space-time surfaces $X^4 \subset M^8$ brings in preferred rest system and reduces the Poincare symmetry further. At the space-time level the assumption that the tangent space of X^4 contains fixed M^2 or at least integral distribution of $M^2(x) \subset M^4$ is necessary for $M^8 - H$ duality [L1].

The representations $SO(4)$ and $SU(3)$ could provide alternative description of physics so that one would have what I have called $SO(4) - SU(3)$ duality [K2]. This duality could manifest in the description of strong interaction physics in terms of hadrons and quarks respectively (conserved vector current hypothesis and partially conserved axial current hypothesis based on $Spin(SO(4)) =$

$SU(2) \times SU(2)_R$. The challenge is to understand in more detail this duality. This could allow also to understand better how the two twistor descriptions might relate.

$SO(4) - SU(3)$ duality implies two descriptions for the states and scattering amplitudes.

Option I: One uses projection of 8-momenta to a fixed $M_T^4 \supset M^2$.

Option II: One assumes that $M_L^4 \supset M^2$ defines the frame in which quaternionic octonion momentum is parallel to M_L^4 : this M_L^4 depends on particle state and describes this dependence in terms of wave function in CP_2 .

1.2 Option I: fixed $M_T^4 \supset M^2$

For Option I the description would be in terms of a *fixed* $M_T^4 \subset M^8 = M_T^4 \times E^4$ and $M^2 \subset M_T^4$ fixed for both options. For given quaternionic light-like M^8 momentum one would have projection to M_T^4 , which is in general massive. E^4 momentum would have same the length squared by light-likeness.

De-localization M_T^4 mass squared equal to $p^2(M_T^4) = m^2$ in E^4 can be described in terms of $SO(4)$ harmonics at sphere having $p^2(E^4) = m^2$. This would be the description applied to hadrons and leptons and particles treated as massive particles. Particle mass would be due to the fixed choice of M_T^4 . What dictates this choice is an interesting question. An interesting question is how these descriptions relate to QFT Higgs mechanism as (in principle) alternative descriptions: the choice of fixed M_T^4 could be seen as analog for the generation of vacuum expectation of Higgs selecting preferred direction in the space of Higgs fields.

1.3 Option II: varying $M_L^4 \supset M^2$

For Option II the description would use $M_L^4 \supset M^2$, which is *not fixed* but chosen so that it contains light-like M^8 momentum. This would give light-like momentum in M_L^4 identifiable as quaternionic sub-space of complexified octonions.

1. One could assign to the state wave function function for the choices of M^4 and by quaternionicity of 8-momenta this would correspond to a state in super-conformal representation with vanishing M_L^4 mass: CP_2 point would code the information about E^4 component light-like 8-momentum. This description would apply to the partonic description of hadrons in terms of massless quarks and gluons.
2. For this option one could use the product of ordinary M^4 twistors and CP_2 twistors. One challenge would be the generalization of the twistor description to the case of CP_2 twistors.

The natural question is what this means from the point of view of p-adic particle massivation [K1]. The basic new result is that $M^8 - H$ duality allows to see particles in two manners. In M^8 picture particles are massive and correspond to a fixed $M^4 \subset M^8$: in this case symmetry group is $SO(4)$: this could correspond to low energy hadron physics. In $M^4 \times CP_2$ picture particles are massless and symmetry group is $SU(3)$: this picture would correspond to high energy hadron physics with massless quarks and gluons. It is shown that p-adic mass calculations performed $M^4 \times CP_2$ picture are consistent with the massless of the particles: in zero energy ontology (ZEO) it is possible to have quantum superpositions of particles with different mass and this is consistent with the description of the situation in terms of p-adic thermodynamics.

2 p-Adic particle massivation and ZEO

At first glance the two pictures about description of light-like M^8 momenta do not seem to be quite consistent with the recent view about TGD in which H -harmonics describe massivation of massless particles.

2.1 The problem

What looks like a problem is following.

1. The resulting particles are massive in M^4 . But they should be massless in $M^4 \times CP_2$ description. The non-vanishing mass would suggest the correct description in terms of Option I for which the description in terms of E^4 momenta with length equal to mass and thus identifiable as points of S^3 . Momentum space wave functions at S^3 - essentially rigid body wave functions given by representation matrices of $SU(2)$ could characterize the states rather than CP_2 harmonic.
2. The description based on CP_2 color partial waves however works and this would favor Option II with vanishing M^4 mass. What goes wrong?

To understand what might be involved, consider p-adic mass calculations.

1. The massivation of physical fermion states includes also the action of super-conformal generators changing the mass. The particles are originally massless and p-adic mass squared is generated thermally and mapped to real mass squared by canonical identification map.

For CP_2 spinor harmonics mass squared is of order CP_2 mass squared and thus gigantic. Therefore the mass squared is assumed to contain negative tachyonic ground state contribution due to the negative half-odd integer valued conformal weight $h_{vac} < 0$ of vacuum. The origin of this contribution has remained a mystery in p-adic thermodynamics but it makes possible to construct massless states. h_{vac} cancels the spinorial contributions and the contribution from positive conformal weights of super-conformal generators so that the particle states are massless before thermalization. This would conform with the idea of using varying M_L^4 and thus CP_2 description.

2. What does the choice of M^4 mean in terms of super-conformal representations? Could the mysterious vacuum conformal weight h_{vac} provide a description for the effect of the needed $SU(3)$ rotation of M^4 from standard orientation on super-conformal representation. The effect would be very simple and in certain sense reversal to the effect of Higgs vacuum expectation value in that it would cancel mass rather than generate it.

An important prediction would be that heavy states should be absent from the spectrum in the sense that mass squared would be p-adically of order $O(p)$ or $O(p^2)$ (in real sense of order $O(1/p)$ or $O(1/p^2)$). The trick would be that the generation of h_0 as a representation of $SU(3)$ rotation of M^4 makes always the dominating contribution to the mass of the state vanishing.

Remark: If the canonical identification I mapping the p-adic mass integers to their real numbers is of the simplest form $m = \sum_n x_n p^n \rightarrow I(m) = \sum_n x_n p^{-n}$, it can happen that the image of rational m/n with p-adic norm not larger than 1 represented as p-adic integer by expanding it in powers of p , can be near to the maximal value of p and the mass of the state can be of order CP_2 mass - about 10^{-4} Planck masses. If the canonical identification is defined as $m/n \rightarrow I(m)/I(n)$ the image of the mass is small for small values of m and n .

3. Unfortunately, it is easy to get convinced that this explanation of h_{vac} is not physically attractive. Identical mass spectra at the level of M^8 and H looks like a natural implication of $M^8 - H$ -duality. $SU(3)$ rotation of M^4 in M^8 cannot however preserve the general form of $M^4 \times CP_2$ mass squared spectrum for the M^4 projections of M^8 momenta at level of M^8 .

Remark: For $H = M^4 \times CP_2$ the mass squared in given representation of Super-conformal symmetries is given as a sum of CP_2 mass squared for the spinor harmonic determining the ground state and of a Virasoro contribution proportional to a non-negative integer. The masses are required to independent of electroweak quantum numbers.

One can imagine two further identifications for the origin of h_{vac} .

1. Take seriously the possibility of complex momenta allowed by the complexification of M^8 by commuting imagine unit i and also suggested by the generalization of the twistorialization. The real and imaginary parts of light-like complex 8-momenta $p_8 = p_{8,Re} + ip_{8,Im}$ are orthogonal to each other: $p_{8,Re} \cdot p_{8,Im} = 0$ so that both real and imaginary parts of p_8 are light-like: $p_{8,Re}^2 = p_{8,Im}^2 = 0$. The M^4 mass squared can be written has $m^2 = m_{Re}^2 - m_{Im}^2$

with $h_{vac} \propto -m_{Im}^2$. The representations of Super-conformal algebra would be labelled by $h_{vac} \propto m_{Im}^2$.

Could the needed wrong sign contribution to CP_2 mass squared mass make sense? CP_2 type extremals having light-like geodesic as M^4 projection are locally identical with CP_2 but because of light-like projection they can be regarded as CP_2 with a hole and thus non-compact. Boundary conditions allow analogs of CP_2 harmonics for which spinor d'Alembertian would have complex eigenvalues.

Does quantum-classical correspondence allow complex momenta: can the classical four-momenta assignable to 6-D Kähler action be complex? The value of Kähler coupling strength allows the action to have complex phase but parts with different phases are not allowed. Could the imaginary part to 4-momentum could come from the CP_2 type extremal with Euclidian signature of metric?

2. Second - not necessarily independent - idea relies on the observation that in TGD one has besides the usual conformal algebra acting on complex coordinate z also its analog acting on the light-like radial coordinate r of light-cone boundary. At light-cone boundary one has both super-symplectic symmetries of $\Delta M_+^4 \times CP_2$ and extension of super-conformal symmetries of sphere S^2 to analogs of conformal symmetries depending on z and r and it seems that one must chose between these two options. Also the extension of ordinary Kac-Moody algebra acts at the light-like orbits of partonic 2-surfaces.

There are two scaling generators: the usual $L_0 = zd/dz$ and the second generator $L_{0,1} = ird/dr$. For $L_{0,1}$ at light-cone boundary powers of z^n are replaced with $(r/r_0)^{ik} = exp(iku)$, $u = log(r/r_0)$. Could it be that mass squared operator is proportional to $L_0 + L_{0,1}$ having eigenvalues $h = n - k$? The absence of tachyons requires $h \geq 0$. Could k characterize given Super-Virasoro representation? Could $k \geq 0$ serve as an analog of positive energy condition allowing to analytically continue $exp(iku)$ to upper u -plane? How to interpret this continuation?

The 3-D generalization of super-symplectic symmetries at light-cone boundary and extended Ka-Moody symmetries at partonic 2-surfaces should be possible in some sense. Could the continuation to the upper u -plane correspond to the continuation of the extended conformal symmetries from light-cone boundary to future light-one and from light-partonic 2-surfaces to space-time interior?

2.2 ZEO forces p-adic particle massivation

Why p-adic massivation should occur at all? Here ZEO comes in rescue.

1. In ZEO one can have superposition of states with different 4-momenta, mass values and also other charges: this does not break conservation laws. How to fix M^4 in this case? One cannot do it separately for the states in superposition since they have different masses. The most natural choices is as the M^4 associated with the dominating contribution to the zero energy state. The outcome would be thermal massivation described excellently by p-adic thermodynamics [K1]. Recently a considerable increase in the understanding of hadron and weak boson masses took place [L3].
2. In ZEO quantum theory is square root of thermodynamics in a well-defined formal sense, and one can indeed assign to p-adic partition function a complex square root as a genuine zero energy state. Since mass varies, one must describe the presence of higher mass excitations in zero energy state as particles in M^4 assigned with the dominating part of the state so that the observed particle mass squared is essentially p-adic thermal expectation value over thermal excitations. p-Adic thermodynamics would thus describe the fact that the choice of M_L^4 cannot not ideal in ZEO and massivation would be possible only in ZEO.
3. Current quarks and constituent quarks are basic notions of hadron physics. Constituent quarks with rather large masses appear in the low energy description of hadrons and current quarks in high energy description of hadronic reactions. That both notions work looks rather paradoxical. Could massive quarks correspond to M_T picture and current quarks to M_L^4

picture but with p -adic thermodynamics forced by the superposition of mass eigenstates with different masses.

The massivation of ordinary massless fermion involves mixing of fermion chiralities. This means that the $SU(3)$ rotation determined by the dominating component in zero energy state must induce this mixing. This should be understood.

REFERENCES

Books related to TGD

- [K1] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: <http://tgdtheory.fi/pdfpool/mless.pdf>, 2006.
- [K2] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Online book. Available at: <http://tgdtheory.fi/pdfpool/visionb.pdf>, 2006.

Articles about TGD

- [L1] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? Available at: http://tgdtheory.fi/public_html/articles/ratpoints.pdf, 2017.
- [L2] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors. Available at: http://tgdtheory.fi/public_html/articles/McKaygeneral.pdf, 2019.
- [L3] Pitkänen M. Two anomalies of hadron physics from TGD perspective. Available at: http://tgdtheory.fi/public_html/articles/twoanomalies.pdf, 2019.