Number Theoretical Feats and TGD Inspired Theory of Consciousness

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.

April 25, 2016

Contents

Abstract

Number theoretical feats of some mathematicians like Ramanujan remain a mystery for those believing that brain acts as a classical computer. Also the ability of idiot savants - lacking even the idea about what prime is - to factorize integers to primes challenges the idea that an algorithm is involved. In this article I discuss some ideas about how various arithmetical feats such as partitioning an integer to a sum of integers and to a product of prime factors might take place. The ideas are inspired by the number theoretic vision about TGD suggesting that basic arithmetics might be realized as naturally occurring processes at quantum level and the outcomes might be "sensorily perceived". I will also discuss how quantum computations could be carried out fast in zero energy ontology (ZEO) using Ramanujan as example.

1 Introduction

Number theoretical feats of some mathematicians like Ramanujan remain a mystery for those believing that brain is a classical computer. Also the ability of idiot savants - lacking even the idea about what prime is - to factorize integers to primes challenges the idea that an algorithm is involved. In this article I discuss ideas about how various arithmetical feats such as partitioning integer to a sum of integers and to a product of prime factors might take place. The ideas are inspired by the number theoretic vision about TGD suggesting that basic arithmetics might be realized as naturally occurring processes at quantum level and the outcomes might be "sensorily perceived". One can also ask whether zero energy ontology (ZEO) could allow to perform quantum computations in polynomial instead of exponential time.

The indian mathematician Srinivasa Ramanujan is perhaps the most well-known example about a mathematician with miraculous gifts. He told immediately answers to difficult mathematical questions - ordinary mortals had to to hard computational work to check that the answer was right. Many of the extremely intricate mathematical formulas of Ramanujan have been proved much later by using advanced number theory. Ramanujan told that he got the answers from his personal Goddess. A possible TGD based explanation of this feat relies on the idea that in zero energy ontology (ZEO) quantum computation like activity could consist of steps consisting quantum computation and its time reversal with long-lasting part of each step performed in reverse time direction at opposite boundary of causal diamond so that the net time used would be short at second boundary.

The adelic picture about state function reduction in ZEO suggests that it might be possible to have direct sensory experience about prime factorization of integers [L2]. What about partitions of integers to sums of primes? For years ago I proposed that symplectic QFT is an essential part of TGD. The basic observation was that one can assign to polygons of partonic 2-surface - say geodesic triangles - Kähler magnetic fluxes defining symplectic invariance identifiable as zero modes. This assignment makes sense also for string world sheets and gives rise to what is usually called Abelian Wilson line. I could not specify at that time how to select these polygons. A very natural manner to fix the vertices of polygon (or polygons) is to assume that they correspond ends of fermion lines which appear as boundaries of string world sheets. The polygons would be fixed rather uniquely by requiring that fermions reside at their vertices.

The number 1 is the only prime for addition so that the analog of prime factorization for sum is not of much use. Polygons with $n = 3, 4, 5$ vertices are special in that one cannot decompose them to non-degenerate polygons. Non-degenerate polygons also represent integers $n > 2$. This inspires the idea about numbers $\{3, 4, 5\}$ as "additive primes" for integers $n > 2$ representable as non-degenerate polygons. These polygons could be associated many-fermion states with negentropic entanglement (NE) - this notion relate to cognition and conscious information and is something totally new from standard physics point of view. This inspires also a conjecture about a deep connection with arithmetic consciousness: polygons would define conscious representations for integers $n > 2$. The splicings of polygons to smaller ones could be dynamical quantum processes behind arithmetic conscious processes involving addition.

2 How Ramanujan did it?

Lubos Motl wrote recently a blog posting (`http://tinyurl.com/zduu72p`) about $P \neq NP$ computer in the theory of computation based on Turing's work. This unproven conjecture relies on a classical model of computation developed by formulating mathematically what the women doing the hard computational work in offices at the time of Turing did. Turing's model is extremely beautiful mathematical abstraction of something very every-daily but does not involve fundamental physics in any manner so that it must be taken with caution. The basic notions include those of algorithm and recursive function, and the mathematics used in the model is mathematics of integers. Nothing is assumed about what conscious computation is and it is somewhat ironic that this model has been taken by strong AI people as a model of consciousness!

1. A canonical model for classical computation is in terms of Turing machine, which has bit sequence as inputs and transforms them to outputs and each step changes its internal state. A more concrete model is in terms of a network of gates representing basic operations for the incoming bits: from this basic functions one constructs all recursive functions. The computer and program actualize the algorithm represented as a computer program and eventually halts - at least one can hope that it does so. Assuming that the elementary operations require some minimum time, one can estimate the number of steps required and get an estimate for the dependence of the computation time as function of the size of computation.

2. If the time required by a computation, whose size is characterized by the number N of relevant bits, can be carried in time proportional to some power of N and is thus polynomial, one says that computation is in class P. Non-polynomial computation in class NP would correspond to a computation time increasing with N faster than any power of N, say exponentially. Donald Knuth, whose name is familiar for everyone using Latex to produce mathematical text, believes on $P = NP$ in the framework of classical computation. Lubos in turn thinks

that the Turing model is probably too primitive and that quantum physics based model is needed and this might allow $P = NP$.

What about quantum computation as we understand it in the recent quantum physics: can it achieve $P = NP$?

1. Quantum computation is often compared to a superposition of classical computations and this might encourage to think that this could make it much more effective but this does not seem to be the case. Note however that the amount of information represents by N qubits is however exponentially larger than that represented by N classical bits since entanglement is possible. The prevailing wisdom seems to be that in some situations quantum computation can be faster than the classical one but that if $P = NP$ holds true for classical computation, it holds true also for quantum computations. Presumably because the model of quantum computation begins from the classical model and only (quantum computer scientists must experience this statement as an insult - apologies!) replaces bits with qubits.

2. In quantum computer one replaces bits with entangled qubits and gates with quantum gates and computation corresponds to a unitary time evolution with respect to a discretized time parameter constructed in terms of fundamental simple building bricks. So called tensor networks realize the idea of local unitary in a nice manner and has been proposed to defined error correcting quantum codes. State function reduction halts the computation. The outcome is non-deterministic but one can perform large number of computations and deduce from the distribution of outcomes the results of computation.

What about conscious computations? Or more generally, conscious information processing. Could it proceed faster than computation in these sense of Turing? To answer this question one must first try to understand what conscious information processing might be. TGD inspired theory of consciousnesss provides one a possible answer to the question involving not only quantum physics but also new quantum physics.

1. In TGD framework Zero energy ontology (ZEO) replaces ordinary positive energy ontology and forces to generalize the theory of quantum measurement. This brings in several new elements. In particular, state function reductions can occur at both boundaries of causal diamond (CD), which is intersection of future and past direct light-cones and defines a geometric correlate for self. Selves for a fractal hierarchy - CDs within CDs and maybe also overlapping. Negentropy Maximization Principle (NMP) is the basic variational principle of consciousness and tells that the state function reductions generate maximum amount of conscious information. The notion of negentropic entanglement (NE) involving p-adic physics as physics of cognition and hierarchy of Planck constants assigned with dark matter are also central elements.

2. NMP allows a sequence of state function reductions to occur at given boundary of diamond-like CD - call it passive boundary. The state function reduction sequence leaving everything unchanged at the passive boundary of CD defines self as a generalized Zeno effect. Each step shifts the opposite - active - boundary of CD "upwards" and increases its distance from the passive boundary. Also the states at it change and one has the counterpart of unitary time evolution. The shifting of the active boundary gives rise to the experienced time flow and sensory input generating cognitive mental images - the "Maya" aspect of conscious experienced. Passive boundary corresponds to permanent unchanging "Self".

3. Eventually NMP forces the first reduction to the opposite boundary to occur. Self dies and reincarnates as a time reversed self. The opposite boundary of CD would be now shifting "downwards" and increasing CD size further. At the next reduction to opposite boundary re-incarnation of self in the geometric future of the original self would occur. This would be re-incarnation in the sense of Eastern philosophies. It would make sense to wonder whose incarnation in geometric past I might represent!

Could this allow to perform fast quantal computations by decomposing the computation to a sequence in which one proceeds in both directions of time? Could the incredible feats of some "human computers" rely on this quantum mechanism (see http://tinyurl.com/hk5baty). The indian mathematician Srinivasa Ramanujan (see https://en.wikipedia.org/wiki/Srinivasa_Ramanujan) is the most well-known example of a mathematician with miraculous gifts. He told immediately answers to difficult mathematical questions - ordinary mortals had to to hard computational work to check that the answer was right. Many of the extremely intricate mathematical formulas of Ramanujan have been proved much later by using advanced number theory. Ramanujan told that he got the answers from his personal Goddess.

Might it be possible in ZEO to perform quantally computations requiring classically non-polynomial time much faster - even in polynomial time? If this were the case, one might at least try to understand how Ramanujan did it although higher levels selves might be involved also (did his Goddess do the job?).

1. Quantal computation would correspond to a state function reduction sequence at fixed boundary of CD defining a mathematical mental image as sub-self. In the first reduction to the opposite boundary of CD sub-self representing mathematical mental image would die and quantum computation would halt. A new computation at opposite boundary proceeding to opposite direction of geometric time would begin and define a time-reversed mathematical mental image. This sequence of reincarnations of sub-self as its time reversal could give rise to a sequence of quantum computation like processes taking less time than usually since one half of computations would take place at the opposite boundary to opposite time direction (the size of CD increases as the boundary shifts).

2. If the average computation time is same at both boundaries, the computation time would be only halved. Not very impressive. However, if the mental images at second boundary - call it A - are short-lived and the selves at opposite boundary B are very long-lived and represent very long computations, the process could be very fast from the point of view of A! Could one overcome the $P \neq NP$ constraint by performing computations during time-reversed re-incarnations?! Short living mental images at this boundary and very long-lived mental images at the opposite boundary - could this be the secret of Ramanujan?

3. Was the Goddess of Ramanujan - self at higher level of self-hierarchy - nothing but a time reversal for some mathematical mental image of Ramanujan (Brahman=Atman!), representing very long quantal computations! We have night-day cycle of personal consciousness and it could correspond to a sequence of re-incarnations at some level of our personal self-hierarchy. Ramanujan tells that he met his Goddess in dreams. Was his Goddess the time reversal of that part of Ramanujan, which was unconscious when Ramanujan slept? Intriguingly, Ramanujan was rather short-lived himself - he died at the age of 32! In fact, many geniuses have been rather short-lived.

4. Why the alter ego of Ramanujan was Goddess? Jung intuited that our psyche has two aspects: anima and animus. Do they quite universally correspond to self and its time reversal? Do our mental images have gender?! Could our self-hierarchy be a hierarchical collection of anima and animi so that gender would be something much deeper than biological sex! And what about Yin-Yang duality of Chinese philosophy and the ka as the shadow of persona in the mythology of ancient Egypt?

3 Symplectic QFT, $\{3, 4, 5\}$ as Additive Primes, and Arithmetic Consciousness

For years ago I proposed that symplectic QFT is an essential part of TGD [K1, K6]. The basic observation was that one can assign to polygons of partonic 2-surface - say geodesic triangles - Kähler magnetic fluxes defining symplectic invariance identifiable as zero modes. This assignment makes sense also for string world sheets and gives rise to what is usually called Abelian Wilson line. I could not specify at that time how to select these polygons in the case of partonic 2-surfaces.

The recent proposal of Maldacena and Arkani-Hamed [B1] (see https://arxiv.org/pdf/1503.08043v1) that CMB might contain signature of inflationary cosmology as triangles and polygons for which the magnitude of n-point correlation function is enhanced led to a progress in this respect. In the proposal of Maldacena and Arkani-Hamed the polygons are defined by momentum conservation. Now the polygons would be fixed rather uniquely by requiring that fermions reside at their vertices and momentum conservation is not involved.

This inspires the idea about numbers $\{3,4,5\}$ as "additive primes" for integers $n > 2$ representable as non-degenerate polygons. Geometrically one could speak of prime polygons not decomposable to lower non-degenerate polygons. These polygons are different from those of Maldacena and Arkani-Hamed and would be associated many-fermion states with negentropic entanglement (NE) - this notion relates to cognition and conscious information and is something totally new from standard physics point of view. This inspires also a conjecture about a deep connection with arithmetic consciousness: polygons would define representations for integers $n > 2$. The splicings of polygons to smaller ones could be dynamical quantum processes behind arithmetic conscious processes involving addition. I have already earlier considered a possible counterpart for conscious prime factorization in the adelic framework [L2].

3.1 Basic ideas of TGD inspired theory of conscious very briefly

Negentropy Maximization Principle (NMP) is the variational principle of consciousness in TGD framework. It says that negentropy gain in state function reduction (quantum jump re-creating Universe) is maximal. State function reduction is basically quantum measurement in standard QM and sensory qualia (for instance) could be perhaps understood as quantum numbers of state resulting in state function reduction. NMP poses conditions on whether this reduction can occur. In standard ontology it would occur always when the state is entangled: reduction would destroy the entanglement and minimize entanglement entropy. When cognition is brought in, the situation changes.

The first challenge is to define what negentropic entanglement (NE) and negentropy could mean.

1. In real physics without cognition one does not have any definition of negentropy: on must define negentropy as reduction of entropy resulting as conscious entity gains information. This kind of definition is circular in consciousness theory.

2. In p-adic physics one can define number theoretic entanglement entropy with same basic properties as ordinary Shannon entropy. For some p-adic number fields this entropy can be negative and this motivates an interpretation as conscious information related to entanglement - rather to the ignorance of external observer about entangled state. The prerequisite is that the entanglement probabilities belong to an an extension of rationals inducing a finite-dimensional extension of rationals. Algebraic extensions are such extensions as also those generate by a root of e (e^p is p-adic number in Q_p).

A crucial step is to fuse together sensory and cognitive worlds as different aspects of existence.

1. One must replace real universe with adelic one so that one has real space-time surfaces and their p-adic variants for various primes p satisfying identical field equations. These are related by strong form of holography (SH) in which 2-D surfaces (string world sheets and partonic 2-surfaces) serve as "space-time genes" and obey equations which make sense both p-adically in real sense so that one can identify them as points of "world of classical worlds" (WCW).

2. One can say that these 2-surfaces belong to intersection of realities and p-adicities - intersection of sensory and cognitive. This demands that the parameters appearing in the equations for 2-surface belong algebraic extension of rational numbers: the interpretation is that this hierarchy of extensions corresponds to evolutionary hierarchy. This also explains imagination in terms of the p-adic space-time surfaces which are not so unique as the real one because of inherent non-determinism of p-adic differental equations. What can be imagined cannot be necessarily realized. You can continued p-adic 2-surface to 4-D surface but not to real one.

There is also second key assumption involved.

1. Hilbert space of quantum states is *same* for real and p-adic sectors of adelic world: for instance, tensor product would lead to total nonsense since there would be both real and

p-adic fermions. This means same quantum state and same entanglement but seen from sensory and various cognitive perspectives. This is the basic idea of adelicity: the p-adic norms of rational number characterize the norm of rational number. Now various p-adic conscious experiences characterize the quantum state.

2. Real perspective sees entanglement always as entropic. For some finite number number of primes p p-adic entanglement is however negentropic. For instance, for entanglement probabilities $p_i = 1/N$, the primes appearing as factors of N are such information carrying primes. The presence of these primes can make the entanglement stable. The total entropy equal to the sum of negative real negentropy + various p-adic negentropies can be positive and cannot be reduced in the reduction so that reduction does not occur at all! Entanglement is stabilized by cognition and the randomness of state function reduction tamed: matter has power over matter!

3. There is analogy with the reductionism-holism dichotomy. Real number based view is reductionistic: information is obtained when the entangled state is split into un-entangled part. p-Adic number based view is holistic: information is inthe negentropic entanglement and can be seen as abstraction or rule. The superposition of state pairs represents a rule with state pairs (a_i, b_i) representing the instance of the rule A↔B. Maximal entanglement defined by entanglement probabilities $p_i = 1/N$ makes clear the profound distinction between these views. In real sector the negentropy is negative and smallest possible. In p-adic sector the negentropy is maximum for p-adic primes appearing as factors of N and total negentropy as their sum is large. NE allows to select unique state basis if the probabilities p_i are different.

   For $p_i = 1/N$ one can choose any unitary related state basis since unit matrix is invariant under unitary transformations. From the real point of view the ignorance is maximal and entanglement entropy is indeed maximal. For instance, in case of Schrödinger cat one could choose the cat's state basis to be any superposition of dead and alive cat and a state orthogonal to it. From p-adic view information is maximal. The reports of meditators, in particular Zen buddhists, support this interpretation. In "enlightened state" all discriminations disappear: it does not make sense to speak about dead or alive cat or anything between these two options. The state contains information about entire state - not about its parts. It is not information expressible using language relying on making of distinctions but silent wisdom.

3.2 How do polygons emerge in TGD framework?

The duality defined by strong form of holography (SH) has 2 sides. Space-time side (bulk) and boundary side (string world sheets and partonic 2-surfaces). 2-D half of SH would suggest a description based on string world sheets and partonic 2-surfaces. This description should be especially simple for the quantum states realized as spinor fields in WCW ("world of classical worlds"). The spinors (as opposed to spinor fields) are now fermionic Fock states assignable to space-time surface defining a point of WCW. TGD extends ordinary 2-D conformal invariance to super-symplectic symmetry applying at the boundary of light-cone: note that given boundary of causal diamond (CD) is contained by light-cone boundary.

1. The correlation functions at imbedding space level for fundamental objects, which are fermions at partonic 2-surfaces could be calculated by applying super-symplectic invariance having conformal structure. I have made rather concrete proposals in this respect. For instance, I have suggested that the conformal weights for the generators of supersymplectic algebra are given by poles of fermionic zeta $\zeta_F(s) = \zeta(s)/\zeta(2s)$ and thus include zeros of zeta scaled down by factor 1/2 [K3]. A related proposal is conformal confinement guaranteeing the reality of net conformal weights.

2. The conformally invariant correlation functions are those of super-symplectic CFT at light-cone boundary or its extension to CD. There would be the analog of conformal invariance associated with the light-like radial coordinate r_M and symplectic invariance associated with CP_2 and sphere S^2 localized with respect to r_M analogous to the complex coordinate in ordinary conformal invariance and naturally continued to hypercomplex coordinate at string

world sheets carrying the fermionic modes and together with partonic 2-surfaces defining the boundary part of SH.

Symplectic invariants emerge in the following manner. Positive and negative energy parts of zero energy states would also depend on zero modes defined by super-symplectic invariants and this brings in polygons. Polygons emerge also from four-momentum conservation. These of course are also now present and involve the product of Lorentz group and color group assignable to CD near its either boundary. It seems that the extension of Poincare translations to Kac-Moody type symmetry allows to have full Poincare invariance (in its interior CD looks locally like $M^4 \times CP_2$).

1. One can define the symplectic invariants as magnetic fluxes associated with S^2 and CP_2 Kähler forms. For string world sheets one would obtain non-integrable phase factors. The vertices of polygons defined by string world sheets would correspond to the intersections of the string world sheets with partonic 2-surfaces at the boundaries of CD and at partonic 2-surfaces defining generalized vertices at which 3 light-like 3-surfaces meet along their ends.

2. Any polygon at partonic 2-surface would also allow to define such invariants. A physically natural assumption is that the vertices of these polygons are realized physically by adding fermions or antifermions at them. Kähler fluxes can be expressed in terms of non-integrable phase factors associated with the edges. This assumption would give the desired connection with quantum physics and fix highly uniquely but not completely the invariants appearing in physical states.

The correlated polygons would be thus naturally associated with fundamental fermions and a better analogy would be negentropically entangled n-fermion state rather than corresponding to maximum of the modulus of n-point correlation function. Hierarchy of Planck constants makes these states possible even in cosmological scales. The point would be that negentropic entanglement assignable to the p-adic sectors of WCW would be in key role.

3.3 Symplectic invariants and Abelian non-integrable phase factors

Consider now the polygons assignable to many-fermion states at partonic 2-surfaces.

1. The polygon associated with a given set of vertices defined by the position of fermions is far from unique and different polygons correspond to different physical situations. Certainly one must require that the geodesic polygon is not self-intersecting and defines a polygon or set of polygons.

2. Geometrically the polygon is not unique unless it is convex. For instance, one can take regular n-gon and add one vertex to its interior. The polygon can be also constructed in several manners. From this one obtains a non-convex $n+1$-gon in $n+1$ manners.

3. Given polygon is analogous with Hamiltonian cycle connecting all points of given graph. Now one does not have graph structure with edges and vertices unless one defines it by nearest neighbor property. Platonic solids provide an example of this kind of situation. Hamiltonian cycles [A1, A2] are key element in the TGD inspired model for music harmony leading also to a model of genetic code [K5] [L1].

4. One should somehow fix the edges of the polygon. For string world sheets the edges would be boundaries of string world sheet. For partonic 2-surfaces the simplest option is that the edges are geodesic lines and thus have shortest possible length. This would bring in metric so that the idea about TGD as almost topological QFT would be realized.

One can distinguish between two cases: single polygon or several polygons.

1. One has maximal entanglement between fundamental fermions, when the vertices define single polygon. One can however have several polygons for a given set of vertices and in this case the coherence is reduced. Minimal correlations correspond to maximal number of 3-gons and minimal number of 4-gons and 5-gons.

2. For large $h_{eff} = n \times h$ the partonic 2-surfaces can have macroscopic and even astrophysical size and one can consider assigning many-fermion states with them. For instance, anyonic states could be interpreted in this manner. In this case it would be natural to consider various decompositions of the state to polygons representing entangled fermions.

The definition of symplectic invariant depends on whether one has single polygon or several polygons.

1. In the case that there are several polygons not containing polygons inside them (if this the case,then the complement of polygon must satisfy the condition) one can uniquely identify the interior of each polygon and assign a flux with it. Non-integrable phase factor is well-defined now. If there is only single polygon then also the complement of polygon could define the flux. Polygon and its complement define fluxes Φ and $\Phi_{tot} - \Phi$.

2. If partonic 2-surface carries monopole Kähler charge Φ_{tot} is essentially $n\pi$, where n is magnetic monopole flux through the partonic 2-surface. This is half integer - not integer: this is key feature of TGD and forces the coupling of Kähler gauge potential to the spinors leading to the quantum number spectrum of standard model. The exponent can be equal to -1 for half-odd integer.

   This problem disappears if both throats of the wormhole contact connecting the space-time sheets with Minkowski signature give their contribution so that two minus-signs give one plus sign. Elementary particles necessarily consist of wormhole contacts through which monopole flux flows and runs along second space-time sheet to another contact and returns along second space-time sheet so that closed monopole flux tube is obtained. The function of the flux must be single valued. This demands that it must reduce to the cosine of the integer multiple of the flux and identifiable as as the real part of the integer power of magnetic flux through the polygon.

The number theoretically deepest point is geometrically completely trivial.

1. Only $n > 2$-gons are non-degenerate and 3-, 4- and 5-gons are prime polygons in the sense that they cannot be sliced to lower polygons. Already 6-gon decomposes to 2 triangles.

2. One can wonder whether the appearance of 3 prime polygons might relate to family replication phenomenon for which TGD suggests an explanation in terms of genus of the partonic 2-surface [K2]. This does not seem to be the case. There is however other three special integers: namely 0,1, and 2.

   The connection with family replication phenomenon could be following. When the number of handles at the parton surface exceeds 2, the system forms entangled/bound states describable in terms of polygons with handles at vertices. This would be kind of phase transition. Fundamental fermion families with handle number 0,1,2 would be analogous to integers 0,1,2 and the anyonic many-handle states with NE would be analogous to partitions of integers $n > 2$ represented by the prime polygons. They would correspond to the emergence of p-adic cognition. One could not assign NE and cognition with elementary particles but only to more complex objects such as anyonic states associated with large partonic 2-surfaces (perhaps large because they have large Planck constant $h_{eff} = n \times h$) [K4].

3.4 Integers $(3,4,5)$ as “additive primes” for integers $n \geq 3$: a connection with arithmetic consciousness

The above observations encourage a more detailed study of the decomposition of polygons to smaller polygons as a geometric representation for the partition of integers to a sum of smaller integers. The idea about integers $(3,4,5)$ as “additive primes” represented by prime polygons is especially attractive. This leads to a conjecture about NE associated with polygons as quantum correlates of arithmetic consciousness.

3.4.1 Motivations

The key idea is to look whether the notion of divisibility and primeness could have practical value in additive arithmetics. 1 is the only prime for addition in general case. $n = 1 + 1 + ...$ is analogous to p^n and all integers are "additive powers" of 1.

What happens if one considers integers $n \geq 3$? The basic motivation is that $n \geq 3$ is represented as a non-degenerate n-gon for $n \geq 3$. Therefore geometric representation of these primes is used in the following. One cannot split triangles from 4-gon and 5-gon. But already for 6-gon one can and obtains 2 triangles. Thus $\{3, 4, 5\}$ would be the additive primes for $n \geq 3$ represented as prime polygons.

The n-gons with $n \in \{3, 4, 5\}$ appear as faces of the Platonic solids! The inclusions of von Neumann algebras known as hyperfinite factors of type II_1 central in TGDs correspond to quantum phases $exp(\pi/n)$ $n = 3, 4, 5....$ Platonic solids correspond to particular finite subgroups of 3-D rotation group, which are in one-one correspondence with simply laced Lie-groups (ADE). There is also a direct connection with the classification of $\mathcal{N} = 2$ super-conformal theories, which seem to be relevant for TGD.

I cannot resist the temptation to mention also a personal reminiscence about a long lasting altered state of consciousness about 3 decades ago. I called it Great Experience and it boosted among other things serious work in order to understand consciousness in terms of quantum physics. One of the mathematical visions was that number 3 is in some sense fundamental for physics and mathematics. I also precognized infinite primes and much later indeed discovered them. I have repeatedly returned to the precognition about number 3 but found no really convincing reason for its unique role although it pops up again and again in physics and mathematics: 3 particle families, 3 colors for quarks, 3 spatial dimensions, 3 quaternionic imaginaryunits, triality for octonions, to say nothing about the role of trinity in mystics and religions. The following provides the first argument for the special role of number 3 that I can take seriously.

3.4.2 Partition of integer to additive primes

The problem is to find a partition of an integer to additive primes $3, 4, 5$. The problem can be solved using a representation in terms of $n > 2$-gons as a geometrical visualization. Some general aspects of the representation.

1. The detailed shape of n-gons in the geometric representation of partitions does not matter: they just represent geometrically a partition of integer to a sum. The partition can be regarded as a dynamical process. n-gons splits to smaller n-gons producing a representation for a partition $n = \sum_i n_i$. What this means is easiest to grasp by imagining how polygon can be decomposed to smaller ones. Interestingly, the decompositions of polytopes to smaller ones - triangulations - appear also in Grassmannian twistor approach to $\mathcal{N} = 4$ super Yang Mills theory.

2. For a given partition the decomposition to n-gons is not unique. For instance, integer 12 can be represented by 3 4-gons or 4 3-gons. Integers $n \in \{3, 4, 5\}$ are special and partitions to these n-gons are in some sense maximal leading to a maximal decoherence as quantum physicist might say.

   The partitions are not unique and there is large number of partitions involving 3-gons,4-gons, 5-gons. The reason is that one can split from n-gons any n_1-gon with $n_1 < n$ except for $n = 3, 4, 5$.

3. The daydream of non-mathematician not knowing that everything has been very probably done for aeons ago is that one could chose n_1 to be indivisible by 4 and 5, n_2 indivisible by 3 and 5 and n_3 indivisible by 3 and 4 so that one might even hope for having a unique partition. For instance, double modding by 4 and 5 would reduce to double modding of $n_1 \times 3$ giving a non-vanishing result, and one might hope that n_1, n_2 and n_3 could be determined from the double modded values of n_i uniquely. Note that for $n_i \in \{1, 2\}$ the number $n = 24 = 2 \times 3 + 2 \times 4 + 2 \times 5$ playing key role in string model related mathematics is the largest integer having this kind of representation. One should numerically check whether any general

orbit characterized by the above formulas contains a point satisfying the additional number theoretic conditions.

Therefore the task is to find partitions satisfying these indivisibility conditions. It is however reasonable to consider first general partitions.

4. By linearity the task of finding general partitions (forgetting divisibility conditions) is analogous to that of finding of solutions of non-homogenous linear equations. Suppose that one has found a partition

$$n = n_1 \times 3 + n_2 \times 4 + n_3 \times 5 \leftrightarrow (n_1, n_2, n_3) \quad . \tag{3.1}$$

This serves as the analog for the special solution of non-homogenous equation. One obtains a general solutions of equation as the sum $(n_1 + k_1, n_2 + k_1, n_3 + k_3)$ of the special solution and general solution of homogenous equation

$$k_1 \times 3 + k_2 \times 4 + k_3 \times 5 = 0 \quad . \tag{3.2}$$

This is equation of plane in N^3 - 3-D integer lattice.

Using $4 = 3 + 1$ and $5 = 3 + 2$ this gives equations

$$k_2 + 2 \times k_3 = 3 \times m \quad , \quad k_1 - k_3 + 4 \times m = 0 \quad , \quad m = 0, 1, 2, ... \tag{3.3}$$

5. There is periodicity of $3 \times 4 \times 5 = 60$. If (k_1, k_2, k_3, m) is allowed deformation, one obtains a new one with same divisibility properties as the original one as $(k_1 + 60, k_2 - 120, k_3 + 60, m)$. If one does not require divisibility properties for all solutions, one obtains much larger set of solutions. For instance $(k_1, k_2, k_3) = m \times (1, -2, 1)$ defines a line in the plane containing the solutions. Also other elementary moves than (1,-2,1) are possible.

One can identify very simple partitions deserving to be called standard partitions and involve mostly triangles and minimal number of 4- and 5-gons. The physical interpretation is that the coherence is minimal for them since mostly the quantum coherent negentropically entangled units are minimal triangles.

1. One starts from n vertices and constructs n-gon. For number theoretic purposes the shape does not matter and the polygon can be chosen to be convex. One slices from it 3-gons one by one so that eventually one is left with $k \equiv n \ mod \ 3 == 0, 1$ or 2 vertices. For $k = 0$ no further operations are needed. For $k = 1$ *resp.* $k = 2$ one combines one of the triangles and edge associated with 1 *resp.* 2 vertices to 4-gon *resp.* 5-gon and is done. The outcome is one of the partitions

$$n = n_1 \times 3 \quad , \quad n = n_1 \times 3 + 4, n = n_1 \times 3 + 5 \tag{3.4}$$

These partitions are very simple, and one can easily calculate similar partitions for products and powers. It is easy to write a computer program for the products and powers of integers in terms of these partitions.

2. There is however a uniqueness problem. If n_1 is divisible by 4 or 5 - $n_1 = 4 \times m_1$ or $n_1 = 5 \times m_1$ - one can interpret $n_1 \times 3$ as a collection of m_1 4-gons or 5-gons. Thus the geometric representation of the partition is not unique. Similar uniqueness condition must apply to n_2 and n_3 and is trivially true in above partitions.

To overcome this problem one can pose a further requirement. If one wants n_1 to be indivisible by 4 and 5 one can transform 2 or or 4 triangles and existing 4-gon or 5-gon or 3 or 6 triangles to 4-gons and 5-gons.

(a) Suppose $n = n_1 \times 3 + 4$. If n_1 divisible by 4 *resp.* 5 or both, $n_1 - 2$ is not and 4-gon and 2 3-gons can be transformed to 2 5-gons: $(n_1, 1, 0) \rightarrow (n_1 - 2, 0, 2)$. If $n_1 - 2$ is divisible by 5, $n_1 - 3$ is not divisible by either 4 or 5 and 3 triangles can be transformed to 4-gon and 5-gon: $(n_1, 1, 0) \rightarrow (n_1 - 3, 2, 1)$.

(b) Suppose $n = n_1 \times 3 + 5$. If n_1 divisible by 4 *resp.* 5 or both, $n_1 - 1$ is not and triangle and 5-gon can be transformed to 2 4-gons: $(n_1, 0, 1) \rightarrow (n_1 - 1, 2, 0)$. If $n_1 - 1$ is divisible by 4 or 5, $n_1 - 3$ is not and 3 triangles and 5-gon can be transformed to 2 5-gons and 4-gon: $(n_1, 0, 1) \rightarrow (n_1 - 3, 1, 2)$.

(c) For $n = n_1 \times 3$ divisible by 4 or 5 or both one can remove only $m \times 3$ triangles, $m \in \{1, 2\}$ since only in these case the resulting $m \times 3$ (9 or 18) vertices can partitioned to a union of 4-gon and 5-gon or of 2 4-gons and 2 5-gons: $(n_1, 0, 0) \rightarrow (n_1 - 3, 1, 1)$ or $(n_1, 0, 0) \rightarrow (n_1 - 6, 2, 2)$.

These transformations seem to be the minimal transformations allowing to achieve indivisibility by starting from the partition with maximum number of triangles and minimal coherence.

Some further remarks about the partitions satisfying the divisibility conditions are in order.

1. The multiplication of n with partition (n_1, n_2, n_3) satisfying indivisibility conditions by an integer m not divisible by $k \in \{3, 4, 5\}$ gives integer with partition $m \times (n_1, n_2, n_3)$. Note also that if n is not divisible by $k \in \{3, 4, 5\}$ the powers of n, n^k has partition $n^{k-1} \times (n_1, n_2, n_3)$ and this could help to solve Diophantine equations.

2. Concerning the uniqueness of the partition satisfying the indivisibility conditions, the answer is negative. $8 = 3 + 5 = 4 + 4$ is the simplest counter example. Also the m-multiples of 8 such that m is indivisible by 2,3,4,5 serve as counter examples. 60-periodicity implies that for sufficiently large values of n the indivisibility conditions do not fix the partition uniquely. (n_1, n_2, n_3) can be replaced with $(n_1 + 60 + n_2 - 120, n_3 + 60)$ without affecting divisibility properties.

3.4.3 Intriguing observations related to 60-periodicity

60-periodicity seems to have deep connections with both music consciousness and genetic code if the TGD inspired model of genetic code is taken seriously code [K5] [L1].

1. The TGD inspired model for musical harmony and genetic involves icosahedron with 20 triangular faces and tetrahedron with 4 triangular faces. The 12 vertices of icosahedron correspond to the 12 notes. The model leads to the number 60. One can say that there are 60 +4 DNA codons and each 20 codon group is 60=20+20+20 corresponds to a subset of aminoacids and 20 DNAs assignable to the triangles of icosahedron and representing also 3-chords of the associated harmony. The remaining 4 DNAs are associated with tetrahedron.

   Geometrically the identification of harmonies is reduced to the construction of Hamiltonian cycles - closed isometrically non-equivalent non-self-intersecting paths at icosahedron going through all 12 vertices. The symmetries of the Hamiltonian cycles defined by subgroups of the icosahedral isometry group provide a classification of harmonies and suggest that also genetic code carries additional information assignable to what I call bio-harmony perhaps related to the expression of emotions - even at the level of biomolecules - in terms of "music" defined as sequences 3-chords realized in terms of triplets of dark photons (or notes) in 1-1 correspondence with DNA codons in given harmony.

2. Also the structure of time units and angle units involves number 60. Hour consists of 60 minutes, which consists of 60 seconds. Could this accident somehow reflect fundamental aspects of cognition? Could we be performing sub-conscious additive arithmetics using partitions of n-gons? Could it be possible to "see" the partitions if they correspond to NE?

3.4.4 Could additive primes be useful in Diophantine mathematics?

The natural question is whether it could be number theoretically practical to use "additive primes" $\{3, 4, 5\}$ in the construction of natural numbers $n \geq 3$ rather than number 1 and successor axiom. This might even provide a practical tool for solving Diophantine equations (it might well be that mathematicians have long ago discovered the additive primes).

The most famous Diophantine equation is $x^n + y^n = z^n$ and Fermat's theorem - proved by Wiles - states that for $n > 2$ it has no solutions. Non-mathematician can naively ask whether the proposed partition to additive primes could provide an elementary proof for Fermat's theorem and continue to test the patience of a real mathematician by wondering whether the parition for a sum of powers $n > 2$ could be always different from that for single power $n > 2$ perhaps because of some other constraints on the integers involved?

3.4.5 Could one identify quantum physical correlates for arithmetic consciousness?

Even animals and idiot savants can do arithmetics. How this is possible? Could one imagine physical correlates for arithmetic consciousness for which product and addition are the fundamental aspects? Is elementary arithmetic cognition universal and analogous to direct sensory experience. Could it reduce at quantum level to a kind of quantum measurement process quite generally giving rise to mental images as outcomes of quantum measurement by repeated state function reduction lasting as long as the corresponding sub-self (mental image) lives?

Consider a partition of integer to a product of primes first. I have proposed a general model for how partition of integer to primes could be experienced directly [L2]. For negentropically entangled state with maximal possible negentropy having entanglement probabilities $p_i = 1/N$, the negentropic primes are factors of N and they could be directly "seen" as negentropic p-adic factors in the adelic decomposition (reals and extensions of various p-adic number fields defined by extension of rationals defined the factors of adele and space-time surfaces as preferred extremals of Kähler action decompose to real and p-adic sectors).

What about additive arithmetics?

1. The physical motivation for n-gons is provided symplectic QFT [K1, K6], which is one aspect of TGD forced by super symplectic conformal invariance having structure of conformal symmetry. Symplectic QFT would be analogous to conformal QFT. The key challenge is to identify symplectic invariants on which the positive and negative energy parts of zero energy states can depend. The magnetic flux through a given area of 2-surface is key invariant of this kind. String world sheet and partonic 2-surfaces are possible identifications for the surface containing the polygon.

   Both the Kähler form associated with the light-cone boundary, which is metrically sphere with constant radius r_M (defining light-like radial coordinate) and the induced Kähler form of CP_2 define these kind of fluxes.

2. One can assign fluxes with string world sheets. In this case one has analog of magnetic flux but over a surface with metric signature (1,-1). Fluxes can be also assigned as magnetic fluxes with partonic 2-surfaces at which fundamental fermions can be said to reside. n fermions defining the vertices at partonic 2-surface define naturally an n-gon or several of them. The interpretation would be as Abelian Wilson loop or equvalently non-integrable phase factor.

3. The polygons are not completely unique but this reflect the possibility of several physical states. n-gon could correspond to NE. The imaginary exponent of Kähler magnetic flux Φ through n-gon is symplectic invariant defining a non-integrable phase factor and defines a multiplicative factor of wave function. When the state decomposes to several polygons, one can uniquely identify the interior of the polygon and thus also the non-integrable phase factor.

   There is however non-uniqueness, when one has only single n-gon since also the complement of n-gon at partonic 2-surface containing now now polygons defines n-gon and the corresponding flux is $\Phi_{tot} - \Phi$. The flux Φ_{tot} is quantized and equal to the integer valued magnetic charge times 2π. The total flux disappears in the imaginary exponent and the non-integrable phase factor for the complementary polygon reduces to complex conjugate of that for polygon. Uniqueness allows only the cosine for an integer multiple of the flux.

The non-integrable phase factor assignable to fermionic polygon would give rise to a correlation between fermions in zero modes invariant under symplectic group. The correlations defined by the n-gons at partonic 2-surfaces would be analogous to that in momentum space implied by the momentum conservation forcing the momenta to form a closed polygon but having totally different origin.

Could it be that the wave functions representing collections of n-gons representing partition of integer to a sum could be experienced directly by people capable of perplexing mathematical feats. The partition to a sum would correspond to a geometric partition of polygon representing partition of positive integer $n \geq 3$ to a sum of integers. Quantum physically it would correspond to NE as a representation of integer.

This might explain number theoretic miracles related to addition of integers in terms of direct "seeing". The arithmetic feats could be dynamical quantum processes in which polygons would decompose to smaller polygons, which would be directly "seen". This would require at least two representations: the original polygon and the decomposed polygon resulting in the state function reduction to the opposite boundary of CD. An ensemble of arithmetic sub-selves would seem to be needed. NMP does not seem to favour this kind of partition since negentropy is reduced but if its time reversal occurs in geometric time direction opposite to that of self it might look like partition for the self having sub-self as mental image.

[L3]

REFERENCES

Mathematics

[A1] Icosahedral graph. Wolfram MathWorld. Available at: http://mathworld.wolfram.com/IcosahedralGraph.html.

[A2] Why are there 1024 Hamiltonian cycles on an icosahedron? Available at: http://tinyurl.com/pmghcwd.

Theoretical Physics

[B1] Arkani-Hamed N and Maldacena J. Cosmological Collider Physics. arXiv:hep-th/1503.08043v1.Available at:https://arxiv.org/pdf/1503.08043v1, 2015.

Books related to TGD

[K1] Pitkänen M. Category Theory and Quantum TGD. In *Towards M-Matrix*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#categorynew, 2006.

[K2] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics*. Onlinebook. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html#elvafu, 2006.

[K3] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? In *Towards M-Matrix*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#fermizeta, 2006.

[K4] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.#anyontgd, 2006.

[K5] Pitkänen M. Quantum Model for Hearing. In *TGD and EEG*. Onlinebook. Available at: http://tgdtheory.fi/public_html//tgdeeg/tgdeeg/tgdeeg.html#hearing, 2006.

[K6] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Onlinebook. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visionb, 2006.

Articles about TGD

[L1] Pitkänen M. Geometric theory of harmony. Available at: http://tgdtheory.fi/public_html/articles/harmonytheory.pdf, 2014.

[L2] Pitkänen M. TGD Inspired Comments about Integrated Information Theory of Consciousness. 2016.

[L3] Pitkänen M. Why Mersenne primes are so special? Available at: http://tgdtheory.fi/public_html/articles/whymersennes.pdf, 2016.