Further comments about classical field equations in TGD framework

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/.

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Abstract

In this article some remarks about field equations defining space-time surfaces in TGD framework are made.

First three dualities at the level of field equations are discussed. These dualities are rather obvious but extremely important concerning the physical interpretation of TGD.

The earlier proposal that external particles correspond to minimal surfaces is strengthened. Also the interaction regions would correspond to minimal surfaces and minimal surface property would break down only at reaction vertices associated with partonic 2-surfaces defining the 2-D counterparts of vertices: this would mean physical exchange of classical conserved charges between volume part of the action and Kähler action just at these points.

This would mean strengthening of the strong form of holography to $M^4 \times CP_2$ counterpart of the proposed number theoretic holography based on the notion of cognitive representation at the level of M^8 and also a justification for the proposed construction of twistor Grassmannian variants of scattering amplitudes involving also data at a discrete set of points.

1 Introduction

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This would mean strengthening of the strong form of holography to $M^4 \times CP_2$ counterpart of the proposed number theoretic holography based on the notion of cognitive representation at the level of M^8 [L1] and also a justification for the proposed construction of twistor Grassmannian variants of scattering amplitudes involving also data at a discrete set of points [L4].

2 Three dualities at the level of field equations

The basic field equations of TGD allow several dualities. There are 3 of them at the level of basic field equations (and several other dualities such as $M^8 - M^4 \times CP_2$ duality).

1. The first duality is the analog of particle-field duality. The spacetime surface describing the particle (3-surface of $H = M^4 \times CP_2$ instead of point-like particle) corresponds to the particle aspect whereas the fields inside it geometrized in terms of sub-manifold geometry correspond to the field aspect. Particle orbit serves as wave guide for field, one might say.

2. Second duality is particle-spacetime duality. Particle identified as 3-D surface means that particle orbit is space-time surface glued to a larger space-time surface by topological sum contacts. It depends on the scale used, whether it is more appropriate to talk about particle or of space-time.

3. The third duality is hydrodynamics- massless field theory duality. Hydrodynamical equations state local conservation of Noether currents. Field equations indeed reduce to local conservation conditions of Noether currents associated with the isometries of H. One the other hand, these equations have interpretation as non-linear geometrization of massless wave equation with coupling to Maxwell fields. This realizes the ultimate dream of theoretician: symmetries dictate the dynamics completely. This is expected to be realized also at the level of scattering amplitudes and the generalization of twistor Grassmannian amplitudes could realize this in terms of Yangian symmetry.

Hydrodynamics-wave equations duality generalizes to the fermionic sector and involves super-conformal symmetry.

1. What I call modified gamma matrices Γ^α are obtained as contractions of the partial derivatives of the action defining space-time surface with respect to the gradients of imbedding space coordinate with imbedding space gamma matrices [K3]. The divergence $D_\alpha \Gamma^\alpha$ vanishes by field equations for the space-time surface and this is necessary for the internal consistency the Dirac equation ($\overline{\Psi}$ satisfies essentially the same equation as Ψ). Γ^α reduce to ordinary ones if the space-time surface is M^4 and one obtains ordinary massless Dirac equation.

2. Modified Dirac equation [K3] expressess conservation of super current and actually infinite number of super currents obtained by contracting second quantized induced spinor field with the solutions of modified Dirac. This corresponds to the super-hydrodynamic aspect. On the other hand, modified Dirac equation corresponds to fermionic analog of massless wave equation.

3 Are space-time surfaces minimal surfaces everywhere except at 2-D interaction vertices?

The action S determining space-time surfaces as preferred extremals follows from twistor lift [K2, K5, K4, L4] and equals to the sum of volume term Vol multiplied by the TGD counterpart of cosmological constant and Kähler action S_K. The field equation is a geometric generalization of d'Alembert (Laplace) equation in Minkowskian (Eucidian) regions of space-time surface coupled with induced Kähler form analogous to Maxwell field. Generalization of equations of motion for particle by replacing it with 3-D surface is in question and the orbit of particle defines a region of space-time surface.

1. Zero energy ontology (ZEO) suggests that the external particles arriving to the boundaries of given causal diamond (CD) are like free massless particles and correspond to minimal surfaces as a generalization of light-like geodesic. This dynamic reduces to mere algebraic conditions and there is no dependence on the coupling parameters appearing in S. In contrast to this, in the interaction regions inside CDs there could be a coupling between Vol and S_K due to the non-vanishing divergences of energy momentum currents associated with the two terms in action cancelling each other.

2. Similar algebraic picture emerges from M^8-H duality [L1] at the level of M^8 and from what is known about preferred extremals of S assumed to satisfy infinite number of super-symplectic gauge conditions at the 3-surfaces defining the ends of space-time surface at the opposite boundaries of CD.

   At M^8 side of M^8-H duality associativity is realized as quaternionicity of either tangent or normal space of the space-time surface. The condition that there is 2-D integral distribution of sub-spaces of tangent spaces defining a distribution of complex planes as subspaces of octonionic tangent space implies the map of the space-time surface in M^8 to that of H.

Given point m_8 of M^8 is mapped to a point of $M^4 \times CP_2$ as a pair of points (m_4, s) formed by $M^4 \subset M^8$ projection m_4 of m_8 point and by CP_2 point s parameterizing the tangent space or the normal space of $X^4 \subset M^8$.

Remark: The assumption about integrable distribution of $M^2(x)$ defining string world sheet in M^4 might be too general: $M^2x)$ could not depend on x.

If associativity or even the condition about the existence of the integrable distribution of 2-planes fails, the map to $M^4 \times CP_2$ is lost. One could cope with the situation since the gauge conditions at the boundaries of CD would allow to construct preferred extremal connecting the 3-surfaces at the boundaries of CD if this kind of surface exists at all. One can however wonder whether giving up the map $M^8 \rightarrow H$is necessary.

3. Number theoretic dynamics in M^8 involves no action principle and no coupling constants, just the associativity and the integrable distribution of complex planes $M^2(x)$ of complexified octonions. This suggests that also the dynamics at the level of H involves coupling constants only via boundary conditions. This is the case for the minimal surface solutions suggesting that $M^8 - H$ duality maps the surfaces satisfying the above mentioned conditions to minimal surfaces. The universal dynamics conforms also with quantum criticality.

4. One can argue that the dependence of field equations on coupling parameters of S leading to a perturbative series in coupling parameters in the interior of the space-time surface inside CD spoils the extremely beautiful purely algebraic picture about the construction of solutions of field equations using conformal invariance assignable to quantum criticality. Classical perturbation series is also in conflict with the vision that the TGD counterparts twistorial Grassmannian amplitudes do not involve any loop contributions coming as powers of coupling constant parameters [L4].

To sum up, both $M^8 - H$ duality, number theoretic vision, quantum criticality, twistor lift of TGD reducing dynamics to the condition about the existence of induced twistor structure, and the proposal for the construction of twistor scattering amplitudes suggest an extremely simple picture about the situation. The divergences of the energy momentum currents of Vol and S_K would be non-vanishing delta function type singularities only at discrete points at partonic 2-surfaces defining generalized vertices so that minimal surface equations would hold almost everywhere as the original proposal indeed stated.

1. The fact that all the known extremals of field equations for S are minimal surfaces conforms with the idea. This might be due to the fact that these extremals are especially easy to construct but could be also true quite generally apart from singular points. The divergences of the energy momentum currents associated with S_K and Vol vanish separately: this follows from the analog of holomorphy reducing the field equations to purely algebraic conditions.

   It is essential that Kähler current j_K vanishes or is light-like so that its contraction with the gradients of the imbedding space coordinates vanishes. Second condition is that in transversal degrees of freedom energy momentum tensor is tensor of form (1,1) in the complex sense and second fundamental form consists of parts of type (1,1) and (-1-1). In longitudinal degrees of freedom the trace H^k of the second fundamental form $H^k_{\alpha\beta} = D_\beta \partial_\alpha h^k$ vanishes.

2. Minimal surface equations are a non-linear analog of massless field equation but one would like to have also the analog of massless particle. The 3-D light-like boundaries between Minkowskian and Euclidian space-time regions are indeed analogs of massless particles as are also the string like word sheets, whose exact identification is not yet fully understood. In any case, they are crucial for the construction of scattering amplitudes in TGD based generalization of twistor Grassmannian approach. At M^8 side these points could correspond to singularities at which Galois group of the extension of rationals has a subgroup leaving the point invariant. The points at which roots of polynomial as function of parameters co-incide would serve as an analog.

   The intersections of string world sheets with the orbits of partonic 2-surface are 1-D light-like curves X^1_L defining fermion lines. The twistor Grassmannian proposal [L4] is that the ends of the fermion lines at partonic 2-surfaces defining vertices provide the information needed to construct scattering amplitudes so that information theoretically the construction of scattering amplitudes would reduce to an analog of quantum field theory for point-like particles.

3. Number theoretic vision discretizes coupling constant evolution: the values of coupling constants are labelled by parameters of extension of rationals and p-adic primes. This implies that twistor scattering amplitudes for given discrete values of coupling constants involve no radiative corrections [L4]: the construction of twistor Grassmannian amplitudes would be extremely simple. Note that infinite perturbation series would break the expression of scattering amplitudes as rational functions with coefficients int he extension of rationals defining the adele [L2, L3]. The cuts for the scattering amplitudes would be replaced by sequences of poles. This is unavoidable also because there is number theoretical discretization of momenta from the condition that their components belong to an extension of rationals defining the adele.

What could the reduction of cuts to poles for twistorial scattering amplitudes at the level of momentum space [L4] mean at space-time level?

1. Poles of an analytic function are co-dimension 2 objects. d'Alembert/Laplace equations holding true in Minkowskian/Euclidian signatures express the analogs of analyticity in 4-D case. Co-dimension 2 rule forces to ask whether partonic 2-surfaces defining the vertices and string world sheets could serve analogs of poles at space-time level? In fact, the light-like orbits X_L^3 of partonic 2-surfaces allow a generalization of 2-D conformal invariance since they are metrically 2-D so that X_L^3 and string world sheets could serve in the role of poles.

   X_L^3 could be seen as analogs of orbits of bubbles in hydrodynamical flow in accordance with the hydrodynamical interpretations. Particle reactions would correspond to fusions and decays of these bubbles. Strings would connect these bubbles and give rise to tensor networks and serve as space-time correlates for entanglement. Reaction vertices would correspond to common ends for the incoming and outgoing bubbles. They would be analogous to the lines of Feynman diagram meeting at vertex: now vertex would be however 2-D partonic 2-surface.

2. What can one say about the singularities associated with the light-like orbits of partonic 2-surfaces? The divergence of the Kähler part T_K of energy momentum current T is proportional to a sum of contractions of Kähler current j_K with gradients ∇h^k of H coordinates. j_K need not be vanishing: it is enough that its contraction with ∇h^k vanishes and this is true if j_K is light-like. This is the case for so called massless extremals (MEs). For the other known extremals j_K vanishes.

   Could the Kähler current j_K be light-like and non-vanishing and singular at X_L^3 and at string world sheets? This condition would provide the long sought-for precise physical identification of string world sheets. This would also induce to the modified Dirac action a 2-D contribution. Minimal surface equations would hold true also at these two kinds of surfaces apart from possible singular points. Even more: j_K could be non-vanishing and thus also singular only at the 1-D intersections X_L^1 of string world sheets with X_L^3 - I have called these curves fermionic lines.

   What it means that j_K is singular - that is has 2-D delta function singularity at string world sheets? j_K is defined as divergence of the induced Kähler form J so that one can use the standard definition of derivative to define j_K at string world sheet as the limiting value $j_K^\alpha = (Div_{+-}J)^\alpha = lim_{\Delta x^n \rightarrow 0}(J_+^{\alpha n} - J_-^{\alpha n})/\Delta x^n$, where x^n is a coordinate normal to the string world sheet. If J is discontinuous, this gives rise to a singular current located at string world sheet. This current should be light like to guarantee that energy momentum currents are divergenceless. If J is not light-like, it gives rise to isometry currents with non-vanishing divergence at string world sheet. This is guaranteed if the isometry currents $T^{\alpha A}$ are continuous through the string world sheet.

3. If the light-like j_K at partonic orbits is localized at fermionic lines X_L^1, the divergences of isometry currents could be non-vanishing and singular only at the vertices defined at partonic 2-surfaces at which fermionic lines X_L^1 meet. The divergences $DivT_K$ and $DivT_{Vol}$

would be non-vanishing only at these vertices. They should of course cancel each other: $DivT_K = -DivT_{Vol}$.

4. $DivT_K$ should be non-vanishing and singular only at the intersections of string world sheets and partonic 2-surfaces defining the vertices as the ends of fermion lines. How to translate this statement to a more precise mathematical form? How to precisely define the notions of divergence at the singularity?

   The physical picture is that there is a sharing of conserved isometry charges of the incoming partonic orbit $i = 1$ determined T_K between 2 outgoing partonic orbits labelled by $j = 2, 3$. This implies charge transfer from $i = 1$ to the partonic orbits $j = 2, 3$ such that the sum of transfers sum up to to the total incoming charge. This must correspond to a non-vanishing divergence proportional to delta function. The transfer of the isometry charge for given pair i, j of partonic orbits that is $Div_{i\rightarrow j}T_K$ must be determined as the limiting value of the quantity $\Delta_{i\rightarrow j}T_K^{\alpha,A}/\Delta x^\alpha$ as Δx^α approaches zero. Here $\Delta_{i\rightarrow j}T_K^{\alpha,A}$ is the difference of the components of the isometry currents between partonic orbits i and j at the vertex. The outcome is proportional delta function.

5. Similar description applies also to the volume term. Now the trace of the second fundamental form would have delta function singularity coming from $Div_{i\rightarrow j}T_K$. The condition $Div_{i\rightarrow j}T_K = -Div_{i\rightarrow j}T_{Vol}$ would bring in the dependence of the boundary conditions on coupling parameters so that space-time surface would depend on the coupling constants in accordance with quantum-classical correspondence. The manner how the coupling constants make themselves visible in the properties of space-time surface would be extremely delicate.

This picture conforms with the vision about scattering amplitudes at both M^8 and H sides of $M^8 - H$ duality.

1. M^8 dynamics based on algebraic equations for space-time surfaces [L1] leads to the proposal that scattering amplitudes can be constructed using the data only at the points of space-time surface with M^8 coordinates in the extension of the rationals defining the adele [L3, L2]. I call this discrete set of points cognitive representation with motivations coming from TGD inspired theory of consciousness [K1].

2. At H side the information theoretic interpretation would be that all information needed to construct scattering amplitudes would come from points at which the divergences of the energy momentum tensors of S_K and Vol are non-vanishing and singular.

Both pictures would realize extremely strong form of holography, much stronger than the strong form of holography that stated that only partonic 2-surfaces and string world sheets are needed.

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