Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory

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December 18, 2020

Abstract

The understanding of the unitarity of the S-matrix has remained a major challenge of Topological Geometrodynamics (TGD) for 4 decades. It has become clear that some basic principle is still lacking. Assigning S-matrix to a unitary evolution works in non-relativistic theory but fails already in the generic quantum field theory (QFT). The solution of the problem turned out to be extremely simple. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space would replace unitary S-matrix.

An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of Teichmueller matrix. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong from of holography (SH), the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the imbedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmueller condition guaranteeing non-negative probabilities.

Equivalence Principle generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

Contents

1 Introduction

I have worked with the problem of understanding the construction of scattering amplitudes in the framework provided by Topological Geometrodynamics (TGD) for about four decades. It soon became clear that the naïve generalization of the path integral approach to a path integral over space-time surfaces did not work because of the horrible non-linearities involved. Around 1985 I started to work with the notion that I later called the "world of classical worlds" (WCW). Eventually I apprehended that the realization of general coordinate invariance (GCI) forces to assign to a 3-surface possibly unique space-time surface (X^4) at which the general coordinate transformations act [K3, K2]. Holography would reduce to GCI. The intuitive expectation is that either space-like 3-surfaces or light-like partonic orbits defining boundaries between Minkowskian and Euclidian space-time regions should be enough to determine X^4 as an analog of Bohr orbit. This leads to strong form of holography (SH) stating that data at partonic 2-surfaces and string world sheets code for X^4.

It should be possible to geometrize the entire quantum physics in terms of WCW geometry and associated spinor structure identifying WCW spinors as fermionic Fock states. A geometrization of the hermitian conjugation essential in quantum theory is needed. This fixed the WCW geometry to be Kähler geometry determined by Kähler function and defining Kähler form providing a realization of the imaginary unit as an antisymmetric tensor [K3]. The existence of Riemann connection fixes the Kähler geometry uniquely already in the case of loop spaces [A10]: maximal isometry group is required. In TGD framework it would correspond supersymplectic transformations of $\delta M^4_\pm \times CP_2$, where $\delta M^4_\pm$,denotes future or past light-cone [K2].

Classical physics becomes an exact part of quantum physics if the space-time surfaces are preferred extremals for some action and therefore analogous to Bohr orbits. Spinor fields should obey the modified Dirac equation (MDE). Modified Dirac action (MDA) is determined by the bosonic action via supersymmetry condition. Kähler function identified as the action for the preferred extremal associated with the 3-surface defines in complex coordinates the Kähler metric and Kähler form via its second derivatives of type $(1,1)$.

The natural looking identification of the action was as Kähler action - a non-linear generalization of Maxwell action replacing Maxwell field and metric with induced Kähler form and metric. It possessed a huge vacuum degeneracy interpreted as spin glass degeneracy and for a long time I looked this feature as something positive despite the fact that the WCW metric becomes degenerate at the vacuum extremals and classical determinism is lost. The addition of volume term having interpretation in terms of cosmological constant would have been a possible cure but would have broken conformal invariance bringing in an *ad hoc* dimensional coupling.

Decades later the proposal for a twistor lift of TGD led to the identification of fundamental action as an analog of Kähler action for 6-D twistor spaces having X^4 as base space and S^2 as fiber [L9]. The induction of the twistor structure from that for the 6+6-D product of twistor spaces of M^4 and CP_2 (these spaces are the only 4-spaces allowing twistor space with Kähler structure [A12] so that TGD is unique) to the 6-surface forces a dimensional reduction reducing 6-D Kähler action to a sum of 4-D Kähler action and volume them. The counterpart of the cosmological constant emerges dynamically. Λ depends on the p-adic length scale characterizing space-time surfaces and approaches to zero in long length scales [L9].

The ontology of standard quantum theory in which 3-D $t = constant$ slice of space-time contains the quantum states, does not fit nicely with TGD framework. Space-time surfaces in 1-1 correspondence with 3-surfaces are more natural objects to consider. This conforms also with the notion of holography implied by GCI: actually SH is highly suggestive and means that 2-D data at partonic 2-surfaces and string world sheets determined the X^4 as a preferred extremal. In particular, various anomalies suggest that the arrow of time need not be fixed.

Eventually this led to zero energy ontology (ZEO) [L7] in which quantum states are essentially superpositions of preferred extremals inside causal diamond (CD): space-time surfaces have ends at the boundaries of CD and these pairs of 3-surfaces or equivalently the 4-surfaces are the basic objects. CDs form a hierarchy: there are CDs with CDs and CDs can also intersect. They would form an analog of atlas of coordinate charts. Each CD would serve as a correlate for a conscious entity so that the charts can be said to be conscious.

ZEO leads to a quantum measurement theory and allows avoiding the basic problems of the standard quantum measurement theory. Zero energy states correspond to state pairs at opposite boundaries of CD or equivalently, superpositions of deterministic time evolutions. In state function reduction (SFR) as a superposition of classical deterministic time evolutions is replaced with a new one.

"Big" and "small" state function reduction - BSFR and SSFR - are the basic notions. In SSFRs as analogs of "weak" measurements following a unitary time evolution, the size of CD increases in statistical sense. The members of the state pairs associated with the passive boundary of CD do not change during SSFRs: this gives rise to the analog of Zeno effect. The active boundary and the states at it change. Active boundary also shifts farther from the passive one. BSFRs correspond to ordinary state function reductions and in BSFRs the arrow of time changes. One could speak of a death of a conscious entity in universal sense and reincarnation with an opposite arrow of time. For instance, the findings of Minev et al [L5] provide support for the time reversal [L5].

1.1 How to construct the TGD counterpart of unitary S-matrix?

The concrete construction of scattering amplitudes remained a challenge from very beginning. During years I have proposed several proposals and many important aspects of the problem are understood but simple rules are still lacking.

1. The time evolutions assignable to SSFRs should be describable by a unitary S-matrix or its analog.

2. The counterpart of S-matrix should have the huge super-symplectic algebra (SSA) and Kac-Moody algebras related to isometries of H as symmetries. These symmetries, extended further to Yangian symmetries and quantum groups with both algebra and co-algebra structure, are expected to be a key element in the construction of the counterpart of S-matrix. In particular, product and co-product in the super-symplectic algebra define excellent candidates for vertices. What has been missing was a concrete guiding principle.

3. Feynman (or twistor) diagrammatics should generalize. Point-like particles are replaced with 3-surfaces and topologically incoming and outgoing many-particle states correspond to disjoint unions of 3-surfaces at the boundaries of CD. The first guess is that the vertices correspond to 3-surfaces at which 4-D lines of the analog of Feynman diagram meet. SH and M^8-H duality [L6] however suggest that the lines of the diagrams should correspond to 3-D light-like orbits of partonic 2-surfaces defining boundaries between space-time regions with Euclidian and Minkowskian signature of the induced metric. Also string world sheets connecting them and also serving as carriers of information in SH should be considered. The 1-D light-like intersections of strings world sheets with partonic orbits would define carriers of fermion number.

4. The identification of fermionic anti-commutation relations was a longstanding challenge. It turned out that the induction of second quantized free fermion fields from H to X^4 fixes the anti-commutations of the induced spinor fields and allows to calculate fermionic propagators. Therefore quantum algebra would give what is needed to calculate scattering amplitudes: the interaction vertices assignable to partonic 2-surfaces and fermionic propagators would result from the induction procedure. 8-D fermions have however 7-D delta functions as anti-commutators and normal ordering of fermions can produce divergences already at the level of the MDA.

   The problem disappears if the MDA is made bilocal [L10]: in this article a more detailed discussion is given and leads to a rather detailed picture about MDA.

5. M^8-H duality [L6, L2, L3, L4] allows to concretize this picture. One can regard X^4 either as a surface in the complexified M^8 or in H. M^8-H duality maps space-time surfaces from M^8 to H. Space-time surfaces in the complexified M^8 correspond to algebraic 4-surfaces determined by real polynomials with real (rational if one requires p-adicization) coefficients. Also rational and even analytic functions can be considered, in which case polynomials could be seen as approximations. The roots of the real polynomial dictate the space-time surfaces as quaternionic/associative 4-surfaces in complexified octonionic M^8. Holography becomes discrete.

   The algebraic equations defining space-time surfaces also have special solutions, in particular 6-spheres. These analogs of 6-branes have as M^4 projections in both M^8 and $H = M^4 \times CP_2 t = r_n$ hyperplanes, where r_n corresponds to a root of a real polynomial defining X^4 in complexified M^8 The interpretation of these hyper-planes is in TGD inspired consciousness is as "very special moments in the life of self".

   The solutions of the analog of Dirac equation in M^8 as algebraic equation [L11] are localized to 3-D light-like surfaces and mapped to light-like 3-surfaces in H identifiable as orbits of partonic 2-surfaces. Partonic 2-surfaces serving as vertices of topological analogs of Feynman diagrams would reside at the above described $t = r_n$ hyperplanes of $H = M^4 \times CP_2$. Scattering amplitudes would have partonic 2-surfaces as vertices and their 3-D light-like orbits as lines. The intersections of string world sheets with the partonic orbits would be 1-D lines and could be interpreted as fermion lines so that also the point particle description would be part of the picture.

   CDs inside CD would define the regions inside which particle reactions occur and this suggests a fractal hierarchy of CDs within CDs as a counterpart for the hierarchy of radiative corrections.

What is still missing is the general principle allowing a bird's eye of view about the counterpart of S-matrix.

1. Unitary time evolution is natural in non-relativistic quantum mechanics but is already problematic in quantum field theory (QFT), in particular in twistor Grassmannian approach [B1]. The idea about the reduction of physics to Kähler geometry inspires the question whether Kähler geometry of WCW could provide a general principle for the construction of the scattering amplitudes and perhaps even an explicit formulas for them.

   Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurements. Could the Kähler metric of state space code the counterpart of S-matrix and even unitary S-matrix?

   An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric the analog of a unitary S-matrix by assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied.

   (a) If the probabilities are identified as the real parts of complex analogs $p^c_{i,j} = g_{i,\overline{j}}g^{\overline{j},i}$ of probabilities, it is enough to require $Re(p^c_{i,j}) \geq 0$. The complex analogs of $ip^c_{i,j}$) would define the analog of Teichmueller matrix [A6, A11, A9] (`https://en.wikipedia.org/wiki/Teichmller_space`) for which imaginary parts of matrix elements are non-negative. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By SH, the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

   (b) By positing the condition that $g_{i,\overline{j}}$ and $g^{\overline{j},i}$ have opposite phases, one can assign to Kähler metric a unitary S-matrix but this does not seem to be necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique. These solutions would be special case of Teichmueller solutions: Teichmueller matrix would be purely imaginary. The condition looks too restrictive. For instance, for torus, this would correspond to a metric conformally equivalent with a flat metric.

2. This inspires the idea that quantum physics could be geometrized by the same manner as Einstein geometrized gravitation. Take a flat Hilbert space bundle (in the case of TGD) and replace its flat Kähler metric both base space and fiber with a non-flat Kähler metric. The replacement of flat metric with a curved one would lead from a non-interacting quantum theory to an interacting one. Quantum theory would be gravitation at the level of this Hilbert bundle! This replacement is completely universal.

   In the TGD framework the world of classical worlds (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at X^4 and induced from second quantized spinors of the imbedding space. Scattering amplitudes would be determined by the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in ZEO and satisfying Teichmueller condition guaranteeing non-negative probabilities.

   WCW geometry is also characterized by zero modes corresponding to non-complex coordinates for WCW giving no contribution to WCW metric. This is self-evident from SH. The zero modes would be in 1-1 correspondence with Teichmueller parameters and WCW Kähler metrics.

   Equivalence Principle (EP) generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give a possible interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic QFT [K8].

There is also challenge of constructing the Kähler metric and associated spinor structure for the spinor bundle of WCW. This would mean a specification of the analogs of Feynman rules so that instead of two problems one would have only one problem.

1. WCW gamma matrices can be identified as superpositions of fermionic oscillator operators associated with quark spinors [L8]. One can consider two approaches to the quantization of these spinors: one studies induced spinor fields obeying MDE and quantizes this or one generalizes the induction of spinors from H to the induction of second quantized spinor fields in H: this would mean simply projecting the spinor fields to X^4. The latter option is extremely simple. It seems possible to avoid divergence problems if the anti-commutators are assigned to different 3-surfaces at different boundaries of CD. This would allow the identification of the Dirac propagator. As a matter of fact, the two approaches are equivalent.

2. WCW gamma matrices would allow the identification as super generators of SSA identified as contractions of gamma matrices SSA with Killing vectors. Quantum states would be created by bosonic and fermionic SSA generators.

3. I have proposed a further supersymmetrization of both H coordinates and spinors by replacing them with expansions in powers of local composites of oscillator operators for quarks and antiquarks [L8]. This however requires Kronecker delta type anti-commutators natural for cognitive representations defining unique discretization of X^4: this allows to avoid normal ordering divergences. Induction of the H spinor fields would lead to 8-D delta function type divergences. This suggests that local composites are not quite local but states consisting of quarks and antiquarks at opposite throats of wormhole contacts identifiable as partonic 2-surfaces. One would obtain leptons as 3-quark states with quarks at the same partonic 2-surface but not at the same point anymore as in the proposal of [L8].

4. The matrix elements of the Kähler metric of WCW Hilbert bundle correspond to scattering amplitudes analogous to Feynman diagrams. What are the Feynman rules? Partonic two surfaces and their orbits correspond to vertices and propagators topologically. The TGD counterpart for $F\overline{F}B$ vertex would correspond to a bosonic wormhole contact with a fermion and antifermion at opposite wormhole throats and representing SCA generator which decomposes to two partonic 2-surfaces carrying fermions at opposite throats representing fermionic SCA generators. This allows avoiding of normal ordering divergences.

   The vertex would correspond to a product or co-product, which can be said to be time reversals of each other. The structure constants of SCA extended to quantum algebra would fix the vertices and thus the analogs of Feynman diagrams completely. Their number is presumably finite for a X^4 with fixed 3-surfaces at its ends and summation over Feynman diagrams would correspond to integration in WCW.

Before discussing them current proposal in detail, the complementary manners to overview TGD as eithyer WCW geometry or as number theory are discussed below. Readers might skip these sections at their first reading and choose to read the section discussing the basic idea in more detail.

In the sequel the basic idea about representation of scattering amplitudes as elements of Kähler metric satisfying what I call "Teichmueller condition", is discussed in TGD framework.

The detailed formulation allows a formulation of conditions for the cancellation of normal ordering divergences and also other divergences. The induction of the second quantized free spinor field from H to space-time surface fixes the propagators at the space-time level. If the creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts, divergences are avoided. ZEO suggests an alternative but not exclusive option that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

The fact that the Dirac propagators for massive particles in the TGD sense reduce in a good approximation to massless propagators when the propagation takes place along light-like distances, allows to considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

2 Could Kähler metric of state space replace S-matrix?

In the sequel a more detailed view about the reduction of S-matrix to a non-flat Kähler geometry of Hilbert space consisting of WCW spinor fields is considered. The proposal is novel in the sense

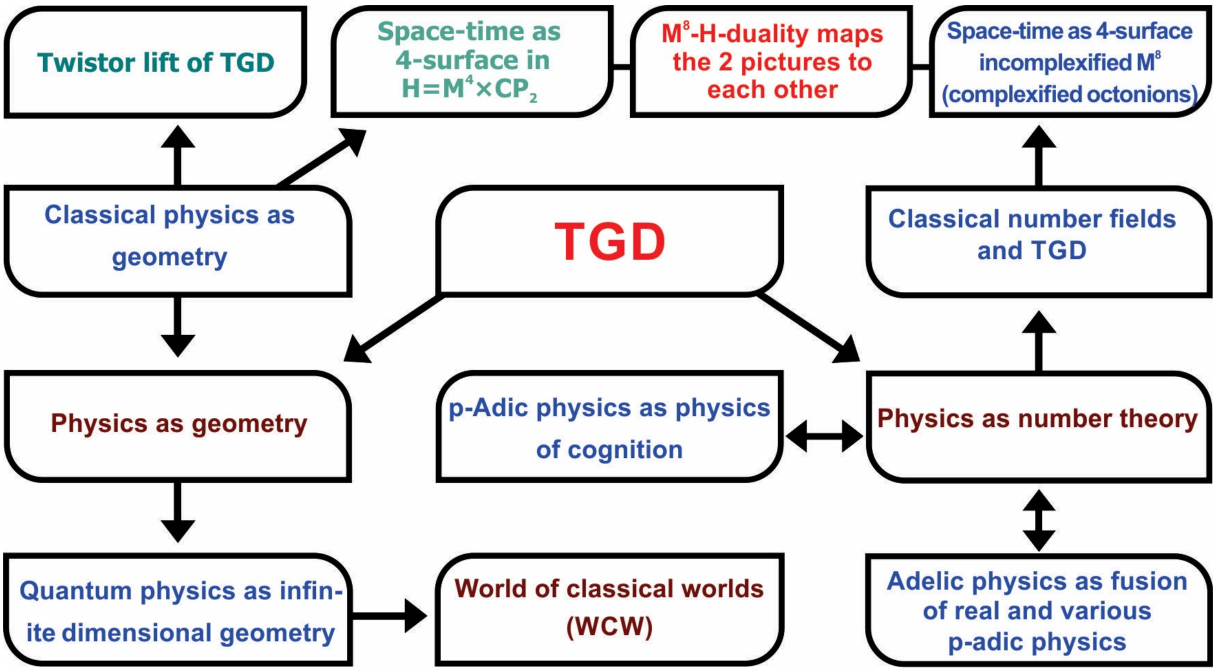

Figure 1: TGD is based on two complementary visions: physcs as geometry and physics as number theory.

that the state space would codes interactions to its geometry just like space-time geometry codes gravitational interaction in general relativity.

2.1 About WCW spinor fields

2.1.1 Induction of second quantized spinor fields from H

There are two approaches to the quantization of induced spinors at space-time surfaces, and these approaches are equivalent.

1. Induction means that gamma matrices are determined by Kähler action as analogs for projections of imbedding space gamma matrices and space-time spinor field χ is simply the restriction of H spinor field Ψ. For a given action determining X^4, supersymmetry allows the identification of the modified Dirac operator D and finding of the modes of the induced H spinor field as solutions of the modified Dirac equation (MDE) $D\chi = 0$. Second quantization would replace their coefficients with oscillator operators. However, it is not clear what the anti-commutation relations for the oscillator operators are.

2. One can generalize the classical induction of spinors Ψ to an induction of second quantized spinor fields in H as a restriction of the second quantized Ψ in H to the X^4. One must however get rid of normal ordering diverges due the fact that the anti-commutators for coinciding points give 7-D delta functions. One gets rid of them, if the Ψ and $\overline{\Psi}$ are assigned to disjoin space-time regions. This leads to bi-local modified Dirac action (MDA), implying automatically the classical field equations for the action determining D.

What does $D\Psi = 0$ really mean when Ψ is quantum field? One can develop the restrictions of the c-valued modes of Ψ in terms of modes of χ satisfying $D\chi = 0$, and obtain an expression for Ψ at X^4 in terms of these modes each satisfying MDE. The operator valued coefficients of Ψ modes contributing to a given mode of χ would define the corresponding oscillator value fermionic oscillator operators at X^4.

Also the generalizations of the variants of MDE restricted to sub-manifolds of X^4 make sense and are needed. The beauty is that there is no need to introduce spinor fields at lower-D surfaces as independent dynamical degrees of freedom. For instance, one only a variant of a modified Dirac action defined by Cherns-Simons analog of Kähler action makes sense at light-like partonic orbits so that one has an analog of a topological quantum field theory (TQFT).

2.1.2 How to avoid normal ordering divergences from fermionic oscillator operators?

The normal ordering divergences due to the anti-commutators of fermionic fields at the same point are really serious since induce spinor fields of 8-D $H = M^4 \times CP_2$ so that normal ordering singularities are proportional to 7-D delta function $\delta^7(0)$. They are encountered already for the ordinary MDA giving rise to bosonic SCA charges as Noether charges, which also are plagued by these divergences. Normal ordering for the oscillators in the Noether charges associated with MDA would allow to get rid of the divergences but is a mere trick. The proposal considered in [L10] is to make MDA bi-local at the space-time level.

Consider the general constraints on bi-locality coming from the cancellation of the normal ordering divergences.

1. Consider first 4-D variant of MDA. The most general option for MDA is that there is an integral over the entire X^4 for both Ψ and $\overline{\Psi}$ separately sothat one has 2 4-D integrations. One obtains potential normal ordering divergences proportional to $\delta^7(0)d^8x$. If one has two space-time sheets which in the generic case intersect transversally at discrete set of points, one obtains a vanishing result. However, the self-pairing of a given space-time sheet gives a divergence as a 4-D volume integral of $\delta^3(0)$. The definition of the self-pairing as a limit of separate space-time sheets approaching each other to get rid of the divergences looks like a trick.

   This suggests that the pairing can occur only between disjoint space-time regions, most naturally space-time sheets. For instance, parallel space-time sheets with overlapping M^4 projections. Allowing pairing only between disjoint regions eliminates also the divergences associated with the bosonic Noether charges deduced from MDA and involving 3+4-D integral instead of 3-D integral.

   What could be the precise definition for these disjoint regions? $M^8 - H$ duality suggests that they correspond to different roots of the octonionic polynomial defined by real polynomials. When 2 roots coincide, one obtains a term of type $\delta^7(0)d^7x$ giving a finite result. What if the number of coinciding roots is higher than 2? This case will be discussed later in number theoretic context.

   What about space-like regions, in particular the wormhole contacts expected to be small deformations of a warped imbedding of CP_2 having light-like M^4 projection but having same Kähler metric and Kähler form as CP_2 [K6]? There is no pairing with a parallel space-time sheet now. It seems that the pairing must be between different wormhole contacts. This pairing could be essential for the understanding of string like entities as paired wormhole contacts providing a model for elementary particles.

2. For the bilinear MDA, the variation of the 4+4-D modified Dirac action with respect to $\overline{\Psi}$ and Ψ yields both the modified Dirac equation $D\Psi = 0$ plus the field equations for the preferred extremal. This gives the modes of the induced spinor field. In the standard picture the hermiticity condition for the Dirac action yields the same outcome and has interpretation as a supersymmetry between classical and fermionic degrees of freedom.

3. Both the phenomenological picture developed during years and $M^8 - H$ duality strongly suggest that spinors can be restricted also to lower-D surfaces. For the lower-D variants

of MDA the normal ordering divergences appear already for tranversal intersections. For instance, for 3-D variant of MDA one has $\delta^7(0)d^7x$ type divergences. The only possible manner to avoid them is to require that paired regions are disjoint. For the 3-D Chern-Simons-Kähler action associated with the light-like partonic orbits the paired space-time sheets are very naturally the opposite wormhole throats so that fermions and antifermions would reside at opposite wormhole throats.

Physical picture also suggests the assignment of actions to 2-D string world sheets and 1-D light-boundaries defining their intersections with partonic orbits.

4. Also 6-D brane-like solutions having the topology of S^6 and $t = r_n$ hyper-plane as intersection with M^4 are of physical interest. Different 4-D space-time surfaces could be glued together along 3-surfaces or 2-D partonic 2-surfaces at S^6. Arguments similar to those already discussed exclude the pairing of various objects with these 6-branes as also their self-pairing.

   Also M^4 and CP_2 define special solutions to the algebraic equations in M^8. MDA reduces to ordinary massless Dirac equation in M^4. In the case of CP_2 one has a massless Dirac equation in CP_2 and only the right-handed neutrino ν_R is possible as a solution. If only quarks are allowed, this solution is excluded. What happens for the deformations of CP_2? Could it be that quarks cannot reside inside wormhole contacts as 4-D entities? Or could one allow solutions of $D\Psi = 0$ as analytic functions of CP_2 coordinates finite in the region in which they are defined - wormhole contact does not span the entire CP_2?

Cognitive representations provide additional insights to the problem of normal ordering divergences, and it could be even argued that they are the only possible manner to define scattering amplitudes as a sequences of improving approximation natural in the approach based on hyperfinite factors of type II_1 (HFFs).

1. For a given extension of rationals determined by the polynomial defining the space-time region in M^8, the space-time surfaces inside CD are replaced with their discretizations consisting of points of M^8 in the extension considered. This surface and cognitive representation are mapped to H by $M^8 - H$ correspondence [L6]. For cognitive representations one can perform discretization by replacing the integrals defining SCA generators with discrete sums over points of the cognitive representation. This replacement is very natural since in the p-adic context the counterpart of the Riemann integral does not exist.

2. The Galois group of extension serves as a symmetry group and one can form analogs of group algebra elements - wave functions in discrete Galois group - acting on the cognitive representation and giving rise to discrete representation of quantum states. This state space has as its dimension the dimension n of the Galois group which for Galois extensions coincides with the dimension of extension [L1, L12]. This group algebra-like structure can be given Kähler metric and also spinor structure and this spinor structure could discretize the spinor structure of WCW if gamma matrices are identified as fermionic oscillator operators.

3. Also now one can avoid divergences if the paired space-time regions, say space-time sheets, in MDA are disjoint. It can however happen that n separate points at the orbit of the Galois group approach each other and coincide: this would correspond to the touching of space-time sheets meaning coinciding roots of the octonionic polynomial. In this situation a subgroup of the Galois group would leave the intersection point invariant.

   The possible normal ordering divergence comes from different pairs of the m points, which coincide. In 4-D case, the situation corresponds to transversal space-time sheets so that the divergence vanishes. For lower-dimensional surfaces, say partonic orbits, the intersections do not occur in the generic situation but if they occur, the divergence is multiplied by a sum over the values of wave function at coinciding branches and vanishes if the representation is *non-singlet*. It would thus seem that the non-singlet character of Galois representations must be posed as an additional condition.

4. This cancellation mechanism works even without discretization since the notions of Galois group and its representations make sense for arbitrary polynomial surfaces without a restriction to rational or algebraic polynomial coefficients so that the cancellation occurs for non-singlet representations when the space-time sheets intersect.

2.1.3 Are fermions 4-D in H but 3-D in M^8?

M^8-H duality suggests the restriction of the induced spinor fields to light-like 3-surfaces having 2-D partonic surfaces as ends. M^8-H duality reduces space-time surfaces in M^8 to algebraic surfaces defined by polynomials of real variable. The coefficients can be complex. Concerning p-adicization real rationals defines the most attractive option. This leads to a picture in which a hierarchy of extensions of rationals defines evolutionary and cognitive hierarchies. The extensions provide cognitive representations as unique discretizations of the X^4 with imbedding space coordinates in extension of rationals and the one can formulate quantum TGD in finite measurement resolution at least using these representations.

The fermionic variant of M^8-H duality [L11] leads to the conclusion that spinor modes in M^8 are restricted at 3-D light-like surfaces obeying an algebraic equations analogs to the momentum space variant of massless Dirac equation. Are H fermions also always restricted to the 3-D light-like orbits of the partonic 2-surfaces at which the signature of the induced metric changes?

On the other hand, the picture deduced at the level of H from the cancellation of the normal ordering divergences allows 4-D fermions, and also implies field equations for X^4 itself. Can one say that free fermions can reside in 4-D space-time but reside only at the 3-D mass shell in momentum space. M^8-H duality would be analogous to the duality between space-time and momentum space descriptions of particles.

Even more, string world sheets have light-like boundaries at the parton orbits. Also fermions in H would be naturally located at string boundaries and behave like point-like particles. One would obtain a picture resembling that provided by twistor Grassmannian approach. Also the cancellation of normal ordering divergences supports this picture and leads to a detailed form of bi-linear modified Dirac action. Also strong form of holography (SH) stating that 2-surfaces carry all information needed to construct the X^4 supports this view. This is actually the same as the phenomenological picture that has been applied.

M^8-H duality predicts also "very special moments in the life self" to have as correlates 6-branes with M^4 time defining in M^8 octonionic real axis (unique rest system) having as values roots of the polynomial defining the space-time surfaces. These surfaces should contain the partonic 2-surfaces defining the reaction vertices. If there is a non-determinism associated with these surfaces it should preserve classical charges and also SSA charge.

2.1.4 Is the proposed counterpart of QFT supersymmetry only an approximate symmetry?

The proposal for the cancellation of the normal ordering divergences allows overviewing leptons as three quark composites with 3 quarks at the same wormhole throat. This option is strongly suggested by the conceptual economy since quarks are enough for WCW spinor structure.

An interesting question is whether TGD allows a counterpart of QFT supersymmetry (SUSY). This was proposed in [L8]. The idea was that both imbedding space coordinates and spinors can be expanded as polynomials in the local compositesof quark and antiquark oscillator operators - rather than anticommuting hermitian theta parameters leading to problems with fermion number conservation - with a well-defined quark number.

The proposal was that leptons are purely local 3-quark-composite analogous to a superpartner of quark: note however that quark superspinor would have quark number one so that precise spartner interpretation fails. This option and only its slightly local variant is possible only for the TGD view about color as angular momentum rather than spin-like quantum number.

This proposal was based on discrete cognitive representations as unique discretizations of the X^4 and on the crucial assumption that fermionic oscillator operators obey Kronecker delta type anticommutations rather than the 8-D anticommutations giving $\delta^7(0)$ anti-commutator singularities for the induced second quantized quark field in H. Can the notion of super-field based on local composites of quarks and antiquarks with a definite fermion number avoid normal ordering divergences for the induced anticommutation relations? One can of course think of a normal ordering of monomials but one expects problems with vertices.

This suggests that the super coordinates of H and superspinors can be only approximate notions. Superfield components would correspond to states with a fixed quark number but quarks and antiquarks would reside at opposite wormhole throats rather than forming exactly local composites. Since the throat is expected to have CP_2 size, these states would be for all practical purposes strictly local composites.

2.2 Kähler metric as the analog of S-matrix

Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurement. Could the Kähler metric define unitary S-matrix? Under simple additional conditions this is true!

2.2.1 The analogs of unitarity conditions

The following little arguments show that given Kähler metric defines an analog of unitary S-matrix giving rise positive transition probabilities, and under additional conditions also a unitary S-matrix between states with quantum numbers labeling basis of complex vectors or of complexified gamma matrices. This defines an S-matrix like entity and under some additional conditions even an unitary S-matrix.

1. The defining conditions for unitary S-matrix and Kähler metric are very similar. S and $S^\dagger$ would correspond to the covariant metric $g_{m\overline{n}}$ and contravariant metric $g_{\overline{m}n}$. Unitary for S-matrix corresponds to the conditions

$$S_{mr}S^\dagger_{rn} = S_{mr}S_{nr} = \delta_{m,n} \quad .$$

(there is summation over repeated indices). The rows of S-matrix are orthonormalized. The definition of the contravariant metriccorresponds the conditions

$$g_{m\overline{r}}g^{\overline{r}n} = \delta_{m,n} \quad .$$

The complex rows of metric tensor and contravariant metric are orthonormalized also now and rows are orthonormal

2. For S-matrix the probabilities are given by $p_{mn} = S_{mn}S^\dagger_{nm} = S_{mn}S_{mn}*$ and are real and non-negative and their sum is equal to one. Also for the Kähler metric the complex analogs of probabilities defined by

$$p^c_{mn} = g_{m\overline{r}}g^{\overline{r}n}$$

sum up to unity. Hence the real parts $Re(p^c_{mn})$ of p^c_{mn} sum up to unity whereas the imaginary parts sum up to zero.

3. p^c_{mn} are not however automatically real and non-negative and it is not clear how to interpret complex or even real but negative probabilities physically. One can however pose the positivity of the real parts of p^c_{mn} as an additional condition on the phase factors $U_{m\overline{n}} = exp(\Phi_{m\overline{n}})$ and $V_{m\overline{n}} = exp(\Psi_{m\overline{n}})$ associated with $g_{m\overline{n}} = R_{mn}U_{m\overline{n}}$ and $g^{\overline{n}m} = S_{nm}V_{\overline{n}m}$. The condition for positivity is

$$U_{m\overline{n}}V_{\overline{n}m} = cos(\Phi_{\overline{n}m} - \Psi_{\overline{n}m} \geq 0$$

and is rather mild requiring the angle difference to be in the range $(-\pi/2, \pi/2)$. This is true of the angles are in the range $(\pi/4, \pi/4)$. The condition $Re(p^c_{mn}) \geq 0$ is equivalent with the condition $Im(ip^c_{mn}) \geq 0$, and characterizes the coefficients of Teichmueller matrices [A6, A11, A9] [K1]: the meaning of this condition will be discussed below.

4. Under what conditions p^c_{mn} reduce to non-negative real numbers? One can express the probabilities as $p_{mn} = g_{m\overline{n}} \times cof(g_{m\overline{n}})/det(g)$. Note that $Z = det(g)$ is constant depending only on the point of the Kähler manifold. One can express g_{mn} as $g_{m\overline{n}} = A_{mn}U_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ as $cof(g_{m\overline{n}}) = B_{mn}V_{m\overline{n}}$. The reality condition implies

$$U_{m\overline{n}} = \overline{V_{m\overline{n}}} \ .$$

The phases of $g_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ are opposite.

This gives additional conditions. Kähler metric involves $N_{tot} = 2N^2$ real parameters There are $(N^2-N)/2$ elements in say upper diagonal and by hermiticity they are complex conjugates of the lower diagonal. This is the number N_{cond} of conditions coming from the reality. There is also one additional condition due to the fact that the probabilities do not depend on the normalization of g. The total number of real parameters is

$$N_{param} = N_{tot} - N_{cond} - 1 = N(N-1) - 1 \ .$$

For instance, for $N \in \{2,3,4\}$ one has $N_{param} =\in \{1,5,11\}$. Unitary matrix allows $N_{unit} = N^2$ real parameters and the ratio $N_{param}/N_{unit} = (N(N-1)-1)/N^2$ approaches unity for large values of N. Note that a unitary matrix with real diagonals has $N^2 - N$ parameters so that the number of parameters is the same as for a hermitian metric with unit determinant.

5. Could one transform the metric defining non-negative probabilities to a unitary matrix by a suitable scaling? One can indeed define a matrix S as a matrix $S_{mn} = \sqrt{A_{mn}B_{mn}/Z U_{mn}}$. One has $S_{mn}S_{mn}* = A_{mn}B_{mn}/Z$ given also by the product of $g_{m\overline{n}}g^{\overline{n}m}$ so that the probabilities are the same. The unitarity conditions reduce to $g_{m\overline{r}}g^{\overline{r}n} = \delta^n_m$.

   In infinite dimensional case problems might be produced by the appearance of the square root of determinant expected to be infinite. However, also the cofactors are expected to diverge, and one can express them as partial derivatives of the metric determinant with respect to the corresponding element of the metric. This is expected to give a finite value for the elements of the contravariant metric. Note that the ratios of the probabilities do not depend on the metric determinant.

2.2.2 Can one distinguish between the descriptions based on Kähler metric and S-matrix?

For the Teichmueller option the proposed analog for S-matrix involves imaginary part. Does it have some physically observable consequences?

Could one imagine a physical situation allowing to test whether the S-matrix description or its TGD variant is nearer to truth? One can indeed imagine an analog of a Markov process characterized by a matrix p of transition probabilities p_{mn} at a given step. For a two-step process the transition matrix would be p^2_{mn}.

In the TGD context one would have $p_{mn} = Re(p^c_{mn})$. What happens in a two-step process? Should one use use p^2_{mn} or $Re((p^c)^2)_{mn} = Re((p^c)2)_{mn} - Im(p^c)^2_{mn}$? If both options are possible, what could distinguish physically between them?

Could the correct interpretation be that p^2_{mn} describes the process when the outcome is measured in both steps, and $Re((p^c)^2)_{mn}$ the process in which only initial and final states are measured? This picture would generalize to n-step processes and predict a deviations from the ordinary Markov process and perhaps allow to compare the predictions of the TGD view and standard view and deduce $Im(p^c)$.

S-matrix and its Hermitian conjugate correspond in standard physics to situations related by CPT symmetry defined as the product of charge conjugation C, spatial reflection P and time reversal T. The transition probabilities would remain invariant in this transformation although transition amplitudes are replaced with their complex conjugates.

What happens to CPT in TGD framework? In TGD framework CPT induces a hermitian conjugation $g_{m\overline{n}} \rightarrow g_{\overline{n}m} = \overline{g_{m\overline{n}}}$ inducing $p^c(mn) \rightarrow \overline{p^c(mn)}$. Transition probabilities $p_{mn} = Re(p^c_{mn})$ are not changed. The probabilities for an n-step Markov process and its CPT transform would be the same since only even powers of $Im(p^c)$ appear in the transition probabilities. In this sense CPT would remain exact symmetry also for generalized Markov processes.

2.2.3 Teichmueller parameters

For Riemann surfaces, Teichmueller parameters Ω_{mn} define the matrix elements of Teichmueller matrix Ω [A6, A11, A9]. Their definition and basic properties are as follows.

1. A Riemann surface with genus g has a homology basis consisting of $2g$ elements. One can find a canonical homology basis (a_i, b_i) in which intersection matrix J_{mn} for the homology basis is diagonal $J(a_i, b_j) = \delta_{i,j}$. One can assign to the g elements a_i g holomorphic forms ω_i as their duals satisfying $\omega_i(a_j) = \delta_{i,j}$. The elements of the Teichmueller matrix Ω are defined as

$$\Omega_{mn} = \int_{b_j} \omega_i \ . \tag{2.1}$$

2. The basic properties of Teichmueller parameters are the following [A6, A11, A9]:

   (a) The $g \times g$ matrix Ω is symmetric.

   (b) The imaginary part of Ω is positive: $Im(\Omega) > 0$. The space of the matrices satisfying these conditions is known as Siegel upper half plane.

   (c) The space of Teichmueller parameters can be regarded as a coset space $Sp(2g, R)/U(g)$ [A9]: the action of $Sp(2g, R)$ is of the same form as the action of $Sp(2g, Z)$ and $U(g) \subset Sp(2g, R)$ is the isotropy group of a given point of Teichmueller space.

   (d) Teichmueller parameters are conformal invariants as is clear from the holomorphy of the defining one-forms.

   (e) Teichmueller parameters completely specify the conformal structure of Riemann surface [A11].

2.2.4 The analog of Teichmueller condition in TGD framework

The analog of Teichmueller condition looks highly attractive option in TGD framewokr.

1. The components ip^c_{mn} would be in the upper complex plane, which is highly interesting from the point of view of complex analysis. Teichmueller spaces (see `https://en.wikipedia.org/wiki/Teichmller_space` and `https://en.wikipedia.org/wiki/Quadratic_differential`) and their possibly existing generalizations therefore provide candidates for Kähler manifolds allowing probability interpretation. Teichmueller space parameterizes the complex structures of space: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices p^c_{mn} correspond to complex structures for some space?

2. The correspondence between $g_{m\overline{n}}$ and p^c_{mn} need not be 1-to-1. One can multiply $g_{m\overline{n}}$ and $cof(g)g)_{m\overline{n}}$ with opposite phases without changing p^c_{mn}. Also a more general unitary transformation $g_{m\overline{r}} \rightarrow g_{n\overline{s}} U^{\overline{s}}_{\ \overline{r}}$ and $g^{\overline{r}n} \rightarrow U^{\overline{r}}_{\ \overline{s}} g^{\overline{s}n}$ preserves p^c_{mn}. If this transformation is induced by a unitary change of basis, it brings no new physics. If this transformation is induced by SSA acting as isometry, it would have physical content.

3. By SH, the most natural candidate would be a Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and of string world sheets. This would mean a huge reduction of degrees of freedom for WCW. SSA conditions indeed reduce WCW to effectively finite-D space. Also $M^8 - H$ duality assuming that space-time surfaces correspond to polynomials predict similar reduction since the roots of the polynomial code for the X^4 once CD is fixed,

4. Teichmueller spaces parameterizing the conformal equivalence classes of Riemann surfaces appearing in the TGD based explanation for family replication phenomenon in terms of the topology of partonic 2-surfaces [K1] have Teichmueller matrices as coordinates.

   Elementary particle vacuum functions in the space of Teichmueller parameters would be functionals in the space of WCW Kähler metrics restricted to fixed space-time-topology.

Could the Teichmueller parameters of partonic 2-source code for single fermion scattering probabilities? As proposed earlier, the ends of the boundaries of string world sheets at partonic orbits carrying fermion number would correspond to marked points of the partonic 2-surfaceas a Riemann surface.

5. What about Teichmueller parameters of partonic 2-surfaces appearing as vertices? The intuitive expectation is that their presence increases the number of WCW degrees of freedom? It seems that also their Teichmueller spaces must be included. Could the strict Bohr orbitology fail in the sense that partonic two-surfaces at both ends of space-time surfaces at boundaries of CD and also in the interior of X^4 serve as causal determinants? M^8-H duality suggests that the roots $t=r_n$ hyperplanes of M^4 (r_n is a root of polynomials determining X^4 in M^8) at which strict determinism fails: this would correspond to the gluing together of partonic orbits as lines of a topological Feynman diagram along their ends.

6. Without SH the degrees of freedom associated with the complement of string world sheets and partonic 2-surfaces would represent additional degrees of freedom, which would not contribute to the WCW metric. For preferred extremals these degrees of freedom would reduce to the interior degrees of freedom for 3-surfaces at either boundary of CD. However, in its most stringent form SH implies that the zero modes are in 1-1 correspondence with Teichmueller parameters and therefore WCW Kähler metrics. Zero modes would characterize the conformal equivalence classes of partonic 2-surfaces.

7. It is not clear what the physical interpretation for the condition

$$\sum_n Im(p^c_{mn}) = 0$$

could be: geometrically the sum of complex vectors defined by the probabilities would sum up to a vector from 0 to 1. The modification of Markov processes already mentioned could provide a possible physical interpretation.

The above observations inspire the idea that quantum physics could be geometrized in the same manner as Einstein geometrized gravitation posing only the Teichmueller condition on the components of the Kähler metric. One would start from a flat Hilbert space bundle and replace its flat Kähler metric with a non-flat one satisfying the Teichmueller conditions. In the TGD framework one must consider a Hilbert space bundle with WCW as a base space. Quantum theory would involve gravitation at the level of Hilbert space and could lead from a non-interacting quantum theory to interacting one. This replacement is completely universal and would solve the unitary problem that has been plaguing - not only TGD but also twistor Grassmann approach [B1].

In the sequel a concrete realization of this idea is considered in the TGD context, where the Hilbert space would consist of WCW spinor fields in the framework provided by the WCW geometry. In infinite-D context the existence of Kähler metric poses extremely strong conditions as Freed demonstrated for loop spaces [A10]: in this case the Kähler metric is unique and has maximal isometry group to guarantee the existence of vielbein connection. This actually led to the TGD vision that the existence of WCW Kähler geometry fixes physics uniquely.

2.3 WCW metric is not enough for the analog of S-matrix

The construction of single particle WCW gamma matrices $\Gamma_{\pm,A}$ assigned to 3-surfaces with a single component is straightforward as a fermionic SCA supercharges determined as super Noether charges for the 3-surface at either boundary of WCW labelled here by $\pm$. Supercharge has the form $u_n J^k_A \gamma_k \Psi$, where u_n is a mode of the induced spinor field satisfying modified Dirac equation, $J^k_A \gamma_k$ is the contraction of SSA Killing vector J_A with H gamma matrices and Ψ is the second quantized free spinor field of H.

The component $G(A, \overline{B}$ the WCW metric is vacuum expectation of the product

$$G(A, \overline{B}) = \langle vac|\{\Gamma_{+A}\Gamma_{-\overline{B}}\}|vac\rangle \ .$$

In the vacuum expectation the anti-commutator can be replaced with the product.

It is important to notice that $G(A,\overline{B})$ is not the fermionic propagator but the symmetric $\Delta function$. Fermion propagator emerges as the c-number part of the decomposition of the time order product of $A(x_1) = \Psi^\dagger(x_1)\gamma^0$ and $B(x_2) = \Psi(x_2)$ to a c- number part defined as its vacuum expectation and q-number part defined as the normal ordered product $: A(x_1)B(x_2) :$. Let t_1 and t_2 be the time coordinates associated with the 4-coordinates x_1 and x_2.

One has for the time ordered product

$$A(x_1)B(x_2)\theta(t_1-t_2) - B(x_2)A(x_1)\theta(t_2-t_1) = \Delta_F(x_1,x_2) + : A(x_1)B(x_2) : \ ,$$
$$\Delta_F(x_1,x_2) = \langle vac|A(x_1)B(x_2)(\theta(t_1-t_2) - B(x_2)A(x_1)\theta(t_2-t_1)|vac\rangle \ .$$

$\theta(t)$ is the step function. The appearance of γ^0 is crucial: $\Psi^\dagger$ appearing in the anticommutator is replaced with $\overline{\Psi}$. In Feynman diagrams γ^0 comes from vertices defined by currents of type $\overline{\Psi}\gamma^k\Psi$. In TGD the SSA co-products appearing in the 3-vertices must contain γ^0. The SCA charges defined by MDA indeed have the same general form as gauge charges.

The interpretation of anticommutator would be as an analog of single particle S-matrix. The Kähler metric of WCW is a single particle notion and can define the analog of S-matrix only for single fermion states. The metric for the Hilbert space bundle WCW spinor fields assignable to the boundaries of CD is needed. This bundle is in the first approximation (no interactions) a tensor power of spinor bundle of single particle WCW consisting of pairs of connected 3-surfaces at the boundaries of CD. The Kähler metric for the WCW spinors - Fock states of fermions and antifermions - would also be a tensor product and in ZEO determined by the complex inner products of states at opposite boundaries of CD. The challenge is to understand how the interactions force this metric to deviate from a mere tensor product of spinor bundle metrics.

2.4 Reduction of the analog of S-matrix to the Kähler metric in the space of WCW spinor fields

Geometrically, the inner product codes for distances and angles but in quantum theory Hilbert space inner products code for probabilities given by the moduli squared. Riemannian geometry geometrizes classical physics. Could infinite-D Kähler geometry of WCW inducing the Kähler geometry of WCW spinor fields geometrize quantum physics and allow to identify the counterpart of unitary S-matrix as Kähler metric? If so, then the construction of scattering amplitudes would reduce to that of geometry for the Hilbert bundle of WCW spinor fields, and basically for the WCW itself.

1. What makes this idea so attractive is that in TGD framework complexified gamma matrices of WCW giving as anti-commutators the WCW Kähler metric are expressible in terms of the fermionic oscillator operators of second quantized spinor field in $H = M^4 \times CP_2$ restricted to X^4 so that the anticommutators can be calculated without information about the solutions of MDE. Second nice point is that WCW gamma matrices would correspond to fermionic generators of SSA so that the construction of scattering amplitudes could reduce to infinite-D group theory.

2. The hypothesis that hyper-finite factors of type II_1 (HFFs) [K9, K7] are involved would make finite-D approximation an excellent one. This would be realized in terms of the condition that states are annihilated by a subalgebra SSA_n of SSA with conformal weights coming as n-multiples of those for the entire algebra $n = 1, 2, ..$ and by the commutator $[SSA_n, SSA]$ so that SS group would reduce effectively to a finite-D dynamical symmetry group. Note that SSA_n is isomorphic to SSA so that one has fractality.

3. The notion of Connes tensor product [A8, A5] playing a role in the inclusions of HFFs to accompany inclusions of extensions of rationals suggests itself. For ordinary matrices the Connes tensor product for $m \times k$ and $k \times n$ matrices is $m \times n$ matrix and reduces to the ordinary matrix product for $k = 1$. The definition of Connes tensor product should involve the partonic 2-surfaces and light-like partonic orbits connecting them and serving as lines of a generalized Feynman diagram.

   Quantum groups [A4, A7, A14] are closely related to the inclusions of HFFs. Besides ordinary product these algebras possess co-algebra structure characterized by co-product [A2, A13], which is in certain sense the time reversal of the product.

4. Kähler metric can be also expressed in terms of second derivatives of Kähler function so that one would have two approaches: the purely bosonic and that given super-algebra generators. A further nice point would be uniqueness: non-flat infinite-D Kähler geometries are highly unique as the construction of loop space geometry demonstrated [A10]. The infinite-D character of state space would make physics unique. A further beautiful element would be the interpretation of non-negative probabilities in terms of the Teichmueller property of the complex probabilities defined by the Kähler metric.

2.4.1 Inner products for both 3-D states at boundaries of CD and for zero energy states are needed

My new proposal states that there should be inner products for both the 3-D states at the boundaries of CD and the zero energy states defined as their pairs: the interpretation of the inner product for zero energy states has remained open question although I have before suggested a generalization of S-matrix to what I called U-matrix [K5]. Besides this, there would be the Kähler metric defined by the pairing between the 3-D states at opposite boundaries of CD. A more detailed consideration shows that non-triviality of the inner products requiring the cancellation of normal ordering divergences forces ZEO.

1. Anti-commutator is for 8-D spinor fields extremely singular being proportional to a 7-D delta function, and normal ordering divergences can be avoided only if the anti-commutators are for pairs of points at disjoint regions of the X^4 - different space-time sheets carrying either quarks or antiquarks, in fact. This would however imply vanishing of the products at $t = constant$ hyperplanes so that standard ontology would fail!

   This provides a motivation for ZEO [L7]. In ZEO, the quantum states in 3-D sense appearing as building bricks of zero energy states are defined at the opposite light-like boundaries of CD. 3-surfaces have an Euclidian signature of the induced metric but since the spinor modes are those of H spinor fields, light-like M^4 distance is possible for the points involved and the inner product defined by the anticommutator can be non-vanishing. This inner product could be used to define a basis of states at boundaries of CD - not only for single fermion states but also many-fermion states. Between bras and kets the inner products would give Kronecker deltas.

2. Zero energy states allow definition of Kähler metric $K_{K\overline{L}}$ of WCW as a hermitian bilinear form between their members at opposite boundaries of CD in terms of the scattering amplitudes.

3. One can define also an inner product for zero energy states as the inner product of a state pair defining a component in zero energy state with another similar pair. The inner product would be induced by the Kähler form and the inner product at the light-like boundary. For zero energy states $c^{1)}_M K^{K\overline{L}}\langle K|\langle\overline{L}|$ (sum over M) and $c^{2)}_M K_{M\overline{N}}|M\rangle|\overline{N}\rangle$, $|c^{i)}_M|^2 = \sum p^{i)}_M = 1$, the inner product between these building bricks would be given by

   $$p^{1)}_M p^{2)}_N K_{M\overline{N}} K^{\overline{N}M} \delta_{M,N}\delta_{N,L} = p^{1)}_M p^{2)}_N p^c M, N \delta_{M,N}\delta_{N,L}$$

   since the inner products between bras and kets give Kronecker deltas.

   The real parts of the complex probabilities $p^c_{M,N}$ define conserved positive probabilities if the Teichmueller condition is satisfied. This matrix would be diagonal in the space of zero energy states but its elements would sum up to unity. A complex analog of a diagonalized density matrix (having real elements) assignable to the initial state $c^{1)}_M|M\rangle$ as probabilities of the final states $c^{1)}_N|N\rangle$ would be in question. Also the real part would define density matrix.

2.4.2 Quantum algebra structure and the Kähler metric in the space of WCW spinor fields

One should construct the Kähler metric in the space of WCW spinor fields.

1. In the first approximation, the WCW for 3-surfaces with n components corresponds to a Cartesian power of single particle WCW (this as long as intersections of 3-surfaces are neglected). Spinor space would be an n^{th} tensor power of WCW spinor spaces - essentially Fock space for fermions - and the inner products would be products of matrix elements of single particle Kähler metric. Single fermion states are created by gamma matrices or equivalently fermionic SCA generators so that they correspond in 1-1 manner to each other. Hence it natural to define tensor power of single particle gamma-matrix algebra.

   By physical intuition, this is expected to work if the X^4 represents the topological analog of a trivial Feynman diagram: no partonic 2-surfaces acting as vertices.

   Also bosonic generators of SCA create states at the boundaries of CD but for the purpose of constructing a unitary S-matrix it is enough to have Hilbert space metric having SCA as maximal group of isometries - or even the Yangian generalization of SCA.

2. One can use perturbative QFT as a guideline. Identify the partonic 2-surfaces at which n lines meet as n-vertices decomposing to 3-vertices. Assign to these n-vertices superpositions of products of fermionic and bosonic SCA generators multiplied by the analogs of couplings. Scattering matrix would be the vacuum expectation of the operator constructed as a product of operators at various partonic 2-surfaces assignable to boundaries of CD and to the interior of the X^4.

3. The states assigned to the partonic surface defining the vertex cannot be chosen arbitrarily. In accordance with the QFT picture, the states should form a superposition over all possible states which can appear as intermediate states. Quantum variant of SCA has both product and co-product having interpretation as time reversals of each other. Annihilation of boson to fermion pair would correspond to co-product and the annihilation ofof a fermion pair to a boson to the product. It looks like a matter of taste whether to use only product or co-product with both arrows of time or both product and co-product with a single arrow of time.

   In $B \rightarrow F\overline{F}$ vertex co-product creates this kind of state: the analog of 3-vertex as the co-product assignable to generator A is a superposition of all pairs $B \otimes C$ of generators, whose product $B \circ C$ contains A. The states associated with the propagator line from the boundary of CD to the interior should correspond to the 3 SCA generators in co-product. The iteration of a co-product defining boson-fermion-antifermion vertex could allow building of n-vertex uniquely. By the previous arguments the co-product involves $gamma^0$ making it possible to assign to the quark line quark propagator rather than anti-commutator.

Bilocality of the bosonic SCA generators allows to get rid of normal ordering divergences but brings in additional complications. On the other hand, one can say that vertices disappear alltogether to the SCA, which is definitely a simplication.

1. FFB interaction vertex, which would look like a point-like 3-vertex in rough enough resolution, decomposes in a resolution allowing allowing seeing of the individual fermions at partonic 2-surfaces to partonic 2-surfaces and propagators.

2. If bosonic SCA generators were not bi- local, one would have 3 partonic 2-surfaces. Normal ordering divergences are cancelled, if the bosonic SCA generators as Noether charges are bi-local and assigned with pairs of 2 disjoint partonic 2-surfaces, which can reside also in the interior of CD: most naturally this pair corresponds to the two throats of wormhole contact. In the phenomenological picture fundamental fermions are associated with either throat of a wormhole contact.

   The simplest bosons identifiable as SCA Noether charges would correspond to wormhole contacts with fermion and antifermion at opposite throats. This conforms with the view about bosons as bound states of fundamental fermions.

3. The bi-local modified Dirac action can also pair the wormhole throats of separate wormhole contacts - call them A and B. Elementary particles would naturally correspond to closed flux tube carrying monopole flux starting connecting the upper throats of A and B, going to

lower space-time sheet through B, and connecting lower throats of B and A and returning to the upper throat of A along A. This configuration would look like a very long highly flattened rectangle with sides having lengths of order CP_2 size and Compton length of particle. The condition that field lines of the monopole flux close would force this arrangement. If gauge bosons and elementary fermions (to be distinguished from fundamental fermion fields) correspond to states of this kind, the quantal notion of Compton length would also have a geometric interpretation. The bi-local Dirac operator could pair the upper throat of A (B) with the lower throat of B (A) at distance of order Compton length.

4. In the vertex 3 wormhole contacts each forming pair of worm-hole throats are glued together at the wormhole contact with two wormhole throats containing only quarks or antiquarks since the elimination of normal ordering divergences does not allow quark and antiquark at same throat. Bosonic propagators would be composites of fermionic propagors. The vacuum expectation value would give rise to fermionic propagators connecting various partonic 2-surfaces. The propagator lines would not enter the same point as in QFT but only to the same partonic 2-surface.

5. The TGD counterpart for $F\overline{F}B$ vertex would correspond to a bosonic wormhole contact with a fermion and antifermion at opposite wormhole throats and representing SCA generator which decomposes to two partonic 2-surfaces carrying fermions at opposite throats representing fermionic SCA generators. The vertex would correspond to a product or co-product, which can be said to be time reversals of each other. The structure constants of SCA extended to quantum algebra would fix the vertices and thus the analogs of Feynman diagrams completely. Their number is presumably finite for a X^4 with fixed 3-surfaces at its ends and summation over Feynman diagrams would correspond to integration in WCW.

The SCA gauge conditions replace WCW with finite-D space. Same is achieved in M^8 picture as a representation of space-time time surfaces as "roots" for octonionic continuations of real polynomials. For the points of cognitive representations of X^4 the imbedding space coordinates are in an extension of rationals at M^8 level and possibly even at the level of H. This brings in an additional simplification. The discretization by the cognitive representation would replace the SCA charges defined as integrals over pairs of partonic 2-surfaces with sums over contributions over a discrete set of pairs of points. Cognitive representations effectively approximate sub-WCW defined by polynomials with given degree with a finite-D discrete Kähler manifold and replace WCW spinor space with a finite-D spinor space [L1, L12]: this is discussed in more detail in the section "How to avoid normal ordering divergences from fermionic oscillator operators?". Quarks at partonic surfaces would be naturally associated with the points of cognitive representation.

2.4.3 The analogs of Feynman rules

It is interesting to formulate more concretely the analogs of Feynman rules completely generally for a single X^4 in the superposition defining spinor field in WCW. One must perform WCW integration to obtain the full amplitude.

1. One assigns to partonic 2-surfaces identified as the opposite throats of wormhole contacts many-fermion states consisting of quarks and antiquarks respectively. Same throat cannot contain both quarks and antiquarks. At boundaries of CD a given quark/antiquark corresponds to a supergenerator of SCA. Super-generators with $Q = 1$ (Q is quark number) and supergenerators with $Q = -1$ at opposite wormhole throat possibly associated with different wormhole contact of the same space-time sheet can combine to form a bosonic SCA generator as a Noether charge. These operators at boundaries of CD create a many-quark state.

2. One assigns many-quark states to the partonic 2- surfaces in the interior of the X^4 by using the same rules. There is basically no difference between the two cases. One must however sum over all possible intermediate states created by SCA. In particular, over the states at partonic 2-surfaces with arbitrary fermion/antifermion numbers. The number of quarks/antiquarks at a given partonic 2-surface can be arbitrarily high but the anti-commutativity at a given throat makes the contributions coming from high quark numbers small.

Assuming that wave functions of quarks at partonic 2-surfaces are constant, statistics allows only finite quark- and antiquark numbers: the extension of the imbedding space coordinates and spinors to superfields consisting of local composites of oscillator operators proposed in [L8] could approximate this situation.

3. An important implication is that for the partonic 2-surfaces at $t = r_n$ planes associated with 6-branes, all anticommutators vanish since fermions and anti-fermions reside at different space-time sheets. The time-ordered products also vanish since one has $t_1 = t_2$. Therefore the numbers of external fermion and antifermion lines at the partonic 2-surfaces must be the same as the number of fermions and antifermions at the partonic 2-surface (bosonic generators involve fermion and antifermion). All radiative corrections coming from a surplus of fermions and antifermions at partonic 2-surface are absent. This simplifies the construction enormously since the number of contributions is finite.

   Only the radiative corrections having interpretation in terms of topological loops for partonic 2-surface can be present and it might well be that they are not realized as preferred extremals. In any case, they would be finite.

4. By $M^8 - H$ duality, the partonic 2-surfaces are associated with $t = r_n$ hyperplanes of M^4, where r_n is a root of the real polynomial extending to octonionic polynomial defining X^4. This analog of a Feynman graph can be regarded as a sequence of intermediate states at hyperplanes labelled by r_n, whose number is finite.

   In the path integral approach as Feynman originally formulated it, this sequence would consist of an infinite number of hyper-planes of M^4 infinitesimally near to each other. This picture could follow as an approximation as the degree of polynomial becomes infinite and algebraic extension becomes algebraic numbers.

5. The conservation of monopole flux (monopoles are not allowed but closed monopole fluxes are possible). Forces elementary particles to involve at least 2 wormhole contacts at the ends of a string-like object in the manner already explained. These are both at boundaries of CDs and in the interior and their throats are populated either by quarks or antiquarks.

6. Generalized Feynman amplitude is obtained as a vacuum expectation of this operator. The expectation value reduces to a quantity involving quark propagators as contractions connecting quarks and anti-quarks at wormhole throats at opposite sheets of double-sheeted structure. All contractions are allowed but always connected external fermions to the partonic 2-surface.

   An important distinction from the QFT picture is that the vertices are not local but quark lines and the quark lines associated with bosons have ends at distinct points at partonic 2-surface. This makes it possible to avoid the standard divergences of QFTs. For instance, the analog of $BF\overline{F}$ at the second end of a stringy object representing an elementary particle has 2 quark line ends at the upper throat and 2 antiquark line ends at the lower throat.

   Also more general contractions - even between different pairs of space-time sheets - are possible and are completely determined by the anti-commutation relations of quark fields in H. Their contribution is expected to be small since the propagators become very small at large distances. For a given X^4 the number of partonic 2-surfaces is finite.

7. The contractions of from quarks at opposite boundaries of CD - external particles - is finite and the remaining contractions between throats of same partonic 2-surface or distinct partonic 2-surfaces in the interior of X^4 give rise to the analogs analogs of self-energy corrections and vertex corrections. They are finite because the ends points of propagators are at distance of order CP_2 size scale rather than vanishing as in QFT.

8. For a given X^4, the number of vertices as number of wormhole contacts is finite and one also expects that the number of space-time surfaces with fixed 3-surfaces at boundaries is finite. It is not clear whether WCW integration over 4-surfaces can give contributions with an arbitrarily high number of wormhole contacts as vertices. WCW integration would reduce to an integration over moduli spaces of complex structures for partonic 2-surfaces

with punctures - the end points of propagators - parameterized by Teichmueller parameters and over the moduli spaces of string world sheets.

2.4.4 About the generalization of Equivalence Principle

Equivalence Principle (EP) states that in Gaussian coordinates the Riemann connection coding for what is regarded as gravitational for in Newtonian theory, vanishes.

The generalization of the EP to the level of WCW would state that for preferred coordinates - call them Gaussian coordinates - obtained by a suitable SS rotation WCW gravitation is eliminated for a given 3-surface (pair of 3-surfaces at boundaries of CD) in the sense that the components of WCW Rieman connection and also of WCW spinor connection vanish at this 3-surface. Note that spinor connection lifts the action of this rotation to the level of WCW spinor fields as action of sigma matrices forming the Hilbert space of zero energy states.

The Gaussian coordinates for WCW allow a perturbatic treatment of the WCW spinors associated with 3-surfaces near the Gaussian 3-surface. Metric, vielbein and gamma matrices have vanishing covariant derivatives so that the first derivatives are proportional to these connections vanishing at the Gaussian 3-surface. Therefore the Taylor expansion zero energy involves only spinor curvature and isometry generators associated with the Gaussian coordinates.

WCW is an analog of symmetric space G/H for with tangent space algebra given by symplectic algebra of $\Delta M^4_\pm \times CP_2$ extends to a super-symplectic algebra SSA [K2] with fermionic generators identifiable as WCW gamma matrices [K2]. WCW is actually a union of analogs of symmetric spaces labelled by zero modes. Each symmetric space in the union allows Lie-algebra decomposition $g = t + h + \bar{t}$, and one can introduced complex coordinates assignable to t and $\bar{t}$ vanishing at Gaussian 3-surface, and the commutators of t and $\bar{t}$ give diagonalizable generators of h. Note that CP_2 provides a finite-D analogy for this.

This rotation does not change the X^4 and its topology. WCW Riemann connection coding for the analogs of classical gravitational interactions at WCW level WCW spinor connection however acts on spinors via sigma matrices and affects the zero energy states consisting basically of fermions. Hence also zero energy states are affected in fermionic degrees of freedom. One can say that this rotation eliminates WCW gravitational interaction making itself visible via spinor connection terms for deformed 3-surfaces.

Can one even say that this rotation eliminates all interactions at the level of WCW? This cannot take place if the particle numbers at the opposite boundaries are different so that there are topological particle exchanges: one could interpret them a non-gravitational interactions.

2.4.5 Connection with infinite primes?

Also the Hilbert space of zero energy states should have spinor structure assigning gamma matrices to the state basis. Does this lead to a hierarchy of second quantizations starting from space-time level and leading to gamma matrices at the level of WCW and continuing indefinitely?

Amusingly, there is a connection to infinite primes [K8], which is an old TGD inspired idea. Whether this idea might have a physical application or not, has remained an open question. The construction of infinite primes reduces to a repeated second quantization of an arithmetic quantum theory with states labelled by ordinary primes. One interpretation would be in terms of abstraction hierarchy. The geometric interpretation would be in terms of the hierarchy of space-time sheets.

Could this hierarchy be realized at the level of Hilbert space and gives rise to an infinite hierarchy of second quantizations? In M^8 picture space-time surfaces correspond to polynomial primes as irreducible polynomials. Infinite primes at the first level indeed correspond to irreducible polynomials. The simplest infinite primes can be mapped to rationals. Polynomials are characterized by their roots and the hierarchy of extensions of rationals central in M^8 picture corresponds to the hierarchy of irreducible polynomials.

One can assign to a polynomial defining an extension of rationals a finite number of ramified primes identified as p-adic primes characterizing the p-adic length scales assignable to elementary particles [K4] and also larger systems. Could the ramified primes correspond to the primes defining the finite part of infinite prime at the first level of the hierarchy. What about finite primes defining the lowest level? Could they correspond to the p-adic primes.

3 The role of fermions

In this section the role of fermions (quarks as it seems) is discussed in more detail. In particular, the conditions on the scattering amplitudes from the cancellation of normal ordering divergences and co-associative octonionic spinors at the level of M^8 are discussed. Also the formulation of scattering amplitudes the level of M^8 is briefly considered.

3.1 Some observations about Feynman propagator for fundamental quark field

In the sequel the divergence cancellation mechanism and the properties of Dirac propagator are discussed in detail. The surprise is that the massive propagators with CP_2 mass scale reduce essentially to massless propagators for light-like separations. This allows understanding of why quarks can give rise to light elementary particles.

The second quantized free quark field Ψ in H defines fundamental fermions appearing as a building brick of elementary particles. The Feynman propagator for Ψ appears in the analogs of Feynman diagrams. Apart from the right handed neutrino (present only as a 3 quark composite at partonic 2-surface if only quarks are involved) the modes of Ψ are extremely massive. Elementary particles are light. How can one understand this?

In p-adic thermodynamics the generation of small mass was assumed to involve a generation of a negative, "tachyonic", ground state conformal weight encountered also in string models. M^8-H correspondence allows a more sophisticated description based on the choice of $M^4 \subset M^8$ mapped to $M^4 \subset H$. By 8-D Lorentz invariance the 4-D mass squared of ground state massless in 8-D sense, depends on the choice of $M^4 \subset H$, and with a proper re-choice of M^4 the particle having large M^4 mass becomes massless.

The action of the generators of super-conformal algebra creates states with a well-defined conformal weight, which are massless for a proper choice of $M^4 \subset M^8$. In p-adic thermodynamics the choice of $M^4 \subset M^8$ would correspond to a generation of negative ground state conformal weight.

The states can however mix slightly with states having higher value of conformal weight, and since one cannot choose M^4 separately for these states, a small mass is generated and described by p-adic thermodynamics.The classical space-time correlate for the almost masslessness is minimal surface property, which provides a non-linear geometrization for massless fields as surfaces. The non-linearity at the classical level leads to a generation of small mass in 4-D sense for which p-adic thermodynamics provides a model.

The propagators for the fundamental quarks in H correspond to CP_2 mass scale. Can this be consistent with the proposed picture? The following simple observations about the properties of predicted fermion propagator and anticommutator for the induced spinor fields lead to a result, which was a surprise to me. The propagators and anti-commutators of massive quarks at light cone boundary are in excellent approximation massless for light-like distances. This makes it possible to understand why elementary fermions are light.

This mechanism does not work in QFT defined in M^4 since inverse propagator is $\gamma^k p_k + m$ so that M^4 chiralities mix for massive states. In TGD picture H-chirality is fixed by 8-D masslessnes and the product of M^4 and CP_2 chiralities for spinors equals to the H chirality. The inverse progator is proportional to the operator $p^k\gamma + D_{CP_2}$, where D_{CP2} is CP_2 part of Dirac operator.

3.1.1 General form of the Dirac propagator in H

Second quantized quark field Ψ restricted to the space-time surface determines the Feynman propagator fundamental quark. The propagator can be expressed as a sum of left- and right-handed propagators as

$$S_F = S_{F,L} + S_{F,R} = D_L G_{F,L} + D_R G_{F,R} \ .$$

Here D_L and D_R are the left- and right-handed parts of a massless (in 8-D sense) Dirac operator D in H involving couplings to CP_2 spinor connection depending on CP_2 chirality in accordance with electroweak parity breaking. $G_{F,L}$ *resp.* $G_{F,R}$ is the propagator for a massless (in 8-D sense) scalar

Laplacian in H coupling to the spinor connection assignable to left *resp.* right handed modes. G_F can be expressed by generalizing the formula from 4-D case

$$G_{F,I}\sum_n \int d^4p \frac{1}{p^2-M^2_{n,I}} exp(ip\cdot(m_1-m_2))\Phi^*_{n,I}(s_1)\Phi_{n,I}(s_2) \ .$$

Here one has $I\in\{L,R\}$ and the mass spectra are different for these modes. Here m_i denote points of M^4 and s_i points of CP_2. n,I, $I\in\{L,R\}$, labels the modes $\Phi_{n,I}$ of a scalar field in CP_2 associated with right and left handed modes having mass squared $M_{n,R}$. Since H-chirality is fixed to be quark chirality, there is a correlation between M^4 - and CP_2 chiralities. Apart from ν_R all modes are massive (ν_R is need not be present as a fundamental fermion) and the mass M_n, which is of order CP_2 mass about 10^{-4} Planck masses, is determined by the CP_2 length scale and depends on CP_2 chirality.

$G_{F,I}$ reduces to a superposition over massive propagators with mass $M_{n,I}$:

$$G_{F,I}=\sum_n G_F(m_1-m_2|M_n)\Phi^*_{n,I}(s_1)\Phi_{n,I}(s_2)P_I \ .$$

Here P_I, $I\in\{L,R\}$ is a projector to the left/right handed spinors. One can express $S_{F,I}$ as a sum of the free M^4 part and interaction term proportional to the left - or right-handed part of CP_2 spinor connection:

$$S_{F,I}=D(M^4)G_{F,I}+A_IG_{F,I} \ .$$

A_I, $I\inf\{L,R\}$ acts either on s_1 or s_2 but the outcome should be the same. The first term gives sum over terms proportional to massive free Dirac propagator in M^4 allowing to get a good idea about the behavior of the propagator.

3.1.2 About the behavior of the quark propagator

The quark propagator reduces to left- and right-handed contributions corresponding to various mass values $M_{n,I}$. To get view about the behaviour of the quark propagator it is useful to study the behavior of $G_F(x,y|M)$ for a given mass as well as the behaviors of free and interacting parts of S_F its free part

From the explicit expression of $G_F(m_1-m_2|M_n)$ one can deduce the behavior of the corresponding contribution to the Feynman propagator. Only ν_R could give a massless contribution to the progator. Explicit formula for G_F can be found from Wikipedia [A3](`https://en.wikipedia.org/wiki/Propagator#Feynman_propagator`):

$$G_F(x,y|m)=\left\{\begin{array}{ll} -\frac{1}{4\pi}\delta(s)+\frac{m}{8\pi\sqrt{s}}H_1^{(1)}(m\sqrt{s}), & s\geq 0\\ -\frac{im}{4\pi^2\sqrt{-s}}K_1^{(1)}(m\sqrt{-s}), & s\leq 0 \ . \end{array}\right.$$

Here $H_1^{(1)}(x)$ is Hankel function of first kind and $K_1^{(1)}$ is modified Bessel function [A1](`https://en.wikipedia.org/wiki/Bessel_function`). Note that for massless case the Hankel term vanishes.

Consider first Hankel function.

1. Hankel function $H_\alpha^{(1)}(x)$ [A1, A3] obeys the defining formula

   $$H_\alpha^{(1)}(x)=\frac{J_{-\alpha}(x)-exp(i\alpha\pi i)J_\alpha(x)}{isin(\alpha\pi)} \ .$$

   For integer values of α one has $J_{-n}(x)=(-1)^nJ_n(x)$ so that $\alpha=n$ case gives formally 0/0 and the limit must be obtained using Hospital's rule.

2. Hankel function $H_1^{(1)}(x)$ can be expressed as sum of Bessel functions of first and second kind

   $$H_1^{(1)}(x)=J_1(x)+iY_1(x) \ .$$

   J_1 vanishes at origin whereas Y_1 diverges like $1/x$ at origin.

3. The behaviors of Bessel functions and their variants near origin and asymptotically are easy to understand by utilizing Schrödinger equation inside a cylinder as a physical analogy. The asymptotic behaviour of Hankel function for large values of x is

$$H_\alpha^{(1)}(x) = \frac{2}{\pi x} exp(i(x - 3\pi/4)) \ ,$$

4. The asymptotic behavior of Hankel function implies that the massive Feynman propagator an oscillatory behavior as a function of $m\sqrt{s}$. Modulus decreases like $1/\sqrt{m\sqrt{s}}$. The asymptotic behavior for the real and imaginary parts corresponds to that for Bessel functions of first kind (J_1) and second kind (Y_1). At origin $H_\alpha^{(1)}(x)$ diverges like $Y_1(x) \sim \frac{(x/2)^{-n}}{\pi}$ near origin. For large values of x $K_1(x)$ decreases exponentially like $exp(-x)\frac{\sqrt{\pi}}{2x}$. At origin $K_1(x)$ diverges.

5. In the recent case the quark propagator would oscillate extremely rapidly leaving only the $\delta(s)$part so that the propagator behaves like massless propagator!

   The localization of quarks to the partonic surfaces with a size scale of CP_2 radius implies thatthat the oscillation does not lead to a vanishing of the Hankel contribution to the scattering amplitudes. For induced spinor fields in the interior of space-time surfaces destructive interference is however expected to occur so that behavior is like that for a massless particle. This should explain why the observed particles are light although the fundamental fermions are extremely massive. The classical propagation would be essentially along light-like rays.

   The long range correlations between quarks would come from the $\delta(s)$ part of the propagator, and would not depend on quark mass so that it would effectively behave like a massless particle. Also the action of Dirac operator on $G_F(x,y)$ in M^4 degrees of freedom is that of a massless Dirac propagator coupling to induced gauge potentials. The quarks inside hadrons and also elementary particles associated with the wormhole throats of flux tubes could be understood as quarks at different partonic 2-surfaces at the boundary of CD having light-like distance in an excellent approximation.

6. The above argument is for the Feynman propagator but should generalize also for anti-commutator. The anticommutator for Dirac operator D in M^4 can be expressed as $D\Delta(x,y)$, where D is a scalar field propagator.

$$\Delta(x,y|m) \propto \left\{ \begin{array}{ll} \frac{m}{8\pi\sqrt{s}} H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{m}{\sqrt{-s}} K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \ . \end{array} \right.$$

   Apart from possible proportionality constants the behavior is very similar to that for Feynman propagator except that the crucial $\delta(s)$ term making possible effectively massless propagation is absent. At light-cone boundary however $\sqrt{s}$ is zero along light rays, and this gives long range correlations between fermions at different partonic 2-surfaces intersected by light rays from the origin. Hence one could have a non-vanishing Hermitian inner product for 3-D states at boundaries of CD.

   Rather remarkably, these results provide a justification for twistor-diagrams identified as polygons consisting of light-like segments.

3.1.3 Possible normal ordering divergences

Concerning the cancellation of normal ordering divergences the singularities of the propagators G_F are crucial. The bi-linearity of the modified Dirac action forcing anticommuting quark and antiquark oscillator operators at different throats of wormhole contacts but this need not guarantee the absence of the divergence since the the free quark propagator in M^4 contains mass independent $\delta(s)$ part plus the divergent part from Hankel function behaving like $1/\sqrt{sm}$. For the massless propagator assignable to $\nu_R$the propagator would reduceto M^4 propagator and only the $\delta(s)$ would contribute.

$s = 0$ condition tells that the distance between fermion and anti-fermion is light-like and is possible to satisfy at the light-like boundary of CD. Paired quark and antiquark at the wormhole throats must reside at the same light-like radial ray from the tip of cd (cd corresponds to causal diamond in M^4). Since partonic surfaces are 2-D this condition selects discrete pairs of points at the pair of the partonic surfaces. The integration over the position of the end of the propagator line over paired partonic 2-surfaces should smooth out the divergences and yield a finite result. This would be crucial for having an inner product for states at the boundary of the light-cone.

This applies also to the point pairs at opposite throats of wormhole contact. Time-ordered product vanishing for $t_1 = t_2$ so that the points must have different values of t and this is possible. The two 2-D integrations are expected to smooth out the singularities and eliminate divergences also now.

4 Conclusions

TGD predicts revolution in quantum theory based on three new principles.

1. ZEO solving the basic paradox of quantum measurement theory. Ordinary ("big") state function reduction involves time reversal forcing a generalization of thermodynamics and leading to a theory of quantum self-organization and self-organized quantum criticality (homeostasis in living matter).

2. Phases of ordinary matter labelled by effective Planck constant $h_{eff} = nh_0$ identified as dark matter and explaining the coherence of living matter in terms of dark matter at magnetic body serving as a master, and predicting quantum coherence in all scales at the level of magnetic bodies. $h_{eff}/h_0 = n$ has interpretation as the dimension for an extension of rationals and is a measure of algebraic complexity. Evolution corresponds to the increase of n.

   Extensions of rationals are associated with adelic physics providing description of sensory experience in terms of real physics and of cognition in terms of p-adic physics. Central notion is cognition representation providing unique discretization of X^4 in terms of points with imbedding space coordinates in the extension of rationals considered $M^8 - H$ duality realizes the hierarchy of rational extensions and assigns them to polynomials defining space-time regions at the level of M^8 and mapped to minimal surfaces in H by $M8 - H$ duality.

3. The replacement of the unitary S-matrix with the Kähler metric of the Kähler space defined by WCW spinor fields satisfying the analog of unitarity and predicting positive definite transition probabilities defining matrix in Teichmueller space. Einstein's geometrization of classical physics extends to the level of state space, Equivalence Principle generalizes, and interactions are coded by the geometry of the state space rather than by an *ad hoc* unitary matrix. Kähler geometry for the spinor bundle of WCW has Riemann connection only for a maximal group of isometries identified as super-symplectic transformations (SS). This makes the theory unique and leads to explicit analogs of Feynman rules and to a proof that theory is free of divergences.

In this work the third principle, which is new, is formulated and some of its consequences are discussed. The detailed formulation allows understanding of how normal ordering divergences and other divergences cancel. The key idea is to induce the second quantized free spinor field from H to space-time surface. This determines the propagators at the space-time level. The condition that creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts is enough. An alternative but not exclusive option suggested by ZEO is that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

A further observation is that the Dirac propagators for particles reduce in a good approximation to massless propagators when the propagation takes place along light-like distances: this provides a considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

Acknowledgements: I am grateful for Reza Rastmanesh for a generous help in the preparation of the manuscript.

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