Could hyperbolic 3-manifolds and hyperbolic lattices be relevant in zero energy ontology?

M. Pitkänen
Email: matpitka@luukku.com.
http://tgdtheory.com/public_html/.

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Abstract

In zero energy ontology (ZEO) lattices in the 3-D hyperbolic manifold defined by H^3 ($t^2-x^2-y^2-z^2=a^2$) (and known as hyperbolic space to distinguish it from other hyperbolic manifolds emerge naturally. The interpretation of H^3 as a cosmic time=constant slice of space-time of sub-critical Robertson-Walker cosmology (giving future light-cone of M^4 at the limit of vanishing mass density) is relevant now. ZEO leads to an argument stating that once the position of the "lower" tip of causal diamond (CD) is fixed and defined as origin, the position of the "upper" tip located at H^3 is quantized so that it corresponds to a point of a lattice H^3/G, where G is discrete subgroup of $SL(2,C)$ (so called Kleinian group). There is evidence for the quantization of cosmic redshifts: a possible interpretation is in terms of hyperbolic lattice structures assignable to dark matter and energy. Quantum coherence in cosmological scales would be in question. This inspires several questions. How does the crystallography in H^3 relate to the standard crystallography in Eucdlidian 3-space E^3? Are there general results about tesselations H^3? What about hyperbolic counterparts of quasicrystals? In this article standard facts are summarized and some of these questions are briefly discussed.

1 Introduction

In zero energy ontology (ZEO) lattices in the 3-D hyperbolic manifold defined by H^3 ($t^2-x^2-y^2-z^2 = a^2$) (and known as hyperbolic space to distinguish it from other hyperbolic manifolds [A3]) emerge naturally. The interpretation of H^3 as a cosmic time=constant slice of space-time of sub-critical Robertson-Walker cosmology (giving future light-cone of M^4 at the limit of vanishing mass density) is relevant now.

1.1 Hyperbolic lattices in H^3 from zero energy ontology

In TGD framework zero energy ontology (ZEO) indeed predicts the hyperbolic lattices if one accepts the following argument.

1. Causal diamond CD is basic element of ZEO. It is defined as the intersection of a pair of future and past directed light-cones and looks like double pyramid Cartesian product with CP_2 makes it 8-D region off $M^4 \times CP_2$ but the presence of CP_2 as Cartesian factor is not relevant. Its opposite light-like boundaries contain positive and negative energy parts of zero energy states with opposite total quantum numbers. In the usual positive energy ontology zero energy states corresponds to physical events consisting of initial and final states. ZEO is consistent with the crossing symmetry of QFTs. ZEO leads to a generalization of S-matrix concept. The time-like entanglement coefficients between positive and negative energy parts of zero energy state define M-matrix identifiable as a "complex square root" of density matrix and expressible as a product of Hermitian square root of density matrix and unitary S-matrix. One can say that quantum theory corresponds to a square root of thermodynamics in ZEO.

2. The "lower" tip of CD can have any position in M^4: one can argue that these degrees of freedom give rise to 4-momentum. The "upper" tip is at M^4 proper time distance a assumed to be integer multiple of CP_2 size. The assumption motivated by number theoretical considerations (the goal is to fuse real and p-adic physics and real continuum must be effectively replaced by rationals or at most their algebraic extension). One can of course consider also the discretization for the position of the lower tip in M^4 and interpret it in terms of finite measurement resolution for four-momentum.

3. One can perform for CD Lorentz boosts preserving the fixed position of "lower" tip but one cannot allow all possible transformations since one would have two separate 3-D continuous degrees of freedom in this case (here is the crux of argument). Therefore I assume that "upper" tip which lies on the hyperbolic space H^3 - hyperboloid - defined by $t^2-x^2-y^2-z^2 = a^2$, $a = n$ in proper units defined by the size scale of CP_2, can have only discrete positions corresponding to a discrete subgroup G of $SL(2, C)$ (double covering of Lorentz group). Recall that H^3 has negative constant sectional curvature.

4. The discrete subgroup G defining G-coset as points of H^3/G is in the most general case discrete subgroup of $SL(2, C)$. It could be also modular subgroup $SL(2, Z)$ or its. Quite generally, one obtains a tesselation of H^3 with a lattice characterizing positions of unit cells H^3/G, which are closed hyperbolic manifolds in absence of singular points known as cusp points and giving rise to punctures and effectively holes. Physically unit cell or fundamental domain corresponds to an open set and effective identification of boundary points comes through "G-periodic" boundary conditions for physical fields analogous to periodic boundary conditions in the case of condensed matter physics. H^3/G has constant negative curvature metric.

1.2 Some examples of hyperbolic manifolds

In order to make things more concrete it is good to have some examples about hyperbolic manifolds.

1. Examples about hyperbolic manifolds are provided by compactifications of tedrahedron and dodecahedron. It is possible to remove the vertices of tedrahedron and identify the faces of tedrahedron in a pairwise manner to get a compact manifold with boundary having the topology of Klein bottle (non-orientable torus). This manifold is known as Gieseking manifold [A2]. This space has finite volume, is non-orientable, and the boundary corresponds to the cusp. Gieseking

manifold is a double cover of the knot complement of figure eight knot which explains why the boundary has genus $g = 1$.

2. The so called Seifert-Weber space [A11] is a closed hyperbolic manifold obtained by gluing each face of a dodecahedron with its opposite. So called Weeks manifold [A16] has smallest volume among closed hyperbolic 3-manifolds. If the volume of the hyperbolic manifolds surfaces as the analog of energy in topological thermodynamics, Weeks manifold might be one of the favored 3-manifold topologies.

3. Thurston's geometrization conjecture [A14] (actually a theorem thanks to the work of Grigori Perelman) implies that all knot complements except those of satellite knots (they include composites of prime knots and torus knots!) and torus knots (trefoil is the simplest example) are hyperbolic manifolds.

4. Kleinian groups [A5] identified as a discrete subgroups G of $PSL(2C)$ acting as isometries of H^3 and conformal symmetries of Riemannian sphere (Möbius transformations) define hyperbolic manifolds as quotients H^3/G. The fundamental group of any hyperbolic manifold is Kleinian group acting also as group of symmetries of a tesselation of H^3.

1.3 Questions

Could hyperbolic lattices and crystals and hyperbolic manifolds have some physical role in TGD?

1. The points of hyperbolic lattices could label astrophysical (possibly dark matter) objects. The indications for the existence of astrophysical objects at lines of sight and coming with quantized redshift [E1, E2] supports this picture [K3]. In cosmology redshift for small distances r is from Hubble law given by $v = Hr$ so that the recession velocity - or equivalently cosmic redshift - serves as a natural measure for the distance.

   If dark matter objects corresponds to CDs with upper vertices at the points of H^3/G, both the directions and magnitudes of the recession velocities would be quantized. The quantization for the velocities would follow from the quantization of the hyperbolic angle η defining Lorentz boosts as integer multiples of basic value: $\eta = n\eta_0$ giving $v/\sqrt{1-v^2} = sinh(\eta) = sinh(n\eta_0)$ $(c = 1)$ reducing for non-relativistic velocities to $v \simeq n\eta_0$.

2. 3-surface is a fundamental dynamical object in TGD. Hyperbolic 3-manifolds are central in the theory of 3-manifolds, and very many 3-manifolds are hyperbolic. Note that also 2-D manifolds with $g > 1$ are hyperbolic. For instance, knot complements of prime knots are hyperbolic apart from some exceptions, and also surface bundles over circle [A13] are hyperbolic. Thurston's theorem [A15] states that the volume of the hyperbolic manifold defines a topological invariant so that continuous deformations of 3-surfaces would correspond to the same hyperbolic volume, which could thus appear as a counterpart of energy in topological thermodynamics telling which hyperbolic 3-manifold topologies contribute significantly to the physical states (in ZEO this thermodynamics is replaced with its "square root").

3. In TGD framework elementary particles correspond to closed flux tube like structures carrying monopole flux. The solutions of the modified Dirac equation [K4] assign to them closed stringy curves, which can get knotted [K2] and in general case when several flux tubes are associated with the elementary particle (say in case of boson) even braiding becomes possible. The homological non-triviality of the knot brings in additional quantum numbers.

   It is natural to assign to the flux tube the geometry $X^2 \times S^1$ corresponding to trivial surface bundle over sphere. The two wormhole contacts associated with the ends of the flux tube allow gluing of X^2 from upper space-time sheet with that associated with the lower space-time sheet and this would transform $X^2 \times S^1$ to a non-trivial bundle. Hence the topology of the flux tube could be characterized by hyperbolic volume. The induced metric of course need not be hyperbolic metric.

4. What is interesting that the isometry group of H^3 has $SL(2, C)$ as a double covering and H^2 realized as upper half-plane has $SL(2, C)$ as conformal isometries. Could this mean some kind of duality analogous to AdS-CFT duality? The hyperbolic manifolds H^3/G have 2-D boundary:

could there be a duality between 2-D conformal field theory at the boundary and string theory in the interior. This is suggested by the strong form of holography (equivalently strong form of general coordinate invariance) stating that partonic 2-surfaces and their 4-D tangent space data code for quantum physics in TGD Universe.

This raises several questions.

1. What happens to 3-D Euclidian crystallography when E^3 is replaced with H^3? How the negative constant sectional curvature affects the character of lattices obtained?

2. Can one build a rough overview about hyperbolic manifolds? Under what conditions the fundamental domain regarded as an open manifold analogous to lattice cell can be compactified by G-periodic boundary conditions to a closed 3-manifold? To me this is not obvious since the compactified manifold could have singularities known as cusps points and represent punctures.

3. Does one obtain also hyperbolic quasicrystals? One can imagine also 2-D hyperbolic quasicrystals analogous to Penrose tilings [A8] defined by the imbedding of 2-D hyperbolic manifold H^2 to H^3 (or higher dimensional hyperbolic space) and by projecting the points of H^3 to H^2 along geodesic lines orthogonal to H^3. One can also imagine 3-D hyperbolic quasicrystals as analogs of Penrose tilings obtained by imbedding H^3 to H^4 or H^5 and performing similar projection.

It turns out that a visit to Wikipedia allows to answer the first two questions.

2 Comparing crystallographies in E^3 and H^3

Consider first crystallography in E^3. There exists a large number of lattice like structures depending on detailed definition used and it is good to summarized first the basic notions.

2.1 Some definitions

Consider first some basic notions.

1. The difference between crystal and lattice is that crystal structure assigns to a given point of lattice some structure, which can be rather complex. In the simplest case this structure is a Platonic solid - a polyhedron which can be regarded as an orbit of a discrete group generated by reflections and rotations.

2. Lattice [A1] in 3-D case can be defined group theoretically in terms of the group leaving the lattice invariant. This group - call it G - is generated by the elements of two groups, the chrystallographic point group [A9] and space-group [A12].

   Point group leaves at least single point of the lattice fixed and defines the symmetries of the structure attached to the lattice point identified as the center point of the structure. There are 32 point groups and they contain reflections across plane, rotations, inversions (3-D reflecting with respect to origin), and improper rotations (rotations followed by inversion).

   Space group contains pure translations, screw transformations rotating around axing and translating along it, and gliding transformation consistent of reflection with respect to plane followed by a translation. There are 230 distinct space groups. The lattice is defined as the set of cosets E^3/G, where G is so called space-group leaving the lattice invariant.

3. The lattice points are in the general case linear combinations of three - in general non-orthogonal - basis vectors (a, b, c) generating the discrete subgroup of translations. The condition that one has crystal consisting of say tedrahedrons as unit cells - poses additional conditions. The duals of the lattice vectors defined by their cross products generate dual lattice.

2.2 Tesselation

Tesselation or tiling is second key notion and there are many different variants of this notion. The most stringent definition of tesselations considered in following is in terms of by a $n+1$-dimensional regular polytope in n-dimensional sphere, Euclidian space, or hyperbolic space.

1. Polytopes are constructed of regular p-polygons in turn defining the 2-D faces of 3-D polyhedrons in defining the 4-D polychrones.

2. n-dimensional tesselations can be defined as boundaries of $n+1$-dimensional polygons. Schläfli symbol [A10] allows to represent n-dimensional tesselations in terms of integer n-tuple o fintegers. In 3-D case one has triple (p,k,r). p is the number of vertices of 2-polygon defining the face of 3-D polyhedron (p,k) and k is the number of faces associated with a given vertex of the polyhedron. r is the number of 3-D polyhedra associated with a given edge of the tesselation.

3. In the case of 2-sphere tesselation in E^3 contains finite number of identical faces projected to the sphere. Tesselations can make sense also if the n-D space is non-compact and the replacement of sphere S^3 of E^4 with hyperbolic space H^3 gives rise to infinite tesselation of H^3. Also tesselations in hyperbolic manifolds H^3/G are possible and in closed case contain a finite number of basic elements.

   Tesselations by regular polytopes [A6] satisfy strong constraints and there are only four tesselations by regular polytopes in H^3 and one in E^3.The list of tesselations is following.

   (a) E^2 allows three regular tesselations by squares, triangles and hexagons: the Schäfli symbols for them are (4,4), (3,6), (6,3).

   (b) H^2 is exceptional and allows infinite number of tesselations.

   (c) E^3 allows single tesselation by cubes: the Schläfli symbol is $(4,3,4)$.

   (d) H^3 allows four tesselations. The Schläfli symbols are (3,5,3), (4,3,5), (5,3,4), (5,3,5). Second and third tesselation are dual tesselations by cubes and dodecahedra. First and fourth tesselation correspond to self-dual tesselations by icosahedra and dodecadedra. For instance, for (5,3,5) means each edge has 5 dodecahedrons around it.

   The large voids with size of order 10^8 ly give rise to honeycomb like structures. Could they correspond to ordinary matter condensed around dark matter honeycomb consisting of dodecahedra?

   (e) For $n>4$ there are three regular tesselations by convex polyhedra in Euclidian space. There are no regular hyperbolic tesselations by convex polyhedra in dimensions $n>5$.

4. If an infinite n-D tesselation is induced by $n+1$-D regular polytope, it seems obvious that the polygon must have infinite number of basic units. There indeed exists this kind of infinite polytopes known as infinite skew polytopes [A4]. 1-D lattice requires 2-D zigzag curve reflected from the real axis at the lattice points. In 1-D cases zigzag curve actually gives two parallel lines carrying lattices and the parallel lines together define a boundary of a stripe. Similar doubling is expected in higher dimensions since it is the boundaries of polytopes, which must give rise to H^n or E^n.

5. The tesselations having E^3/G as a unit cell are obtained by assuming G to be a subgroup of translations. As already noticed this subgroup in question is generated by 3 generators represented by - in general non-orthogonal vectors - and the fundamental domain is parallelpiped generated by these vectors. When the vectors are orthogonal and have same length one obtains the regular tesselation by cubes. The four tesselations by regular polytopes must be distinguished from the infinite number of tesselations defined by the orbit of discrete subgroup $G \subset PSL(2,C)$ in H^3 with fundamental domain H^3/G replacing the polyhedron as a basic unit. The case of E^3 suggests that these tesselations give as a special case the 4-tesselations using regular polytopes. A good first guess is that G is generated by Lorentz boosts with same velocity in 3 orthogonal directions.

2.3 Tesselations of H^3

Consider now the case of H^3 more closely.

1. In the case of H^3 a discrete subgroup G of Lorentz group $SL(2,C)$ with infinite number of elements representing Lorentz boots replaces discrete subgroup of translations in E^3. G is known as Kleinian group [A5]. G can be also restricted to be a subgroup of the modular group $SL(2,Z)$. Note that $G = SL(2,Z)$ is braid group for 3-braid divided by its center and isomorphic to the knot group of trefoil as one learns from Wikipedia [A7]. Therefore the subgroups of the knot group of trefoil are very interesting concerning lattices in H^3. The complement of trefoil and any torus knot however fails to defined hyperbolic 3-manifold. For larger subgroups of $SL(2,C)$ one obtains smaller fundamental domain and more lattice points.

2. For non-compact discrete subgroups of $SL(2,Z)$ (and also $SL(2,C)$!) the lattice consists in the language of cosmologist of locations of astrophysical objects (possibly consisting of dark matter) with quantized redshifts and direction angles. The counterparts of parallelepipeds are interiors of hyperbolic 3-manifolds and there are very many of them. For prime knot complements which very often are hyperbolic 3-manifolds, the boundary is torus and allows a constant sectional curvature metric with vanishing sectional curvature. This motivates the question whether $g > 1$ negative constant sectional curvature 2-surfaces could appear as boundaries of hyperbolic 3-manifolds.

3. It is not completely obvious how to define the edges and faces of hyperbolic polygons. Edges are naturally defined as geodesic lines but what about faces. In E^3 they are pieces of plane which are minimal surfaces but also geodesic sub-manifolds with vanishing second fundamental form meaning that all geodesics of these surfaces are also geodesics of E^3. Minimal 2-surfaces are by definition manifolds with a negative curvature and this seems to fit with the negative curvature property of H^3. H^3, E^3, and S^3 are very closely related (they define the 3 constant sectional curvature Robertson Walker cosmologies) In the case of S^3 spheres S^2 are geodesic sub-manifolds. In the similar manner H^2 defines a geodesic sub-manifold of H^3. If so, the faces would be 2-D hyperbolic manifolds with boundary, and having constant negative sectional curvature.

4. One can wonder what is the 4-D space used to define H^3 tesselations. Is it Minkowski space M^4 or is it H^4? The first problem is that tesselation is infinite. Second problem is that H^3 should but cannot play the same role as sphere S^2 in E^3. The problem is that H^3 can be thought of as having boundary at infinity, and therefore is not itself a boundary unlike S^2. It is the boundary property of S^2, which allows to assign Platonic solid with the vertices of tedrahedron at the surface of S^2.

   Infinite tesselation requires infinite polytope as already noticed. For $1-D$ tesselation one has zigzag curve in planar stripe, and one obtains two copies of the tesselation defining a boundary of 2-D stripe. Are the segments of zigzag curve replaced by a 4-D object having as boundary cube, icosahedron, or dodecahedron of H^3? Does the boundary property require that there are two lattices at hyperboloids $a = a_1$ and $a = a_2$ of M^4. These hyperboloids define a boundary and one can speak about the interior and boundary of 4-D polytope.

   An interesting question is how this relates to zero energy ontology, where CD plays a key role. Can one imagine that the pair of H^3:s is replaced with a pairs of hyperboloids with opposite time orientation so that their intersection consists of temporal mirror images of part of H^3 glued together along 2-sphere (this could be seen as a generalization of CD)? The boundaries of CD would correspond to the limiting case $a = 0$ for H^3 giving light-cone boundary for which radial coordinate does not contribute to metric so that metrically one has 2-D sphere (this makes possible huge extension of conformal invariance in TGD Universe). How could one define tesselations of light-cone boundary?

5. For Platonic solids boundary is always topologically a sphere. For prime knot complements the boundary is 2-torus $S^1 \times S^1$. What does this mean geometrically in the gluing of fundamental domains together? Also 2-surface bundles over spheres are hyperbolic manifolds and are obtained by identifying the ends of $X^2 \times D^1$ by a homeomorphism. The homotopy equivalence class of

the map $X^2 \to X^2$ characterizes the bundle structure. In this case one should fill the twisted torus like surface by polygon lattice.

3 Quasicrystals

One can also ask whether hyperbolic quasicrystals are possible. In the following some basic facts about quasicrystals are summarized and some questions relating to the dynamics of quasicrystals are considered before brief comments on hyperbolic quasicrystals.

3.1 Basic facts about quasicrystals

Quasicrystals are lattices, which do not have translational symmetries. Quasicrystals can be finite or infinite and only in special cases local matching rules give rise to infinite quasicrystal instead of finite local empire (to be defined later). The so called empire problem for Penrose tilings has been solved by Laura Effinger-Dean [A17] .

1. Especially interesting example about quasiperiodic 2-D lattices are Penrose tilings [A8] for which basic objects have 5-fold local rotation symmetry: this is not allowed in ordinary crystallography. They are also self-similar. Their number is uncountably infinite. There is a theorem [A8] stating that Penrose tilings are obtained as projections of 5-dimensional lattices to 2-D plane imbedded in 5-D Euclidian space. If the parameters characterizing the plane have irrational values one obtains quasicrystal. This theorem generalizes to Euclidian spaces E^n imbbedded to higher-dimensional Euclidian spaces E^{n+k} carrying lattice structure.

2. In the case of Penrose tiling the plane is characterized by its normal space characterizing the orientation of the plane: for rational values of the "slope" of the plane one obtains periodic lattices with finite number of points projected to same point at E^2. For irrationals slopes just one point is projected to a given point of E^2. One can regard the space of the plane imbeddings containing also Penrose tilings as a coset space $SO(5)/SO(2)\times SO(3)$ having dimension $D = 10-1-3 = 6$. The space for Penrose tilings (with crystals excluded) is rather delicate mathematical notion and represents basic example of a non-commutative geometry [A18].

3. An important concept related to Penrose tilings is the notion of empire already mentioned [A17]. One starts from a given "seed" for a quasicrystal, and builds a larger quasicrystal using local matching rules forbidding gaps. Local empire is the largest quasicrystal obtained in this manner and is a connected structure. Empire in turn is the largest set of tiles shared by all tilings containing the "seed" and is in general non-connected and can be even infinite. For ordinary crystals single unit cell fixes the lattice completely as its empire.

3.2 About dynamics of quasicrystals

Consider next possible dynamics of quasicrystals.

1. The fact that the local matching rules are not enough to construct infinite quasicrystal uniquely and that there is no guarantee that a given seed leads to infinite quasicrystal led Penrose to ask whether the formation of quasicrystal involves macroscopic quantum phase transition in which quasicrystal is created in single quantum leap rather than being a result of growth process. Experimentalist can of course argue that real quasicrystals are always infinite and this is just because the growth process stops because local matching rules fail at some step.

2. The conditions that quasicrystal property is preserved in the dynamics of quasicrystal is extremely strong. One manner to satisfy it would be the reduction of the dynamics to dynamics in the space of quasicrystals and crystals. The rigid body dynamics associated with the rotation of E^n in E^{n+k} containing the mother crystal would induce the variation of the projection of the crystal to E^n containing also quasicrystal configurations. In the case of imbeddings $E^2 \subset E^5$ containing also Penrose tilings, the analog of rigid body motion would take place in $SO(5)/SO(3)\times SO(2)$. This dynamics can be solved both classically and quantum mechanically. The special feature of the dynamics would be correlation between short and long scale aspects

of the dynamics since both local consistency rules and global consistency rules are automatically satisfied.

3. Quasicrystal excitations are known as phasons [D1]. The intriguing observation is that they can be described using hydrodynamics (long length scale description) and microscopically as re-arrangements of nearby atoms. There is a strong correlation between short and long length scales. If quasicrystal property is preserved by the dynamics, this is expected. The reduction to rigid body dynamics with only 6 degrees of freedom might of course be quite too restrictive an assumption and it is quite possible that the excitations have nothing to do with quasicrystallinity. Macroscopic quantum transitions can be also considered. The most mundane explanation would be in terms of thermodynamics: in ZEO square root of thermodynamics could unify quantal and thermodynamical explanations.

3.3 What about hyperbolic quasicrystals?

Hyperbolic 2-D quasicrystals are of special interest in TGD since they can be assigned to the spaces H^2 imbedded to H^3. Could one generalize the construction of Penrose tilings to a construction recipe for hyperbolic quasicrystals? For the hyperbolic counterparts of Penrose tilings one could imagine isometric imbedding of $H^2 \subset H^n$, $n > 2$. H^3 is the physically preferred option in TGD. Imbedding would represent 2-D hyperboloid $H^2 = SO(1,2)/SO(2)$ of M^3 as constant sectional curvature sub-manifold of n-dimensional hyperboloid in $H^n = SO(1,n)/SO(n)$. There is a continuum of this kind of imbeddings. In the compact case one has imbeddings of S^2 to S^3 and the space of imbeddings is $SO(3)/SO(1) \times SO(2) = S^1$. Same holds true in the hyperbolic case. For $H^n \subset H^{n+k}$ one has $SO(n+k)/SO(n) \times SO(k)$. One can consider also 3-D hyperbolic quasicrystals and the imbedding $H^3 \rightarrow H^n$, $n > 3$ might gives this kind of quasicrystals. This imbedding would not however have a concrete geometric interpretation in TGD framework.

Could hyperbolic 2-planes or finite pieces of them allow a physical interpretation as 2-D physical systems in cosmological scales? Certainly the existence of quasicrystals and even more that of crystals in cosmological scales requires quantum coherence in cosmological scales, and dark matter and dark energy as phases with large and even gigantic value of Planck constant [K1] [L1] could give rise this kind of structures.

4 Some considerations relating to the dynamics of quasicrystals

The dynamics of quasicrystalslooks to me very interesting because it shares several features of the dynamics of Kähler action defining the basic variational principle of classical TGD and defining the dynamics of space-time surfaces. In the following I will compare the basic features of the dynamics of quasicrystals to the dynamices of preferred extremals of Kähler action.

Magnetic body carrying dark matter is the fundamental intentional agent in TGD inspired quantum biology and the cautious proposal is that magnetic flux sheets could define the grid of 3-planes defining quasiperiodic background fields favoring 4-D quasicrystals in TGD Universe. Quite recently it has been reported that 3-D curved orthogonal grids characterize the architecture of neural wring so that this hypothesis might make sense.

4.1 They non-determism for the dynamics of quasicrystals contra non-determinism of Kähler action

The dynamics of quasicrystals is non-deterministic in the sense that one cannot construct a unique quasicrystal by starting from a finite portion or even lower-dimensional section of quasicrystal. Four-dimensional quasicrystals would therefore define a non-deterministic dynamics which could serve as simple geometric correlate for quantum dynamics: this of course only in the sense of quantum classical correspondence. The global empires of the 4-D quasicrystal could be interpreted as self-organization patterns and the global empires would give rise to long range correlations representing the effects of intentional action.

This is very much analogous to 4-D spin glass degeneracy in TGD framework.

1. In TGD framework the preferred extrremals of so called Kähler action define the dynamics of space-time surfaces. Kähler action is Maxwell action for the gauge field induced from the Kähler form of CP_2. Symlectic transformations of CP_2 act as abelian gauge transformations and therefore leave the induced Kähler form invariant. They do not however leave the induced metric invariant so that the action changes by a contribution assignable to classical gravitation. For vacuum extremals however the symplectic transformations act as symmetries.

2. This implies huge vacuum degeneracy. Every space-time surface for which CP_2 projection is Lagrangian manifold and thus having at most 2-D CP_2 projection has vanishing induced Kähler form and is therefore vacuum extremal: there is infinite number of 6-D vacuum sectors labelled by Lagrangian sub-manifolds of CP_2 transformed to each other by symplectic transformations. These vacuum extremals behave non-deterministically which means an analogy with quasicrystal dynamics and suggests that quasicrystals might define a simplified model for quantal self-organization.

3. Small deformations of these define non-vacuum extremals and It is very conceivable that part of the vacuum degeneracy remains and is manifested as multifurcations. The number n of branches for a multifurcation has interpretation in terms of effective Planck constant $\hbar_{eff} = n\hbar$ to which dark matter is assinged in TGD framework. This degeneracy is very much analogous to a 4-dimensional spin glass degeneracy meaning that space-time decomposes to deterministically behaving regions just like spin glass decomposes to magnetized regions with varying direction of magnetization.

4. The interpretation for the situation in TGD framework is in terms of quantum classical correspondence: not only quantum states correspond to space-time geometries as analogs of Bohr orbits but also quantum jump sequences defining contents of consciousness have non-deterministic space-time geometries as geometric correlates. Space-time geometry and topology are like written text telling about contents of consciousness.

5. Also p-adic topology as effective topology of space-time surfaces and natural topology for the landscape of extrema of Kähler action emerges naturally from this degeneracy. In physics obeying effective p-adic topology the counterpart would be short range chaos with long range correlations in the sense that one would have periodicity in the sense that physical states at time time at and $t + kp^n$, n large enough, would be very near to each other. The interpretation in terms of intentional action would be natural. One could also imagine of define the analogs of empires as connected deterministic regions of space-time surface and the analogs of empires would be unions of disconnected components perhaps understable in terms of p-adicity. Self-organization patterns would naturally correspond to these regions. Many-sheeted space-time would imply fractal hierarchy of self-orgnization patterns within self-organization patterns.

4.2 The dynamics of quasicrystals as a model for fundamental dynamics or high level symbolic dynamics?

Stephen Wolfram has suggested that cellular automatons could define the fundamental dynamics. It is not difficult to invent grave objections against this view. One of the objections is that this kind of dynamics is based on simple and reather ad hoc rules and applies to a society rather than to elementary particles. It is difficult to invent objections to this counter argument. One can however ask in what scale the symblic dynamics emerges? My answer for few years ago would have been: in biological length scales. TGD Universe is however fractal and this forces to ask whether this symbolic dynamics emerges already at fundamental scales. In any case, even in this case this dynamics would not be identifiable as the fundamental dynamics but as analogous to rules of behavior in society.

The dynamics of quasicrystals indeed suggests an identification as dynamics of self organization patterns obeyd at relatively high level of dynamical hierarchy. One could speak of symbolic dynamics which prevails at the level of biomolecules (genetic code) and at higher levels. This dynamics is dynamics for a society of conscious entities, which can decide whether to obey the rules or not. Rules as such do not matter too much: what is important that they make possible to predict the behavior of individuals and therefore make possible co-operation and formation of coherent and synchronous large scale structures making possible collective consciousness. In our society moral rules, laws, traffic rules, grammatic rules of language, etc... are examples about symbolic dynamics having very little to do with laws of physics at fundamental level. They are also rather arbitrary and often even irrational but this not essential.

A natural question is whether the non-unique rules for building quasicrystals could provide a simplified model for this dynamics - or even semi-realistic model at the molecular level? These rules would be like rules of society and invididuals- say molecular clusters - could also refuse to follow them: this would lead to a breakdown of the quasicrystal growth and isolate the individual from the society. If this interpretation is correct, the quasicrystals can be seen as idealized structures having maximal complexity and resulting only when the dynamics in question is very coherent. (Quantum) Critical systems have universal dynamics and there is large number of models making same predictions for given system: this is used to find the simplest possible model to simplify the mathematical description. From this point of view quasicrystals could be seen as an especially simple model possibly able to catch the universal properties of a real world system.

Does this self-organization dynamics then emerge at and above biomolecular scales or in all scales? In TGD framework the dynamics at fundamental level would be the dynamics of space-time surfaces and that of WCW ("world of classical worlds") spinor fields. The fractality of TGD Universe however suggests that self-organization occurs in all length scales above CP_2 scale which is about 10^4 Planck scale. If so structures analogous to finite pieces quasicrystals should appear in all scales down to CP_2 scale. I have also proposed method to construct preferred extremals of Kähler action and this recipe leads to an iteration procedure. Quite generally, iteration is known to lead to fractals as fixed sets of iteration. Therefore space-time surfaces could be seen as space-time correlates of self-organization patterns and fractals.

Fractality would mean that even inanimate matter should share some aspects assigned to living matter and that also systems like species and biosphere should behave like living organisms. Sheldrake has proposed the not only the notion of memory at the level of entire species but that even inanimate systems could have "habits". For instance, minerals would have adopted a habit to crystallize to a particular crystal form. In this framework living matter would differ from solids in that its habits would be much more flexible.

For instance, water forms multilayered lattice like structures around biomolecule known as quasi-lattices (to be distinguished from quasicrystals!). These quasilattices around molecules are analogous to ice coverings. Could these quasilattices be actually deformed quasicrystals having water molecule as a basic tedrahedral building blocks giving rise to icosahedral blocks (as suggested in discussions): the 4 electron pairs of water molecule are indeed located at the vertices of tedrahedron and for lattice like structures a regular tedrahedron is in question.

This molecular ice would form a quasicrystal, which could store a lot of information about environment via its structural degeneracy. In the presence of energy feed inducing "molecular summer" the molecular ice would melt, globular proteins would open and self-organize to form molecular aggregates as a reaction to the energy feed. After the energy feed stops, molecules would fold back to globular form but the memory from the "molecular summer" would be stored by the molecular aggregates, perhaps also carried by the quasicrystal like structure surrounding them as ice.

4.3 What could be the variational principle behind self-organization?

Quasicrystals (say Penrose tilings) have a huge ground state degeneracy: given region of quasicrystal can be completed to infinite number of quasicrystals. For crystals the situation is different: empire is the entire infinite crystal. Quasicrystals clearly analogous to spin glass systems possessing also large ground state degeneracy.

TGD Universe is a 4-D spin glass, and this degeneracy would imply non-determinism analogous to the non-determinism of quasi-crystal dynamics in 4-D 4-D Minkowski space)with local empires interpreted as self-organization patterns and global empiress reflecting the long range correlations due to intentional action and obeyance of social rules. In human society the ability to predict what person probably does next year at given day only by knowing his title, would be example about this kind of long range correlation due to intentional action and willingness to obey social dynamics.

4.3.1 Wy Negentropy Maximization Principle would favor quasicrystals?

In TGD inspired theory of consciousness Negentropy Maximization Principle (NMP) is the basic variational principle. Therefore entanglement negentropy is expected to be the fundamental quantity.

1. Since conscious entities forming larger coherent structures (societies) are in question, it seems that one should characterize the quasilattice by a negentropy, which should be maximized (purely mathematically negentropy is very similar to entropy which is maximed for a closed system). This negentropy would *not* correspond to the negative of the ordinary thermodynamical entropy which characterizes ensemble of particles rather than single coherent unit of them.

2. In TGD framework this negentropy could be a counterpart for the number theoretic negentropy characterizing negentropic entanglement identified as a measure for conscious information. This information measure is assigned with the magnetic flux tubes connecting biomolecules and other units of living organism and even living organisms to larger coherent structures. In the case of quasicrystals flux tubes or flux sheets give rise to the long range constraints binding the units of quasi-crystal to each other.

3. The maximization of negentropy characterizing information content of conscious experience should be equivalent with the maximation of complexity as the number of almost degenerate ground states of quasicrystal. It is intuitively clear why quasicrystals would be favored over crystals. But how quasicrystals could maximize entanglement negentropy? Why the entanglement negentropy would be large for quasicrystals? Why the negentropic entanglement would grow with the number of quasicrystal configurations? Is the entanglement is between two different quasicrystals it means formation of quantum superposition of pairs of quasicrystal configurations and the large the number of quasicrystal degeneracy the large the maximal entanglement negentropy.

4.3.2 Maximal capacity to represent information with minimal metabolic energy costs as a basic variational principle?

The interpretation as symbolic dynamics assignable to conscious entities would suggest that the maximation of the capacity to represent information (perhaps with minimal metabolic costs) could be the variational principle behind this dynamics. The number of different quasicrystals formed using the given rules should be maximal. This would give rise to very large number of states with nearly same energy allowing to represent the states of the external world (primitive sensory system). The larger the size of quasicrystal, the larger the number of degenerate configurations. Here of physical constraints would pose an upper limit of the size.

But can one really assume rigid rules of construction giving rise to only quasicrystals? If the basic dynamical units are conscious entities they refuse to obey strict rules although they can decide to do so under "social pressures" (absence of metabolic energy feed can transform a sinner a saint!). Should also the rules be an outcome of the variational principle alone? Or are they forced by some minimization principle - say minimization of metabolic energy feed - in presence of background field configuration regarded as an external field inducing the quasiperiodicity and thus favoring quasicrystals?

To my opinion all configurations of basic units must be accepted a priori: even random spatial configurations of the basic units. For random configurations complexity would be maximal but co-operation minimal, long range correlations would be absent, and the ability to represent information would be minimal. For crystals long range correlations and co-operation would be maximal but crystal would have minimal capacity to represent and mimic. The natural manner to achieve long range correlations is to assume slowly varying quasiperiodic fields configurations.

4.3.3 A possible realization for 4-D dynamics favoring quasicrystals

Can one imagine a physical realization of 4-D quasicrystal dynamics in TGD framework? The basic problem is to understand how the rules for the formation of quasicrystals are forced. Certainly the hyper-plane grids associated with the basic polytope defining the quasicrystal force the long range correlations. But how to realize these grids physically?

1. In TGD Universe magnetic body acts as intentional agent using ordinary living matter as a motor organ and sensory receptor. This suggests that the plane grids parallel to the faces of icosahedron in the case of 3-D quasicrystal could in TGD Universe be realized as thin (and thus effectively 2-D) magnetic flux sheets forming the magnetic body around which the ordinary matter would self-organize to form a quasicrystal as a configuration sustainable by using minimum metabolic energy feed. The grids would form the magnetic body. Rather remarkably, quite recent findings strongly suggest that brain involves an orthogonal grid of curved planes. Maybe this grid correspond to a quasilattice associated with a cubic basic unit serving as a basic information processing unit.

2. Maybe the basic variational principle could be minimization of metabolic energy feed in presence of fixed grid structure formed by flux sheets representing the slow dynamics to which the molecular dynamics would rapidly respond. The motor activities of the magnetic body itself would deform the quasicrystals: the flux sheets could be deformed and the distances between the flux sheets could also vary. This would lead to a new quasicrystal configurations distinguished by their high negentropic content. Maybe also the paradoxixal properties of phasons could be understood in this framework as being induced from the dynamics of flux sheets.

3. Also other configurations would be possible but would require higher metabolic energy feed to preserve entangelement negentropy (amount of conscious information). In 4-D case one would have similar grids of thin and effectively 3-D magnetic flux sheets associated with the 3-D faces (maybe icosahedrons) of 4-D building brick of quasicrystals. Magnetic flux sheets would carry dark matter and give rise to negentropic entanglement between the units of the quasicrystal.

To sum up, the basic variational principle of quasicrystal dynamics might be minimization of metabolic energy feed in presence of fixed configuration of the magnetic body obeying a relatively slow dynamics (note that the time scale of EEG is in the range .01-.1 seconds to be compared to the time scale of 10^{-10} seconds of conformational dynamics biomolecules). This would mean constraints coming from the existence of grids of thin 3-planes parallel to the basic units of the 3-faces of 4-D basic unit of quasicrystal.

To show that this picture makes sense, one should be able to estimate reliably the metabolic energy feed needed to preserve a given negentropic entanglement entropy for a given configuration of the basic units (say clusters of water molecules) and to show that it is minimized for quasicrystal configurations in presence of the grid structure formed by flux sheets. This is probably relatively easy since the first guess for the equilibrium configurations corresponds to the highly symmetric crossing points for three 3-planes.

Mathematics

[A1] Crystal structure. `http://en.wikipedia.org/wiki/Crystal_structure`.

[A2] Gieseking manifold. `http://en.wikipedia.org/wiki/Gieseking_manifold`.

[A3] Hyperbolic manifold. `http://en.wikipedia.org/wiki/hyperbolic_manifold`.

[A4] Infinite skew polyhedron. `http://en.wikipedia.org/wiki/Infinite_skew_polyhedron`.

[A5] Kleinian group. `http://en.wikipedia.org/wiki/Kleinian_group`.

[A6] List of regular polytopes. `http://en.wikipedia.org/wiki/List_of_regular_polytopes`.

[A7] Modular group. `http://en.wikipedia.org/wiki/Modular_group`.

[A8] Penrose tilings. `http://en.wikipedia.org/wiki/Penrose_tilings`.

[A9] Point group. `http://en.wikipedia.org/wiki/Point_group`.

[A10] Schlfli symbol. `http://en.wikipedia.org/wiki/Schlfli_symbol`.

[A11] SeifertWeber space. `http://en.wikipedia.org/wiki/SeifertWeber_space`.

[A12] Space group. http://en.wikipedia.org/wiki/Space_group.

[A13] Surface bundle over the circle. http://en.wikipedia.org/wiki/Surface_bundle_over_thecircle.

[A14] Thurston's theorem. http://en.wikipedia.org/wiki/Thurston's_geometrization_conjecture.

[A15] Thurston's theorem. http://en.wikipedia.org/wiki/Thurston's_theorem.

[A16] Weeks manifold. http://en.wikipedia.org/wiki/Weeks_manifold.

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[K2] M. Pitkänen. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.com/public_html/tgdgeom/tgdgeom.html#knotstgd, 2006.

[K3] M. Pitkänen. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.com/public_html/tgdclass/tgdclass.html#cosmo, 2006.

[K4] M. Pitkänen. The Recent Vision About Preferred Extremals and Solutions of the Modified Dirac Equation. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.com/public_html/tgdgeom/tgdgeom.html#dirasvira, 2012.

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