Could brain be represented as a hyperbolic geometry?

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May 1, 2020

Abstract

There are proposals that the lattice-like structures formed by neurons in some brain regions could be mapped to discrete sets of 2-D hyperbolic space H^2, possibly tesselations analogous to lattices of 2-D plane. The map is rather abstract: the points of tesselation would correlate with the statistical properties of neurons rather than representing their geometric positions as such.

In TGD framework zero energy ontology (ZEO) suggests a generalization of replacing H^2 with 3-D hyperbolic space H^3. The magnetic body (MB) of any system carrying dark matter as $h_{eff} = nh_0$ provides a representation of any system (or perhaps vice versa). Could MB provide this kind of representation as a tesselation at 3-D hyperboloid of causal diamond (cd) defined as intersection of future and past directed light-cones of M^4? The points of tesselation labelled by a subgroup of $SL(2,Z)$ or it generalization replacing Z with algebraic integers for an extension of rationals would be determined by its statistical properties.

The positions of the magnetic images of neurons at H^3 would define a tesselation of H^3. The tesselation could be mapped to the analog of Poincare disk - Poincare ball - represented as $t = T$ snapshot (t is the linear Minkowski time) of future light-cone. After $t = T$ the neuronal system would not change in size. Tesselation could define cognitive representation as a discrete set of space-time points with coordinates in some extension of rationals assignable to the space-time surface representing MB. One can argue that MB has more naturally cylindrical instead of spherical symmetry so that one can consider also a cylindrical representation at $E^1 \times H^2$ so that symmetry would be broken from $SO(1,3)$ to $SO(1,2)$.

$M^8 - H$ duality would allow to interpret the special value $t = T$ in terms of special 6-D brane like solution of algebraic equations in M^8 having interpretation as a "very special moment of consciousness" for self having CD as geometric correlate. Physically it could correspond to a (biological) quantum phase transition decreasing the value of length scale dependent cosmological constant Λ in which the size of the system increase by a factor, which is power of 2. This proposal is extremely general and would apply to cognitive representations at the MB of any system.

Contents

1 Introduction

There are proposals that neuronal systems in brain could have hyperbolic geometry [J1] (http://tinyurl.com/ybghux6d) in the sense that neurons could be mappable to a 2-D lattice like structure representable in terms of to 2-D hyperbolic geometry H^2. A concrete identification as a lattice-like structure in H^2 would not be in question.

1.1 A concrete representation of hyperbolic geometry is not in question

The tesselations of P^2 represented as Poincare disk have large density of points near the boundary. The concrete geometry of the cortex could very roughly correlate with the geometry of near the boundary of Poincare disk or even boundary sphere of 3-D Poincare ball representing 3-D hyperbolic space H^3. A rather abstract representation based on statistical properties of the network formed by the neurons would be in question. If a genuine geometric representation as a tesselation of hyperbolic space exist it must be realized somewhere else than brain.

To see what is involved, note that the line element of Poincare disk is given by

$$ds^2 = d\eta^2 + sinh^2(\eta)d\phi^2 \quad .$$

to be compared with the line element of ordinary disk given by

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 \quad .$$

For given neuron the size of the radial coordinate η of Poincare disk would correspond roughly to the number of connections it has, kind of popularity. For large values of radial coordinate η the circles of Poincare disk have radius proportional to η and circumference proportional to $sinh(\eta)$ increasing exponentially for large values of η whereas for ordinary disk both radial distance circumference would be proportional to ρ.

For the neurons of cortex, in particular pyramidal neurons, the image points would have large distance from the origin of hyperbolic space. The image points for neurons resembling each other would have small distance with respect to the angular coordinate of the Poincare disk. Since similar neurons can have large distances from each other at the level of brain, the representation must involve a map taking them close to each other.

1.2 Hyperbolic geometry and its symmetries

The standard representations for 2-D hyperbolic geometry are 2-D Poincare plane (http://tinyurl.com/y8tnklz6) and Poincare disk (http://tinyurl.com/y8bcd6cv). Poincare disk is claimed to be natural representation space for the lattice like structure of neutrons. These lattice structures of H^2 are known as tesselations.

Remark: There is a painting of Escher visualizing Poincare disk. From this painting one learns that the density of points of the tesselation increases without limit as one approaches the boundary of the Poincare disk.

The group $SL(X)$, $X = C, R$, consists of matrices $[a, b; c, d]$ with $a, b, c, d \in X$ satisfying $ad - bc = 1$. The modular group $SL(2, Z)$ acts subgroup of both $SL(2, C)$ and $SL(2, R)$. $SL(2, C)$ *resp.* $SL(2, R)$ forms a double covering of Lorentz group $SO(1, 3)$ *resp.* $SO(1, 2) = SL(2, R)$. $SL(2, C)/SU(2) = SO(1, 3)/SO(3)$ defines 3-D hyperbolic geometry H^3 realized as $a = \sqrt{t^2 - x^2 - y^2 - z^2} = constant$ hyperboloid of future light-cone M^4_+ having $SO(1, 3)$ as isometries. $SL(2, R) = SO(1, 2)$ acts as isometries of H^2 realizes as hyperboloid of M^3_+. $SL(2, C)$ *resp.* $SL(2, R)$ acts as complex *resp.* real Möbius (conformal) transformations $z \rightarrow (az + b)/(cz + d)$, $ad - bc = 1$, of complex plane *resp.* upper half plane.

The modular group $SL(2, Z)$ acting as the subgroup of $SL(2, R) \subset SL(2, C)$ consists of matrices $[a, b; c, d]$ having integer valued elements satisfying $ad - bc = 1$. Alternative definition identifies the elements differing by sign (https://en.wikipedia.org/wiki/Modular_group) is a basic example of infinite discrete sub-group.

Modular group is representable as a free product $Z_2 * Z_3$ with generators S *resp.* T subject to relations $S^2 = I$ and $(ST)^3 = I$. Modular group has braid group B_3 of 3 braids as a universal covering group. Modular group has an infinite number of congruence subgroups $\Gamma(N)$ as subgroups.

The diagonal elements of $\Gamma(N)$ satisfy $a \ mod \ N = d \ mod \ N = \pm 1$ and $c \ mod \ N = d \ mod \ N = 0$ so that the matrices are equal to $\pm I$ modulo N. There is also a hierarchy of subgroups $\Gamma_0(N)$ for which matrices are upper triangular matrices modulo N.

In TGD one has also p-adic length scale hierarchy with preferred p-adic primes $p \simeq 2^k$. Therefore the groups $\Gamma(p^n)$ are of special interest in TGD framework.

If replaces N with an extension of rationals, one obtains huge hierarchy of subgroups expected to be relevant in TGD framework. One can define the notion of integer also for the extensions of rationals. Algebraic integer is defined as a root of a monic polynomial $P_n = x^n + ...$ with integer coefficients. Also the counterparts of the groups $\Gamma(N)$ can be defined, in particular those associated with $N = p^n$.

H^n, $n = 2, 3$ allows infinite number of tesselations as left coset spaces $G\backslash H^n$ of $H^n = SO(1,n)/SO(1,1)$. G is here some infinite discrete subgroup $G \subset SO(1,n)$ of $SO(1,n)$ such as $\Gamma(N)$. For ordinary sphere S^2 the analogs of tesselations are finite lattices and correspond to Platonic solids - tetrahedron, octahedron and cube, and icosahedron and dodecahedron. Tesselations would therefore define hyperbolic analogs of Platonic solids.

The groups $SL(2,Z)/Z_N$ are finite groups. For $N = 3$ one obtains tetrahedral group and $N = 5$ gives icosahedral group. Both groups play central role in TGD inspired model of genetic code [L1, L7] but their origin has remained unclear. $\Gamma(N)$ is a normal subgroup $SL(2,Z)$ so that the coset space is group too: $SL(2,Z)/\Gamma(N) = SL(2,Z_N)$. One can represent the elements of group algebra $G(SL(2,Z))$ of $SL(2,Z)$ as entangled elements in the tensor product of $G(SL(2,Z)/\Gamma(N))$ and $G(SL(2,Z_N))$. Number theoretic state function reduction as a "small" state function reduction (SSFR) for elements of $G(SL(2,Z)))$ would project them to unentangled products of elements of $G(SL(2,Z)/\Gamma(N))$ and $G(SL(2,Z_N))$. Maybe genetic code could relate with $\Gamma(N)$ with $N = 3$ and $N = 5$.

1.3 Could magnetic body provide a concrete geometric representation for the tesselation of hyperbolic space?

In TGD framework magnetic body (MB) having an onion-like structure and carrying dark matter as ordinary matter labelled by effective Planck constant $h_{eff} = nh_0$, where n corresponds to the dimension of extension of rationals serving as a kind of IQ. Various quantum scales, in particular quantum coherence length are expected to be proportional to n so that algebraic extensions of rationals define an evolutionary hierarchy with levels labelled by the dimension of extension. Space-time surface for given value of n can be regarded as a covering spaces with n sheets related by the action of Galois group of Galois extension acting as symmetry.

The question is whether one could generalize the hypothesis [J1] (`http://tinyurl.com/ybghux6d`) in TGD framework. In the sequel such a generalization replacing 2-D hyperbolic space with its 3-D counterpart and assuming that the hyperbolic tesselation is associated with MB of brain or of its subsystem considered. This generalization reduces to P^2 if one restricts P^3 to subspace P^2 and restricts $SL(2,C)$ ($SO(1,3)$) as symmetry to cylindrical symmetry $SL(2,R)$ ($SO(1,2)$). Cylindrical symmetry is natural to magnetic flux tubes and cylindrical magnetic flux sheets so that P^2 option might be more natural.

The notion of MB is extremely general and makes sense in all scales, and one can consider the possibility that the hyperbolic tesselations could provide a kind of universal for the MB of system responsible for cognitive representations.

2 Could regions of brain be mapped to tesselations in 3-D hyperbolic space?

The question is whether some 3-D lattice-like structures formed by neurons of brain or its subsystem could correspond to tesselations of 2-D or 3-D hyperbolic space H^3 realization as cognitive representations at the MB of brain having hierarchical onion-like structure correlating with hierarchical structure of brain. The tesselation would be defined by an infinite discrete subgroup G of $SL(2,C)$ such that elements are algebraic integers in the extension of rationals. The unit cells of the tesselation would be labelled by elements of G and would therefore define cognitive representation.

One can consider two basic options. Brain or its substructure as 3-D structure is mapped

1. either to a tesselation of H^3 at which $SL(2,C)$ acts as isometries,

2. or to a cylindrically to a tesselation of H^2 at which $SL(2,R)$ acts as isometries represented as upper half-plane or as Poincare disk where the action is as conformal transformation. One can consider also mapping to a complex plane compactified to Riemann sphere at which $SL(2,C)$ acts: now the action is however not as isometries but conformal transformations.

   The interpretation could be in terms of symmetry breaking selecting time axis and spin quantization axis as direction of cylinder.

2.1 Some basic facts

Consider first some basic facts about the possible role of 3-D hyperbolic space and its tesselations in TGD.

1. 3-D hyperbolic space H^3 representable as hyperboloid $t^2 - x^2 - y^2 - z^2 \equiv t^2 - r_M^2 = a^2$. a has interpretation as light-cone proper time and in TGD inspired cosmology it corresponds to cosmic time. 2-D hyperbolic space could be seen as subspace of H^3. Now infinite discrete subgroups of $SO(1,3)$ would define tesselations as lattice-like structures. They would serve as 3-D analogs of Platonic solids. I have proposed [K1] that they could explain the astrophysical objects a located along lines with redshifts coming as multiples of a basic redshift in terms of lattice-like structures in cosmic scales.

2. Brain region itself cannot correspond in any manner to a region of H^3 represented as $a = constant = a_0$ hyperboloid. MB of brain region might however do so. The mapping of brain region to the hyperboloid $a = a_0$ could be mediated by gravitational magnetic flux tubes which can be radial since the Kähler flux vanishes in good approximation and there is no conserved monopole flux. Only the cognitive representation as discrete points in extension of rationals would correspond to points of the hyperboloid.

   If MB participates in cosmological expansion assignable to CD, its size would scale up like a as also the cognitive representation associated with the tesselation, whose points would be labelled by discrete infinite subgroup G - say congruence group $\Gamma(N)$ for extension of rationals. In ZEO this means that the part of tesselation inside CD would approach to the boundary of CD (or cd). The finite size of CD would however prevent the expansion to values of $a > T$, T is the size of CD define as the maximal radius of the intersection light-cones involved. It would also prevent MB from reaching the boundary of CD. One cannot therefore exclude cosmic expansion of MB.

3. One can challenge the assumption about cosmic expansion of MB. Quite generally, all known astrophysical objects participate in cosmological expansion by receding from each other as the cosmic redshifts show but do not experience cosmological expansion themselves. TGD solves this paradox by the assumption that cosmic expansion takes place as quantum phase transitions in which expansion occurs in rapid jerks, which correspond to reductions of length scale dependent cosmological constant Λ by a power of 2 if p-adic length scale hypothesis is accepted [L9] .

   There is evidence that even Earth has experienced this kind of expansion during Cambrian Explosion, which would have increased the radius of Earth by factor 2 [L6]. This would have been also a giant step in biological evolution as the multicellular life developed in the Earth's interior would have bursted to the surface of Earth and oceans would have formed. An interesting question inspired by the fractality of TGD Universe is whether one could see also the biological growth and development of organs and organelles as sequences of this kind of phase transitions.

   This situation might hold true also for MB so that also it should evolve by rapid jerks as the value of Λ is reduced.

4. In TGD space-times are surfaces in $M^4 \times CP_2$. In zero energy ontology (ZEO) they are 4-surfaces in causal diamond (CD), where one has $= cd \times CP_2$, where cd is diamond-like intersection of future and past directed light-cones.

   For light-cone M^4_+ one has a natural slicing is by using the hyperboloids $a = constant$. This slicing would define a natural time coordinate as analog of cosmic time. The usual linear Minkowski coordinates define a second natural natural slicing by $t = constant$ sections, where t is the linear Minkowski time.

   One can define the standard hyperbolic coordinates of M^4_+ by the line element

$$ds^2 = da^2 - a^2(d\eta^2 + sinh^2(\eta)d\Omega^2) \ .$$

   $d\Omega^2 = d\theta^2 + sin^2(theta)d\phi^2$ is the line element of unit sphere S^2. η is the hyperbolic angle identifiable as analog of ordinary angle and having expression

$$tanh(\eta) = \frac{r_M}{t} \equiv \beta$$

   having an interpretation as velocity $\beta = v/c$ n radial direction satisfying $\beta \leq 1$: one has $t = acosh(\eta)$ and $r_M = asinh(\eta)$.

2.2 About the precise correspondence between 3-D surfaces and H^3

What could the precise correspondence between 3-D surface giving rise to a cognitive representation of MB and tesselation of H^3 be?

1. The space-time surface representing MB is not hyperbolic space itself but could in some sense have discrete subgroup of $G \subset H^3$ as its symmetries: a possible interpretation would be as cognitive representations [L12, L8] consisting of points of H with coordinates in extension of rationals defining the adele [L3, L4]. The lattice-like structure associated with 3-surfaces could be mappable to this kind of hyperboloid for some value of a.

   Could the part of MB representing sub-system of brain in question be seen as an intersection of the with $t = T$ section of M^4_+ with the slicing of M^4_+ by $a = constant$ hyperboloids such that magnetic images of neurons as points of the tesselation of H^3 defining cognitive representation would belong to the intersection? For $t > T$ the 3-D structure would be preserved in good approximation.

2. The usual time=constant snapshot in M^4_+ satisfying $t = T$ intersects the hyperboloids with $0 \leq a \leq T$. The condition $t = acosh(\eta) = T$ gives $a = T/cosh(\eta)$ so that a indeed varies in this range. This gives for the radial M^4 coordinate $r_M = asinh(\eta) = Ttanh(\eta)$ giving $r_M \leq T$.

   It seems that this projection is 3-D analog of Poincare disk as a "Poincare ball" of radius $r_M \leq T$ with at least analog of hyperbolic geometry. At least the density of intersections with hyperboloids increases as one approaches light-cone boundary since the density of hyperboloids increases.

3. A tesselation of H^3 corresponds to the points $\{(asinh(\eta_n), \Omega_n)\}$. The lattice-like structure in $E^3$for $t = T$ would correspond to points (r_M, Ω) in $\{Ttanh(\eta_n), \Omega_n\}$. The difference from the representation hyperbolic geometry as H^3 is that instead of $r_M = asinh(\theta_n)$ for H^3 one has $r_M = Ttanh(\eta_n)$ for the analog of Poincare disk. For small values of η one has $sinh(\eta) \simeq \tanh(\eta)$ but not for large values so that E^3 is compressed to Poincare ball B^3.

   Neurons with large number of connections would correspond to points of tesselation with large values of η_n and similar neurons even if far away from each other would be mapped near to each other at spheres $\eta_n =$ constant surfaces (spheres for H^3 or circles for H^2).

   The discrete geometries for the magnetic image of neural sub-system as tesselations would naturally correspond to discrete subgroups of $G \subset SO(1,3)$ as analogs $G\backslash H^3$ of Platonic solids. As found, there is infinite number of them and concordance groups $\Gamma(N)$ ore of special interest. One obtains also their 2-D variants as 2-D planar slices consistent with the symmetries just like one can have 2-D lattices as sub-lattices of 3-D lattices in E^3.

Remark: The elements of subgroup $G \subset SL(2,C)$ for given extension of rationals provide natural coordinates for the unit cells of tesselation, and can be used instead of $\{\eta_n, \Omega_n\}$.

4. The system could have a finite size due to finite light-velocity if it has resulted in an event analogous to Big Bang like event (TGD predicts a hierarchy of cosmologies within cosmologies and cd is geometrically analogous to Big Bang followed by Big Crunch). This option does not however look plausible at the level of visible bio-matter. At the level of MB this could be make sense and correspond to the emergence of a new onion-like layers to MB bringing in new scale of quantum coherence as CD.

   In the case of MB one can estimate the T from the assumption that EEG corresponds to communications between brain and particular layer of its MB. Schumann frequency 7.8 Hz corresponds to wavelength of $\lambda = 2\pi R_E$, R_E Earth radius. EEG alpha band is around 10 Hz and corresponds to a slightly shorter wave length lengths. If this frequency is realized as cyclotron frequency the corresponding part of MB should be of the order of Earth size. This would give $R \sim R_E$ and $T \leq R/c \leq .1$ s. The part of neuronal system considered could be the above described intersection corresponding to time $t = T$. After this no expansion would take place and the 3-D analog of Poincare ball would be preserved.

   Note that if MB would participate in cosmic expansion, one would expect that the frequency scale of EEG scales down like $1/a$, which is not observed. Different bands of EEG could however correspond to different values of $a = a_0$ defining different layers of MB.

The neuronal network has been assumed to be accompanied by flux tube network with flux tubes parallel to axons defining the "small" part of MB with size of order body size [L2, L5]. How the topology of this network correlates with the topology of the "large" part of MB with layers having size scales even larger than Earth size? Could the "small" networks at the level of biological body be representations of the "large" networks at the level of MB - or vice versa.

The higher level representations would re-organize the nodes of "small" flux tube networks by various criteria such as the number of connections to other nodes. Similar nodes - even distant ones - would correspond to points near to each other. Therefore similar neurons could be treated as coherent units with coherence induced from that at higher level. Synchronous firing would be the signature for nearness at the higher level. The hierarchy of layers of MB would perform basically classification of the objects of the system at the lowest level.

There is a huge number of possibilities for the cognitive representations corresponding to various values of N (in particular powers preferred prime p) labeling $\Gamma(N)$, to hierarchy of extensions of rationals and the values of T possibly identifiable as roots of polynomials defining representation of layer of MB in M^8. Therefore one can hope that this vision could provide universal view about the anatomy of MB in relation to that of biological body (in very general sense).

2.3 The interpretation of the hyperbolic tesselations of neurons in terms of ZEO, $M^8 - H$ duality, and cognitive representations

This picture suggests an interesting connection to TGD based view about quantum measurement theory [L13], which actually extends physics to a theory of consciousness. Causal diamonds (CDs) have a key role in ZEO and hyperbolic geometry is very naturally associated with them. The notions $M^8 - H$ duality [L11, L10] could provide an explanation for the special value $t = T$, and tesselations could correspond to a particular cognitive representation [L12].

1. In zero energy ontology (ZEO) replacing ordinary ontology of quantum theory the notion of causal diamond (CD) plays a central role. CDs for a length scale hierarchy and CDs have sub-CDs. Space-time surfaces for given CD have ends at the upper and lower boundary of CD. In this picture the appearance of hyperbolic geometry at the the level of MB would be very natural.

2. $M^8 - H$ duality [L11] states that space-time surfaces could be regarded either as algebraic surfaces in M^8 or as preferred extremals of action in $H = M^4 \times CP_2$ reducing to minimal

surface satisfying infinite number of additional conditions. Otherwise the consistency of dynamics in H dictated by partial differential equations with algebraic dynamics in M^8 dictated by algebraic equations would not be possible.

One can say that space-time surfaces are roots of an octonionic polynomial obtained as an algebraic continuation of a real polynomial with rational coefficients to octonionic polynomial. This in the sense that either imaginary or real part of P in quaternionic sense vanishes and gives rise to 4-D surface in the generic case.

3. A special prediction of M^8 picture is that besides 4-D surfaces as roots of algebraic equations also 6-D special brane-like solutions with topology of 6-sphere S^6 are possible. For these solutions both real and imaginary parts vanish. These solutions have counterparts in H, and their intersection with cd is $t = r_n$ ball, where r_n is the root of P.

4. I have called the moments $t = r_n$ "very special moments in the life of self" identified as evolution of zero energy state of self by "small" state function reductions (SSFRs) as analogs of weak measurements. Also the size of CD increases in this process in statistical sense and corresponds to the increase of clock time as a natural correlate of subjective time defined by the sequence of SSFRs.

5. Could the state of neuron system at $t = T$ correspond to $T = r_n$ as a root of polynomial P? Could these special moments correspond to rapid jerks in the cosmological expansion so that also the development of living organism would involve a sequence of them increasing the value of Λ. Presumably these jerks would occur at the level of MB and possibly induce those at the level of biological body. At the level o MB they could also correspond to a phase transition like events in the evolution of consciousness involving scaling up the size of MB.

To summarize, the tesselations of H^3 or $E^1 \times H^2$ suggest a universal cognitive representations realized at the MB of the system. One would have hierarchy of p-adic length scales and extensions of rationals giving rise to hierarchies of tesselations defining cognitive representations at corresponding layers of MB. Living matter would be only a special case. In living matter EEG would define important hierarchies of tesselations but also other frequency ranges would do so.

REFERENCES

Neuroscience and Consciousness

[J1] Cacciola A et al. Coalescent embedding in the hyperbolic space unsupervisedly discloses the hidden geometry of the brain. Available at:https://arxiv.org/pdf/1705.04192.pdf, 2017.

Books related to TGD

[K1] Pitkänen M. More about TGD Inspired Cosmology. In *Physics in Many-Sheeted Space-Time: Part II.* Available at: http://tgdtheory.fi/pdfpool/cosmomore.pdf, 2019.

Articles about TGD

[L1] Pitkänen M. Geometric theory of harmony. Available at: http://tgdtheory.fi/public_html/articles/harmonytheory.pdf, 2014.

[L2] Pitkänen M. DMT, pineal gland, and the new view about sensory perception. Available at: http://tgdtheory.fi/public_html/articles/dmtpineal.pdf, 2017.

[L3] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.

[L4] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[L5] Pitkänen M. Emotions as sensory percepts about the state of magnetic body? Available at: http://tgdtheory.fi/public_html/articles/emotions.pdf, 2018.

[L6] Pitkänen M. Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life. Available at: http://tgdtheory.fi/public_html/articles/expearth.pdf, 2018.

[L7] Pitkänen M. An overall view about models of genetic code and bio-harmony. Available at: http://tgdtheory.fi/public_html/articles/gcharm.pdf, 2019.

[L8] Pitkänen M. Are fundamental entities discrete or continous and what discretization at fundamental level could mean? Available at: http://tgdtheory.fi/public_html/articles/zetaspecu.pdf, 2019.

[L9] Pitkänen M. Cosmology in crisis. Available at: http://tgdtheory.fi/public_html/articles/cosmocrisis.pdf, 2019.

[L10] Pitkänen M. $M^8 - H$ duality and consciousness. Available at: http://tgdtheory.fi/public_html/articles/M8Hconsc.pdf, 2019.

[L11] Pitkänen M. New results related to $M^8 - H$ duality. Available at: http://tgdtheory.fi/public_html/articles/M8Hduality.pdf, 2019.

[L12] Pitkänen M. Scattering amplitudes and orbits of cognitive representations under subgroup of symplectic group respecting the extension of rationals . Available at: http://tgdtheory.fi/public_html/articles/symplorbsm.pdf, 2019.

[L13] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: http://tgdtheory.fi/public_html/articles/zeoquestions.pdf, 2019.