Some questions related to the twistor lift of TGD

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Abstract

In this article I consider some questions related to the twistor lift of TGD.

1. Can the analog of Kähler form assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications?
2. How does gravitational coupling emerge at fundamental level?
3. Could one regard localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space.

1 Introduction

In this section I consider further questions related to the twistor lift.

1. Can the analog of Kähler form $J(M^4)$ assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory? What would be the physical implications? The basic objection is the loss of Poincare invariance is lost. This can be however avoided by introducing the moduli space for Kähler forms. This moduli space is actually the moduli space of causal diamonds (CDs) forced in any case by zero energy ontology (ZEO) and playing central role in the generalization of quantum measurement theory to a theory of consciousness and in the explanation of the relationship between geometric and subjective time. $J(M^4)$ corresponds to parallel constant electric and magnetic fields in given direction. One implication is transversal localization of imbedding space spinor modes. Also CP breaking is implied by the electric field and the question is whether this could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$. Whether stationary spherically symmetric metric as minimal surface allows a sensible physical generalization is a killer test for the hypothesis.

2. How does gravitational coupling emerge at fundamental level? The answer is obvious: string area action is scaled by $1/G$ as in string models. The objection is that p-adic mass calculations suggest that string tension is determined by CP_2 size R: the analog of string tension appearing in mass formula given by p-adic mass calculations would be by a factor about 10^{-8} smaller than that estimated from string tension. The discrepancy evaporates by noticing that p-adic mass calculations rely on p-adic thermodynamics at imbedding space level whereas string world sheets appear at space-time level.

3. Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface having by definition vanishing induced Kähler form: $J(M^4)+J(CP_2)=0$. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. Could string world sheets be minimal surfaces satisfying $J(M^4)+J(CP_2)=0$. The Lagrangian condition allows also more general solutions - even 4-D space-time surfaces

and one obtains analog of brane hierarchy. Could one allow spinor modes also at these analogs of branes. Is Lagrangian condition equivalent with the original condition that induced W boson fields making the em charge of induced spinor modes ill-defined vanish and allowing also solution with other dimensions. How Lagrangian property relates to the idea that string world sheets correspond to complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space.

2 Can the Kähler form of M^4 appear in Kähler action?

I have already earlier considered the question whether the analog of Kähler form assignable to M^4 could appear in Kähler action. Could one replace the induced Kähler form $J(CP_2)$ with the sum $J = J(M^4) + J(CP_2)$ such that the latter term would give rise to a new component of Kähler form both in space-time interior at the boundaries of string world sheets regarded as point-like particles? This could be done both in the Kähler action for the interior of X^4 and also in the topological magnetic flux term $\int J$ associated with string world sheet and reducing to a boundary term giving couplings to U(1) gauge potentials $A_\mu(CP_2)$ and $A_\mu(M^4)$ associated with $J(CP_2)$ and $J(M^4)$. The interpretation of this coupling is an interesting challenge.

Consider first the objections against introducing $J(M^4)$ to the Kähler action at imbedding space level.

1. $J(M^4)$ would would break translational and Lorentz symmetries at the level of imbedding space since $J(M^4)$ cannot be Lorentz invariant. For imbedding space spinor modes this term would bring in coupling to the self-dual Kähler form in M^4. The simplest choice is $A = (A_t = z, A_z = 0, A_x = y, A_y = 0)$ defining decomposition $M^4 = M^2 \times E^2$. For Dirac equation in M^4 one would have free motion in preferred time-like (t,z)-plane plane M^2 in whereas in x- and y-directions (E^2 plane) would one have harmonic oscillator potentials due to the gauge potentials of electric and magnetic fields. One would have something very similar to quark model of hadron: quark momenta would have conserved longitudinal part and non-conserved transversal part. The solution spectrum has scaling invariance $\Psi(m^k) \rightarrow \Psi(\lambda m^k)$ so that there is no preferred scale and the transversal scales scale as $1/E$ and $1/k_x$.

2. Since $J(M^4)$ is not Lorentz invariant Lorentz boosts would produce new $M^2 \times E^2$ decomposition. If one assumes above kind of linear gauge as gauge invariance suggests, the choices with fixed second tip of causal diamond (CD) define finite-dimensional moduli space $SO(3,1)/SO(1,1) \times SO(2)$ having in number theoretic vision an interpretation as a choice of preferred hypercomplex plane and its orthogonal complement. This is the moduli space for hypercomplex structures in M^4 with the choices of origins parameterized by M^4. The introduction of the moduli space would allow to preserve Poincare invariance.

3. If one generalizes the condition for Kähler metric to $J^2(M^4) = -g(M^4)$ fixing the scaling of J, the coupling to $A(M^4)$ is also large and suggests problems with the large breaking of Poincare symmetry for the spinor modes of the imbedding space for given moduli. The transversal localization by the self-dual magnetic and electric fields for $J(M^4)$ would produce wave packets in transversal degrees of freedom: is this physical?

   This moduli space is actually the moduli space introduced for causal diamonds (CDs) in zero energy ontology (ZEO) forced by the finite value of volume action: fixing of the line connecting the tips of CD the Lorentz boost fixing the position for the second tip of CD parametrizes this moduli space apart from division with the group of transformations leaving the planes M^2 and E^2 having interpretation a plane defined by light-like momentum and polarization plane associated with a given CD invariant.

4. Why this kind of symmetry breaking for Poincare invariance? A possible explanation proposed already earlier is that quantum measurement involves a selection of quantization axis. This choice necessarily breaks the symmetries and $J(M^4)$ would be an imbedding space correlate for the selection of rest frame and quantization axis of spin. This conforms with the fact that CD is interpreted as the perceptive field of conscious entity at imbedding space level:

the contents of consciousness would be determined by the superposition of space-time surfaces inside CD. The choice of $J(M^4)$ for CD would select preferred rest system (quantization axis for energy as a line connecting tips of CD) via electric part of $J(M^4)$ and quantization axis of spin (via magnetic part of $J(M^4)$. The moduli space for CDs would be the space for choices of these particular quantization axis and in each state function reduction would mean a localization in this moduli space. Clearly, this reduction would be higher level reduction and correspond to a decision of experimenter.

To summarize, for $J(M^4) = 0$ Poincare symmetries are realized at the level of imbedding space but obviously broken slightly by the geometry of CD. The allowance of $J(M^4) \neq 0$ implies that both translational and rotational symmetries are reduced for a given CD: the interpretation would be in terms of a choice of quantization axis in state function reduction. They are however lifted to the level of moduli space of CDs and exact in this more abstract sense. This is nothing new: already the introduction of ZEO and CDs force by volume term in action forced by twistor lift of TGD implies the same. Also the view about state function reduction requires wave functions in the moduli space of CDs. This is also essential for understanding how the arrow of geometric time is inherited from that of subjective time in TGD inspired theory of consciousness [K1, K7].

What about the situation at space-time level?

1. The introduction of $J(M^4)$ part to Kähler action has nice number theoretic aspects. In particular, J selects the preferred complex and quaternionic sub-space of octonionic space of imbedding space. The simplest possibility is that the Kähler action is defined by the Kähler form $J(M^4) + J(CP_2)$.

   Since M^4 and CP_2 Kähler geometries decouple it should be possible to take the counterpart of Kähler coupling strength in M^4 to be much larger than in CP_2 degrees of freedom so that M^4 Kähler action is a small perturbation and slowly varying as a functional of preferred extremal. This option is however not in accordance with the idea that entire Kähler form is induced.

2. Whether the proposed ansätze for general solutions make still sense is not clear. In particular, can one still assume that preferred extremals are minimal surfaces? Number theoretical vision strongly suggests - one could even say demands - the effective decoupling of Kähler action and volume term. This would imply the universality of quantum critical dynamics. The solutions would not depend at all on the coupling parameters except through the dependence on boundary conditions. The coupling between the dynamics of Kähler action and volume term would come also from the conservation conditions at light-like 3-surfaces at which the signature of the induced metric changes.

3. At space-time level the field equations get more complex if the M^4 projection has dimension $D(M^4) > 2$ and also for $D(M^4) = 2$ if it carries non-vanishing induced $J(M^4)$. One would obtain cosmic strings of form $X^2 \times Y^2$ as minimal surface extremals of ordinary Kähler action or X^2 Lagrangian manifold of M^4 as also CP_2 type vacuum extremals and their deformations with M^4 projection Lagrangian manifold. Thus the differences would not be seen for elementary particle and string like objects. Simplest string worlds sheet for which $J(M^4)$ vanishes would correspond to a piece of plane M^2.

   M^4 is the simplest minimal surface extremal of Kähler action necessarily involving also $J(M^4)$. The action in this case vanishes identically by self-duality (in Euclidian signature self-duality does not imply this). For perturbations of M^4 such as spherically symmetric stationary metric the contribution of M^4 Kähler term to the action is expected to be small and the come mainly from cross term mostly and be proportional to the deviation from flat metric. The interpretation in terms of gravitational contribution from M^4 degrees of freedom could make sense.

4. What about massless extremals (MEs)? How the induced metric affects the situation and what properties second fundamental form has? Is it possible to obtain a situation in which the energy momentum tensor T^α and second fundamental form $H^k_{\alpha\beta}$ have in common components which are proportional to light-like vector so that the contraction $T^{\alpha\beta}H^k_{\alpha\beta}$ vanishes?

Minimal surface property would help to satisfy the conditions. By conformal invariance one would expect that the total Kähler action vanishes and that one has $J^{\alpha}_{\gamma} J^{\gamma\beta} \propto a g^{\alpha\beta} + b k^{\alpha} k^{\beta}$. These conditions together with light-likeness of Kähler current guarantee that field equations are satisfied.

In fact, one ends up to consider a generalization of MEs by starting from a generalization of holomorphy. Complex CP_2 coordinates ξ^i would be functions of light-like M^2 coordinate $u_+ = k \cdot m$, k light-like vector, and of complex coordinate w for E^2 orthogonal to M^2. Therefore the CP_2 projection would 3-D rather than 2-D now.

The second fundamental form has only components of form $H^k_{u_+ w}$, $H^k_{u_+ \overline{w}}$ and H^k_{ww}, $H^k_{\overline{w}\overline{w}}$. The CP_2 contribution to the induced metric has only components of form $\Delta g_{u_+ w}$, $\Delta g_{+\overline{w}}$, and $g_{\overline{w}w}$. There is also contribution $g_{u_+ u_-} = 1$, where v is the light-like dual of u in plane M^2. Contravariant metric can be expanded as a power series for in the deviation ($\Delta g_{u_+ w}$, $\Delta g_{u_+ \overline{w}}$) of the metric from ($g_{u_+ u_-}, g_{w\overline{w}}$). Only components of form g^{u_+, u_i} and $g^{w,\overline{w}}$ are obtained and their contractions with the second fundamental form vanish identically since there are no common index pairs with simultaneously non-vanishing components. Hence it seems that MEs generalize!

I have asked earlier whether this construction might generalize for ordinary MEs. One can introduce what I have called Hamilton-Jacobi structure for M^4 consisting of locally orthogonal slicings by integrable 2-surfaces having tangent space having local decomposition $M^2_x \times E^2_x$ with light-like direction depending on point x. An objection is that the direction of light-like momentum depends on position: this need not be inconsistent with momentum conservation but would imply that the total four-momentum is not light-like anymore. Topological condensation for MEs and at MEs could imply this kind modification.

5. There is also a topological magnetic flux type term for string world sheet. Topological term can be transformed to a boundary term coupling classical particles at the boundary of string world sheet to CP_2 Kähler gauge potential (added to the equation for a light-like geodesic line). Now also the coupling to M^4 gauge potential would be obtained. The condition $J(M^4) + J(CP_2) = 0$ at string world sheets [K6] is very attractive manner to identify string world sheets as analogs of Lagrangian manifolds but does not imply the vanishing of the net $U(1)$ couplings at boundary since the induce gauge potentials are in general different.

   Also topological term including also M^4 Kähler magnetic flux for string world sheet contributes also to the modified Dirac equation since the gamma matrices are modified gamma matrices required by super-conformal symmetries and defined as contractions of canonical momentum densities with imbedding space gamma matrices [K3]. This is true both in space-time interior, at string world sheets and at their boundaries. CP_2 (M^4) term gives a contribution proportional to CP_2 (M^4) gamma matrices.

   At imbedding space level transversal localization would be the outcome and a good guess is that the same happens also now. This is indeed the case for M^4 defining the simplest extremal. The general interpretation of M^4 Kähler form could be as a quantum tool for transversal dynamical localization of wave packets in Kähler magnetic and electric fields of M^4. Analog for decoherence occurring in transversal degrees of freedom would be in question. Hadron physics could be one application.

How to test this idea?

1. It might be possible to kill the idea by showing that one does not obtain spherically symmetric Schwartschild type metric as a minimal surface extremal of generalized Kähler action: these extremals are possible for ordinary Kähler action [L1] [K5]. For the canonical imbedding of M^4 field equations are satisfied since energy momentum tensor vanishes identically. For the small deformations the presence of $J(M^4)$ would reduce rotational symmetry to cylindrical symmetry.

2. $J(M^4)$ could make its presence manifest in the physics of right-handed neutrino having no direct couplings to electroweak gauge fields. Mixing with left handed neutrino is however

induced by mixing of M^4 and CP_2 gamma matrices. The transversal localization of right-handed neutrino in a background, which is a small deformation of M^4 could serve as an experimental signature.

3. CP breaking in hadronic systems is one of the poorly understood aspects of fundamental physics and relates closely to the mysterious matter-antimatter asymmetry. The constant electric part of self dual $J(M^4)$ implies CP breaking. I have earlier consider that Kähler electric fields could cause this breaking but now the electric field is not constant. Second possibility is that matter and antimatter correspond to different values of h_{eff} and are dark relative to each other. The question is whether $J(M^4)$ could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$ and whether this breaking could explain hadronic CP breaking and matter anti-matter asymmetry. Could M^4 part of Kähler electric field induce different $h_{eff}/h = n$ for particles and antiparticles.

3 About string like objects

String like objects and partonic 2-surfaces carry the information about quantum states and about space-time surfaces as preferred extremals if strong form of holography (SH) holds true. SH has of course some variants. The weakest variant states that fundamental information carrying objects are metrically 2-D. The light-like 3-surfaces separating space-time regions with Minkowskian and Euclidian signature of the induced metric are indeed metrically 2-D, and could thus carry information about quantum state.

An attractive possibility is that this information is basically topological. For instance, the value of Planck constant $h_{eff} = n \times h$ would tell the number sheets of the singular covering defining this surface such that the sheets co-incide at partonic 2-surfaces at the ends of space-time surface at boundaries of CD. In the following some questions related to string world sheets are considered. The information could be also number theoretical. Galois group for the algebraic extension of rationals defining particular adelic physics would transform to each other the number theoretic discretizations of light-like 3-surfaces and give rise to covering space structure. The action is trivial at partonic 2-surfaces should be trivial if one wants singular covering: this would mean that discretizations of partonic 2-surfaces consist of rational points. $h_{eff}/h = n$ could in this case be a factor of the order of Galois group.

The original observation was that string world sheets should carry vanishing W boson fields in order that the em charge for the modes of the induced spinor field is well-defined. This condition can be satisfied in certain situations also for the entire space-time surface. This raises several questions. What is the fundamental condition forcing the restriction of the spinor modes to string world sheets - or more generally, to surface of given dimension? Is this restriction dynamical. Can one have an analog of brane hierarchy in which also higher-D objects can carry modes of induced spinor field [K4]? Could the analogs of Lagrangian sub-manifolds of $X^4 \subset M^4 \times CP_2$ satisfying $J(M^4) + J(CP_2) = 0$ define string world sheets and their variants with varying dimension? The additional condition would be minimal surface property.

3.1 How does the gravitational coupling emerge?

The appearance of $G = l_P^2$ has coupling constant remained for a long time actually somewhat of a mystery in TGD. l_P defines the radius of the twistor sphere of M^4 replaced with its geometric twistor space $M^4 \times S^2$ in twistor lift. G makes itself visible via the coefficients $\rho_{vac} = 8\pi\Lambda/G$ volume term but not directly and if preferred extremals are minimal surface extremals of Kähler action ρ_{vac} makes itself visible only via boundary conditions. How G appears as coupling constant?

Somehow the M^4 Kähler form should appear in field equations. $1/G$ could naturally appear in the string tension for string world sheets as string models suggest. p-Adic mass calculations identify the analog of string tension as something of order of magnitude of $1/R^2$ [K2]. This identification comes from the fact that the ground states of super-conformal representations correspond to imbedding space spinor modes, which are solutions of Dirac equation in $M^4 \times CP_2$. This argument is rather convincing and allows to expect that the p-adic mass scale is not determined by string tension and it can be chosen to be of order $1/G$ just as in string models.

3.2 Non-commutative imbedding space and strong form of holography

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

General Coordinate Invariance poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in imbedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 imbedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Kächtion also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing imbedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive. Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4) + J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions. This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4) + J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K4].

It is relatively easy to construct am infinite family of Lagrangian string world sheets satisfying $J(M^4) + J(CP_2) = 0$ using generalized symplectic transformations of M^4 and CP_2 as Hamiltonian flows to generate new ones from a given Lagrangian string world sheets. One must pose minimal surface property as a separate condition. Consider a piece of M^2 with coordinates (t, z) and homologically non-trivial geodesic sphere S^2 of CP_2 with coordinates $(u = cos(\Theta), \Phi)$. One has $J(M^4)_{tz} = 1$ and $J_{u\Phi} = 1$. Identify string world sheet via map $(u, \Phi) = (kz, \omega t)$ from M^2 to S^2. The induced CP_2 Kahler form is $J(CP_2)_{tz} = k\omega$. $k\omega = -1$ guarantees $J(M^4) + J(CP_2) = 0$. The strings have necessarily finite length from $L = 1/k \leq z \leq L$. One can perform symplectic transformations of CP_2 and symplectic transformations of M^4 to obtain new string world sheets. In general these are not minimal surfaces and this condition would select some preferred string world sheets.

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in imbedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of imbedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces would be enough to construct quantum states. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD.

To sum up, one cannot exclude the possibility that $J(M^4)$ is present implying a universal transversal localization of imbedding space spinor harmonics and the modes of spinor fields in the interior of X^4: this could perhaps relate to somewhat mysterious de-coherence interaction producing locality and to CP breaking and matter-antimatter asymmetry. The moduli space for M^4 Kähler structures proposed by number theoretic considerations would save from the loss of Poincare invariance and the number theoretic vision based on quaternionic and octonionic structure would have rather concrete realization. This moduli space would only extend the notion of WCW.

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