Some questions related to the twistor lift of TGD

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Abstract

In this article I consider some questions related to the twistor lift of TGD.

1. What does the induction of the twistor structure really mean?
2. Can the analog of Kähler form assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications?
3. How does gravitational coupling emerge at fundamental level?
4. Could one regard localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space.

1 Introduction

In this article I consider further questions related to the twistor lift.

1. What does the induction of the twistor structure really mean? What is meant with twistor space. For instance, is the twistor sphere for M^4 time-like or space-like. The induction procedure involves dimensional reduction forced by the condition that the projection of the sum of Kähler forms for the twistor spaces $T(M^4)$ and $T(CP_2)$ gives Kähler form for the twistor sphere of X^4. Better understanding of the details is required.

2. Can the analog of Kähler form $J(M^4)$ assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory? What would be the physical implications?

   The basic objection is the loss of Poincare invariance. This can be however avoided by introducing the moduli space for Kähler forms. This moduli space is actually the moduli space of causal diamonds (CDs) forced in any case by zero energy ontology (ZEO) and playing central role in the generalization of quantum measurement theory to a theory of consciousness and in the explanation of the relationship between geometric and subjective time [K5].

   Why $J(M^4)$ would be needed? $J(M^4)$ corresponds to parallel constant electric and magnetic fields in given direction. Constant E and $B = E$ fix directions of quantization axes for energy (rest system) and spin. One implication is transversal localization of imbedding space spinor modes: imbedding space spinor modes are products of harmonic oscillator Gaussians in transversal degrees of freedom very much like quarks inside hadrons.

   Also CP breaking is implied by the electric field and the question is whether this could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$. The estimate for the mass splitting of neutral kaon and anti-kaon is of correct order of magnitude.

   Whether stationary spherically symmetric metric as minimal surface allows a sensible physical generalization is a killer test for the hypothesis that $J(M^4)$ is covariantly constant. The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.

3. How does gravitational coupling emerge at fundamental level? The first naive guess is obvious: string area action is scaled by $1/G$ as in string models. The objection is that p-adic mass calculations suggest that string tension is determined by CP_2 size R: the analog of string tension appearing in mass formula given by p-adic mass calculations would be by a factor about 10^{-8} smaller than that estimated from string tension. The discrepancy evaporates by noticing that p-adic mass calculations rely on p-adic thermodynamics at imbedding space level whereas string world sheets appear at space-time level. Furthermore, if the action assignable to string world sheets is effective action expressing 4-D action in 2-D form as strong form of holography (SH) suggests string tension is expected to be function of the parameters appearing in the 4-D action.

4. Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface having by definition vanishing induced Kähler form: $J(M^4)+J(CP_2)=0$. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. Could string world sheets be minimal surfaces satisfying $J(M^4)+J(CP_2)=0$. The Lagrangian condition allows also more general solutions - even 4-D space-time surfaces and one obtains analog of brane hierarchy. Could one allow spinor modes also at these analogs of branes. Is Lagrangian condition equivalent with the original condition that induced W boson fields making the em charge of induced spinor modes ill-defined vanish and allowing also solution with other dimensions. How Lagrangian property relates to the idea that string world sheets correspond to complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space.

2 More details about the induction of twistor structure

The notion of twistor lift of TGD [K14] [L3] has turned out to have powerful implications concerning the understanding of the relationship of TGD to general relativity. The meaning of the twistor lift really has remained somewhat obscure. There are several questions to be answered. What does one mean with twistor space? What does the induction of twistor structure of $H = M^4 \times CP_2$ to that of space-time surface realized as its twistor space mean?

2.1 What does one mean with twistor space?

The notion of twistor space has been discussed in [K14] from TGD point of view.

1. In the case of twistor space of M^4 the starting point of Penrose was the isomorphism between the conformal group of Spin(4,2) of 6-D Minkowski space $M^{4,2}$ and the group SU(2,2) acting on 2+2 complex spinors.

   6-D twistor space could be identified as 6-D coset space $SU(2,2)/SU(2,1)\times U(1)$. For E^6 this would give projective space $CP_3 = SU(4)/SU(3)\times U(1)$ and in twistor Grassmann approach this definition is indeed used. It is thought that the problems caused by Euclidization are not serious.

2. One can think $SU(2,2)$ as 4×4 complex matrices with orthogonal complex row vector $Z_i = (Z_{i1}, ..., Z_{i4})$, and norms $(1,1,-1-1)$ in the metric $s^2 = \sum \epsilon_i |z_i|^2$, $\epsilon_i \leftrightarrow (1,1,-1,-1)$. The sub-matrices defined by (Z_{k2}, Z_{k3}, Z_{k4}), $k = 2,3,4$, can be regarded apart from normalization elements of $SU(1,2)$. The column vector with components Z_{i1} with $Z_{1,1} = \sqrt{1+\rho^2}$, $\rho^2 = |Z_{21}|^2 - |Z_{31}|^2 - |Z_{41}|^2$ corresponds to a point of the twistor space. The S^2 fiber for given values of ρ and (Z_{31}, Z_{41}) could be identified as the space spanned by the values of Z_{21}. Note that S^2 would have time-like signature and the signature of twistor space would be (3,3), which conforms with the existence of complex structure. There would be dimensional democracy at this level.

3. The identification of 4-D base of the twistor space is unclear to me. The base space of the this twistor space should correspond to the conformal compactification M^4_c of M^4 having metric defined only apart from conformal scaling. The concrete realization M^4_c would be in terms of $M^{4,2}$ light-cone with points projectively identified. As a metric object this space is ill-defined and can appear only at the level of scattering amplitudes in conformally invariant quantum field theories in M^4.

4. Mathematicians define also a second variant of twistor space with S^2 fiber and this space is just $M^4 \times S^2$ [B8] (see `http://arxiv.org/pdf/1308.2820.pdf`). This space has a well-defined metric and seems to be the only possible one for the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces. Whether the signature of S^2 is time-like or space-like has remained an open question but time-like signature looks natural. The radius R_P of S^2 has been proposed to be apart from a numerical constant equal to Planck length l_P. Note that the isometry group is 9-D $SO(3,1) \times SU(2)$ rather than 15-D $SU(2,2)$. In TGD light-likeness in 8-D sense replaces light-likeness in 4-D sense: does this somehow replace the conformal symmetry group $SO(4,2)$ with $SO(3,1) \times SO(3)$? Could $SU(2)$ rotate the direction of spin quantization axis.

2.2 Twistor lift of TGD

In TGD one replaces imbedding space $H = M^4 \times CP_2$ with the product $T = T(M^4) \times T(CP_2)$ of their 6-D twistor spaces, and calls $T(H)$ the twistor space of H. For CP_2 the twistor space is the flag manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ consisting of all possible choices of quantization axis of color isospin and hypercharge.

1. The basic idea is to generalize Penrose's twistor program by lifting the dynamics of space-time surfaces as preferred extremals of Kähler action to those of 6-D Kähler action in twistor space $T(H)$. The conjecture is that field equations reduce to the condition that the twistor structure of space-time surface as 4-manifold is the twistor structure induced from $T(H)$.

   Induction requires that dimensional reduction occurs effectively eliminating twistor fiber $S^2(X^4)$ from the dynamics. Space-time surfaces would be preferred extremals of 4-D Kähler action plus volume term having interpretation in terms of cosmological constant. Twistor lift would be more than an mere alternative formulation of TGD.

2. The reduction would take place as follows. The 6-D twistor space $T(X^4)$ has S^2 as fiber and can be expressed locally as a Cartesian product of 4-D region of space-time and of S^2. The signature of the induced metric of S^2 should be space-like or time-like depending on whether the space-time region is Euclidian or Minkowskian. This suggests that the twistor sphere of M^4 is time-like as also standard picture suggests.

3. Twistor structure of space-time surface is induced to the allowed 6-D surfaces of $T(H)$, which as twistor spaces $T(X^4)$ must have fiber space structure with S^2 as fiber and space-time surface X^4 as base. The Kähler form of $T(H)$ expressible as a direct sum

   $$J(T(H)) = J(T(M^4)) \oplus J(T(CP_2))$$

   induces as its projection the analog of Kähler form in the region of $T(X^4)$ considered.

   There are physical motivations (CP breaking, matter antimatter symmetry, the well-definedness of em charge) to consider the possibility that also M^4 has a non-trivial symplectic/Kähler form of M^4 obtained as a generalization of ordinary symplectic/Kähler form [L3]. This requires the decomposition $M^4 = M^2 \times E^2$ such that M^2 has hypercomplex structure and E^2 complex structures.

   This decomposition might be even local with the tangent spaces $M^2(x)$ and $E^2(x)$ integrating to locally orthogonal 2-surfaces. These decomposition would define what I have called Hamilton-Jacobi structure [K9]. This would give rise to a moduli space of M^4 Kähler forms allowing besides covariantly constant self-dual Kähler forms with decomposition (m^0, m^3) and (m^1, m^2) also more general self-dual closed Kähler forms assignable to integrable local decompositions. One example is spherically symmetric stationary self-dual Kähler form corresponding to the decomposition (m^0, r_M) and (θ, ϕ) suggested by the need to get spherically symmetric minimal surface solutions of field equations. Also the decomposition of Robertson-Walker coordinates to (a, r) and (θ, π) assignable to light-cone M^4_+ can be considered.

The moduli space giving rise to the decomposition of WCW to sectors would be finite-dimensional if the integrable 2-surfaces defined by the decompositions correspond to orbits of subgroups of the isometry group of M^4 or CD. This would allow planes of M^4, and radial half-planes and spheres of M^4 in spherical Minkowski coordinates and of M^4_+ in Robertson-Walker coordinates. These decomposition could relate to the choices of measured quantum numbers inducing symmetry breaking to the subgroups in question. These choices would chose a sector of WCW [K5] and would define quantum counterpart for a choice of quantization axes as distinct from ordinary state function reduction with chosen quantization axes.

4. The induced Kähler form of S^2 fiber of $T(X^4)$ is assumed to reduce to the sum of the induced Kähler forms from S^2 fibers of $T(M^4)$ and $T(CP_2)$. This requires that the projections of the Kähler forms of M^4 and CP_2 to $S^2(X^4)$ are trivial. Also the induced metric is assumed to be direct sum and similar conditions holds true.These conditions are analogous to those occurring in dimensional reduction.

   Denote the radii of the spheres associated with M^4 and CP_2 as $R_P = kl_P$ and R and the ratio R_P/R by ϵ. Both the Kähler form and metric are proportional to R_p^2 resp. R^2 and satisfy the defining condition $J_{kr}g^{rs}J_{sl} = -g_{kl}$. This condition is assumed to be true also for the induced Kähler form of $J(S^2(X^4)$.

   Let us introduce the following shorthand notations

$$\begin{array}{llll} S_1^2 = S^2(X^4) \ , & S_2^2 = S^2(CP_2) \ , & S_3^2 = S^2(M^4) \ , \\ J_i = \frac{J(S_i^2)}{R^2} \ , & g_i = \frac{g(S_i^2))}{R^2} \ . & \end{array} \tag{2.1}$$

   This gives the following equations.

$$J_1 = J_2 + \epsilon J_3 \ , \quad g_1 = g_2 + \epsilon g_3 \ , \quad J_1 g_1 J_1 = -g_1 \ . \tag{2.2}$$

   Projections to $S_1^2 = S^2(X^4)$ are assumed at r.h.s.. The product of the third equation is defined as tensor contraction and involves contravariant form of g.

2.3 Solutions to the conditions defining the twistor lift

Consider now solutions to the conditions defining the twistor lift.

1. The simplest solution type corresponds to the situation in which either S_2^2 (S_3^2) equals to S_1^2) and S_3^2 (S_2^2) projection of $T(X^4)$ is single point. In this case the conditions of Eq. are trivially satisfied. These two solutions could correspond to Euclidian and Minkowskian space-time regions. Also the solution for which twistor sphere degenerates to a point must be considered and form $J(M^4) = 0$ this would correspond to the reduction of dimensionally reduced action to Kähler action defining the original variant of TGD. Note that preferred extremals are conjectured to be minimal surfaces extremals of Kähler action always [L1].

2. One can consider also more general solutions. Depending on situation, one can use for $S^2(X^4)$ either the coordinates of S_2^2 or S_3^2. Let us choose S_2^2. One can of course change the roles of the spheres.

   Consider an ansatz for which the projections of J_2 and J_3 to S_1^2 are in constant proportionality to each other. This is guaranteed if the spherical coordinates $(u = cos(\Theta), \Phi)$ of S_2^2 and S_3^2 are related by $(u(M^4), \Phi(M^4)) = (u(CP_2), n\Phi(CP_2))$ so that the map between the two spheres has winding number n. With this assumption one has

$$\begin{aligned} J_1 &= (1+\epsilon n)J_2 \ , \\ g_1 &= (1+\epsilon n^2)g_2 \ , \end{aligned} \tag{2.3}$$

The third condition of Eq. 1 equation gives

$$(1+n\epsilon)^2 \quad = \quad (1+n^2\epsilon)^2 \ . \tag{2.4}$$

This in turn gives

$$1+n\epsilon = \delta(1+n^2\epsilon) \ , \quad \delta = \pm 1 \ . \tag{2.5}$$

The only solution for $\delta = +1$ is $n = 0$ or $n = 1$. For $\delta = -1$ there are no solutions.

One has 3+1 different solutions corresponding to the degenerate solution $(n_1, n_2) = (0, 0)$ and 3 solutions with (n_1, n_2) equal $(1, 0)$, $(0, 1)$ or $(1, 1)$. The conditions are very stringent and it is not clear whether there are any other solutions.

3. The further conditions implying locally direct sum for g and J pose strong restrictions on space-time surfaces. The conjecture that the solutions of these conditions correspond to preferred extremals of 6-D Kähler action leads by dimensional reduction to the conclusion that the 4-D action contains besides 4-D Kähler action also a volume term coming from S^2 Kähler actions and giving rise to cosmological constant.

   What is of special interest is that for the degenerate solution the volume term vanishes, and one has mere 4-D Kähler action with induced Kähler form possibly containing also $J(M^4)$, which leads to a rather sensible cosmology having interpretation as infinite volume limit for causal diamond (CD) inside which space-time surfaces exist. This limit could be appropriate for QFT limit of TGD, which indeed corresponds to infinite-volume limit at which cosmological constant approaches zero.

What could be the physical interpretation of the solutions?

1. Physical intuition suggests that S^2_1 must be space-like for Euclidian signature of space-time region $[(n_1, n_2) = (1, 0)]$ and time-like for Minkowskian signature $[(n_1, n_2) = (0, 1)]$.

2. By quantum classical correspondence one can argue that the non-vanishing of space-time projection of $J(M^4)$ resp. $J(CP_2)$ is necessary to fix local quantization axis of spin resp. weak isospin. If so, then $n_1 = 1/0$ resp. $n_2 = 1/0$ would tell that the projection of $J(CP_2)$ resp. $J(M^2)$ is non-vanishing/vanishes. If both contributions vanish $[(n_1, n_2) = (0, 0)]$ one has generalized Lagrangian 4-surface, which would be vacuum extremal. The products of 2-D Lagrangian manifolds for M^4 and CP_2 would be vacuum extremals. One can wonder whether there exist 4-surfaces representable as a graph of a map $M^4 \rightarrow CP_2$ such that the induced Kähler form vanishes. This picture allows only the imbeddings of trivial Robertson-Walker cosmology as vacuum extremal of Kähler action since both M^4 contribution to Kähler action and volume term would be non-vanishing $[(n_1, n_2) = (0, 1)]$.

2.4 Twistor lift and the reduction of field equations and SH to holomorphy

It has become clear that twistorialization has very nice physical consequences. But what is the deep mathematical reason for twistorialization? Understanding this might allow to gain new insights about construction of scattering amplitudes with space-time surface serving as analogs of twistor diatrams.

Penrose's original motivation for twistorilization was to reduce field equations for massless fields to holomorphy conditions for their lifts to the twistor bundle. Very roughly, one can say that the value of massless field in space-time is determined by the values of the twistor lift of the field over the twistor sphere and helicity of the massless modes reduces to cohomology and the values of conformal weights of the field mode so that the description applies to all spins.

I want to find the general solution of field equations associated with the Kähler action lifted to 6-D Kähler action. Also one would like to understand strong form of holography (SH). In TGD fields in space-time are are replaced with the imbedding of space-time as 4-surface to H. Twistor lift imbeds the twistor space of the space-time surface as 6-surface into the product of twistor spaces of M^4 and CP_2. Following Penrose, these imbeddings should be holomorphic in some sense.

Twistor lift $T(H)$ means that M^4 and CP_2 are replaced with their 6-D twistor spaces.

1. If S^2 for M^4 has 2 time-like dimensions one has 3+3 dimensions, and one can speak about hyper-complex variants of holomorphic functions with time-like and space-like coordinate paired for all three hypercomplex coordinates. For the Minkowskian regions of the space-time surface X^4 the situation is the same.

2. For $T(CP_2)$ Euclidian signature of twistor sphere guarantees this and one has 3 complex coordinates corresponding to those of S^2 and CP_2. One can also now also pair two real coordinates of S^2 with two coordinates of CP_2 to get two complex coordinates. For the Euclidian regions of the space-time surface the situation is the same.

Consider now what the general solution could look like. Let us continue to use the shorthand notations $S_1^2 = S^2(X^4); S_2^2 = S^2(CP_2); S_3^2 = S^2(M^4)$.

1. Consider first solution of type $(1,0)$ so that coordinates of S_2^2 are constant. One has holomorphy in hypercomplex sense (light-like coordinate $t-z$ and $t+z$ correspond to hypercomplex coordinates).

   (a) The general map $T(X^4)$ to $T(M^4)$ should be holomorphic in hyper-complex sense. $S_1^2$is in turn identified with S_3^2 by isometry realized in real coordinates. This could be also seen as holomorphy but with different imaginary unit. One has analytical continuation of the map $S_1^2 \rightarrow S_3^2$ to a holomorphic map. Holomorphy might allows to achieve this rather uniquely. The continued coordinates of S_1^2 correspond to the coordinates assignable with the integrable surface defined by $E^2(x)$ for local $M^2(x) \times E^2(x)$ decomposition of the local tangent space of X^4. Similar condition holds true for $T(M^4)$. This leaves only $M^2(x)$ as dynamical degrees of freedom. Therefore one has only one holomorphic function defined by 1-D data at the surface determined by the integrable distribution of $M^2(x)$ remains. The 1-D data could correspond to the boundary of the string world sheet.

   (b) The general map $T(X^4)$ to $T(CP_2)$ cannot satisfy holomorphy in hyper-complex sense. One can however provide the integrable distribution of $E^2(x)$ with complex structure and map it holomorphically to CP_2. The map is defined by 1-D data.

   (c) Altogether, 2-D data determine the map determining space-time surface. These two 1-D data correspond to 2-D data given at string world sheet: one would have SH.

2. What about solutions of type $(0,1)$ making sense in Euclidian region of space-time. One has ordinary holomorphy in CP_2 sector.

   (a) The simplest picture is a direct translation of that for Minkowskian regions. The map $S_1^2 \rightarrow S_2^2$ is an isometry regarded as an identification of real coordinates but could be also regarded as holomorphy with different imaginary unit. The real coordinates can be analytically continued to complex coordinates on both sides, and their imaginary parts define coordinates for a distribution of transversal Euclidian spaces $E_2^2(x)$ on X^4 side and $E^2(x)$ on M^4 side. This leaves 1-D data.

(b) What about the map to $T(M^4)$? It is possible to map the integrable distribution $E_2^2(x)$ to the corresponding distribution for $T(M^4)$ holomorphically in the ordinary sense of the word. One has 1-D data. Altogether one has 2-D data and SH and partonic 2-surfaces could carry these data. One has SH again.

3. The above construction works also for the solutions of type $(1,1)$, which might make sense in Euclidian regions of space-time. It is however essential that the spheres S_2^2 and S_3^2 have real coordinates.

SH thus would thus emerge automatically from the twistor lift and holomorphy in the proposed sense.

1. Two possible complex units appear in the process. This suggests a connection with quaternion analytic functions [K14] suggested as an alternative manner to solve the field equations. Space-time surface as associative (quaterionic) or co-associate (co-quaternionic) surface is a further solution ansatz.

   Also the integrable decompositions $M^2(x) \times E^2(x)$ resp. $E_1^2(x) \times E_2^2(x)$ for Minkowskian resp. Euclidian space-time regions are highly suggestive and would correspond to a foliation by string wold sheets and partonic 2-surfaces. This expectation conforms with the number theoretically motivated conjectures [K12].

2. The foliation gives good hopes that the action indeed reduces to an effective action consisting of an area term plus topological magnetic flux term for a suitably chosen stringy 2-surfaces and partonic 2-surfaces. One should understand whether one must choose the string world sheets to be Lagrangian surfaces for the Kähler form including also M^4 term. Minimal surface condition could select the Lagrangian string world sheet, which should also carry vanishing classical W fields in order that spinors modes can be eigenstates of em charge.

   The points representing intersections of string world sheets with partonic 2-surfaces defining punctures would represent positions of fermions at partonic 2-surfaces at the boundaries of CD and these positions should be able to vary. Should one allow also non-Lagrangian string world sheets or does the space-time surface depend on the choice of the punctures carrying fermion number (quantum classical correspondence)?

3. The alternative option is that any choice produces of the preferred 2-surfaces produces the same scattering amplitudes. Does this mean that the string world sheet area is a constant for the foliation - perhaps too strong a condition - or could the topological flux term compensate for the change of the area?

   The selection of string world sheets and partonic 2-surfaces could indeed be also only a gauge choice. I have considered this option earlier and proposed that it reduces to a symmetry identifiable as U(1) gauge symmetry for Kähler function of WCW allowing addition to it of a real part of complex function of WCW complex coordinates to Kähler action. The additional term in the Kähler action would compensate for the change if string world sheet action in SH. For complex Kähler action it could mean the addition of the entire complex function.

3 How does the twistorialization at imbedding space level emerge?

An objection against twistorialization at imbedding space level is that M^4-twistorialization requires 4-D conformal invariance and massless fields. In TGD one has towers of particle with massless particles as the lightest states. The intuitive expectation is that the resolution of the problem is that particles are massless in 8-D sense as also the modes of the imbedding space spinor fields are.

To explain the idea, let us select a fixed decomposition $M^8 = M_0^4 \times E_0^4$ and assume that the momenta are complex - for motivations see below.

1. With inspiration coming from $M^8 - H$ duality [K7] suppose that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed, and corresponding to real octonionic unit and preferred imaginary unit. Obviously 8-D light-likeness for $M^8 = M_0^4 \times E_0^4$ reduces to 4-D light-likeness for a preferred choice of $M^8 = M^4 \times CP_2$ decomposition.

2. This suggests that in the case of massive M^4_0 momenta one can apply twistorialization to the light-like M^4-momentum and code the information about preferred M^4 by a point of CP_2 and about 8-momentum in $M^8 = M^4_0 \times E^4_0$ by an $SU(3)$ transformation taking M^4_0 to M^4. Pairs of twistors and $SU(3)$ transformations would characterize arbitrary quaternionic 8-momenta. 8-D masslessness gives however 2 additional conditions for the complex 8-momenta probably reducing $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2! This would also solve the basic problem of twistor approach created by the existence of massive particles.

The assumption of complex momenta in previous considerations might raise some worries. The space-time action of TGD is however complex if Kähler coupling strength is complex, and there are reasons to believe that this is the case. Both four-momenta and color quantum numbers - all Noether charges in fact - could be complex. A possible physical interpretation for complex momenta could be in terms of the natural width of states induced by the finite size of CD. Also in twistor Grassmannian approach one encounters complex but light-like four-momenta. Note that complex light-like space-time momenta correspond in general to massive real momenta. It is not clear whether it makes sense to speak about width of color quantum numbers: their reality would give additional constraint. The emergence of M^4 mass in this manner could be involved with the classical description for the emergence of the third helicity.

The observation that octonionic twistors make sense and their restriction to quaternionic twistors produce ordinary M^4 twistors provides an alternative view point to the problem. Also $M^8 - H$ duality proposed to map quaternionic 4-D surfaces in octonionic M^8 to (possibly quaternionic) 4-D surfaces in $M^4 \times CP_2$ is expected to be relevant. The twistor lift of $M^8 - H$ duality would give $T(M^8) - T(H)$ duality.

Twistor Grassmann approach [B4, B3, B2, B5, B6, B1] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ whereas the twistor lift of classical TGD uses $M^4 \times S^2$. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identified as surfaces in $T(M^4) \times T(CP_2)$ requires the mapping of these twistor spaces to each other - the incidence relations of Penrose indeed realize this map.

3.1 $M^8 - H$ duality at space-time level

Twistors emerge as a description of massless particles with spin [B7] but are not needed for spin zero particles. Therefore one can consider first mere momenta.

1. Consider first space-time surfaces of M^8 with Minkowskian signature of the induced metric so that the tangent space is M^4. $M^8 - H$ duality [K7] implies that CP_2 points parameterize quaternionic sub-spaces M^4 of octonions containing fixed $M^2_0 \subset M^4$. Using the decomposition $1+1+3+\overline{3}$ of complexified octonions to representations of $SU(3)$, it is easy to see that this space is indeed CP_2. M^4 correspond to the sub-space $1+1+2$ where 2 is $SU(2) \subset SU(3)$ doublet.

   CP_2 spinor mode would be spinor mode in the space of quaternionic sub-spaces $M^4 \subset M^8$ with $M^2_0 \subset M^4$ with real octonionic unit defining preferred time like direction and imaginary unit defining preferred spin quantization axis. $M^8 - H$ duality allows to map quaternionic 4-surfaces of $M^4 \supset M^2_0$ to 4-surfaces in H. The latter could be quaternionic but need not to.

2. For Euclidian signature of the induced metric tangent space is E^4. In this case co-associative surfaces are needed since the above correspondence make sense only if the tangent space corresponds to M^4. For instance, for CP_2 type exremals tangent space corresponds to E^4. M^4 and E^4 change roles. Also now the space of co-associative tangent spaces is CP_2 since co-associative tangent space is the octonionic orthogonal complement of the associative tangent space. One would have Euclidian variant of the associative case.

$M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the level, which deserves to be called fundamental. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4? In any case, one should demonstrate that the spectrum of states with $M^4 \times E^4$ with quaternionic light-like 8-momenta is equivalent with the spectrum of states for $M^4 \times CP_2$

3.2 Parametrization of light-like quaternionic 8-momenta in terms of $T(CP_2)$

The following argument shows that the twistor space $T(CP_2)$ emerges naturally from M^8-H correspondence for quaternionic light-like M^8 momenta.

1. Continue to assume a fixed decomposition $M^8 = M_0^4 \times E_0^4$, and that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed. Light-like quaternionic 8-momentum in $M^8 = M_0^4 \times E_0^4$ can be reduced to light-like M^4 momentum and vanishing E^4 momentum for some preferred $M^8 = M^4 \times E^4$ decomposition.

   One can therefore describe the situation in terms of light-like M^4-momentum and $U(2)$ transformation (as it turns out) mapping this momentum to 8-D momentum in given frame and giving the M_0^4 and E_0^4 momenta. The alternative description is in terms M_0^4 massive momentum and the E_0^4 momentum. The space of light-like complex M^4 momenta with fixed M_0^2 part and non-vanishing E^2 part is given by CP_2 as also the space of quaternionic planes. Given quaternionic plane is in turn characterized by massless M^4-momentum.

2. The description of M^4-massive momentum should be based on twistor associated with the light-like M^4 momentum plus something describing the $SU(3)$ transformation leaving the preferred imaginary unit of M_0^2 un-affected. The transformations leaving unaffected the M^4 part of M^8-momentum coded by the $SU(2)$ doublet 2 of color triplet 3 in the color decomposition of complex 8-momentum $1+1+3+\overline{3}$ but acting on E^4 part $1+\overline{3}$ non-trivially correspond to $U(2)$ subgroup. $U(2)$ element thus codes for the E^4 part of the light-like momentum and $SU(3)$ code for quaternionic 8-momenta, which can be also massive. Massless and complex M^4 momenta are coded by $SU(3)/U(2) = CP_2$ as also the tangent spaces of Minkowskian space-time regions (by M^8-H duality).

3. General complex quaternionic momenta with fixed M^4 part are parameterized by $SU(3)$. Complex light-like 8-momenta satisfy two additional constraints from light-likenes condition, and one expects the reduction of $SU(3)$ to $SU(3)/U(1)\times U(1)$ - the twistor space of CP_2. Therefore the light-like 8-momentum is coded by a twistor assignable to massless M^4-momentum by an point of $SU(3)/U(1)\times U(1)$ giving $T(M^4)\times T(CP_2)$.

By the previous arguments, the inclusion of helicities and electroweak charges gives twistor lift of M^8-H correspondence.

1. In the case of E^4 the helicities would correspond to two $SO(4)$ spins to be mapped to right and left-handed electroweak spins or weak spin and weak charges. Twistor space $T(CP_2)$ gives hopes about a unified description of color - and electro-weak quantum numbers in terms of partial waves in the space $SU(3)/U(1)\times U(1)$ for selections of quantization axes for color quantum numbers.

2. A possible problem relates to the particles massive in M^4 sense having more helicity states than massless particles. How can one describe the presence of additional helicities. Should one introduce the analog of Higgs mechanism providing the missing massless helicities? Quantum view about twistors describes helicity as a quantum number - conformal weight - of a wave function in the twistor sphere S^2. In the case of massive gauge bosons which would require the introduction of zero helicity as a spin 0 wave function in twistor space.

3. One should relate the description in terms of M^8 momenta to the description in terms of $M^4\times CP_2$ color partial waves massless in 8-D sense. The number of partial waves for given CP_2 mass squared is finite and this should be the case for quaternionic E^4 momenta. How color quantum numbers determining the M^4 mass relate to complex E^4 momenta parameterized by $U(2)$ plus two constraints coming from complex light-likeness. The number of degrees of freedom is 2 for given $U(2)$ orbit and the quantization suggests dramatic reduction in the number of 8-momenta. This strongly suggests that it is only possible to talk about wave functions in the space of allowed E^4 momenta - that is in the twistor space $T(CP_2)$. Fixing the M^4-part of 8-momentum parameterized by a point of CP_2 leaves only a wave function in the fiber S^2.

The discussion leaves some questions to ponder.

1. $M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the fundamental level. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4?

2. What about more general $SO(1,7)$ transformations? Are they needed? One could consider the possibility that $SO(1,7)$ acts in the moduli space of octonion structures of M^8. If so, then these additional moduli must be included. Otherwise given 8-D momenta have M_0^2 part fixed and orbit of given M^4 momentum is the smaller, the smaller the E^2 part of M^4 momentum is. It reduces to point if M^4 momentum reduces to M_0^2.

3.3 A new view about color, color confinement, and twistors

To my humble opinion twistor approach to the scattering amplitudes is plagued by some mathematical problems. Whether this is only my personal problem is not clear.

1. As Witten shows in [B4], the twistor transform is problematic in signature (1,3) for Minkowski space since the the bi-spinor μ playing the role of momentum is complex. Instead of defining the twistor transform as ordinary Fourier integral, one must define it as a residue integral. In signature (2,2) for space-time the problem disappears since the spinors μ can be taken to be real.

2. The twistor Grassmannian approach works also nicely for (2,2) signature, and one ends up with the notion of positive Grassmannians, which are real Grassmannian manifolds. Could it be that something is wrong with the ordinary view about twistorialization rather than only my understanding of it?

3. For M^4 the twistor space should be non-compact $SU(2,2)/SU(2,1) \times U(1)$ rather than $CP_3 = SU(4)/SU(3) \times U(1)$, which is taken to be. I do not know whether this is only about short-hand notation or a signal about a deeper problem.

4. Twistorilizations does not force SUSY but strongly suggests it. The super-space formalism allows to treat all helicities at the same time and this is very elegant. This however forces Majorana spinors in M^4 and breaks fermion number conservation in $D = 4$. LHC does not support $\mathcal{N} = 1$ SUSY. Could the interpretation of SUSY be somehow wrong? TGD seems to allow broken SUSY but with separate conservation of baryon and lepton numbers.

In number theoretic vision something rather unexpected emerges and I will propose that this unexpected might allow to solve the above problems and even more, to understand color and even color confinement number theoretically. First of all, a new view about color degrees of freedom emerges at the level of M^8.

1. One can always find a decomposition $M^8 = M_0^2 \times E^6$ so that the possibly complex light-like quaternionic 8-momentum restricts to M_0^2. The preferred octonionic imaginary unit represent the direction of imaginary part of quaternionic 8-momentum. The action of G_2 to this momentum is trivial. Number theoretic color disappears with this choice. For instance, this could take place for hadron but not for partons which have transversal momenta.

2. One can consider also the situation in which one has localized the 8-momenta only to $M^4 = M_0^2 \times E^2$. The distribution for the choices of $E^2 \subset M_0^2 \times E^2 = M^4$ is a wave function in CP_2. Octonionic $SU(3)$ partial waves in the space CP_2 for the choices for $M_0^2 \times E^2$ would correspond ot color partial waves in H. The same interpretation is also behind $M^8 - H$ correspondence.

3. The transversal quaternionic light-like momenta in $E^2 \subset M_0^2 \times E^2$ give rise to a wave function in transversal momenta. Intriguingly, the partons in the quark model of hadrons have only precisely defined longitudinal momenta and only the size scale of transversal momenta can be specified. This would of course be a profound and completely unexpected connection! The

introduction of twistor sphere of $T(CP_2)$ allows to describe electroweak charges and brings in CP_2 helicity identifiable as em charge giving to the mass squared a contribution proportional to Q_{em}^2 so that one could understand electromagnetic mass splitting geometrically.

The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4) + J(CP_2)$. Em charge is the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere is natural.

4. In general case the M^2 component of momentum would be massive and mass would be equal to the mass assignable to the E^6 degrees of freedom. One can however always find $M_0^2 \times E^6$ decomposition in which M^2 momentum is light-like. The naive expectation is that the twistorialization in terms of M^2 works only if M^2 momentum is light-like, possibly in complex sense. This however allows only forward scattering: this is true for complex M^2 momenta and even in M^4 case.

   The twistorial 4-fermion scattering amplitude is however *holomorphic* in the helicity spinors λ_i and has no dependence on $\tilde{\lambda}_i$. Therefore carries no information about M^2 mass! Could M^2 momenta be allowed to be massive? If so, twistorialization might make sense for massive fermions!

M_0^2 momentum deserves a separate discussion.

1. A sharp localization of 8-momentum to M_0^2 means vanishing transversal E^2 momentum so that the action of $U(2)$ would becomes trivial. Neither M_0^2 localization nor localization to single M^4 (localization in CP_2) looks implausible physically - consider only the size scale of CP_2).

   For the preferred extremals of twistor lift of TGD either M^4 or CP_2 twistor sphere can effectively collapse to a point. This would mean disappearence of the degrees of freedom associated with M^4 helicity or electroweak quantum numbers.

2. The localization to $M^4 \supset M_0^2$ is possible for the tangent space of quaternionic space-time surface in M^8. This could correlate with the fact that neither leptonic nor quark-like induced spinors carry color as a spin like quantum number. Color would emerge only at the level of H and M^8 as color partial waves in WCW and would require de-localization in the CP_2 cm coordinate for partonic 2-surface. Note that also the integrable local decompositions $M^4 = M^2(x) \times E^2(x)$ suggested by the general solution ansätze for field equations are possible.

3. Could it be possible to perform a measurement localization the state precisely in fixed M_0^2 always so that the complex momentum is light-like but color degrees of freedom disappear? This does not mean that the state corresponds to color singlet wave function! Can one say that the measurement eliminating color degrees of freedom corresponds to color confinement. Note that the subsystems of the system need not be color singlets since their momenta need not be complex massless momenta in M_0^2. Classically this makes sense in many-sheeted space-time. Colored states would be always partons in color singlet state.

4. At the level of H also leptons carry color partial waves neutralized by Kac-Moody generators, and I have proposed that the pion like bound states of color octet excitations of leptons explain so called lepto-hadrons [K8]. Only right-handed covariantly constant neutrino is an exception as the only color singlet fermionic state carrying vanishing 4-momentum and living in all possible M_0^2:s, and might have a special role as a generator of supersymmetry acting on states in all quaternionic subs-spaces M^4.

5. Actually, already p-adic mass calculations performed for more than two decades ago [K4, K3, K6], forced to seriously consider the possibility that particle momenta correspond to their projections o $M_0^2 \subset M^4$. This choice does not break Poincare invariance if one introduces moduli space for the choices of $M_0^2 \subset M^4$ and the selection of M_0^2 could define quantization axis of energy and spin. If the tips of CD are fixed, they define a preferred time direction assignable to preferred octonionic real unit and the moduli space is just S^2. The analog of

twistor space at space-time level could be understood as $T(M^4) = M^4 \times S^2$ and this one must assume since otherwise the induction of metric does not make sense.

What happens to the twistorialization at the level of M^8 if one accepts that only M_0^2 momentum is sharply defined?

1. What happens to the conformal group $SO(4,2)$ and its covering $SU(2,2)$ when M^4 is replaced with $M_0^2 \subset M^8$? Translations and special conformational transformation span both 2 dimensions, boosts and scalings define 1-D groups $SO(1,1)$ and R respectively. Clearly, the group is 6-D group $SO(2,2)$ as one might have guessed. Is this the conformal group acting at the level of M^8 so that conformal symmetry would be broken? One can of course ask whether the 2-D conformal symmetry extends to conformal symmetries characterized by hyper-complex Virasoro algebra.

2. Sigma matrices are by 2-dimensionality real (σ_0 and σ_3 - essentially representations of real and imaginary octonionic units) so that spinors can be chosen to be real. Reality is also crucial in signature $(2,2)$, where standard twistor approach works nicely and leads to 3-D real twistor space.

   Now the twistor space is replaced with the real variant of $SU(2,2)/SU(2,1) \times U(1)$ equal to $SO(2,2)/SO(2,1)$, which is 3-D projective space RP^3 - the real variant of twistor space CP_3, which leads to the notion of positive Grassmannian: whether the complex Grassmannian really allows the analog of positivity is not clear to me. For complex momenta predicted by TGD one can consider the complexification of this space to CP_3 rather than $SU(2,2)/SU(2,1) \times U(1)$. For some reason the possible problems associated with the signature of $SU(2,2)/SU(2,1) \times U(1)$ are not discussed in literature and people talk always about CP_3. Is there a real problem or is this indeed something totally trivial?

3. SUSY is strongly suggested by the twistorial approach. The problem is that this requires Majorana spinors leading to a loss of fermion number conservation. If one has $D = 2$ only effectively, the situation changes. Since spinors in M^2 can be chosen to be real, one can have SUSY in this sense without loss of fermion number conservation! As proposed earlier, covariantly constant right-handed neutrino modes could generate the SUSY but it could be also possible to have SUSY generated by all fermionic helicity states. This SUSY would be however broken.

4. The selection of M_0^2 could correspond at space-time level to a localization of spinor modes to string world sheets. Could the condition that the modes of induced spinors at string world sheets are expressible using real spinor basis imply the localization? Whether this localization takes place at fundamental level or only for effective action being due to SH, is a question to be settled. The latter options looks more plausible.

To sum up, these observation suggest a profound re-evalution of the beliefs related to color degrees of freedom, to color confinement, and to what twistors really are.

3.4 How do the two twistor spaces assignable to M^4 relate to each other?

Twistor Grassmann approach [B4, B3, B2, B5, B6, B1] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$. Twistor lift of classical TGD uses $M^4 \times S^2$: this seems to be necessary since $T_1(M^4)$ does not allow M^4 as space-space. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identifed as surfaces in $T(M^4) \times T(CP_2)$ is an attractive idea suggesting a very close correspondence with twistor string theory of Witten and construction of scattering amplitudes in twistor Grassmann approach.

One should be able to relate these two twistor spaces and map the twistor spaces $T(X^4)$ identified as surfaces in $T(H) = T(M^4) \times T(CP_2)$ to those in $T_1(H) = T_1(M^4) \times T(CP_2)$. This map is strongly suggested also by twistor string theory. This map raises hopes about the analogs of twistor Grassmann amplitudes based on introduction of $T(CP_2)$.

At least the projections of 2-surfaces to $T(M^4)$ should be mappable to those in $T_1(M^4)$. A stronger condition is that $T(M^4)$ is mappable to $T_1(M^4)$. Incidence relations for twistors $Z = (\lambda, \mu)$ assigning to given M^4 coordinates twistor sphere, are given by

$$\mu_{\dot{\alpha}} = m_{\alpha\dot{\alpha}}\lambda^{\alpha} \quad .$$

This condition determines a 2-D sub-space - complex light ray - of complexified Minkowski space M^4_c. Also complex scaling of Z determines the same sub-space. Therefore twistor sphere corresponds to a complex light ray M^4_c, whose points differ by a shift by a complex light-like vector (λ is null bi-spinor annihilated by light-like m).

Since twistor line (projective sphere) determines a point of M^4_c, two points of twistor sphere labelled by A and B are needed to determined m:

$$m_{\alpha\dot{\alpha}} = \frac{\lambda_{A,\alpha}\mu_{B,\dot{\alpha}}}{\langle\lambda_A\lambda_B\rangle\rangle} + \frac{\lambda_{B,\alpha}\mu_{A,\dot{\alpha}}}{\langle\lambda_B\lambda_A\rangle} \quad .$$

The solutions are invariant under complex scalings $(\lambda, \mu) \to k(\lambda, \mu)$. Therefore co-incidence relations allow to assign projective line - sphere S^2 - to a point of M^4 in $T(M^4)$. This sphere naturally corresponds to S^2 in $T(M^4) = M^4 \times S^2$. This allows to assign pairs $(m \times S^2)$ in $T(M^4)$ to spheres of $T_1(M^4)$ and one can map the projections of 2-surfaces to $T(M^4)$ to $T_1(M^4)$.

Thus one cannot assign M^4 point to single twistor but can map any pair of points at twistor sphere of $T_1(M^4)$ to the same point of M^4 in $T(M^4) = M^4 \times S^2$ and also identify the twistor sphere with S^2. Twistor spheres are labelled by the base space of $T_1(M^4)$ and therefore base space can be mapped to M^4.

Two M^4 points separated by light-like distance correspond to twistor spheres intersecting at one point as is clear from the fact that the difference $m_1 - m_2$ of the points annihilates the twistor λ. $T_1(M^4)$ is singular as fiber bundle over M^4 since the same point of fiber is projected to two different points of M^4.

Could one replace $T(M^4)$ with $T_1(M^4)$ by modifying the induction procedure suitable?

1. $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ has $SU(2,2)$ invariant metric and $SU(2,2)$ corresponds to the 15-D spin covering group of $SO(4,2)$ having $SO(3,1)$ as sub-group. What does one obtain if one induces the metric of the base space of $T_1(M^4)$ to M^4 via the above identification?

   The induced metric would depend on the choice of the base space, and one would have analog of gauge invariance since for a given point of the base the point of the fiber sphere can be chosen freely. A reasonable guess is that the induced metric is determined apart from conformal scaling. One could fix the gauge by - say - assuming that the S^2 point is constant but it is not clear whether this allows to get the flat M^4 metric with any choice.

2. If the twistor sphere of $T_1(M^4)$ has radius of order Planck length l_P, the overall scaling factor of the metric of $T_1(M^4)$ is of order l_P^2. Also the induced M^4 metric would have this scaling factor. For $T_1(M^4)$ one could not perform this scaling. This need not be a problem in $T(M^4)$ since one scale up the flat metric of M^4 by scaling the coordinates. This kind of scaling would in fact smooth out the possible deviations from flat M^4 metric very effectively. In any case, it seems that one must assume that imbedding space corresponds to $T(M^4)$.

4 Can the Kähler form of M^4 appear in Kähler action?

I have already earlier considered the question whether the analog of Kähler form assignable to M^4 could appear in Kähler action. Could one replace the induced Kähler form $J(CP_2)$ with the sum $J = J(M^4) + J(CP_2)$ such that the latter term would give rise to a new component of Kähler form both in space-time interior at the boundaries of string world sheets regarded as point-like particles? This could be done both in the Kähler action for the interior of X^4 and also in the topological magnetic flux term $\int J$ associated with string world sheet and reducing to a boundary term giving couplings to U(1) gauge potentials $A_\mu(CP_2)$ and $A_\mu(M^4)$ associated with $J(CP_2)$ and $J(M^4)$. The interpretation of this coupling is an interesting challenge.

What conditions one can pose on $J(M^4)$?

1. The simplest possibility is that $J(M^4)$ is covariantly constant and self-dual. $J(M^4)$ would define a global decomposition $M^4 = M^2 \times E^2$ in terms of parallel constant electric and magnetic fields of equal magnitude. CD with this variant of $J(M^4)$ would be naturally associated with planewave like radiative solutions.

2. One could however give up the covariant constancy. In this case spherically symmetric variants of $J(M^4)$ naturally associated with sperically symmetric stationary metric and possible analogs of Robertson-Walker metrics. $J(M^4)$ would be closed except at the world line connecting the tips of CD and carry identical magnetic and electric charges.

3. $J(M^4)$ would define Hamilton Jacobi-structure and an attractive idea is that the orthogonal 2-surfaces associated with the foliation of M^4 are orbits of a subgroup of Poincare group. This structure would characterize quantum measurement at the level of WCW and quantum measurement would involve selection of a sector of WCW characterized by $J(M^4)$ [K5].

Consider now the objections against introducing $J(M^4)$ to the Kähler action at imbedding space level.

1. $J(M^4)$ would would break translational and Lorentz symmetries at the level of imbedding space since $J(M^4)$ cannot be Lorentz invariant. For imbedding space spinor modes this term would bring in coupling to the self-dual Kähler form in M^4. The simplest choice is $A = (A_t = z, A_z = 0, A_x = y, A_y = 0)$ defining decomposition $M^4 = M^2 \times E^2$. For Dirac equation in M^4 one would have free motion in preferred time-like (t,z)-plane plane M^2 in whereas in x- and y-directions (E^2 plane) would one have harmonic oscillator potentials due to the gauge potentials of electric and magnetic fields. One would have something very similar to quark model of hadron: quark momenta would have conserved longitudinal part and non-conserved transversal part. The solution spectrum has scaling invariance $\Psi(m^k) \rightarrow \Psi(\lambda m^k)$ so that there is no preferred scale and the transversal scales scale as $1/E$ and $1/k_x$.

2. Since $J(M^4)$ is not Lorentz invariant, Lorentz boosts would produce new $M^2 \times E^2$ decomposition (or its local variant). If one assumes above kind of linear gauge as gauge invariance suggests, the choices with fixed second tip of causal diamond (CD) define finite-dimensional moduli space $SO(3,1)/SO(1,1) \times SO(2)$ having in number theoretic vision an interpretation as a choice of preferred hypercomplex plane and its orthogonal complement. This is the moduli space for hypercomplex structures in M^4 with the choices of origins parameterized by M^4. The introduction of the moduli space would allow to preserve Poincare invariance.

3. If one generalizes the condition for Kähler metric to $J^2(M^4) = -g(M^4)$ fixing the scaling of J, the coupling to $A(M^4)$ is also large and suggests problems with the large breaking of Poincare symmetry for the spinor modes of the imbedding space for given moduli. The transversal localization by the self-dual magnetic and electric fields for $J(M^4)$ would produce wave packets in transversal degrees of freedom: is this physical?

   This moduli space is actually the moduli space introduced for causal diamonds (CDs) in zero energy ontology (ZEO) forced by the finite value of volume action: fixing of the line connecting the tips of CD the Lorentz boost fixing the position for the second tip of CD parametrizes this moduli space apart from division with the group of transformations leaving the planes M^2 and E^2 having interpretation a plane defined by light-like momentum and polarization plane associated with a given CD invariant.

4. Why this kind of symmetry breaking for Poincare invariance? A possible explanation proposed already earlier is that quantum measurement involves a selection of quantization axis. This choice necessarily breaks the symmetries and $J(M^4)$ would be an imbedding space correlate for the selection of rest frame and quantization axis of spin. This conforms with the fact that CD is interpreted as the perceptive field of conscious entity at imbedding space level: the contents of consciousness would be determined by the superposition of space-time surfaces inside CD. The choice of $J(M^4)$ for CD would select preferred rest system (quantization axis for energy as a line connecting tips of CD) via electric part of $J(M^4)$ and quantization axis of spin (via magnetic part of $J(M^4)$. The moduli space for CDs would be the space for choices of these particular quantization axis and in each state function reduction would mean

a localization in this moduli space. Clearly, this reduction would be higher level reduction and correspond to a decision of experimenter.

To summarize, for $J(M^4) = 0$ Poincare symmetries are realized at the level of imbedding space but obviously broken slightly by the geometry of CD. The allowance of $J(M^4) \neq 0$ implies that both translational and rotational symmetries are reduced for a given CD: the interpretation would be in terms of a choice of quantization axis in state function reduction. They are however lifted to the level of moduli space of CDs and exact in this more abstract sense. This is nothing new: already the introduction of ZEO and CDs force by volume term in action forced by twistor lift of TGD implies the same. Also the view about state function reduction requires wave functions in the moduli space of CDs. This is also essential for understanding how the arrow of geometric time is inherited from that of subjective time in TGD inspired theory of consciousness [K2, K15].

What about the situation at space-time level?

1. The introduction of $J(M^4)$ part to Kähler action has nice number theoretic aspects. In particular, J selects the preferred complex and quaternionic sub-space of octonionic space of imbedding space. The simplest possibility is that the Kähler action is defined by the Kähler form $J(M^4) + J(CP_2)$.

   Since M^4 and CP_2 Kähler geometries decouple it should be possible to take the counterpart of Kähler coupling strength in M^4 to be much larger than in CP_2 degrees of freedom so that M^4 Kähler action is a small perturbation and slowly varying as a functional of preferred extremal. This option is however not in accordance with the idea that entire Kähler form is induced.

2. Whether the proposed ansätze for general solutions make still sense is not clear. In particular, can one still assume that preferred extremals are minimal surfaces? Number theoretical vision strongly suggests - one could even say demands - the effective decoupling of Kähler action and volume term. This would imply the universality of quantum critical dynamics. The solutions would not depend at all on the coupling parameters except through the dependence on boundary conditions. The coupling between the dynamics of Kähler action and volume term would come also from the conservation conditions at light-like 3-surfaces at which the signature of the induced metric changes.

3. At space-time level the field equations get more complex if the M^4 projection has dimension $D(M^4) > 2$ and also for $D(M^4) = 2$ if it carries non-vanishing induced $J(M^4)$. One would obtain cosmic strings of form $X^2 \times Y^2$ as minimal surface extremals of ordinary Kähler action or X^2 Lagrangian manifold of M^4 as also CP_2 type vacuum extremals and their deformations with M^4 projection Lagrangian manifold. Thus the differences would not be seen for elementary particle and string like objects. Simplest string worlds sheet for which $J(M^4)$ vanishes would correspond to a piece of plane M^2.

   M^4 is the simplest minimal surface extremal of Kähler action necessarily involving also $J(M^4)$. The action in this case vanishes identically by self-duality (in Euclidian signature self-duality does not imply this). For perturbations of M^4 such as spherically symmetric stationary metric the contribution of M^4 Kähler term to the action is expected to be small and the come mainly from cross term mostly and be proportional to the deviation from flat metric. The interpretation in terms of gravitational contribution from M^4 degrees of freedom could make sense.

4. What about massless extremals (MEs)? How the induced metric affects the situation and what properties second fundamental form has? Is it possible to obtain a situation in which the energy momentum tensor T^{α} and second fundamental form $H^k_{\alpha\beta}$ have in common components which are proportional to light-like vector so that the contraction $T^{\alpha\beta}H^k_{\alpha\beta}$ vanishes?

   Minimal surface property would help to satisfy the conditions. By conformal invariance one would expect that the total Kähler action vanishes and that one has $J^{\alpha}_{\gamma}J^{\gamma\beta} \propto ag^{\alpha\beta} + bk^{\alpha}k^{\beta}$. These conditions together with light-likeness of Kähler current guarantee that field equations are satisfied.

In fact, one ends up to consider a generalization of MEs by starting from a generalization of holomorphy. Complex CP_2 coordinates ξ^i would be functions of light-like M^2 coordinate $u_+ = k \cdot m$, k light-like vector, and of complex coordinate w for E^2 orthogonal to M^2. Therefore the CP_2 projection would 3-D rather than 2-D now.

The second fundamental form has only components of form $H^k_{u_+w}$, $H^k_{u_+\overline{w}}$ and H^k_{ww}, $H^k_{\overline{w}\overline{w}}$. The CP_2 contribution to the induced metric has only components of form Δg_{u_+w}, $\Delta g_{+\overline{w}}$, and $g_{\overline{w}w}$. There is also contribution $g_{u_+u_-} = 1$, where v is the light-like dual of u in plane M^2. Contravariant metric can be expanded as a power series for in the deviation (Δg_{u_+w}, $\Delta g_{u_+\overline{w}}$) of the metric from ($g_{u_+u_-}, g_{w\overline{w}}$). Only components of form g^{u_+,u_i} and $g^{w,\overline{w}}$ are obtained and their contractions with the second fundamental form vanish identically since there are no common index pairs with simultaneously non-vanishing components. Hence it seems that MEs generalize!

I have asked earlier whether this construction might generalize for ordinary MEs. One can introduce what I have called Hamilton-Jacobi structure for M^4 consisting of locally orthogonal slicings by integrable 2-surfaces having tangent space having local decomposition $M^2_x \times E^2_x$ with light-like direction depending on point x. An objection is that the direction of light-like momentum depends on position: this need not be inconsistent with momentum conservation but would imply that the total four-momentum is not light-like anymore. Topological condensation for MEs and at MEs could imply this kind modification.

5. There is also a topological magnetic flux type term for string world sheet. Topological term can be transformed to a boundary term coupling classical particles at the boundary of string world sheet to CP_2 Kähler gauge potential (added to the equation for a light-like geodesic line). Now also the coupling to M^4 gauge potential would be obtained. The condition $J(M^4) + J(CP_2) = 0$ at string world sheets [K14] is very attractive manner to identify string world sheets as analogs of Lagrangian manifolds but does not imply the vanishing of the net $U(1)$ couplings at boundary since the induce gauge potentials are in general different.

   Also topological term including also M^4 Kähler magnetic flux for string world sheet contributes also to the modified Dirac equation since the gamma matrices are modified gamma matrices required by super-conformal symmetries and defined as contractions of canonical momentum densities with imbedding space gamma matrices [K10]. This is true both in space-time interior, at string world sheets and at their boundaries. CP_2 (M^4) term gives a contribution proportional to CP_2 (M^4) gamma matrices.

   At imbedding space level transversal localization would be the outcome and a good guess is that the same happens also now. This is indeed the case for M^4 defining the simplest extremal. The general interpretation of M^4 Kähler form could be as a quantum tool for transversal dynamical localization of wave packets in Kähler magnetic and electric fields of M^4. Analog for decoherence occurring in transversal degrees of freedom would be in question. Hadron physics could be one application.

How to test this idea?

1. It might be possible to kill the assumption that $J(M^4)$ is covariantly constant by showing that one does not obtain spherically symmetric Schwartschild type metric as a minimal surface extremal of generalized Kähler action: these extremals are possible for ordinary Kähler action [L1] [K13]. For the canonical imbedding of M^4 field equations are satisfied since energy momentum tensor vanishes identically. For the small deformations the presence of $J(M^4)$ would reduce rotational symmetry to cylindrical symmetry.

   The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.An attractive proposal is that the the pairs of orthogonal 2-surface correspond to Hamilton-Jacobi structures for which the two surfaces are orbits of subgroups of Poincare group.

2. $J(M^4)$ could make its presence manifest in the physics of right-handed neutrino having no direct couplings to electroweak gauge fields. Mixing with left handed neutrino is however

induced by mixing of M^4 and CP_2 gamma matrices. The transversal localization of right-handed neutrino in a background, which is a small deformation of M^4 could serve as an experimental signature.

3. CP breaking in hadronic systems is one of the poorly understood aspects of fundamental physics and relates closely to the mysterious matter-antimatter asymmetry. The constant electric part of self dual $J(M^4)$ implies CP breaking. I have earlier consider that Kähler electric fields could cause this breaking but now the electric field is not constant. Second possibility is that matter and antimatter correspond to different values of h_{eff} and are dark relative to each other. The question is whether $J(M^4)$ could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$ and whether this breaking could explain hadronic CP breaking and matter anti-matter asymmetry. Could M^4 part of Kähler electric field induce different $h_{eff}/h = n$ for particles and antiparticles.

5 About string like objects

String like objects and partonic 2-surfaces carry the information about quantum states and about space-time surfaces as preferred extremals if strong form of holography (SH) holds true. SH has of course some variants. The weakest variant states that fundamental information carrying objects are metrically 2-D. The light-like 3-surfaces separating space-time regions with Minkowskian and Euclidian signature of the induced metric are indeed metrically 2-D, and could thus carry information about quantum state.

An attractive possibility is that this information is basically topological. For instance, the value of Planck constant $h_{eff} = n \times h$ would tell the number sheets of the singular covering defining this surface such that the sheets co-incide at partonic 2-surfaces at the ends of space-time surface at boundaries of CD. In the following some questions related to string world sheets are considered. The information could be also number theoretical. Galois group for the algebraic extension of rationals defining particular adelic physics would transform to each other the number theoretic discretizations of light-like 3-surfaces and give rise to covering space structure. The action at partonic 2-surfaces should be trivial if one wants singular covering: this would mean that discretizations of partonic 2-surfaces consist of rational points. $h_{eff}/h = n$ could in this case be a factor of the order of Galois group.

The original observation was that string world sheets should carry vanishing W boson fields in order that the em charge for the modes of the induced spinor field is well-defined. This condition can be satisfied in certain situations also for the entire space-time surface. This raises several questions. What is the fundamental condition forcing the restriction of the spinor modes to string world sheets - or more generally, to surface of given dimension? Is this restriction dynamical. Can one have an analog of brane hierarchy in which also higher-D objects can carry modes of induced spinor field [K11]? Could the analogs of Lagrangian sub-manifolds of $X^4 \subset M^4 \times CP_2$ satisfying $J(M^4) + J(CP_2) = 0$ define string world sheets and their variants with varying dimension? The additional condition would be minimal surface property.

5.1 How does the gravitational coupling emerge?

The appearance of $G = l_P^2$ has coupling constant remained for a long time actually somewhat of a mystery in TGD. l_P defines the radius of the twistor sphere of M^4 replaced with its geometric twistor space $M^4 \times S^2$ in twistor lift. G makes itself visible via the coefficients $\rho_{vac} = 8\pi\Lambda/G$ volume term but not directly and if preferred extremals are minimal surface extremals of Kähler action ρ_{vac} makes itself visible only via boundary conditions. How G appears as coupling constant?

Somehow the M^4 Kähler form should appear in field equations. $1/G$ could naturally appear in the string tension for string world sheets as string models suggest. p-Adic mass calculations identify the analog of string tension as something of order of magnitude of $1/R^2$ [K4]. This identification comes from the fact that the ground states of super-conformal representations correspond to imbedding space spinor modes, which are solutions of Dirac equation in $M^4 \times CP_2$. This argument is rather convincing and allows to expect that the p-adic mass scale is not determined by string tension.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One expects long gravitational flux tubes mediating gravitational interactions. The hypothesis $\hbar_{eff} = n\hbar = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is a parameter with dimensions of velocity, suggests that the string tension assignable to the flux tubes mediating gravitational interaction between masses M and m is apart from a numerical factor equal to Λ_{gr}^{-2}, where gravitational Compton length is $\Lambda_{gr} = h_{gr}/m = GM/v_0$ so that the length of the flux tubes is of order Λ_{gr}.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One would like to have long gravitational flux tubes mediating gravitational interactions. Strong form of holography (SH) indeed suggests that stringy action appears as effective action expressing 4-D space-time action and modified Dirac action as 2-D actions assignable to string world sheets [L2] (see `http://tinyurl.com/zylrd7w`). This view would allow to understand the localization of spinor modes to string world sheets carrying vanishing W fields in terms as an effective description implying well-defineness of classical em charge and conservation of em charge at the level of scattering amplitudes. In fact that the introduction of the Kähler form $J(M^4)$ would allow to understand string world sheets as analogs of Lagrangian sub-manifolds.

5.2 Non-commutative imbedding space and strong form of holography

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

General Coordinate Invariance poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in imbedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 imbedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Käction also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing imbedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive. Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4) + J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions.

This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4)+J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K11].

Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would couple only to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [K1] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of electro-weak group to electromagnetic gauge group.

It is relatively easy to construct am infinite family of Lagrangian string world sheets satisfying $J(M^4)+J(CP_2) = 0$ using generalized symplectic transformations of M^4 and CP_2 as Hamiltonian flows to generate new ones from a given Lagrangian string world sheets. One must pose minimal surface property as a separate condition. Consider a piece of M^2 with coordinates (t, z) and homologically non-trivial geodesic sphere S^2 of CP_2 with coordinates $(u = cos(\Theta), \Phi)$. One has $J(M^4)_{tz} = 1$ and $J_{u\Phi} = 1$. Identify string world sheet via map $(u, \Phi) = (kz, \omega t)$ from M^2 to S^2. The induced CP_2 Kahler form is $J(CP_2)_{tz} = k\omega$. $k\omega = -1$ guarantees $J(M^4) + J(CP_2) = 0$. The strings have necessarily finite length from $L = 1/k \leq z \leq L$. One can perform symplectic transformations of CP_2 and symplectic transformations of M^4 to obtain new string world sheets. In general these are not minimal surfaces and this condition would select some preferred string world sheets.

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in imbedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of imbedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces plus number theoretic discretization of space-time surface would be enough to construct quantum states. Therefore SH would be thus slightly broken. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD.

To sum up, one cannot exclude the possibility that $J(M^4)$ is present implying a universal transversal localization of imbedding space spinor harmonics and the modes of spinor fields in the interior of X^4: this could perhaps relate to somewhat mysterious de-coherence interaction producing locality and to CP breaking and matter-antimatter asymmetry. The moduli space for M^4 Kähler structures proposed by number theoretic considerations would save from the loss of Poincare invariance and the number theoretic vision based on quaternionic and octonionic structure would have rather concrete realization. This moduli space would only extend the notion of WCW.

5.2.1 Two options for fundamental variational principle

One ends up to two options for the fundamental variational principle.

Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K11].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If the induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes!

If string world sheets are generalized Lagrangian sub-manifolds, only the induced em field would be non-vanishing and electroweak symmetry breaking would be a fundamental prediction. This however requires that M^4 has the analog of symplectic structure suggested also by twistorialization. This in turn provides a possible explanation of CP breaking and matter-antimatter asymmetry. In this case 4-D spinor modes do not define space-time super-symmetries.

The latter option conforms with number theoretically broken SH and would mean that the theory is amazingly simple. String world sheets together with number theoretical space-time discretization meaning small breaking of SH would provide the basic data determining classical and quantum dynamics. The Galois group of the extension of rationals defining the number-theoretic space-time discretization would act as a covering group of the covering defined by the discretization of the space-time surface, and the value of $h_{eff}/h = n$ would correspond to dimension of extension dividing the order of its Galois group. The phase transitions reducing n would correspond to spontaneous symmetry breaking leading from Galois group to a subgroup and the transition would replace n with its factor.

The ramified primes of the extension would be preferred primes of given extension. The extensions for which the number of p-adic space-time surfaces representable also as a real algebraic continuation of string world sheets to preferred extrenal is especially large would be physically favored as also corresponding ramified primes. In other words, maximal number of p-adic imaginations would be realizable so that these extensions and corresponding ramified primes would be winners in the number-theoretic fight for survival. Whether this conforms with p-adic length scale hypothesis, remains an open question.

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