A more detailed view about the construction of scattering amplitudes

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.

February 4, 2015

Abstract

This article summarizes the recent state of art in the construction of scattering amplitudes. The basic framework is provided by Zero Energy Ontology leading to a generalization of S-matrix so that one has U-matrix having as its rows M-matrices. Given M-matrix is product of hermitian square root of density matrix and unitary S-matrix. "World of Classical Worlds" serves as the arena of quantum physics and its huge symmetries give excellent hopes about well-defined scattering amplitudes. Twistor approach with generalized Yangian symmetry is highly suggestive in TGD framework since imbedding space is completely unique in twistorial sense. The topological view about elementary particles is in a key role as also the localization of the modes of the induced spinor fields to string world sheets from the condition that electromagnetic charge is well-defined. The Feynman graphics in fermionic degrees of freedom is extremely simple because the only vertices is analogous to mass insertion vertex and corresponds to the discontinuity of the modified Dirac operator at partonic vertices. One can also understand the emergence of the counterpart of Higgs vacuum expectation value from first principles.

1 Basic principles

In order to facilitate the challenge of the reader I summarize basic ideas behind the construction of scattering amplitudes.

1.1 Zero energy ontology

In Zero Energy Ontology (ZEO) quantum theory as hermitian square root of thermodynamics, which leads to a generalization of the notion of S-matrix to a unitary U-matrix between zero energy states having as rows M-matrices which are products of hermitian square roots of density matrices with a common unitary matrix S for given CD. Number theoretical considerations suggests that CD size comes as integer multiples of CP_2 size so that one obtain a hierarchy U-matrices having interpretation in terms of length scale evolution. For given CD also sub-CDs contribute down to some minimal scale defining UV scale. The largest CD defines the IR cutoff.

Scattering amplitudes would characterize the modes of WCW spinor field as time-like entanglement coefficients between positive and negative energy states associated with zero energy states.

The construction of scattering amplitudes - or M-matrix elements in ZEO - reduces at basic level to the construction of the Feynman diagram like entities for fundamental fermions, which serve basic building bricks of elementary particles.

1.2 Construction of scattering amplitudes as functional integrals in WCW

The decomposition of space-time surface to Minkowskian and Eucldian regions is the basic distinction from ordinary quantum field theories since it replaces path integral with mathematically well-defined functional integral over WCW.

1. Space-time surface decomposes to regions with Minkowskian or Euclidian signature of the induced metric. The regions with Euclidian metric are identified as lines of generalized Feynman diagrams. The boundaries between two kinds of regions - to be called parton orbits - can be regarded as carriers of elementary particle quantum numbers such as fermion number assignable to the boundaries of string world sheets at them. Induced spinor fields are localized at them from the well-definedness of electromagnetic charge requiring that induced W boson fields vanish. Hence strings emerge from TGD. Note that at boundary between Euclidian and Minkowskian regions the metric determinant vanishes.

2. Weak form of electric magnetic duality together with the assumption that the term $j^\alpha A_\alpha$ in Kähler action vanishes imply that Kähler action reduces to 3-D Chern-Simons term. This hypothesis is inspired by TGD as almost topological quantum field theory conjecture. In Minkowskian regions this conjecture is very natural. In the Euclidian region the contribution to Kähler action need not reduce to a mere Chern-Simons term associated with its boundary. This would be due to the non-triviality of the U(1) bundle defined by Kähler form giving also Chern-Simons terms inside the CP_2 type vacuum extremal.

3. Scattering amplitude is a functional integral over space-time surfaces: the data about these space-time surfaces are coded by their ends about the opposite light-like boundaries of causal diamond (CD) of given scale. The weight function in the functional integral is exponential of Kähler function of "world of classical worlds" coming from Euclidian regions of the space-time surface representing lines of generalized Feynman diagram and being deformation of CP_2 type vacuum extremals representing wormhole contacts connecting two space-time sheets with Minkowskian signature of induced metric. Kähler function is the exponent of Kähler action from Euclidian regions. The real exponent takes care that the functional integral is obtained instead of path integral so that the outcome is mathematically well-defined.

4. Euclidian region would give only the analog of thermodynamics but there is also an imaginary exponential coming from the exponential of the imaginary

Kähler action from Minkowskian regions. Space-time surfaces are extremals of Kähler action and for very general ansatz Minkowskian contribution to Kähler action reduces to imaginary Chern-Simons term at the light-like 3-D boundary between regions at which the 4-D metric is degenerate. This term makes possible interference of different contributions to the functional integral which is absolutely essential in quantum field theory.

1.3 Why it might work?

There are many reasons encouraging the hopes about calculable theory.

1. The theory has huge super-conformal symmetries dramatically reducing the dynamical degrees of freedom by the choice of conformal gauge. This implies that both the space-like 3-surfaces at the ends of space-time surface and partonic orbits satisfy classical Super conformal conditions for generalizations of ordinary super-conformal algebras perhaps extending to multilocal Yangian with locus identified as single partonic 2-surface at the light-like boundary of CD.

   Yangian symmetry in turn gives excellent hopes about twistorialization: in fact, $M^4 \times CP_2$ is completely unique choice for the imbedding space by twistorial considerations and the product of the twistor spaces of M^4 and CP_2 allows to constructed the twistor spaces of space-time surfaces as liftings of the extremals of Kähler action to 6-D sphere bundles over space-time surface.

2. The integrand in the functional integral represents the analog of ordinary Feynman diagrams involving only fermions and 1-D lines. Indeed, by bosonic emergence all bosons (in fact all elementary particles) can be regarded as composites of fundamental fermions. The only fermionic vertices are 2-fermion vertices since 3-vertices correspond to space-time surfaces meeting along common 3-surface and are thus purely topological. This is of course excellent news from the point of view of finiteness. The fermionic vertices are represented by the discontinuity of the modified Dirac operator associated with the string boundary line at partonic 2-surface so that there are no coupling constants involved. The only fundamental coupling parameter is Kähler strength whose value is dictated by quantum criticality as the analog of critical temperature.

One must have a view about what elementary particles - as opposed to fundamental fermions - are, how the ordinary view about scattering based on exchanges of elementary particles emerges from this picture and how say BFF vertex reduces to a diagram at for fundamental fermions involving only 2-fermion vertices.

2 Elementary particles in TGD framework

The notion of elementary particles involves two aspects: elementary particles as space-time surfaces and elementary particles as many-fermion states with fundamental fermions localized at the wormhole throats and defining elementary particles as their bound states (including physical fermions).

2.1 Elementary particles as space-time surfaces

Let us first summarize what kind of picture ZEO suggests about elementary particles.

1. Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [K1].

2. The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.

3. A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

   The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the modified Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is sparticle of left-handed neutrino.

   It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

3 Localization of the induced spinor fields at string world sheets and fermionic propagators

The localization of induced spinors to string world sheets emerges from the condition that electromagnetic charge is well-defined for the modes of induced spinor fields. There is however an exception: covariantly constant right handed neutrino spinor ν_R: it can be de-localized along entire space-time surface. Right-handed neutrino has no couplings to electroweak fields. It couples however to left handed neutrino by induced gamma matrices except when it is covariantly constant. Note that standard model does not predict ν_R but its existence is necessary if neutrinos develop Dirac mass. ν_R is indeed something which must be considered carefully in any generalization of standard model.

It has turned out that covariantly constant right-handed neutrino very probably corresponds to a pure gauge degree of freedom. Non-covariantly constant right-handed neutrino however mixes with left handed neutrino since the modified gamma matrices involve both M^4 and CP_2 gamma matrices and latter mix M^4 chiralities. These right-handed neutrinos localized to partonic 2-surfaces would generate broken SUSY. There are however good reasons to expect that the mass scale for SUSY breaking corresponds to that for the mixing of right and left handed neutrinos inducing neutrino massivation and that the p-adic mass scale of particle and sparticle are same. The only manner to avoid conflict with experimental facts is that sparticles are dark in TGD sense that is having Planck constant $h_{eff} = n \times h$. This would conform with the idea that the hierarchy of Planck constants corresponds to a hierarchy of breakings of conformal invariance, which is indeed behind the massivation.

The localization has powerful consequences since it gives in fermionic degrees of freedom what looks like ordinary Feynman diagrams but with only 2-fermion vertex. Space-time topology describes the vertices, say BFF vertex.

Fermion lines correspond to boundaries of string world sheets at the parton orbits. At these lines one must pose a boundary condition and the boundary condition is that the action of the modified Dirac operator associated with the boundary equals to the action of massless Dirac operator in momentum space representation. This reduces the fermionic propagator to massless Dirac propagator and simplify the construction decisively. If residue integral applied in twistor approach makes sense for the fermionic virtual momenta, this in turn gives inverse of Dirac propagator contracted between massless spinors with non-physical helicities.

What is the correct choice for the modified Dirac operator?

1. Chern-Simons-Dirac operator modified gamma matrix consists of CP_2 gamma matrices only and has square which does not vanish identically. Hence the covariant derivative of the induced spinor field along the string boundary must vanish if one wants that that fermion four-momentum is light-like and it must annihilate the spinors at its ends. The helicity of fermion would be physical and one would have incoming or outgoing on mass-shell fermion. This is certainly highly undesirable.

   Covariant constancy of the spinor mode has however the nice implication that it gives the familiar non-integrable phase factor for the dependence of the induced spinor at the fermion line behaving like Wilson line. Wilson lines are known to lead in string model picture to twistor diagrams so that it seems that we are in correct track.

This forces to ask whether fermion propagator are needed at all in the construction of fermionic Feynman diagrams. Dimensional arguments force them - at least if the scattering is just fermion scattering. The exchanged particles consist however of wormhole contacts: it is wormhole contact with propagates. Boson exchange corresponds to the exchange of wormhole contact with fermion and antifermion at throats. The four-momenta of fermion and antifermion making the virtual boson are tightly correlated reducing the pair of fermionic propagators to bosonic propagator and eliminating one virtual momentum integration. This conforms with the fact that in twistor approach the integration over bosonic virtual momenta over boson cancels the bosonic propagator.

2. What about Kähler-Dirac operator, which is indeed the most natural first guess. For Kähler-Dirac operator one could have light-like K-D gamma matrix at the string boundary and formally one can have modes, which are not covariantly constant. Unfortunately, the definition of the projection of Kähler-Dirac operator to the line is problematic since the determinant of the induced metric vanishes at partonic orbit and the component of canonical momentum density along the string boundary can diverge.

   One could of course consider the possibility of defining the condition as a limit. The infinitesimal value of the covariant derivative would compensate for the divergence of Kähler-Dirac gamma matrix Γ^t at the limit and one would obtain light-like momenta and non-physical fermion helicities of TGD based variant of twistor approach. For Kähler-Dirac term Γ^t can be infinite at the limit. If $g_{ti} = 0$ holds true then simple matrix algebra shows that g^{tt} becomes infinite whereas g^{ti} are limiting values of form 0/0. If $J_{ti} = 0$ holds true for Kähler form, manifest infinity transforms to 0/0 type limit and one has hope of finite limiting value. Weak form of electric magnetic duality could in factor guarantee $J_{ti} = 0$ by forcing J_{tn} and J_{xy} to be the only non-vanishing components of induced Kähler form.

   Remarkably, also now one would obtain the non-integrable phase factor characterizing Wilson line. The graph would depend on space-time surface only through this phase factor appearing at 2-fermion vertices and by the discontinuity of the modified Dirac operator at partonic 2-surface defining the most natural candidate for the 2-fermion vertex.

3. If the modified Dirac operator is defined by the induced metric at the string boundary and if its is light-like. This is trivial manner to solve the problem but now one loses the non-integrable phase factor and Wilson line picture.

4 Vertices

Vertices can be considered at both space-time level and fermionic level.

1. At space-time level vertices correspond to the fusion of space-surfaces representing particles along common 3-surface defining the vertex. At the parton level 3-light-like parton orbits fuse together along partonic 2-surface. In these vertices particle number changes this change correspond the change of particle number for elementary particles.

2. At fermion level vertices are localized at the partonic 2-surfaces and vertex corresponds to the discontinuity of the Kähler Dirac operator at the corner of the line representing the boundary of string world sheet. The creation of fermion pair from vacuum corresponds to an corner of string boundary at which the boundaries of string world sheets associated with two outgoing or incoming sheets meet. The creation/annihilation of a fermion pair is essential for the realization of say tree diagrams describing fermion scattering by virtual boson exchange.

The discontinuity of the modified Dirac operator is the key quantity. If one assumes continuity of the time derivative the discontinuity reduces to the discontinuity of $D = \Gamma^t D_t$ acting on say incoming spinor. ∂t is continuous as operator. The continuity of Γ^t could be posed as a condition and would state that the canonical momentum density at the corner of string is continuous. This might well be achieved with the proposed conditions for $J_{t\alpha} = 0$ and $g_{t\alpha} = 0$. The outcome is non-trivial since $D(out/in)$ need not annihilate $\Psi(in/out)$ so that with the assumptions just listed one obtains $\Gamma^t \Delta A_t$, where ΔA_t is the discontinuity of the induced spinor connection and is gauge invariant quantity. Also the non-integrable phase factor associated with the line entering to the vertex appears in the vertex and comes from covariant constancy of the induced spinor field along the line.

As already noticed, the definition of Γ_t might be problematic. For Chern-Simons term Γ_t is finite but in this case one looses the justification for M^4 propagator of fermion as coming from the boundary condition. For Kähler-Dirac action there are hopes of obtaining a finite limiting value for Γ^t and therefore of $\Gamma^t D_t$ if the induced Kähler form satisfies condition $J_{ti} = 0$ in the case that one has $g_{t\alpha} = 0$. Furthermore, the continuity of Γ^t could be posed as condition for vertices. Note that one obtains also the non-integrable phase factor allowing Wilson line interpretation.

Nothing has been said about the discontinuity of the modified Dirac operator through the wormhole contact as one traverses from Euclidian to Minkowskian side. This discontinuity might be also relevant.One could consider also the difference of the above discussed discontinuity between Euclidian and Minkowskian regions.

5 What one should obtain at QFT limit?

After functional integration over WCW of one should obtain a scattering amplitude in which the fermionic 2- vertices defined as discontinuities of the modified Dirac operator at partonic 2-surfaces should boil down to a contraction of an M^8 vector with gamma matrices of M^8. This vector has dimension of mass. This basic parameter should characterize many different physical situations. Consider only the description of massivation of elementary particles regarded as bound states of fundamental massless fermions and the mixing of left and right-handed fermions. Also

CKM mixing should involve this parameter. These vectors should also appear in Higgs couplings, which in QFT description contain Higgs vacuum expectation as a factor.

In twistor approach virtual particles have complex light-like momenta. Fundamental fermions have most naturally real and light-like momenta. $\mathcal{N} = 4$ SUSY describes gauge bosons which correspond to bound states of fundamental fermions in TGD. This suggests that the four-momenta of bound states of massless fermions - be they hadrons, leptons, or gauge bosons - can be taken to be complex.

There is an intriguing connection with TGD based notion of space-time. In TGD one obtains at space-time level complexified four-momenta since the four-momentum from Minkowskian/Euclidian region is real/imaginary. In the case of physical particle necessary involving two wormhole contacts and two flux tubes connecting them the total complexifies four momentum would be sum of two real and two imaginary contributions. Every elementary particle should have also imaginary part in its four-momentum and would be massless in complexified sense allowing mass in real sense given by the length of the imaginary four-momentum.

TGD predicts Higgs boson although Higgs expectation does not have any role in quantum TGD proper. Higgs vacuum expectation is however a necessary part of QFT limit (Higgs decays to WW pairs require that vacuum expectation is non-vanishing). Higgs vacuum expectation must correspond in TGD framework to a quantity with dimensions of mass. In TGD Higgs cannot be scalar but a vector in CP_2 degrees of freedom. The problem is that CP_2 does not allow covariantly constant vectors. The imaginary part of classical four-momentum gives a parameter which has interpretation as a vector in the tangent space of which is same as that of $M^4 \times CP_2$. Could $M^8 - H$ duality be realized at the level of tangent space and for relate four-momentum and color quantum numbers to 8-momentum?

Elementary particles of course need not be eigenstates of the imaginary part of four-momentum. For a fixed mass one can have wave functions in the space of imaginary four-momentum analogous to S^3 spherical harmonics at the sphere of E^4 with radius defined by the length of imaginary four-momentum (mass). These harmonics are characterized by SO(4) quantum numbers. Could one interpret this complexification in terms of M^8-duality and say that SO(4) defines the symmetries for the low energy dual of WCW defining high energy description of QCD based on SU(3) symmetry. SO(4) would corresponds to the symmetry group assigned to hadrons in the approach based on conserved vector currents and partially conserved axial currents. SO(4) would be much more general and associated also with leptons.

The anomalous color hyper-charge of leptonic spinors would imply that one can have also in the case of leptons a wave function in S^3. Higher harmonics would correspond to color excitations of leptons and quarks. If one considers gamma matrices, complexification of M^4 means introduction of gamma matrix algebra of complexified M^4 requiring 8 gamma matrices. This suggests a connection with $M^8 - H$ duality. All elementary particles have also imaginary part of four-momentum and the 8-momentum can be interpreted as M^8-momentum combining the four-momentum and color quantum numbers together.

REFERENCES

Books related to TGD

[K1] M. Pitkänen. Does the QFT Limit of TGD Have Space-Time Super-Symmetry? In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public_html/tgdquant/tgdquantum.html#susy, 2006.