# TGD based view about ferromagnetism

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August 11, 2020

**Abstract**

Yi Li et al have proposed an interesting exact result, which might help to understand the transition to ferromagnetism. This finding involves as an auxiliary concept Hamiltonian curve in a lattice of $N$ points with single vacant point. The observation was that by moving the vacancy around , one can induce arbitrary permutation of the non-vacant lattice points carrying spins and leading to the Hamiltonian curve containing the spins ordered from 1 to $N$ . The problem is that the result holds true only for single vacancy.

The finding however inspired the question about how ferromagnetism could be described in TGD framework. The notion of flux tube is the key concept and besides Maxwellian flux tubes there are also monopole flux tubes. The observation that magnetic moments having formal identification as infinitesimal pairs of magnetic monopole and anti-monopole leads to ask whether their sequences parallel to local magnetization M could correspond to magnetic moments residing at possibly dark (in TGD sense) monopole flux tubes as classical counterparts by quantum-classical correspondence. $h_{eff} = nh_0 > h$ would give rise to quantum coherence at these flux tubes in ascale longer than atomic length scale.

The monopole flux lines must be closed, which suggests that the return fluxes for the strands of a braid formed by $N$ flux tubes parallel to $M$ and containing the magnetic moments along it, fuse to a monopole return flux at larger parallel space-time sheet identifiable in terms of field $H$ so that one would have $B = M + H$ for the magnetic field experienced by a test particle touching both space-time sheets. General magnetized state would correspond to a non-trivial braid with non-parallel flux tubes providing a representation for the Maxwellian counterpart of $M$. For ferromagnetism the braid would be trivial. Also closed flux tubes without return flux and thus defining Hamiltonian cycle can be considered and could give rise to spin glass phase when the lattice (graph) allows a large number of Hamiltonian cycles.

The transition to ferromagnetism could be described as time-like braid for the space-like braid defined by the flux tubes of $M$. It would correspond to so-called 2-braid. Quantum groups and also 2-braid groups could be natural notion in their mathematical description. The transition to ferromagnetism would correspond to braid opening and if return flux arrives along single sheet, is possible for arbitrary braid. If the return flux is by separate flux tubes, braid opening must involve re-connections.

## Contents



# 1 Introduction

I received a link to a highly interesting popular article (https://tinyurl.com/y8b86df3) about ferromagnetism. According to the article, Yi Li, a physicist working at John Hopkins

University and his two graduate students, Eric Bobrow and Keaton Stubis, seem to have made a considerable progress in understanding how the system of electron spins in lattice ends up to a ferromagnetic sate [D1] (https://tinyurl.com/y9ycj3nt). This ferromagnetism is known as itinerant ferromagnetism and involves vacancies, sites without electron, which can be moved freely without affecting the energy of the state. This article inspired train of thought allowing to develop TGD view about ferromagnetism.

## 1.1 The ideas related to the work of Li et al

The problem considered by Li et al is how the ferromagnetic state could emerge from an arbitrary state with some numbers of spin up and spin down states at lattice sites connected by edges.

1. Permutation of electrons with same spin leave the ferromagnetic state invariant and does not cost energy while permutations in arbitrary configuration can do so.
2. Li et al [D1] considered a simple $4 \times 4$ lattice with single vacancy and noticed a connection with so called 15-puzzle involving 15 tiles and single vacancy with neighboring tiles of vacancy able to move to its position. The observation is following. If one has spin lattice containing single vacancy, one can number the sites by a number running from 1 to $N$ (now 15) in arbitrary manner. If so called connectedness condition holds true one can realize any permutation of these numbers. This means that 15-puzzle has always a solution. In particular, one can arrange the situation that the numbers form an ordered sequence from 1 to $N$ so that numbers $n$ and $n + 1$ are nearest neighbors.

   The result found by Li et al first for 2-D $4 \times 4$ lattice with single vacancy generalizes to lattices, which are non-separable in the sense the removal of a lattice site does not separate any pair of spins - they are still connected by an edge-loop.
3. The curve solving the 15-puzzle goes through all points of the $4 \times 4$ lattice and is generally known as Hamiltonian curve. It becomes Hamiltonian cycle if the numbers 1 and $N$ are nearest neighbors.
4. The basic problem of this approach is that the theorem is true only for single vacancy and does not allow generalization to a larger number of vacancies. It is however known that ferromagnetism is possible up to fraction 1/3 for vacancies. The challenge is to generalize the result of Li et al.

## 1.2 Some reasons to get interested

In TGD framework there are good reasons of getting interested on these results.

1. The result of Li et al states that ferromagnetic phase transition might be understood in terms of shifting of lattice vacancy if the lattice with single defect allows deformation of any configuration of spin labelled by numbers $N$ running from 1 to $N$ to a closed curve connecting nearest neighbors along which $N$ increases. Could there be a connection with Hamiltonian curves making sense for lattice like structures (actually all graphs)? Could Hamiltonian curve have some deeper physical meaning or is it only an auxiliary notion useful for representing the possibility to realize all permutations for the points of a lattice with vacancy by shifting it suitably?

   Hamiltonian curve connects neighboring points of a lattice and goes through all points without self-intersections. Icosahedral geometry appears in biology and one can ask whether this kind of cycles could be actually realized physically - say as flux tubes at icosahedron and tetrahedron, which play key role in TGD inspired biology. Flux tube are actually fundamental objects in TGD Universe in all scales. For instance, final states of stars could correspond to flux tube spaghettis consisting of single volume filling flux tube [L2] (https://tinyurl.com/tkkyyd2).
2. If the Hamiltonian cycle is something physical it could correspond to flux tube. The notion of magnetic flux tube central in TGD might allow application to ferromagnetism. TGD predicts two kinds of flux tubes: Maxwellian ones and monopole flux tubes with magnetic fields requiring no currents to generate them: they are not not allowed by Maxwell's theory.

   The preservation of the Earth's magnetic field predicted to decay rather rapidly as currents generating it dissipate supports the view that it contains monopole flux part which from biological input would correspond to endogenous magnetic field $B_{end}$, which is a fraction 2/5 about the nominal value of $B_E = .5$ Gauss. The presence of magnetic fields in cosmological scales is also a mystery finding a solution in terms of monopole flux tubes.

3. Monopole flux tubes must be closed. Closed non-intersecting flux tubes connecting nearest neighbors in lattice would correspond to Hamiltonian cycles. In TGD inspired biology Hamiltonian cycles associated with icosahedron and tetrahedron provide a realization of the vertebrate genetic code [L1] ( `https://tinyurl.com/yad4tqwl`) but it is still somewhat of mystery why the points of icosahedron and tetrahedron, which are lattices (tesselations) at sphere, would be connected by a curve. Quantum classical correspondence suggests that magnetization corresponds to flux tubes connecting magnetic dipoles as formal analogs of monopole-antimonopole pairs. Could magnetic flux tubes provide a concrete realizaon for these Hamiltonan cycles?
4. Closed monopole flux tubes seem to be unrealistic for the description of ferromagnetism, which suggests the presence of $N$ parallel flux tubes carrying magnetization $M$ and defining a braid connecting opposite ends of ferromagnet. The monopole fluxes could arrive as single flux along parallel space-time sheet carrying field $H$ defined by single thick flux tube. Test particle would experience $B = M + H$.

The following considerations are not much more than first impressions and probably require updating.

# 2 TGD based view

Flux tubes are the new element of condensed matter physics predicted by TGD. Could they provide insights into ferromagnetism?

## 2.1 Starting from text book picture about ferromagnetism

To develop TG view about ferromagnetism it is best to start from the text book picture.

1. In the standard model of ferromagnetism one assumes the presence of field $B$ identified as sum $B = M + H$ of magnetization and field $H$ equal to $B$ outside the magnet. $M$ is due to magnetic dipoles besides magnetic field $B$ and the interaction of spins with $H$ is important. $B$ is usually regarded as the fundamental field $M$ and and $H$ appear as auxiliary notions and their relation to $B$ requires a model for the system: typically $H$, $M$, and $B$ are assumed to be linearly related.
2. The field $M$ could be naturally assigned with a flux tube connecting the spins - perhaps at nearest neighbor lattice points. What about H? In standard model $H$ and $B$ are parallel for the ferromagnetic configuration. If $B$ is assigned with the flux tube connecting the magnetic moments and $B$ is parallel to $H$, this would suggest a flux tube consisting of long straight portions parallel to each other.

   In the many-sheeted space-time of TGD $M$ and $H$ can reside at different space-time sheets, which are parallel so that they are on top of each other in $H = M^4 \times CP_2$. $H$ is at large space-time sheet including also the environment. The decomposition to sum would have representation as a set theoretic union.

   The test particle would experience the sum of the magnetic fields associated with the two sheets. Could $M$ and $H$ as the return flux associated with $M$ and superposing with the external contribution to $H$ correspond to these two space-time sheets so that particle would experience their sum as $B = M + H$? If so, ferromagnetism could be seen as a direct signature of many-sheeted space-time.

## 2.2 Could also monopole flux tubes be important?

There is still one important aspect related to the TGD view about magnetic field which might play important role. TGD predicts two kind of flux tubes. The first kind of flux tubes could be called Maxwellian and the corresponding magnetic fields require current to generate them. There are also flux tubes having closed cross section and carrying monopole fluxes. No currents are required to generate corresponding magnetic fields. Could also these flux tubes having no current as sources be present? This would mean new physics.

1. The first thing to notice is that the interpretation of magnetization $M$ is as a magnetic field generated by magnetic moments. The usual interpretation is that spins are

analogous to magnetic moments created by currents consisting of rotating charge. For spin there is no such rotating charge. Second interpretation is as magnetic moments identifiable as infinitesimal monopole pairs.

2. Could one think that the flux tubes containing sequence magnetic moments correspond to monopole flux and that closing this loop could give rise to monopole magnetic field? Ordinary Maxwellian part could be also present and have current as source. How $M$ and $H$ would relate to these. Could $M$ correspond to the monopole part and $H$ the Maxwellian part?

   Are spins necessary for the existence of a monopole flux tube? Could quantum classical correspondence require this? Could dark charged matter assigned with the monopole flux tubes correspond to the magnetic moments of say dark valence electrons with non-standard value of $h_{eff} = nh_0$ so that $M$ would be represented by monopole flux tubes classically? If the return flux represented by $H$ is absent, flux tube must give rise to a Hamiltonian cycle. If $H$ is present, it would be enough to have flux tubes representing $N$ braid strands fusing to single monopole flux carrying the return flux.

   Formation of a flux loop defining Hamiltonian cycle would be a new kind of phenomenon analogous to spontaneous magnetization requiring no external field $H$. Spontaneous magnetization would be however something different. A trivial braid consisting of $N$ parallel strands representing $M$ and parallel to it locally with return flux arriving along single large flux tube carrying $H$ would be formed in ferromagnetic transition and also in spontaneous magnetization.

## 2.3 Bringing in thermodynamics

One can try to make this more concrete by bringing in thermodynamics.

1. Assume that there exists single flux tube - connecting all the lattice points (magnetic moments) or possibly $N$ flux tubes parallel to local magnetization $M$ and giving rise to a braid like structure representing the topology of flux lines of $M$ connecting opposite boundaries of magnet.

2. In the general case, the points of the lattice could be connected by a flux tubes connecting points, which need not be nearest neighbors. The first guess is that the magnetic interaction energy of spins at the ends of the flux tube portion connecting them decreases with the distance between spins. There should be also magnetic energy associated with the field $H$ at the space-time sheet carrying the return flux. Thermodynamics would bring in entropy and free energy $F = E - TS$ would be mimimized. Entropy maximization would favor long random flux tubes and energy minimization short flux tubes.

   One expects that flux tube has free energy F increasing with flux tube length. If one does not allow self-intersections - as suggested by repulsive Coulomb interaction and Fermi statistics - the flux tube could be either Hamiltonian cycle or consist of analogs of braid strands: in the case of ferromagnetism the strands would be parallel to each other. The interaction energy would be same for all Hamiltonian cycles if determined by nearest neighbour interactions.

3. In the general case with lattice replaced by graph one expects that a large number of Hamiltonian cycles not related by rotation to each other exists so that one would have large number of states with same minimum energy. Could this somehow correspond to spin glass state allowing large number of degenerate states? The flux tube need not be closed. In ferromagnetic configuration this would be the case.

4. How would the assignment of spin direction to the lattice points affect the situation? Could the numbers $N_+$ and $N_-$ of spin up and spin down electrons determine the flux tube configuration (say braid) by (Gibbs) energy minimization?

## 2.4 Could 2-braid describe the transition to ferromagnetism?

In the work of Li et al discussed in the article, the permutations of lattice points are induced by moving the vacancy around. This picture inspired the considerations above but is too limited. In fact the work of Li et al only directed attention to Hamiltonian cycles and braids formed by the non-closed analogs.

1. TGD picture brings in mind braid-knot connection. One can replace the braid associated with $M$ with a knot by connecting the magnetic moments at the opposite ends of the

braid by strands of a trivial braid at parallel space-time sheet. This trivial braid would carry the return flux having interpretation in terms of field $H$.

The flux tubes of trivial braid could also fuse to single thicker flux tube or flow to a larger space-time sheet carrying the total return flux associated with $M$. This would conform with the idea that $H$ provides a description of the system in longer length scale being analogous to a smoothed out total magnetic field acting as self-consistent background.

**Remark**: Could one assume that only $H$ assignable to big flux tube has constant direction and magnitude and that $M$ is represented as flux tubes connecting dipoles can in principle correspond to any permutation of atoms? For this option the spontaneous magnetization would correspond to a superposition of different configurations with same weights and would be invariant under permutations as in the argument of Li et al involving no flux tubes. This option does not look attractive.

2. What braid picture allows to say about the transition to ferromagnetism? Could the transition be realized by deforming the flux tubes associated with $M$ and forming a non-trivial braid be induced by permutation of the lattice points taking the non-trivial braid to trivial one? This would be like opening the braid. The lattice points in the initial and final state would correspond to the ends of a dynamical evolution. The permutation would be realized as a time-like braiding with braid strands in time direction.

   Mathematically braid group corresponds to the covering group of permutation group and quantum group representations correspond to the representations of braid groups. The description of the transition could provide a new application of quantum groups.

The description as time-like braiding is not however complete since there is an additional structure involved: the flux tubes connecting the magnetic dipoles in lattice and defining a braid or even more complex configuration having flux tube connections between non-neighboring magnetic moments.

1. If there is no return flux assignable to $H$, $M$ corresponds to a closed flux tube carrying monopole flux the dynamical time-like dynamical braiding would lead to a Hamiltonian cycle in this case and the number of final state configurations would be finite, there is degeneracy. Could spin glass phase correspond to this situation?

2. In ferromagnetism final state would contain $N$ parallel strands carrying the monopole flux assignable to $M$ and the return flux $H$ would arrive along parallel thick flux tube. In general configuration these strands can be braided. The transition to ferromagnetism would represent time-like braiding of an ordinary 3-D braiding of flux tube strands connecting the opposite boundaries of ferromagnetic. In the initial state braid would be non-trivial and the flux tubes of braid would not have minimal length and minimum energy. In the final ferromagnetic state braid would be trivial with parallel flux tubes.

   Mathematically this process would correspond to what is called 2-braiding: I have proposed that 2-braidings are important in TGD inspired biology as a topological description of dynamical processes. An interesting interpretation is as a topological analog for problem solving. I have also proposed that in bio-systems topological quantum computation programs are represented as this kind 2-braidings for flux tubes [K1, K2] (`https://tinyurl.com/ycvgjccq` and `https://tinyurl.com/ydylud6c`.)

   Ferromagnetism would correspond to an opening of a non-trivial braid. If the return flux arrives along flux tubes this is possible smoothly only if the knot defined in this manner is trivial. To achieve opening, the 2-braiding must involve reconnections, which correspond to cutting the knot strand and reconnecting the pieces in new manner: this is how Alexander opened his knot. Fermi statistics and repulsive Coulomb interaction do not favour this mechanism. If the return flux arrives along single flux tube, the opening could correspond to a smooth deformation without reconnections transferring the braiding to the parallel space-time sheet, where it is "neutralized" by fusing the flux tubes to single flux tube.

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