E_8 symmetry, harmony, and genetic code

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.

March 8, 2016

Contents

Abstract

This article is motivated by a bundle of intriguing connections. First, there is a connection between the symmetries of icosahedron and root systems of Lie groups F_4 and E_8. One starts from the symmetries of 3-D icosahedron and ends up with 4-D root systems of F_4 and E_8. E_8 defines a lattice in 8-D Euclidian space: what is intriguing that the dimensions 3,4, 8 fundamental in TGD emerge. Secondly, the TGD based model for music harmonies with 12-note scale modelled as Hamiltonian cycle at icosahedron and 3-chords of harmony as triangles of the icosahedron, led also to a model explaining the degeneracies of genetic code and to an extension of genetic code to explain also the so called 21st and 22nd amino-acid and deviations from genetic code. Thirdly, the McKay correspondence assigns to finite subgroups of 3-D rotation group all simply laced Lie groups. In particular, icosahedral symmetries correspond to Lie group E_8. Finally, the vision about finite measurement resolution and hyper-finite factors in turn strongly suggests that these Lie groups act as dynamical gauge or Kac-Moody type symmetries of quantum TGD. These connections are discussed in the sequel from TGD point of view.

1 Introduction

Bee gave in Facebook a link to an article about a connection between icosahedron and E_8 root system [B1] (see http://tinyurl.com/zotpm4b). The article (I have seen an article about the same idea earlier but forgotten it!) is very interesting.

The article talks about a connection between icosahedron and E_8 root system (see https://en.wikipedia.org/wiki/Root_system). Icosahedral group has 120 elements and its double covering $2 \times 120 = 240$ elements. Remarkably, E_8 root system has 240 roots. E_8 Lie algebra is 248 complex-dimensional contains also the 8 commuting generators of Cartan algebra besides roots: it is essential that the fundamental representation of E_8 co-incides with its adjoint representation. The double covering group of icosahedral group acts as the Weyl group E_8. A further crucial point is that the Clifford algebra in dimension $D = 3$ is 8-D.

One starts from the symmetries of 3-D icosahedron and ends up with 4-D root system F_4 assignable to Lie group and also to E_8 root system. E_8 defines a lattice in 8-D Euclidian space: what is intriguing that dimensions 3,4, 8 fundamental in TGD emerge. To me this looks fascinating - the reasons will be explained below.

1.1 What I might have understood

I try to explain what I have possibly understood.

1. The notion of root system is introduced. The negatives of roots are also roots but not other multiples. Root system is crystallographic if it allows a subset of roots (so called simple roots) such that all roots are expressible as combinations of these simple roots with coefficients having the same sign. Crystallographic root systems are special: they correspond to the fundamental weights of some Lie algebra. In this case the roots can be identified essentially as the quantum numbers of fundamental representations from which all other representations are obtained as tensor products. Root systems allow reflections as symmetries taking root system to itself. This symmetry group is known as Coxeter group and generalizes Weyl group. Both H_3 and H_4 are Coxeter groups but not Weyl groups.

2. 3-D root systems known as Platonic roots systems (A_3, B_3, H_3) assignable to the symmetries of tetrahedron, octahedron (or cube), and icosahedron (or dodecahedron) are constructed. The root systems consist of 3 suitably chosen unit vectors with square equal to 1 (square of reflection equals to one) and the Clifford algebra elements generated by them by standard Clifford algebra product. The resulting set has a structure of discrete group and is generated by reflections in hyper-planes defined by the roots just as Weyl group does. This group acts also on spinors and one obtains a double covering SU(2) of rotation group SO(3) and its discrete subgroups doubling the number of elements. Platonic symmetries correspond to the Coxeter groups for a "Platonic root system" generated by 3 unit vectors defining the basis of 3-D Clifford algebra. H_3 is not associated with any Lie algebra but A_3 and B_3 are.

   Pinors (spinors) correspond to products of arbitrary/even number of Clifford algebra elements. Spinors induced orientation preserving transformations and pinos also orientation reversing ones. They mean something else than usually a bein identified as elements of the Clifford algebra acting and being acted on from left or right by multiplication so that they always behave like spin 1/2 objects since only the left(right)-most spin is counted. The automorphisms involve both right and left multiplication reducing to SO(3) action and see the entire spin of the Clifford algebra element.

3. The 3-D root systems (A_3, B_3, H_3) are shown to allow an extension to 4-D root systems known as (D_4, F_4, H_4) in terms of 3-D spinors. D_4 and F_4 are root systems of Lie algebras (see `https://en.wikipedia.org/wiki/Semisimple_Lie_algebra`). F_4 corresponds to non-simply-laced Lie group related to octonions. H_4 is not a root system of any Lie algebra.

4. The observation that the dimension of Clifford algebra of 3-D space is $2^3 = 8$ and thus allows imbedding of at most 8-D root system must have inspired the idea that it might be possible to construct the root system of E_8 in 8-D Clifford algebra from 240 pinors of the double covering the 120 icosahedral reflections. Platonic solids would be behind all exceptional symmetry groups since E_6 and E_7 are subgroups of E_8 and the construction should give their root systems also as low-dimensional root systems.

1.2 Mc Kay correspondence

The article explains also McKay correspondence stating that the finite subgroups of rotation group SU(2) correspond to simply laced affine algebras assignable with ADE Lie groups.

1. One considers the irreducible representations of a finite subgroup of the rotation group. Let the number of non-trivial representations be m so that by counting also the trivial representation one has $m + 1$ irreps altogether. In the Dynkin diagram of affine algebra of group with m-D Cartan algebra the trivial representation corresponds to the added node. One decomposes the tensor product of given irrep with the spin 2 representation into direct sum of irreps and constructs a diagram in which the node associated with the irrep is connected to those nodes for which corresponding representation appears in the direct sum. One can say that going between the connected nodes corresponds to forming a tensor product with the fundamental representation. It would be interesting to know what happens if one constructs

analogous diagrams by considering finite subgroups of arbitrary Lie group and forming tensor products with the fundamental representation.

2. The surprising outcome is that the resulting diagram corresponds to a Dynkin diagram of affine (Kac-Moody) algebra of ADE group with Cartan algebra, whose dimension is m. Cartan algebra elements correspond to tensor powers of fundamental representation: can one build any physical picture from this? For $m = 6, 7, 8$ one obtains E_6 , E_7 , E_8. The result of the article implies that these 3 Lie-groups correspond to basis of 3 3-D unit identified as units of Clifford algebra: could this identification have some concrete meaning as preferred non-orthogonal 3-basis?

3. McKay correspondence emerges also for inclusions of hyper-finite factors of type II_1 [K2] The integer m characterizing the index of inclusion corresponds to the dimensions of Cartan algebra for ADE type Lie group. The inclusions of hyperfinite factors (HFFs) are characterized by integer $m \geq 3$ giving the dimension of Cartan algebra of ADE Lie groups (there are also C, F and G type Lie groups). m= 6,7,8 corresponds to exceptional groups E_6 , E_7 , E_8 on one hand and to the discrete symmetry groups of tetrahedron, octahedron, icosahedron on the other hand acting as symmetries of corresponding 3-D non-crystallographic systems and not allowing interpretation as Weyl group of Lie group.

2 TGD inspired interpretation

In the following the TGD inspired interpretation of the E_8-icosahedron correspondence is related to the TGD inspired model of harmony relying on the symmetries of icosahedron.

2.1 Connection with the model of harmony

These findings become really exciting from TGD point of view when one recalls that the model for bioharmony [K1] [L1] (see) for 12-note harmonies central in classical music in general relies on icosahedral geometry. Bioharmonies would add something to the information content of the genetic code: DNA codons consisting of 3 letters A,T,C,G would correspond to 3-chords defining given harmony realized as dark photon 3-chords and maybe also in terms of ordinary audible 3-chords. This kind of harmonies would be roughly triplets of 3 basic harmonies and there would be 256 of them (the number depends on counting criteria). The harmonies could serve as correlates for moods and emotional states in very general sense: even biomolecules could have "moods". This new information should be seen in biology. For instance, different alleles of same gene are known to have different phenotypes: could they correspond to different harmonies? In epigenetics the harmonies could serve as a central notion and allow to realize the conjectured epigenetic code and histone code. Magnetic body and dark matter at them would be of course the essential additional element.

The inspiring observations are that icosahedron has 12 vertices - the number of notes in 12-note harmony and 20 faces- the number of amino-acids and that DNA codons consist of three letters - the notes of 3-chord.

1. Given harmony would be defined by a particular representation of Pythagorean 12-note scale represented as self-non-intersecting path (Hamiltonian cycle) connecting the neighboring vertices of icosahedron and going through all 12 vertices. One assumes that neighboring vertices differ by one quint (frequency scaling by factor 3/2): quint scale indeed gives full octave when one projects to the basic octave. One obtains several realizations (in the sense of not being related by isometry of icosahedron) of 12-note scale. These realizations are characterized by symmetry groups mapping the chords of harmony to chords of the same harmony. These symmetry groups are subgroups of the icosahedral group: Z_6, Z_4, and two variants of Z_2 (generated by rotation of π and by reflection) appear. Each Hamiltonian cycle defines a particular notion of harmony with allowed 3-chords identified by the 20 triangles of icosahedron.

2. Pythagoras is trying to whisper me an unpleasant message: the quint cycle does not quite close! This is true. Musicologists have been suffering for two millenia of this problem.

One must introduce 13th note differing only slightly from some note in the quint cycle. At geometrical level one must introduce tetrahedron besides icosahedron - only four notes and four chords and gluing along one side to icosahedron gives only one note more. One can keep tetrahedron also as disjoint from icosahedron as it turns out: this would give 4-note harmony with 4 chords something much simpler that 12- note harmony.

3. The really astonishing discovery was that one can understand genetic code in this framework. First one takes three different types of 20-chord harmonies with group Z_6, Z_4, and Z_2 defined by Hamiltonian cycles: this can be done in many different maners (there are 256 of them). One has 20+20+20 chords and one finds that they correspond nicely to 20+20+20=60 DNA codons: DNA codons coding for a given amino-acid correspond to the orbit of the triangle assigned with the amino-acid under the symmetry group of harmony in question.

   The problem is that there are 64 codons, not 60. The introduction of tetrahedron brings however 4 additional codons and gives 64 codons altogether. One can map the resulting 64 chord harmony to icosahedron with 20 triangles (aminoacids) and the degeneracies (number of DNA codons coding for given amino-acid in vertebrate code) come out correctly! Even the two additional troublesome amino-acids Pyl and Sec appearing in Nature and the presence of two variants of genetic code (relating to two kinds of Z_2 subgroups) can be understood.

2.2 What could the interpretation of the icosahedral symmetry?

An open problem is the proper interpretation of the icosahedral symmetry.

1. A reasonable looking guess would be that it quite concretely corresponds to a symmetry of some biomolecule: both icosahedral or dodecahedral geometry give rise to icosahedral symmetry. There are a lot of biomolecules with icosahedral symmetry, such as clathrate molecules at the axonal ends and viruses. Note that dodecahedral scale has 20 notes - this might make sense for Eastern harmonies - and 12 chords and there is only single dodecahedral Hamiltonian path found already by Hamilton and thus only single harmony. Duality between East and West might exist if there is mapping of icosahedral notes and to dodecahedral 5-chords and dodecahedral notes to icosahedral 3-chords and different notions of harmony are mapped to different notions of melody - whatever the latter might mean!).

2. A more abstract approach tries to combine the above described pieces of wisdom together. The dynamical gauge group E_8 (or Kac-Moody group) emerging for m=8 inclusion of HFFs is closely related to the inclusions for the fractal hierarchy of isomorphic sub-algebras of super-symplectic subalgebra. $h_{eff}/h = n$ could label the sub-algebras: the conformal weights of sub-algebra are be n-multiples of those of the entire algebra.

   The integers n_i resp. n_f for included resp. including super conformal sub-algebra would be naturally related by $n_f = m \times n_i$. $m = 8$ would correspond to icosahedral inclusion and E_8 would be the dynamical gauge group characterizing dark gauge degrees of freedom. The inclusion hierarchy would allow to realize all ADE groups as dynamical gauge groups or more plausibly, as Kac-Moody type symmetry groups associated with dark matter and characterizing the degrees of freedom allowed by finite measurement resolution.

3. E_8 as dynamical gauge group or Kac-Moody group would result from the super-symplectic group by dividing it with its subgroup representing degrees of freedom below measurement resolution. E_8 could be the symmetry group of dark living matter. Bioharmonies as products of three fundamental harmonies could relate directly to the hierarchies of Planck constants and various generalized super-conformal symmetries of TGD! This convergence of totally different theory threads would be really nice!

2.3 Experimental indications for dynamical E_8 symmetry

Lubos (see `http://tinyurl.com/htjp55h`) (thanks to Ulla for the link to the posting of Lubos) has written posting about experimental finding of E_8 symmetry emerging near the quantum critical point of Ising chain at quantum criticality at zero temperature. Here is the abstract (see `http://tinyurl.com/zulzk9y`):

Quantum phase transitions take place between distinct phases of matter at zero temperature. Near the transition point, exotic quantum symmetries can emerge that govern the excitation spectrum of the system. A symmetry described by the E_8 Lie group with a spectrum of eight particles was long predicted to appear near the critical point of an Ising chain. We realize this system experimentally by using strong transverse magnetic fields to tune the quasione-dimensional Ising ferromagnet CoNb2O6 (cobalt niobate) through its critical point. Spin excitations are observed to change character from pairs of kinks in the ordered phase to spin-flips in the paramagnetic phase. Just below the critical field, the spin dynamics shows a fine structure with two sharp modes at low energies, in a ratio that approaches the golden mean predicted for the first two meson particles of the E8 spectrum. Our results demonstrate the power of symmetry to describe complex quantum behaviors.

Phase transition leads from ferromagnetic to paramagnetic phase and spin excitations as pairs of kinks are replaced with spin flips (shortest possible pair of kinks and loss of the ferromagnetic order). In attempts to interpret the situation in TGD context, one must however remember that dynamical E_8 is also predicted by standard physics so that one must be cautious in order to not draw too optimistic conclusions.

In TGD framework $h_{eff}/h \geq 1$ phases or phase transitions between them are associated with quantum criticality and it is encouraging that the system discussed is quantum critical and 1-dimensional.

1. The large value of h_{eff} would be associated with dark magnetic body assignable to the magnetic fields accompanying the E_8 "mesons". Zero temperature is not a prerequisite of quantum criticality in TGD framework.

2. One should clarify what quantum criticality exactly means in TGD framework. In positive energy ontology the notion of state becomes fuzzy at criticality. For instance, it is difficult to assign the above described "mesons" with either ferromagnetic or paramagnetic phase since they are most naturally associated with the phase change. Hence Zero Energy Ontology (ZEO) might show its power in the description of (quantum) critical phase transitions.

   Quantum criticality could correspond to zero energy states for which the value of h_{eff} differs at the opposite boundaries of causal diamond (CD). Space-time surface between boundaries of CD would describe the transition classically. If so, then E_8 "mesons" would be genuinely 4-D objects - "transitons" - allowing proper description only in ZEO. This could apply quite generally to the excitations associated with quantum criticality. Living matter is key example of quantum criticality and here "transitons" could be seen as building bricks of behavioral patterns. Maybe it makes sense to speak even about Bose-Einstein condensates of "transitons".

   The finding suggests that quantum criticality is associated with the transition increasing $n_{eff} = h_{eff}/h$ by factor $m = 8$ or its reversal - maybe the standard value $n_{eff}(i) = 1$. $n_{eff}(f) = 8$ could correspond to the ferromagnetic phase having long range correlations. Could one say that at the side of criticality (say the "lower" end of CD) the $n_{eff}(f) = 8$ excitations are pure gauge excitations and thus "below measurement resolution" but become real at the other side of criticality (the "upper" end of CD)?

3. The 8 "mesons" associated with spin excitations naturally correspond to the generators of the Cartan algebra of E_8. If the "mesons" belong to the fundamental (= adjoint) representation of E_8, one would expect 120+120 additional particles with non-vanishing E_8 charges. Why only Cartan algebra? Is the reasons that Cartan algebra is in preferred role in the representations of Kac-Moody algebras in that charged Kac-Moody generators can be constructed from Cartan algebra generators by standard construction used also in string models. Could this explain why one expects only 8 "mesons". Are charged "mesons" labelled by the elements of double covering of icosahedral group more difficult to excite?

REFERENCES

Theoretical Physics

[B1] Dechant P-P. The birth of E8 out of the spinors of the icosahedron. *Proc Royal Soc A*. Available at: http://rspa.royalsocietypublishing.org/content/472/2185/20150504., 472(2185), 2016.

Books related to TGD

[K1] Pitkänen M. Quantum Model for Hearing. In *TGD and EEG*. Onlinebook. Available at: http://tgdtheory.fi/public_html//tgdeeg/tgdeeg/tgdeeg.html#hearing, 2006.

[K2] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann, 2006.

Articles about TGD

[L1] Pitkänen M. Geometric theory of harmony. Available at: http://tgdtheory.fi/public_html/articles/harmonytheory.pdf, 2014.