An interesting cosmic co-incidence

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Contents

1 The numerical co-incidence and questions

Sahil Gupta told about an interesting numerical coincidence and asked some questions relating to it. In the following I answer the questions from the TGD point of view.

The co-incidence is following.

The Hubble radius/the radius of the electron mass photon = the proton charge radius/the proton mass black hole radius

Here I interpret the "radius of electron mass photon" as electron Compton length.

Questions:

- What do you make of this coincidence?
- What if we determine H actually is a constant?
- What if the redshift is not caused by galactic recession? (returning to Zwicky's hypothesis)

1.1 The co-incidence

The co-incidence is very interesting.

1.1.1 Some general comments

First some general comments about the co-incidence.

1. The coincidence says states that the ratio $r_p/r_{p,s}$ of the measured proton charge radius $r_p = .84$ fm (rather near to its Compton length $L_p = 1.32$ fm) to its Schwarzschild radius $r_{p,s}$ equals to the ratio D_H/L_e of the Hubble radius $D_H = 1/H$ to what I interpret to be electron Compton length L_e:

$$\frac{r_p}{r_{p,s}} = \frac{D_H}{L_e} \quad . \tag{1.1}$$

   Note that I use units in which $c = 1$ is true. Wikipedia also gives an approximate theoretical formula for r_p.

2. The co-incidence can be written also in the form $r_p/L_{p,s} \simeq D_H/L_e$ and would relate totally different scales. As if L_e would take the role of $L_{p,s}$ and D_H the role of r_p in cosmological scales.

3. D_H is not very precisely defined (recall the debate related to the two possible values of H_0) and that r_p, which is not as fundamental a concept as L_p. Hence the replacement of r_p with L_p can be considered.

4. The co-incidence would hold only for the recent cosmology unless one assumes that H is constant for ever.

1.1.2 What co-incidence allows to say about the value of Λ?

It is interesting to see what the co-incidence allows to conclude about the the value of the cosmological constant.

1. One can transform the co-incidence into the form (units with $\hbar = 1$ $c = 1$ are used and m_P denotes Planck mass)

$$\frac{H_0^2}{8\pi G} = \frac{1}{8\pi} m_e^2 m_p^2 (\frac{m_p}{m_P})^2 \quad . \tag{1.2}$$

   Here m_P is Planck mass.

2. The energy density associated with cosmological constant Λ can be parametrized as

$$\Lambda = 3\Omega_\Lambda H_0^2 \quad , \quad \Omega_\Lambda \simeq .688 \quad . \tag{1.3}$$

3. If one uses the co-incidence, the dark energy density associated with Λ can be expressed as

$$\rho_{dark} = \frac{\Lambda}{8\pi G} = 3\Omega_\Lambda H_0^2 = \frac{3\Omega_\Lambda}{8\pi} m_e^2 m_p^2 (\frac{m_p}{m_P})^2 \quad . \tag{1.4}$$

4. A convenient parametrization of ρ_{dark} is as (here the units with $c = 1$, $\hbar = 1$ are used)

$$\rho_{dark} = \frac{1}{L^4} \quad . \tag{1.5}$$

   Here a numerical factor not far from a unity does not affect the value of L appreciably. This gives for L

$$\frac{1}{L} = (\frac{3\Omega_\Lambda}{8\pi})^{1/4} m_e^{1/2} m_p^{1/2} (m_p/m_P)^{1/2} \quad . \tag{1.6}$$

One obtains $L = 1.11$ mm to be compared with the wavelength of CMB photons of about 1.9 mm at the maximum of wavelength distribution. The corresponding energy is 1.12 meV. This estimate is of course only an order of magnitude estimate but one can ask whether this energy could correspond to a rest mass of some particle. TGD indeed predicts that the existence of pseudoscalars analogous to pions and there are some experimental indications for the existence of a pseudoscalar with mass around 1 meV identified as dark matter candidate.

5. Electropion predicted by TGD as an explanation of old anomaly related to heavy ion scattering at energies near Coulomb wall [K3] has mass very near to $2m_e = 1$ MeV and corresponds to a p-adic length scale $L_p \propto \sqrt{p} \propto 2^{k/2}$ defined by Mersenne prime $M_{127} = 2^{127} - 1$. p-Adic length scale hypothesis states that the primes $p \simeq 2^k$ are physically favoured. The mass 1 meV is assignable to p-adically scaled up electron with $k = 197$, which is prime.

1.2 Tired light and cosmic redshift

There was also a question about tired light as a possible explanation of the cosmic redshift. The cosmic redshift seems to be in conflict with energy conservation. In general relativity energy and momentum are not conserved since the GRT space-time does not have translations as symmetries (Noether's theorem says that symmetries correspond conservation laws and energy conservation corresponds to time translation symmetry).

In TGD energy and momentum are observed since Poincare symmetries are not those of space-time but those of 8-D $H = M^4 \times CP_2$. In TGD 3-space is identified as a 3-surface in 4-D space-time which in turn is 4-surface in H. The orientation of the 4-dimensional tangent space in H varies from point to point and its orientation for the source differs from that for the receiver. The change of the orientation means Lorenz boost changing the energy and momentum: one simply observes the four-momentum in a different rest system. In the far-from-source region the observed four-momentum is different from that in nearby regions and gives rise to cosmic redshift. Energy and momentum are conserved and light is not tired.

1.3 Could Hubble constant be a genuine constant?

Hubble constant $H = (dR/dt)/R$ depends on cosmic time in standard cosmology. Hubble radius corresponds to the distance for which the recession velocity $v_r = HD = c$ holds true. Above Hubble radius $D_H = c/H$ the expansion would exceed light velocity. Below I will consider in the TGD framework a cosmology with genuinely constant H could mean in the TGD Universe. It seems that $H = constant$ cosmology cannot be physical in TGD framework. Rather, a string dominated cosmology for which the expansion velocity dR/dt is constant is more sensible.

2 TGD view about cosmology with genuinely constant H

In the TGD framework, RW cosmology is embeddable as a 4-surface to future light-one and one has $H = (da/dt)/a$, where $a =\equiv R$ is the light-cone proper time [K2, K1]. One has $dt^2 = (dt/da)^2 da^2 = g_{aa} da^2$, $(da/dt)^2 = 1/g_{aa}$.

1. In the standard cosmology, H is usually not a genuine constant. In the TGD framework Hubble constant would characterize a given space-time sheet as a kind of sub-cosmology and would be determined by its p-adic length scale determining its size.

   Cosmic time correlates with the p-adic length scale of a region of space-time with Hubble radius. This is actually a very important distinction. In TGD a discrete spectrum of Hubble constants is predicted as also a spectrum of length scale dependent cosmological constants.

2. From the TGD point of view, the observed variation of H could mean that the light arrives along space-time sheets with different p-adic length scales and sizes. The size scale of the space-time sheet would increase in phase transitions and the value of H would be reduced. This would mean a discrete variant of cosmological expansion occurring in jerks [L1, L5]. I wrote quite recently about the evidence for this in the case of Earth and Venus [L3].

Hubble constant $H = (dR/dt)/R$ depends on the cosmic time t in the standard cosmology. Hubble radius corresponds to the distance for which the recession velocity $v_r = HD = c$ holds true. In the TGD framework, RW cosmology is embeddable as a 4-surface to future light-one and one has $H = 1/\sqrt{g_{aa}}a$, where $a = R$ is light-cone proper time.

What does the assumption that H is constant mean in the TGD Universe?

1. In the TGD Universe, H could be rather naturally constant at a given space-time sheet characterized by a p-adic length scale, which corresponds to a prime p near a power of 2. The finite size is true also in time direction and makes sense in the Zero Energy Ontology of TGD. Assume therefore that H is constant corresponding to discrete p-adic length scale hierarchy.

2. One can embed sub-critical RW cosmologies as 4-surfaces globally to $H = M^4 \times CP_2$ with 1-D CP_2 projection which is a geodesic circle with angular coordinate Φ. For over-critical cosmologies the embedding is possible for a finite cosmic time interval only.

   The embedding is $\Phi = f(a)$, where a light-cone proper time a is identifiable as a scale factor $a = R$. The induced metric in RW coordinates has

$$g_{aa} = 1 - R^2(\frac{df}{da})^2 \equiv 1 - (\frac{du}{da})^2 \quad , \quad u \equiv R\Phi \quad . \tag{2.1}$$

   Other components of the Robertson-Walker metric of the future light-cone are not affected.

3. The condition

$$H^2 = \frac{1}{g_{aa}a^2} = H_0^2 \quad . \tag{2.2}$$

   (H_0 is constant) gives the condition

$$(\frac{du}{da})^2 = 1 - \frac{1}{H_0^2 a^2} \quad . \tag{2.3}$$

   The condition makes sense only if the right-hand side is non-negative. The condition fails for $a \leq T_H = 1/H_0$, which is slightly longer than the estimated age of the Universe. The physical intuition suggests that the failure occurs for $a > T_H$, which suggests that $H = constant$ cosmology is not physical.

2.1 About the dependence of the value of H on p-adic length scale

The sub-cosmology with $H = H_0$ does not seem promising. One can however consider the possibility that H_0 corresponds to say minimal value of H.

1. Concerning the dependence of H_0 on p-adic length scale, the simplest guess is that $H_0(p)$, p p-adic prime, is of the form

$$H_0 = \frac{v_0}{L_p} \quad . \tag{2.4}$$

   Also G could be involved in the formular. $L_p \propto \sqrt{p}$ is p-adic length scale L_p. By the p-adic length scale hypothesis stating that the preferred p-adic primes are near some powers of 2, is proportional one has

$$L_p \propto p^{1/2} = 2^{k/2} \quad , \tag{2.5}$$

   where k is some positive integer. This is of course only the simplest guess. v_0 is a velocity parameter and $D = L_p$ gives $v_r = v_0 < c$. v_0 would be the maximal recession velocity if L_p defines the maximal distance D for the space-time sheet and would be near to c for largest space-time sheets.

2. Interestingly, a varying velocity parameter $v_0 < c$ appears in the Nottale hypothesis $\hbar_{gr} = GMm/v_0$ characterizing a given system [L4, L2]. The generalization of the Nottale hypothesis is central for TGD inspired quantum gravitation. Could v_0 appear as a maximal recession velocity at a given space-time sheet?

   One has $\beta_0 = v_0/c = 1/2$ for Earth [L4] (`http://tgdtheory.fi/public_html/ccheff.pdf`) and would mean that the gravitational Compton length

$$\Lambda_{gr} = \frac{\hbar_{gr}}{m} = \frac{r_s}{2\beta_0} \ , \quad r_s = 2GM \ , \tag{2.6}$$

   which does not depend on the mass m, is equal to Scwartschild radius r_s. For the inner planets of the solar system one has $v_0/c \times 2^{-11}$.

   The object with the largest known redshift has $z = 11.09$ and has a distance $D = 12.39$ Gly (`https://cutt.ly/DQZoKS3`). The approximate formula

$$v_r = \frac{HD}{z+1} \tag{2.7}$$

   with $H = 70\ km/sMpc$, $pc = 3.26$ ly gives $v_r = .08c$. This velocity is below $c/2$ suggested by Nottale's formula as maximal recession velocity at a given space-time sheet.

To sum up, the coincidences could hold true only for "our" cosmological space-time sheet but not generally. Furthermore, the idea about $H = constant$ does not look promising in the TGD framework.

2.2 Sub-cosmology with a genuinely constant Hubble constant

One can look at what a sub-cosmology with constant value of Hubble constant defined by a given space-time sheet identified as a Lorentz invariant 4-surface in $H = M^4 \times CP_2$ would look like. This representability is certainly not necessary since the Einstein-Yang-Mills limit of TGD replaces many-sheeted space-time with a region of M^4 deformed so that it is slightly curved. One might however hope that cosmologis, which allow an embedding to H are physically favoured.

For $\Lambda > 0$, the cosmologies considered do not correspond to preferred extremals, that is minimal surfaces. For $\Lambda = 0$ (mere Kähler action as action) the solutions considered are vacuum extremals with respect to Kähler action and the interpretation of the Einstein tensor as energy momentum tensor is questionable.

With these warnings in mind, one can look at what Friedman equations give. One can assume in the Friedman equations $H = constant = H_0$ giving $da/dt = 1/H_0a$ and $d^2a/dt^2 = -1/H_0^2a^3$. This gives

$$\begin{aligned} &3H_0^2 + \frac{3K}{a^2} = k\rho + \Lambda \ , \\ &\frac{3}{a}\frac{d^2a}{dt^2} = -\frac{3}{a^4H_0^2} = -\frac{k}{2}(\rho + 3p) + \Lambda \ , \\ &k = 8\pi G \ . \end{aligned} \tag{2.8}$$

For hyperbolic metric for which 3-space is hyperbolic 3-space, one has $K = -1$.

Hyperbolic metric is the only possibility since $K = 0$ and $K = -1$, $g_{tt} = (da/dt)^2$ is fixed completely from the condition that 3-space is Euclidean 3-space or 3-sphere [K2]. This forces us to generalize the solution so that CP_2 projection is a geodesic sphere. In these cases H is not constant.

The first Friedman equation gives the conditions

$$\rho = \rho_{max} - \frac{3}{ka^2} \ , \quad \rho_{max} = \frac{3H_0^2 - \Lambda}{k} = \frac{(1-\Omega_\Lambda)H_0^2}{k} \ . \tag{2.9}$$

$\rho_{max} > 0$ is true for $\Omega_\Lambda < 1$ (sub-criticality). For $H_0 = 67.66$ km/sMpc, one has $\Omega_\Lambda = .688$ (https://en.wikipedia.org/wiki/Cosmological_constant). The cosmology is sub-critical as the hyperbolity implies.

From the first condition one finds that $\rho \geq 0$ holds true when the condition

$$a \geq a_{min} = \sqrt{\frac{1}{H_0^2 - \Lambda/3}} = \frac{1}{\sqrt{1-\Omega_\Lambda}} \times T_H \geq T_H \quad (2.10)$$

is satisfied. The recession velocity $v_r = HD$ computed from the naive formula would be superluminal for $a \geq a_{min}$.

This cosmology does not make sense for $a < a_{min}$ if the mass density is required to be positive. A possible interpretation is asymptotic cosmology but superluminal recession velocities suggest that the cosmology is non-physical. The earlier cosmology would correspond to radiation or matter dominated phases. For asymptotic cosmology ρ approaches a constant value ρ_{max}.

The second condition gives pressure p, which is positive and satisfies

$$p = \frac{1}{k} \times [\frac{1}{H_0^2 a^4} + \frac{1}{a^2} + (3\Omega_\Lambda - 1)H_0^2] \ . \quad (2.11)$$

The cautious conclusion is that the proposed cosmology is not physical. The reason is probably that the assumption $H = constant$ has no physical motivation.

2.3 String dominated cosmology

Cosmic strings thickened to flux tubes define the fractal TGD cosmology. TGD also allows cosmic string dominated cosmologies with sub-critical mass density and with $\Lambda = 0$: this is discussed in [K2, K1]. This cosmology has infinite duration. The string dominated cosmology has an interpretation as asymptotic cosmology in which strings are idealizations for flux tubes. Also radiation dominated and mass dominated cosmologies allow embeddings.

For the simplest string dominated cosmology with $\Phi = \omega a$, $da/dt \equiv 1/X$ (rather $H = (da/dt)/a$) is constant. One has $(da/dt)^2 = 1/g_{aa} = 1/(1 - R^2\omega^2) = X$ and H behaves like $1/a$. Friedmann equations give

$$\rho = 3\frac{X^2-1}{ka^2} - \frac{\Lambda}{k} \ , \quad \rho + 3p - \frac{2\Lambda}{k} = 0 \ , \quad k = 8\pi G \ . \quad (2.12)$$

For $\Lambda = 0$ the $\rho + p = 0$ expressing that the energy momentum tensor is traceless holds true. Kähler action as fundamental action has by conformal invariance this property and the vanishing of Λ corresponds to the vanishing of the volume term of the fundamental action so that the prediction makes sense.

The mass density is positive for

$$a \leq a_{max} = \sqrt{\frac{3(X^2-1)}{\Lambda}} = \sqrt{\frac{R^2\omega^2}{\Lambda}} \ . \quad (2.13)$$

This cosmology could serve as a simple model for a primordial cosmology dominated by cosmic strings with 2-D string world sheets as M^4 projection so that Einsteinian space-time as a small metric deformation of M^4 does not make sense in this phase. Einsteinian space-time would emerge in the transition to a radiation dominated cosmology as cosmic strings thicken to flux tubes. This transition is analogous to the inflationary period.

When the mass density becomes negative, this transition should take place. TGD also allows the counterpart of a cosmology with a flat 3-space and accelerating expansion. This cosmology is string dominated for very early times and could describe the TGD analog of the inflationary phase preceding the radiation dominated cosmology.

For $\Lambda = 0$ the mass density is positive for all values of a. This case could correspond to the limit of infinitely large space-time sheets and reflects the vanishing volume term in the action determining space-time surfaces. It is an open question whether one should allow this phase [L6].

In this case, the second equation gives

$$\rho + 3p = 0 \ . \tag{2.14}$$

The equation states that the trace of the energy momentum tensor giving 4-D scalar curvature vanishes. The interpretation is in terms of conformal invariance assignable to YM action (curvature scalar is dimensional quantity and must vanish in the scaling invariant situation).

The action defining space-time surface contains volume term and Kähler term analogous to Maxwell action. $\Lambda > 0$ would correspond to a non-vanishing of the volume term and breaking of the conformal invariance. Note that the situation differs from the inflationary cosmology for which the 3-D rather than 4-D curvature scalar vanishes (3-space is flat E^3 rather than hyperbolic space).

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