Some questions about coupling constant evolution

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Abstract

In this chapter questions related to the hierarchy of Planck constants and p-adic coupling constant evolution (CCE) in the TGD framework are considered.

1. Is p-adic length scale hypothesis (PLS) correct in this recent form and can one deduce this hypothesis or its generalization from the basic physics of TGD defined by Kähler function of the "world of classical worlds" (WCW)? The fact, that the scaling of the roots of polynomial does not affect the algebraic properties of the extension strongly suggests that p-adic prime does not depend on purely algebraic properties of EQ. In particular, the proposed identification of p as a ramified prime of EQ could be wrong.

   Number theoretical universality suggests the formula $exp(\Delta K) = p^n$, where ΔK is the contribution to Kähler function of WCW for a given space-time surface inside causal diamond (CD).

2. The understanding of p-adic length scale evolution is also a problem. The "dark" CCE would be $\alpha_K = g_K^2/2h_{eff} = g_K^2/2nh_0$, and the PLS evolution $g_K^2(k) = g_K^2(max)/k$ should define independent evolutions since scalings commute with number theory. The total evolution $\alpha_K = \alpha_K(max)/nk$ would induce also the evolution of other coupling strengths if the coupling strenghts are related to α_K by Möbius transformation as suggested.

3. The formula $h_{eff} = nh_0$ involves the minimal value h_0. How could one determine it? p-Adic mass calculations for $h_{eff} = h$ lead to the conclusion that the CP_2 scale R is roughly $10^{7.5}$ times longer than Planck length l_P. Classical argument however suggests $R \simeq l_P$. If one assumes $h_{eff} = h_0$ in the p-adic mass calculations, this is indeed the case for $h/h_0 = (R(CP_2)/l_P)^2$. This ratio follows from number theoretic arguments as $h/h_0 = n_0 = (7!)^2$. This gives $\alpha_K = n_0/kn$, and perturbation theory can converge even for $n = 1$ for sufficiently long p-adic length scales. Gauge coupling strengths are predicted to be practically zero at gravitational flux tubes so that only gravitational interaction is effectively present. This conforms with the view about dark matter.

4. Nottale hypothesis predicts gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values. Gravitational fine structure constant is given by $\alpha_{gr} = \beta_0/4\pi$. Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution $\beta^2 = xr_S/2L_N = \beta_{0,max}^2/N^2$, where x is proportionality constant, which can be fixed.

   Phase transitions changing β_0 are possible at $L_N/a_{gr} = N^2$ and these scales correspond to radii for the gravitational analogs of the Bohr orbits of hydrogen. p-Adic length scale hierarchy is replaced by that for the radii of Bohr orbits. The simplest option is that β_0 obeys a CCE induced by α_K.

   This picture conforms with the existing applications and makes it possible to understand the value of β_0 for the solar system, and is consistent with the application to the superfluid fountain effect.

Contents

1 Introduction

In this article questions related to the notions of the p-adic CCE and hierarchy of Planck constants will be considered.

1.1 How p-adic primes are determined?

p-Adic length scale (PLS) hypothesis plays a central role in TGD in all length scales. For instance, it makes it possible to use simple scaling arguments to deduce quantitative predictions for the masses of new particles predicted by TGD.

PLS hypothesis states that the size scales of space-time surfaces correspond to PLSs $L_p = \sqrt{p}R(CP_2)$. The additional hypothesis is $p \simeq m^k$, $m = 2, 3, ...$ a small prime. The success of p-adic mass calculations [K1] supports $p \simeq 2^k$ hypothesis [K7] seriously. There also exists empirical evidence for a possible generalization to small primes, in particular $m = 3$, in biology [I1, I2].

The physical and mathematical identification of the origin of the p-adic prime p defining the PLS is however a problem.

The proposal has been that the p-adic prime p defining the PLS corresponds to a ramified prime of the extension of rationals (EQ) associated with the polynomial defining space-time region in M^8 picture. Ramified primes appear as factors of the discriminant of the polynomial defining EQ. I have not been able to find any really convincing explanation for why p should correspond to a ramified prime so that p-dic prime might emerge in some other manner.

In p-adic thermodynamics Boltzmann weights $exp(-E/T)$ must be replaced with $p^{L_0/T}$, where L_0 is scaling generator. The exponent $\Omega = exp(K)$ of the Kähler function K of WCW defines vacuum functional. Could Ω be number-theoretically universal and thus exist as a p-adic number for some prime p determining naturally the PLS. This is the case if one has $\Omega = p^n$, n integer.

As such, this idea does not make sense but one consider a subsystem defined by sub-CD defining self in zero energy ontology (ZEO) based theory of consciousness [L10, L20] [K13]?

p-Adic prime defines naturally the scale of CD for trivial extension of rationals and this scale is scaled up by factor n for an extension of dimension n. This also conforms with the assumption that p-adic CCE and "dark" CCE are independent.

1.2 Trying to understand p-adic CCE

TGD leads to a number theoretic vision about CCE [L6]. Number theoretic universality plays a key role in this picture. CCE certainly involves the hierarchy of extensions of rationals (EQs) possibly involving non-rational extensions by roots of e, which induce finite extensions of p-adics. It would be nice if the EQ alone would determine the values of the coupling constants.

1. The starting point is that the continuous CCE with respect to length scale reduces to a discrete PLS evolution with respect to L_p, $p \simeq 2^k$. There is also dark evolution with respect to $n = h_{eff}/h_0$. These evolutions are separate since the scaling of the roots of the polynomial do not affect the purely algebraic properties of the extension. The natural assumption is that these evolutions factorize so that one has $\alpha_K = g_K^2(p)/2h_{eff}$.

2. p-Adic CCE would be roughly logarithmic with respect to L_p. The observation that α is near $\alpha = 1/137$ for p-adic length scale $L(137)$ suggests that for α_K defining the fundamental coupling strength one has

$$\alpha_K = \frac{g_K^2(max)}{2kh_{eff}} \quad .$$

Since $1/\alpha_K(137) = 137$ is prime for ordinary matter with $h_{eff} = h$, one must have

$$\frac{g_K^2(max)}{2h} = 1 \quad .$$

giving $h = g_K^2(max)/2$. The value h need not however be the minimal value h_0 of h_{eff} since one can have $h = n_0 h_0$ $\alpha_K(max) = 2n_0$ so that one can write

$$\alpha_K = \frac{1}{knn_0} \quad . \tag{1.1}$$

$n_0 > 1$ would mean that the ordinary matter would be actually dark in the sense that the order of the extension of rationals associated with the ground state would be n_0.

For h_0 that value of α_K could be so large that the perturbation series does not converge except in very long length scales for which k is expected to be large. Exotic phases with $h_{eff} < h$ could become possible in these scales.

1.3 How p-adic prime is defined at the level of WCW geometry?

The p-adic prime p should emerge from the dynamics defined by Kähler function.

1. The Kähler function K of the "world of classical worlds" (WCW), or more generally the generalization of $exp(K)$ to a vacuum functional possibly involving also a genuine state dependent part is a central quantity concerning scattering amplitudes. Suppose that one can consider a subsystem defined by CD and the contribution ΔK from CD to K.

   Number theoretical universality suggests that the exponential $exp(\Delta K)$ or its appropriate generalization exists in all p-adic number fields or at least in an extension of the p-adic number field corresponding to the p-adic prime p. Could this condition fix p dynamically?

2. Suppose that for some prime p one can write

$$e^{\Delta K} = p^{\frac{\Delta K}{log(p)}}$$

   such that $\Delta K/log(p)$ is integer. The exponential would be a power of p just as the p-adic analog of Boltzmann weight in p-adic thermodynamics [K1]. This would select a unique p-adic prime p defining the PLS and this prime need not be a ramified prime. In p-adic thermodynamics [K1] $X = \Delta K/log(p)$ has interpretation as an eigenvalue of the scaling generator L_0 of conformal algebra and one can even consider the possibility that there is a connection.

1.4 What about the evolution of the gravitational fine structure constant?

Nottale hypothesis [E1] predicts gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values so that the above picture fails. Gravitational fine structure constant is given by $\alpha_{gr} = \beta_0/4\pi$.

Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution $\beta^2 = xr_S/2L_N = \beta_{0,max}^2/N^2$, where x is proportionality constant, which can be fixed. Phase transitions changing β_0 are possible at $L_N/a_{gr} = N^2$ and these scales correspond to radii for the gravitational analogs of the Bohr orbits of gravitational Bohr atom. PLS hierarchy is replaced by that for the radii of Bohr orbits.

What could be the interpretation of N? The safest assumption is that the CCE of β_0 is analogous to that of the other coupling constants and induced from that of α_K.

1.5 What is the minimal value of h_{eff}?

The formula $h_{eff} = nh_0$ involves the minimal value h_0 of h_{eff}. The simplest explanation for the findings of Randell Mills [D2] is that one has $h = 6h_0$. h_0 could be also smaller [L1].

What is the value of h_0? A possible answer to this question came from the observation made already during the first 10 years of TGD. The observation was that the imbeddings of spherically symmetric stationary metrics (see the Appendix) suggest that CP_2 radius R is of order Planck length l_P rather than by factor about $10^{7.5}$ longer. Could one have $h = n_0h_0$, $n_0 \sim 10^{7.5}$ so that the ordinary matter would be actually dark?

CP_2 radius would be Planck length apart from numerical constant not far from unity. The p-adic mass calculations would give correct results for $h_{eff} = h_0$. R could be interpreted as $R^2 = n_0 l_P^2$. The perturbative expansion for $h_{eff} < h$ would not converge except in long p-adic length scales, where the p-adic evolution reduces the value of α_K.

Gauge coupling strengths are predicted to be practically zero at gravitational flux tubes with very large h_{eff} so that only gravitational interaction is effectively present. This conforms with the view about dark matter.

2 Number theoretical universality of vacuum functional and p-adic CCE

The Kähler geometry of WCW is defined by a Kähler function $K(X^4(X^3))$ identified as the action of preferred extremal consisting of volume term and Kähler action. The vacuum functional is of form $\Omega = exp(K + iS)$. Here K is the real Kähler function and S is the counterpart of real action in the path integral of QFTs.

$exp(iS)$ could be interpreted as a dynamical part of vacuum functional, which depends on state rather than being "God-given". The reason why this would be the case would be that it is possible. For $exp(K)$ there is no choice since the Kähler geometry of WCW is expected to be unique merely from its existence as already in the case of loop spaces [A2].

Number theoretical universality is a challenge for this general picture.

1. In the p-adic context the notion of WCW geometry is highly questionable. The integration associated with definition of volume term and Kähler action is the tough problem.

   This has inspired the proposal that the exponent of the action completely disappears from the scattering amplitudes. This indeed happens in quantum field theory based on path integral around stationary point.

2. The classical nondeterminism suggests a weaker formulation. The sum over the contributions of stationary points would be replaced by integral over preferred extremals consisting of 3-surfaces at PB plus sum over the paths of the tree resulting from classical non-determinism.

   The sum over the paths of the tree-like structure remains in the superposition of amplitudes for sub-CD and it might be possible to define the deviation $\Delta K + i\Delta S$ of the action for each of them and separate $exp(\Delta K + i\Delta S)$ from the entire exponent of action, which would therefore disappear from the expression of the scattering amplitudes for given X^3 and given CD. Otherwise, the knowledge of the entire WCW Kähler function would be needed.

   A possible interpretation is in terms of a decomposition to an unentangled tensor product corresponding to sub-CD and its environment so that one can separate the physics inside sub-CD from that of environment and code it by $exp(\Delta K + i\Delta S)$.

3. The simplest option, very probably too simple, would be that one has $\Delta K = 0$. Kähler function would be same for all paths of the tree and one would obtain a discretized analog of path integral. This would require that all the branches of the tree have same value of action. This does not however require the same value of volume and Kähler action separately.

   It will be foud that $\Delta K \neq 0$ assuming that $exp(\Delta K)$ reduces to an integer power of p for some prime identifiable as p-adic prime defining the PLS, is more interesting option since it would reduce p-adic thermodynamics to the level of WCW and also allow to the understand of PLS evolution of couping constants.

   The number theoretical existence of the phases $exp(i\Delta S)$ would require that they belong to the EQ defining the space-time region inside CD.

4. This picture suggests that the number theoretically universal part is associated with the sub-CDs and with the discrete physics of the tree-like structure whereas the Kähler function for 3-surfaces would be defined only in real framework. This would neatly separate the physics of sensory and Boolean cognition as something number theoretically universal from the physics proper, so to say.

   Since conscious experience gives all information about physics, one can ask whether the adelic physics associated with various sub-selves could together be enough to represent all that is representable from the physics proper. This could result as somekind of limiting case (EQ approaches algebraic numbers).

If this view is correct, then one expects that various notions shared by QFTs and TGD, in particular CCE, could have number theoretic descriptions as indeed suggested [L6]. In the sequel I will discuss some speculations in this framework.

2.1 The recent view about zero energy ontology

Zero energy ontology (ZEO) [K13] [L10, L20] plays a key role in the formulation of TGD based quantum measurement theory.

1. The concept of causal diamond (CD) is central. CD serves as a correlate for the perceptive field of conscious entity: this in the case that one has sub-CD so that the space-time surfaces inside CD continue outside it.

   The scale size scale of the CD identifiable as the temporal distance T between its tips could be proportional the p-adic prime p at the lowest level of dark matter hierarchy and to np at dark sectors. p-Adic length scales L_p characterizing the sizes scale of 3-surfaces are proportional to $\sqrt{p}$ and the proposal is that the relation between T and L_p is same as the relationship between diffusion time T and the root mean square distance R travelled by diffusion.

2. The twistor lift of TGD predicts that the action principle defining space-time surfaces is the sum of a volume term characterized by length scale dependent cosmological constant Λ and Kähler action and induced from 6-D Kähler action whose existence fixes the imbedding space uniquely to $M^4 \times CP_2$. The reason is that the required Kähler structure exists only for the twistor spaces of M^4, E^4, and CP_2 [A4].

3. The recent progress in the understanding of zero energy ontology (ZEO) [L20] leads to rather detailed view about the dynamics of the space-time surfaces inside sub-CD.

   Space-time surfaces are analogs of soap films spanned by a frame having the 3-surfaces at its ends located at the boundary of CD as fixed part of frame and the dynamically generated parts of frame in the interior of CD. Outside the frame preferred extremal is an analog of a complex surface and a simultaneous extremal of both volume term and Kähler action since the field equations reduce to conditions expressing the analogy of holomorphy [K9, K12]. The field equations reduce to contractions of tensors of type (1,1) with tensors of type (2,0)+(0,2) and are therefore trivially true.

   The minimal surface property fails at the frame, and only the full field equations are true. The divergences of isometry currents associated with volume term have delta function singularities which however cancel each other to guarantee field equations and conservation laws. This is expected to give rise to a failure of determinism, which is however finite in the sense that the space-time surfaces associated with given 3-surface X^3 at the passive boundary of CD (PB) form a finite set which is a tree-like structure (for a full determinism only single space-time surface as analog of Bohr orbit would be realized). Therefore the non-determinism of classical dynamics for a fixed X^3 is extremely simple and quantum dynamics and classical dynamics are very closely related since quantum states are superpositions of the paths of the tree.

4. One also ends up to quite precise identification sub-CD or space-time surface inside sub-CD as a correlate of perceptive field of a conscious entity. The essential element of the picture that for sub-CD the 3-surface X^3 at PB is fixed but due to the non-determinism the end at active boundary (AB) is not completely fixed and there is finite non-determinism in the state space defined by superpositions of the paths of the tree.

   For the highest level in the hierarchy of CDs associated with self, the space-time surfaces inside CD do not continue outside it and this CD God-like entity, whose dynamics is not restricted by the boundary conditions.

This view provides additional perspectives on discreteness of adelic physics unifying the physics of sensory experience and cognition [L4, L5].

1. Discreteness is essential in the number theoretic universality since in these case real structures and their p-adic counterparts correspond naturally to each other. This has led to the notion of cognitive representation as a set of points of space-time surface with preferred imbedding space coordinates having values in the EQ defined by the polynomial defining the space-time surface in complexified M^8 and mapped to H by $M^8 - H$ duality [L12, L13]. The finite-dimensionality of the state space associated with the tree structure conforms with this vision.

2. Discreteness is natural for the dynamics of concious experience and cognition. Mental images as sub-selves correspond to the sub-CDs inside CD. Sub-CDs are naturally located at the loci of non-determinism defined by the fixed part of the frame dynamically and generated frames in the interior and at AB.

   Attention would fix the 3-surfaces at the PB of a sub-CD as a perceptive sub-field and all CDs in the hierarchy would be fixed in this manner. The loci of non-determinism would serve as targets of attention. Sensory perception, memory recall, and other functions would reduce to directed attention inside CD.

   Fermionic degrees of freedom at boundaries of CD are are additional discrete degrees of freedom and responsible for Boolean cognition whereas the discrete dynamics of frame would correspond to sensory experience and sensory aspects of cognition.

3. This picture inspires the question whether the number theoretically universal parts of adelic physics might relate to the physics due to the non-determinism in the interior of sub-CD. This physics would be basically the physics that can be observed. This would mean enormous simplification.

   This idea is not new. The amazing success of p-adic thermodynamics based mass calculations [K1] could be understood if p-adic physics is seen as a physics of cognitive representation of real number based physics.

In the sequel some speculations are discussed by taking the above picture as a basis.

2.2 Number theoretical constraints on $exp(\Delta K)$

Number theoretical universality suggests that the exponents $exp(\Delta K + i\Delta S)$ for X^4 inside sub-CD is well-defined at least for some p-adic number fields or their extensions.

It has been already found that number theoretical universality requires that the phases $exp(i\Delta S)$ belong to the EQ associated with the space-time surfaces considered.

The condition that the phase is a root of unity is more general than the condition of semiclassical approximation of wave mechanics stating that the action is quantized as a multiple of Planck constant h. The analog of this condition would imply $exp(i\Delta S) = 1$. This quantization condition would make S obsolete.

What about the number theoretical universality of $exp(\Delta K)$? One can consider three options.

1. p-Adic exponent function $exp(x)$ exists if the p-adic norm of x is smaller than 1. The problem is that the p-adic exponent function and its real counterpart behave very differently [K6]. In particular, $exp(x)$ is not periodic. Integer powers of e^p are however ordinary p-adic number by its Taylor series and roots of e define finite-D extensions of p-adic number fields. Therefore $exp(\Delta K)$ could make sense as an integer power for a root of e.

   If ΔK is integer, $exp(\Delta K)$ exists p-adically for primes p dividing ΔK.

2. Also $p^{\Delta K/log(p)}$ could exist p-adically if $\Delta K/log(p)$ is integer. This implies strong conditions. ΔK must be of form $\Delta K = log(p)m$, m integer. If ΔK corresponds to Kähler function of WCW, p is fixed and would define the sought-for preferred p-adic prime p defining the PLS.

3. Since the powers p^n converge to zero for $n \to \infty$, one can formally replace $exp(\Delta K)$ with $exp(\Delta K) = p^{\Delta K)/log(p)}$ and require that the exponent is an integer. The replacement of the ordinary Boltzman weights with powers of p is indeed carried out in p-adic thermodynamics [K1]. This suggests that the Boltzman factors of p-adic thermodynamics reduce to exponents $p^{\Delta K}$ at the level of WCW.

3 Hierarchy of Planck constants, Nottale's hypothesis, and TGD

3.1 Nottale's hypothesis

Nottale's hypothesis [E1] and its generalization to TGD [K10, K4] has non-relativistic and relativistic forms.

1. The non-relativistic formula for $\hbar_{gr}$ as given by the Nottale's formula

$$\begin{array}{l} \hbar_{gr} = \frac{GMm}{\beta_0} \ , \\ \alpha_{gr} = \frac{GMm}{4\pi\hbar_{gr}} = \frac{\beta_0}{4\pi} \ . \end{array} \tag{3.1}$$

   The formula makes sense only $\hbar_{gr}/\hbar > 1$.

2. The relativistically invariant formula for $\hbar_{gr}$ reads for four momenta $P = (M, 0)$ $p = (E, p_3)$ as:

$$\hbar_{gr} = \frac{GP \cdot p}{\beta_0} = \frac{GME}{\beta_0} = \frac{r_s E}{2\beta_0} \ , \tag{3.2}$$

   where r_s is Schwartschild radius. Adelic physics implies that momentum components belong to an extension of rationals defining the adele so that the spectrum of E and of $\hbar_{gr}$ are discretized.

3.1.1 Nottale's hypothesis and biology

Nottale's hypothesis involves a lot of uncertainties also at the conceptual level. Hence it is important to see whether basic facts from TGD inspired biology support the Nottale's hypothesis.

1. The cyclotron frequencies in an "endogenous" magnetic field $B_{end} = 2B_E/5$, where $B_E = .5$ Gauss is the nominal value of the Earth's magnetic emerge in the explanation of the findings of Blackman and other [J1] showing that ELF photons have effects on vertebrate brain. B_{end} is assigned with the monopole flux tubes of B_E. Also lower and higher values of B_{end} can be considered and the models of hearing [K2] and genetic code [K3] suggests that the values of B_{end} correspond to the notes of 12-note scale. This suggests that also the Z^0 magnetic field might be involved.

2. Biophoton energies are in visible and UV range and in the TGD based model they are assumed to result in the transformations of dark photons with much smaller frequency but same energy to ordinary photons. For instance, photons with 10 Hz frequency can transform to biophotons. By $E = h_{eff}f$, requires $h_{eff} = h_{gr}$. The implication is that cyclotron energies do not depend on particle mass. Furthermore, Schwartschild radius $r_S = .9$ cm of Earth defines universal gravitational Compton length for $\beta_0 = 1/2$.

Assume that $\hbar_{gr}$ corresponds to Earth mass and $\beta_0 = 1/2$ and consider cyclotron states in $B_{end} = .2$ Gauss.

1. The value of $r = \hbar_{gr}/\hbar$ for proton is given as the ratio r_s/L_p, where L_p is the Compton radius of proton. This gives $r = .833 \times 10^{13}$. For ions with mass number A the value of r is scaled to Ar.

2. What is the cyclotron energy associated with the 10 Hz frequency in this case? The energy of a photon with frequency f is for $\hbar_{gr}(m_p)$ given by $E_c/eV = r \times 1.24 \times (f/(3 \times 10^{14} Hz)$. Proton's cyclotron frequency is $f_c = 300$ Hz in B_{end} and corresponds to 10 eV, which is in the UV region and rather large.

3. All cyclotron frequencies of charged particles correspond to $E_c = 10$ eV cyclotron energy, which seems rather large. If $\hbar_{gr}$ is reduced by factor 1/4 as required to explain the findings of Mills at least partially, the cyclotron energy becomes 2.5 eV, which is in the visible range. Scaling by factor 1/2 gives cyclotron energy 5 eV in UV.

4. Smaller values of E_c would require smaller fields. The Z_0 charge of proton is roughly a fraction 1/50 of its em charge and since Kähler field contributes also to Z^0 field one would obtain energy about .2 eV in the IR region.

10 Hz alpha frequency which is of special interest concerning understanding of conscious experience and it is interesting to look for concrete numbers.

1. $f = 10$ Hz is alpha frequency and the cyclotron frequency $f_c = 10$ eV Fe^{2+} ion with mass number $A = 56$. Fe^{2+} ions play a central role in biology.

2. For $f = 10$ Hz the energy $\hbar_{gr}(m_p)$ (proton) is .333 eV to be compared with the metabolic energy currency $\sim .5$ eV and is below the visible range.

3. In the TGD inspired biology, 3 proton units represent dark genetic codons and for $\hbar_{gr}(3m_p)$ the energy corresponds to $E \times 1$ eV, which is still slightly below the visible range [L22, L15, L17]. In the dark variant of double DNA strand parallel to the ordinary double strand, the 2 dark codons form a pair by the dark variant of the base pairing so that one has effective $A06$ and $E = 2$ eV, which corresponds to red light.

4. The energy $E = 2$ eV of the codon pair for $f = 10$ Hz corresponds formally to $A = 6$ and would characterize ^6Li. Litium's cyclotron frequency is around $f_c = 50$ Hz is known to have biological significance. Li is used in the treatment of depression [K14]. One might imagine that the coupling of Lithium to dark codon pairs might be involved.

5. For higher mass numbers, the energies for 10 Hz and $\hbar_{gr}(Am_p)$ belong to the UV region. For oxygen one with $A = 16$ has $E = 5.3$ eV, which could correspond to some important molecular transition energy. Molecular bond dissociation energies (`https://cutt.ly/3QoZxY9`) vary in the range .03 -10 eV. O-H, O=O ad O=CO bond energies are somewhat above 5 eV. The idea indeed is that the transformation of dark photons to ordinary bio-photons allows a control of molecular biochemistry.

6. DNA codons have charge proportional to mass and in a good approximation one has $f_c(DNA) = 1$ Hz independently of the length of the DNA strand. For $\hbar_{gr}(Fe^{++})$ $f_c(DNA)$ would correspond to $E = 1.86$ eV in the range of visible energies.

3.2 Trying to understand $\hbar_{eff}$ and $\hbar_{gr}$

Although $\hbar_{eff}$and $\hbar_{gr}$ have become an essential part of quantum TGD, there are still many poorly understood aspects related to them.

3.2.1 Should one introduce a hierarchy of poly-local Planck constants?

The ordinary Planck constant is a universal constant and single-particle entity and serves as a quantization unit for local charges. $\hbar_{gr}$ depends on the masses of the members of the interacting systems and a bi-local character. This suggests that one should not mix these notions.

Both $\hbar_{gr}$ and its possible generalization to gauge interactions such as $\hbar_{em}$, would depend on the charges of the interacting particles. If they serve as charge units, the charges must be bilocal.

Should one introduce a hierarchy of poly-local Planck constants? Later a possible interpretation in terms of Yangian symmetries [A1] [B4, B2], which involve poly-local charges, will be considered. Each multi-local contribution to charge would involve its own Planck constant determined number theoretically.

Standard quantization rules for observables use $\hbar$ as a basic unit. Should one modify these rules by replacing $\hbar$ with (say) $\hbar_{em} = q_1q_2e^2/\alpha$ for $q_1q_2\alpha \geq 1$? Could these rules hold true at magnetic flux tubes characterized by $\hbar_{em}$? Could the charge units for the matter in the non-perturbative phase be q_1q_2-multiples of the ordinary basic units? Could one find empirical evidence for the scaling up of the quantization unit in non-perturbative phases?

In order to avoid total confusion, one must distinguish clearly between the single particle Planck constant and its 2-particle and n-particle variants as Yangian picture suggests. One must also distinguish between p-adic CCE as a discrete counterpart of ordinary CCE and dark coupling constant evolution.

3.2.2 The counterpart of $\hbar_{gr}$ for gauge interactions

The gauge couplings g_i for various interactions disappear completely from the basic formulation of TGD since they are automatically absorbed into the definition of the induced gauge potentials. Hence $\beta_0/4\pi \equiv \alpha_K$ appears as a coupling parameter in the perturbative expansion based on the exponent of Kähler function. $\hbar$ or $\hbar_{eff}$ appear as charge unit only in the definition of conserved charges as Noether charges but not in the action exponential.

The generalization of the Nottale formula to other interactions is not quite obvious. Two-particle Planck constant $h_{eff}(2)$ is in question and GMm would be replaced with the product $q_1q_2g_i^2$. Since α_K determines all other coupling strengths so that it is enough to consider it.

The parameter $\beta_0/4\pi$ is analogous to fine structure constant since gravitational perturbative expansion is in powers of it [K10] [L19].

β_0 is the gravitatoinal counterpart of the dimensionless coupling strength α_K defined in the QFT framework as a derived quantity $\alpha_K = g_K^2/4\pi$ but identified in the TGD context as the fundamental parameter appearing in Kähler action.

In TGD e does not appear as gauge coupling at the fundamental level (as opposed to QFT limit) but one can *define* e^2 as $e^2 = 4\pi\alpha\hbar$. α would obey p-adic CCE and $\hbar$ would be universal constant at single particle level. For dark phases, for which one has $h_{eff} > h$, $\alpha(1) \propto 2/n$, n dimension of the extension would hold true.

Consider the analog of the Nottale formula for em interactions. The coupling strength would be $q_1q_2e^2$ and for $q_1q_2e^2\alpha > 1$, one would have

$$\hbar_{em}(2) = \frac{q_1q_2e^2}{\beta_0} \quad . \tag{3.3}$$

This would give $\alpha_{em}(2) = \beta_0$. For $\beta_0 = \alpha$, one would obtain a coupling parameter α instead of $q_1q_2\alpha$ and the interpretation would be in terms of a transition to non-perturbative phase.

Does this phase transition correspond to a transition to dark phase? Could one interpret the phase transition by saying the dimension of extension is scaled by $n = \hbar_{em}(2)/\hbar$ identified as scaling of the dimension of extension of rationals?

Number theoretic vision predicts that in the dark evolution h_{eff} scales as n, the dimension of extension of rationals for all values of particle number in the definition of $h_{eff}(h)$ so that the single particle coupling constant strength would behave like $1/n$.

3.2.3 Charge fractionalization and the value of $\hbar_{eff}$

$\hbar_{eff} < \hbar$ implies charge fractionalization at the level of imbedding space. This inspires the question whether an analog of fractional quantum Hall effect could be in question. This is not the case.

1. The TGD based model for anyons [K8] relies on the observation that the unit for the fractional quantization of transverse conductance in fractional quantum Hall effect (FQHE) as

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \quad , \\ \nu &= \frac{n}{m} \quad . \end{aligned} \tag{3.4}$$

   The proposal is that FQHE could be understood as integer quantum Hall effect corresponding to $n \rightarrow kn$ for $h_{eff} = km\hbar$. $k = 1$ is the simplest possibility. Interestingly, the observed values of m are primes [D1]: they would correspond to simple Galois groups Z_p in the TGD framework.

2. The fractionalization of charges could be understood at space-time level by noticing that n-sheetedness can be realized as analog of analytic function $z^{1/n}$. n full 2π turns are needed to return to the original point at space-time level so that it is possible to have fractional spin as multiples of $\hbar/n$. The many-particle states however have half-integer spin always since they correspond to representations of the Lorenz group as a symmetry group of $M^4 \times CP_2$. The action of rotations by multiples of 2π would correspond to the action of the Galois group.

These two apparently conflicting mechanisms of charge fractionization correspond to two views about symmetries: either their act at the level of the embedding space or of space-time.

3. For $GMm/v_0 < \hbar$ one would have formally $\hbar_{eff} < \hbar$. Could this option make sense and give rise to a charge fractionalization? One can argue that for $\hbar_{gr} < \hbar_0$, $\hbar_0$ serves as the quantization unit and holds at the level of ordinary matter. This would give a condition $GMm \leq \beta_0$ to the product Mm of the masses involved.

   A stronger condition would hold true at single particle level and state $M/M_{Pl} \geq \sqrt{\beta_0}$ (or $M/M(CP_2) \geq \sqrt{\beta_0}$) for both masses involved. Dark quantum gravity would hold true only above Planck masses. In applications to elementary particle level this would require quantum coherent states of particles with total mass not smaller than Planck mass. Interestingly, a water blob with the size of a large cell has this size for $\beta_0 = 1/2$ [L11].

3.2.4 What does the dependence of $\hbar_{gr}$ on particle masses mean?

$\hbar_{gr}$ depends on two masses. How could one interpret this geometrically?

1. The interpretation has been that a particle with energy E (and mass m) experiences the gravitational field of mass M via gravitational flux tubes characterized by $h_{gr} = GME/v_0$ so that every particle has its specific gravitational flux tubes.

2. Could the thickness of the gravitational flux tubes correspond to the ordinary Compton length λ_c or gravitational Compton length $\lambda_{gr} = GM/v_0$? λ_c decreases with mass and λ_{gr} looks a more reasonable option concerning gravitational interaction.

3. At least static gravitational fields are analogous to static electric fields and in many-sheeted space-time the voltages as analogs of gravitational potential difference are the same along different space-time sheets. The same should hold for gravitational potential.

   Could one assume that gravitational potential has almost copies at all parallel sheets of the many-sheeted space-time (parallel with respect to M^4). Could these sheets correspond to different particle masses so that a particle with a given mass would have its own space-time sheet to represent its interactions with the central mass M.

4. These classical fields would be somehow represented by Kähler magnetic flux tubes carrying generalized Beltrami fields [B1, B6, B3, B5] having also an electric part.

   Could these flux tubes somehow also represent the classical gravitational field? Could the electric part for the induced M^4 Kähler form predicted by the twistor lift of TGD [K9, K12] giving rise to CP breaking, give a representation for the gravitational potential? Could this concretely realize the analogy between gravitation and electromagnetism?

5. A possible realization of this picture would be a fractal structure consisting of flux tubes within flux tubes emanating from the central mass. The radii of the flux tubes would decrease with m as long as $GMm/\beta > \hbar$ holds true. For smaller masses, the flux tube radius would correspond to Compton length.

   Fractal structures known as fractons (`https://en.wikipedia.org/wiki/Fracton`) are the recent hot topic of condensed matter physics (`https://cutt.ly/YQjqyjJ`. The explanation requires the replacement of the time evolution as a time translation with a scaling and condensed matter lattice would be replaced with fractal. These phases have exotic properties: in particular, thermal equilibrium need not be possible. There are also long range correlations due to fractality, which makes these phases ideal for quantum computation.

   In the TGD framework, the time evolutions between SSFRs are indeed generated by the scaling operator L_0 of super-conformal algebra and many-sheeted space-time is indeed both p-adic and dark fractal. The hierarchy of Planck constants makes possible quantum coherence in all scales.

3.2.5 Yangian symmetry and poly-local Planck constants

The product structure of $\hbar_{gr}$ and $\hbar_{em}$ has remained a mystery since it suggests that it characterizes the interaction of 2 space-time sheets whereas the ordinary Planck constant serves as a quantization unit for single particle states. Instead of a Galois group for a single space-time sheet, one would have a product of Galois groups for the two space-time sheets determined as roots for the polynomials in the product. Therefore one should write $\hbar_{gr} = \hbar_{eff,2}$ to distinguish it from a single particle Planck constant $\hbar_{eff}(1) \equiv \hbar_{eff}$.

1. In the TGD framework, wormhole contacts connecting two space-time sheets with Minkowskian signature are indeed building bricks of elementary particles and fundamental fermions appearing as building bricks of elementary particles would be associated with the throats of the wormhole contact.

   Could the two Minkowskian sheets be microscopically k-sheeted entities with sheets parallel to M^4 and perhaps determined as roots of a polynomial of degree k and having Galois group with order m? The maximal Galois group would be S_k with $m = k!$.

   The scaling of $\hbar_0 \rightarrow \hbar(2)$ would mean that the pairs of these space-time surface sheets decompose to $\hbar(2)/\hbar_0$ pairs as orbit of $Gal \times Gal$ contributing to various quantum numbers a contribution proportional $\hbar_{gr}(2)/\hbar_0 = n_1 n_2 = k_1 k_2 m^2$.

   The quantization unit would be $\hbar_0(2)$ for 2-particle quantities such as relative angular momentum. Spin is however thought to be single particle observable. The ordinary phase has a single-particle Planck constant as $\hbar(1)/\hbar_0 = m$.

2. There is no obvious reason for excluding the values of single particle $h_{eff}(1)/h_0$, which are considerably smaller than m or even equal to the minimal value $h_{eff}(1) = h_0$: they would correspond to Galois groups with smaller orders than $m = k!$ of say S_k.

   These exotic particles would have charge and spin units considerably smaller than $\hbar = mh_0$. Why have they not been observed (the findings of Mills are a possible exception and anyonic charge fractionization seems to be a different phenomenon)? Are these space-time sheets somehow unstable? Does gravitation somehow select the Galois group of stable ground state space-time surface so that R as a fundamental length scale is replaced with l_P as effective fundamental length?

3. Yangian algebras [A1] [B4, B2] involve besides single particle observables also $n > 1$-particle observables. Conserved charges have poly-local components which depend on n particles. Note that interaction energy represented as a potential energy is the simplest example about non-local 2-particle contribution to conserved energy.

   Yangian algebras are proposed to be central for TGD [L3] and would reflect the replacement of the space-time locality with locality at the level of "world of classical worlds" (WCW) due to the replacement of a point like with a 3-surface, which can also consist of disjoint parts. Yangian picture suggests that single-particle $\hbar$ has n-particle generalization. The possible number theoretical rule could be

   $$\frac{\hbar_{gr,n}}{\hbar_0} = \prod_k n_k \ , \qquad (3.5)$$

   where n_k correspond to the orders of Galois groups associated with the space-time sheets involved.

3.3 h/h_0 as the ratio of Planck mass and CP_2 mass?

Could one understand and perhaps even predict the value of h_0? Here number theory and the notion of n-particle Planck constant $h_{eff}(n)$ suggested by Yangian symmetry could serve as a guidelines.

1. Hitherto I have found no convincing empirical argument fixing the value of $r = \hbar/\hbar_0$: this is true for both single particle and 2-particle case.

   The value $h_0 = \hbar/6$ [L1] as a maximal value of $\hbar_0$ is suggested by the findings of Randell Mills [D2] and by the idea that spin and color must be representable as Galois symmetries so that the Galois group must contain $Z_6 = Z_2 \times Z_3$. Smaller values of h_0 cannot be however excluded.

2. A possible manner to understand the value r geometrically would be following. It has been assumed that CP_2 radius R defines a fundamental length scale in TGD and Planck length squared $l_P^2 = \hbar G = x^{-2} \times 10^{-6} R^2$ defines a secondary length scale. For Planck mass squared one has $m_{Pl}^2 = m(CP_2, \hbar)^2 \times 10^6 x^2$, $m(CP_2, \hbar)^2 = \hbar/R^2$. The estimate for x from p-adic mass calculations gives $x \simeq 4.2$. It is assumed that CP_2 length is fundamental and Planck length is a derived quantity.

   But what if one assumes that Planck length identifiable as CP_2 radius is fundamental and CP_2 mass corresponds the minimal value h_0 of $h_{eff}(2)$? That the mass formula is quadratic and mass is assignable to wormhole contact connecting two space-time sheets suggests in the Yangian framework that $h_{eff}(2)$ is the correct Planck constant to consider.

One can indeed imagine an alternative interpretation. CP_2 length scale is deduced indirectly from p-adic mass calculation for electron mass assuming $h_{eff} = h$ and using Uncertainty Principle. This obviously leaves the possibility that $R = l_P$ apart from a numerical constant near unity, if the value of h_{eff} to be used in the mass calculations is actually $h_0 = (l_P/R)^2\hbar$. This would fix the value of $\hbar_0$ uniquely.

The earlier interpretation makes sense if $R(CP_2)$ is interpreted as a dark length scale obtained scaling up l_P by $\hbar/\hbar_0$. Also the ordinary particles would be dark.

h_0 would be very small and $\alpha_K(\hbar_0) = (\hbar/\hbar_0)\alpha_K$ would be very large so that the perturbation theory for it would not converge. This would be the reason for why $\hbar$ and in some cases some smaller values of h_{eff} such as $\hbar/2$ and $\hbar/4$ [D2] [L1] seem to realized.

For $R = l_P$ Nottale formula remains unchanged for the identification $M_P = \hbar/R$ (note that one could consider also $\hbar_0/R$ used in p-adic mass calculations).

3.3.1 Various options

Number theoretical arguments allow to deduce precise value for the ratio $\hbar/\hbar_0$. Accepting the Yangian inspired picture, one can consider two options for what one means with $\hbar$.

1. $\hbar$ refers to the single particle Planck constant $\hbar_{eff}(1)$ natural for point-like particles.

2. $\hbar$ refers to $h_{eff}(2)$. This option is suggested by the proportionality $M^2 \propto \hbar$ in string models due to the proportionality $M^2 \propto \hbar/G$ in string models. At a deeper level, one has $M^2 \propto L_0$, where L_0 is a scaling generator and its spectrum has scale given by $\hbar$.

   Since M^2 is a p-adic thermal expectation of L_0 in the TGD framework, the situation is the same. This also due the fact that one has In TGD framework, the basic building bricks of particles are indeed pairs of wormhole throats.

One can consider two options for what happens in the scaling $h_{eff} \rightarrow kh_{eff}$.

Option 1: Masses are scaled by k and Compton lengths are unaffected.

Option 2: Compton lengths are scaled by k and masses are unaffected.

The interpretation of $M_P^2 = (\hbar/hbar_0)M^2(CP_2)$ assumes Option 1 whereas the new proposal would correspond to Option 2 actually assumed in various applications.

The interpretation of $M_P^2 = (\hbar/hbar_0)M^2(CP_2)$ assumes Option 1 whereas the new proposal would correspond to Option 2 actually assumed in various applications.

For Option 1 $m_{Pl}^2 = (\hbar_{eff}/\hbar)M^2(CP_2)$. The value of $M^2(CP_2) = \hbar/R^2$ is deduced from the p-adic mass calculation for electron mass. One would have $R^2 \simeq (\hbar_{eff}/\hbar)l_P^2$ with $\hbar_{eff}/\hbar = 2.54\times 10^7$. One could say that the real Planck length corresponds to R.

3.3.2 Quantum-classical correspondence favours Option 2)

In an attempt to select between these two options, one can take space-time picture as a guideline. The study of the imbeddings of the space-time surfaces with spherically symmetric metric carried out for almost 4 decades ago suggested that CP_2 radius R could naturally correspond to Planck length l_P. The argument is described in detail in Appendix and shows that the $l_P = R$ option with $h_{eff} = h$ used in the classical theory to determine α_K appearing in the mass formula is the most natural.

3.3.3 Deduction of the value of $\hbar/\hbar_0$

Assuming Option 2), the questions are following.

1. Could $l_P = R$ be true apart from some numerical constant so that CP_2 mass $M(CP_2)$ would be given by $M(CP_2)^2 = \hbar_0/l_P^2$, where $\hbar_0 \simeq 2.4 \times 10^{-7}\hbar$ ($\hbar$ corresponds to $\hbar_{eff}(2)$) is the minimal value of $\hbar_{eff}(2)$. The value of h_0 would be fixed by the requirement that classical theory is consistent with quantum theory! It will be assumed that $\hbar_0$ is also the minimal value of $\hbar_{eff}(1)$ both $\hbar_{eff}(2)$.

2. Could $\hbar(2)/\hbar_0(2) = n_0$ correspond to the order of the product of identical Galois groups for two Minkowskian space-time sheets connected by the wormhole contact serving as a building brick of elementary particles and be therefore be given as $n_0 = m^2$?

Assume that one has $n_0 = m^2$.

1. The natural assumption is that Galois symmetry of the ground state is maximal so that m corresponds to the order a maximal Galois group - that is permutation group S_k, where k is the degree of polynomial.

   This condition fixes the value k to $k = 7$ and gives $m = k! = 7! = 5040$ and gives $n_0 = (k!)^2 = 25401600 = 2.5401600 \times 10^7$. The value of $\hbar_0(2)/\hbar(2) = m^{-2}$ would be rather small as also the value of $\hbar_0(1)\hbar(1)$. p-Adic mass calculations lead to the estimate $m_{Pl}/m(CP_2) = \sqrt{m}m(CP_2) = 4.2 \times 10^3$, which is not far from $m = 5040$.

2. The interpretation of the product structure $S_7 \times S_7$ would be as a failure of irreducibility so that the polynomial decomposes into a product of polynomials - most naturally defined for causally isolated Minkowskian space-time sheets connected by a wormhole contact with Euclidian signature of metric representing a basic building brick of elementary particles.

   Each sheet would decompose to 7 sheets. $\hbar_{gr}$ would be 2-particle Planck constant $h_{eff}(2)$ to be distinguished from the ordinary Planck constant, which is single particle Planck constant and could be denoted by $h_{eff}(1)$.

   The normal subgroups of $S_7 \times S_7$ $S_7 \times A_7$ and $A_7 \times A_7$, S_7, A_7 and trivial group. A_7 is simple group and therefore does not have any normal subgroups expect the trivial one. S_7 and A_7 could be regarded as the Galois group of a single space-time sheet assignable to elementary particles. One can consider the possibility that in the gravitational sector all EQs are extensions of this extension so that $\hbar$ becomes effectively the unit of quantization and m_{Pl} the fundamental mass unit. Note however that for very small values of α_K in long p-adic length scales also the values of $h_{eff} < h$, even h_0, are in principle possible.

   The large value of $\alpha_K \propto 1/\hbar_{eff}$ for Galois groups with order not considerably smaller than $m = (7!)^2$ suggests that very few values of $h_{eff}(2) < h$ are realized. Perhaps only $S_7 \times S_7$ $S_7 \times A_7$ and $A_7 \times A_7$ are allow by perturbation theory. Now however that in the "stringy phase" for which super-conformal invariance holds true, h_0 might be realized as required by p-adic mass calculations. The alternative interpretation is that ordinary particles correspond to dark phase with R identified dark scale.

3. A_7 is the only normal subgroup of S_7 and also a simple group and one has $S_7/A_7 = Z_2$. $S_7 \times S^7$ has $S_7 \times S^7/A_7 \times A_7 = Z_2 \times Z_2$ with $n = n_0/4$ and $S_7 \times S^7/A_7 \times S_7 = Z_2$ with $n = n_0/2$. This would allow the values $\hbar/2$ and $\hbar/4$ as exotic values of Planck constant.

The atomic energy levels scale like $1/\hbar^2$ and would be scaled up by factor 4 or 16 for these two options. It is not clear whether $\hbar \rightarrow \hbar/2$ option can explain all findings of Randel Mills [D2] in TGD framework [L1], which effectively scale down the principal quantum number n from n to $n/2$.

4. The product structure of the Nottale formula suggests

$$n = n_1 \times n_2 = k_1 k_2 m^2 \ . \tag{3.6}$$

Equivalently, n_i would be a multiple of m. One could say that $M_{Pl} = \sqrt{\hbar/\hbar_0} M(CP_2)$ effectively replaces $M(CP_2)$ as a mass unit. At the level of polynomials this would mean that polynomials are composites $P \circ P_0$ where P_0 is ground state polynomial and has a Galois group with degree n_0. Perhaps S_7 could be called the gravitational or ground state Galois group.

3.4 Connection with adelic physics and infinite primes

The structure of $\hbar_{gr}$ and its electromagnetic counterpart $\hbar_{em}$ characterize 2-particle states whereas $\hbar$ characterizes single particle state. Yangian picture suggests that the notion of $\hbar_{eff}(n)$, $n = 1, 2, ..$ makes sense.

One can decompose a state consisting of N particles in several manners to partitions consisting of m subsets with n_i, $i = 1, ..., n$ in a given subset of particles. Could these subsets correspond to gravitationally bound states so that one can take these sets as basic entities characterized by masses and assume that gravitational interactions reduce to gravitational interactions between them and are quantal for $GM_iM_j/v_0 \geq \hbar$. Same question applied to electromagnetic, weak and color interactions.

3.4.1 Connection with adelic physics

This picture would have analog at the level of adelic physics [L12, L13, L18].

1. In the M^8 picture space-time surfaces correspond to "roots" of complexified octonionic polynomials obtained from irreducible real polynomials with rational (or perhaps even algebraic) coefficients. The dynamics realizes associativity of the normal space of the complexified space-time surface having 4-D space-time surface as real part mapped from M^8 to $H = M^4 \times CP_2$ by $M^8 - H$ correspondence.

2. One can consider irreducible polynomials of several variables such that the additional variables are interpreted as parameters [L16]. The parametrized set of polynomials defines a parametrized set of space-time surfaces and one can have a superposition of quantum states corresponding to irreducible polynomial of degree n and products of irreducible polynomials with sum of degrees n_i equal to n. This kind of parametrized set could define sub-spaces of the "world of classical worlds" (WCW).

3. Irreducibility fails for some parameter values forming lower-dimensional manifolds of the parameter space. The failure of the irreducibility means decomposition to a product of polynomials in which the set of roots decomposes to subsets R_i, which are roots of a rational polynomial with a lower degree n_i. Spacetime surface as a coherent structure decomposes to uncorrelated space-time surfaces with a discrete set of points as intersections. In this manner one obtains a decomposition of the parameter space to subsets of decreasing dimension. The generic situation has maximal dimension and dimension equal to that of the parameter space.

4. The catastrophe theory [A3] founded by Rene Thom studies these situations. In catastrophe theory, the failure of the irreducibility is of very special nature and means that some roots of the polynomial co-incide and become multiple roots. For polynomials with rational coefficients, they would become multiple rational roots so that the degree of the polynomial determining the extension would be reduced by two units. This is discussed in detail from TGD point of view in [L16]. For polynomials with rational coefficients, typically complex conjugate roots become rational and the dimension of the algebraic extension is reduced.

5. The quantum state defined by the polynomial of several variables would be a superposition of space-time surfaces labelled by the points of the parameter space. It would decompose to subsets defining what is known as a stratification. The subsets for which the polynomial fails to be irreducible would have lower dimension. For polynomials with rational coefficients these sets would be discrete and it is not clear whether the lower-dimensional sets are non-empty in the generic case.

6. The decomposition to k irreducible polynomials with degrees n_i, $i = 1, ..., k$ would correspond to a decomposition of the space-time surface to separate space-time surfaces with $h_{gr,i} = n_i h_0 = GM_i m/v_0$ (same applies to h_{em}) satisfying $\sum n_i = n$. These would correspond to different decompositions of the total energy to a sum of energies E_i: $E = \sum E_i$. The irreducible polynomials with degree n_i could be interpreted as bound states for a subset of basic units. Maximal decomposition would correspond to $n_i = 1$ and have interpretation as a set of elementary particles with $h_{eff} = h_0$ (note that $h = 6h_0$ in the proposal inspired by the findings of Randel Mills [L1]).

3.4.2 Connection with infinite primes

The notion of infinite prime [K11] resonates with this picture.

1. The hierarchy of infinite primes has an interpretation as a repeated second quantization of supersymmetric arithmetic QFT. Polynomial primes of variable polynomials of single variable with rational coefficients follow ordinary primes in the hierarchy. Higher levels correspond to polynomial primes for polynomials of several variables and second quantization corresponds to the formation of polynomials of single variable with coefficients as polynomials of $n-1$ variables.

   Irreducible polynomials of higher than first order have interpretation as bound states whereas polynomials reducing to products of monomials correspond to Fock states of free particles.

2. The beatiful feature would be a number theoretic description of also bound states. The description of the particle decays as a failure of the irreducibility of the polynomials corresponding to infinite primes would extend this picture to the dynamics.

3. Second beautiful feature is the number theoretic description of particle reactions. Particle reactions with unentagled final states would naturally correspond to a situation in which the initial (prepared) and final (state function reduced) states are products of polynomials. Interaction period would correspond to an irreducible polynomial.

   This picture conforms with the proposal inspired originally by a model of "cold fusion". unnelling phenomenon crucial for nuclear reactions would correspond to a formation of dark phase in which the value of h_{eff} increases [L9, L2, L14]. This picture generalizes to all particle reactions.

4 How to understand coupling constant evolution?

In this section, the evolutions of Kähler coupling strength α_K and gravitational fine structure constant α_{gr} are discussed. The reason for restricting to α_K is that it is expected to induce the evolution of various gauge couplings, and could also induce the evolution of α_{gr}.

4.1 Evolution of Kähler coupling strength

The evolution of Kähler coupling strength $\alpha_K = g_K^2/2h_{eff}$ gives the evolution of α_K as a function of dimension n of EQ: $\alpha_K = g_K^2/2nh_0$. If g_K^2 corresponds to electroweak U(1) coupling, it is expected to evolve also with respect to PLS so that the evolutions would factorize.

Note that the original proposal that g_K^2 is renormalization group invariant was later replaced with a piecewise constancy: α_K has indeed interpretation as piecewise constant critical temperature

1. In the TGD framework, coupling constant as a continuous function of the continuous length scale is replaced with a function of PLS so that coupling constant is a piecewise constant function of the continuous length scale.

   PLSs correspond to p-adic primes p, and a hitherto unanswered question is whether the extension determines p and whether p-adic primes possible for a given extension could correspond to ramified primes of the extension appearing as factors of the moduli square for the differences of the roots defining the space-time surface.

   In the M^8 picture the moduli squared for differences $r_i - r_j$ of the roots of the real polynomial with rational coefficients associated with the space-time surfaces correspond to energy squared and mass squared. This is the case of p-adic prime corresponds to the size scale of the CD.

   The scaling of the roots by constant factor however leaves the number theoretic properties of the extension unaffected, which suggests that PLS evolution and dark evolution factorize in the sense that PLS reduces to the evolution of a power of a scaling factor multiplying all roots.

2. If the exponent $\Delta K/log(p)$ appearing in $p^{\Delta K/log(p))} = exp(\Delta K)$ is an integer, $exp(\Delta K)$ reduces to an integer power of p and exists p-adically. If ΔK corresponds to a deviation from the Kähler function of WCW for a particular path in the tree inside CD, p is fixed and $exp(\Delta K)$ is integer. This would provide the long-sought-for identification of the preferred p-adic prime. Note that p must be same for all paths of the tree. p need not be a ramified prime so that the trouble-some correlation between n and ramified prime defining padic prime p is not required.

3. This picture makes it possible to understand also PLS evolution if ΔK is identified as a deviation from the Kähler function. $p^{\Delta K/log(p))} = exp(\Delta K)$ implies that ΔK is proportional to $log(p)$. Since ΔK as 6-D Kähler action is proportional to $1/\alpha_K$, $log(p)$-proportionality of ΔK could be interpreted as a logarithmic renormalization factor of $\alpha_K \propto 1/log(p)$.

4. The universal CCE for α_K inside CDs would induce other CCEs, perhaps according to the scenario based on Möbius transformations [L6].

4.1.1 Dark and p-adic length scale evolutions of Kähler coupling strength

The original hypothesis for dark CCE was that $h_{eff} = nh$ is satisfied. Here n would be the dimension of EQ defined by the polynomial defining the space-time surface $X^4 \subset M^8_c$ mapped to H by $M^8 - H$ correspondence. n would also define the order of the Galois group and in general larger than the degree of the irreducible polynomial.

Remark: The number of roots of the extension is in general smaller and equal to n for cyclic extensions only. Therefore the number of sheets of the complexified space-time surface in M^8_c as the number of roots identifiable as the degree d of the irreducible polynomial would in general be smaller than n. n would be equal to the number of roots only for cyclic extensions (unfortunately, some former articles contain the obviously wrong statement $d = n$).

Later the findings of Randell Mills [D2], suggesting that h is not a minimal value of h_{eff}, forced to consider the formula $h_{eff} = nh_0$, $h_0 = h/6$, as the simplest formula consistent with the findings of Mills [L1]. h_0 could however be a multiple of even smaller value of h_{eff}, call if h_0 and the formula $h_0 = h/6$ could be replaced by an approximate formula.

The value of $h_{eff} = nh_0$ can be understood by noticing that Galois symmetry permutes "fundamental regions" of the space-time surface so that action is n times the action for this kind of region. Effectively this means the replacement of α_K with α_K/n and implies the convergence of the perturbation theory. This was actually one of the basic physical motivations for the hierarchy of Planck constants. In the previous section, it was argued that h_0 is given by the ratio l_P/R of Planck length and CP_2 length scale identified as dark scale and equals to $n_0 = (7!)^2$.

The basic challenge is to understand p-adic length scale evolutions of the basic gauge couplings. The coupling strengths should have a roughly logarithmic dependence on the p-adic length scale $p \simeq 2^{k/2}$ and this provides a strong number theoretic constraint in the adelic physics framework.

Since Kähler coupling strength α_K induces the other CCEs it is enough to consider the evolution of α_K.

4.1.2 p-Adic CCE of α from its value at atomic length scale?

If one combines the observation that fine structure constant is rather near to the inverse of the prime $p = 137$ with PLS, one ends up with a number theoretic idea leading to a formula for α_K as a function of p-adic length scale.

1. The fine structure constant in atomic length scale $L(k = 137)$ is given $\alpha(k) = e^2/2h \simeq 1/137$. This finding has created a lot of speculative numerology.

2. The PLS $L(k) = 2^{k/2}R(CP_2)$ assignable to atomic length scale $p \simeq 2^k$ corresponds to $k = 137$ and in this scale α is rather near to 1/137. The notion of fine structure constant emerged in atomic physics. Is this just an accident, cosmic joke, or does this tell something very deep about CCE?

   Could the formula

   $$\alpha(k) = \frac{e^2(k)}{2h} = \frac{1}{k}$$

   hold true?

There are obvious objections against the proposal.

1. α is length scale dependent and the formula in the electron length scale is only approximate. In the weak boson scale one has $\alpha \simeq 1/127$ rather than $\alpha = 1/89$.

2. There are also other interactions and one can assign to them coupling constant strengths. Why electromagnetic interactions in electron Compton scale or atomic length scales would be so special?

The idea is however plausible since beta functions satisfy first order differential equation with respect to the scale parameter so that single value of coupling strength determines the entire evolution.

4.1.3 p-Adic CCE from the condition $\alpha_K(k = 137) = 1/137$

In the TGD framework, Kähler coupling strength α_K serves as the fundamental coupling strength. All other coupling strengths are expressible in terms of α_K, and in [L6] it is proposed that Möbius transformations relate other coupling strengths to α_K. If α_K is identified as electroweak $U(1)$ coupling strength, its value in atomic scale $L(k = 137)$ cannot be far from 1/137.

The factorization of dark and p-adic CCEs means that the effective Planck constant $h_{eff}(n, h, p)$ satisfies

$$h_{eff}(n, h, p) = h_{eff}(n, h) = nh \quad . \tag{4.1}$$

and is independent of the p-adic length scale. Here n would be the dimension of the extension of rationals involved. $h_{eff}(1, h, p)$ corresponding to trivial extension would correspond to the p-adic CCE as the TGD counterpart of the ordinary evolution.

The value of h need not be the minimal one as already the findings of Randel Mills [D2] suggest so that one would have $h = n_0h_0$.

$$h_{eff} = nn_0h \quad , \quad \alpha_{K,0} = \frac{g^2_{K,max}}{2h_0} = n_0 \quad . \tag{4.2}$$

This would mean that the ordinary coupling constant would be associated with the non-trivial extension of rationals.

Consider now this picture in more detail.

1. Since dark and p-adic length scale evolutions factorize, one has

$$\alpha_K(n) = \frac{g_K^2(k)}{2h_{eff}} \ , \quad h_{eff} = nh_0 \ . \tag{4.3}$$

   $U(1)$ coupling indeed evolves with the p-adic length scale, and if one assumes that $g_K^2(k, n_0)$ ($h = n_0 h_0$) is inversely proportional to the logarithm of p-adic length scale, one obtains

$$\begin{aligned} g_K^2(k, n_0) &= \frac{g_K^2(max)}{k} \ , \\ \alpha_K &= \frac{g_K^2(max)}{2kh_{eff}} \ . \end{aligned} \tag{4.4}$$

2. Since $k = 137$ is prime (here number theoretical physics shows its power!), the condition $\alpha_K(k = 137, h_0) = 1/137$ gives

$$\frac{g_K^2(max)}{2h_0} = \alpha_K(max) = (7!)^2 \ . \tag{4.5}$$

   The number theoretical miracle would fix the value of $\alpha_K(max)$ to the ratio of Planck mass and CP_2 mass $n_0 = M_P/M(CP_2) = (7!)^2$ if one takes the argument of the previous section seriously.

   The convergence of perturbation theory could be possible also for $h_{eff} = h_0$ if the p-adic length scale $L(k)$ is long enough to make $\alpha_K = n_0/k$ small enough.

3. The outcome is a very simple formula for α_K

$$\alpha_K(n, k) = \frac{n_0}{kn} \ , \tag{4.6}$$
$$\tag{4.7}$$

   which is a testable prediction if one assumes that it corresponds to electroweak $U(1)$ coupling strength at QFT limit of TGD. This formula would give a practically vanishing value of α_K for very large values of n associated with h_{gr}. Here one must have $n > n_0$.

   For $h_{eff} = nn_0 h$ characterizing extensions of extension with $h_{eff} = h$ one can write

$$\alpha_K(nn_0, k) = \frac{1}{kn} \ . \tag{4.8}$$

4. The almost vanishing of α_K for the very large values of n associated with $\hbar_{gr}$ would practically eliminate the gauge interactions of the dark matter at gravitational flux tubes but leave gravitational interactions, whose coupling strength would be $\beta_0/4\pi$. The dark matter at gravitational flux tubes would be highly analogous to ordinary dark matter.

4.2 The evolution of the gravitational fine structure constant

Nottale [E1] introduced the notion of gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ ($\beta_0 = v_0/c$ is velocity parameter), which has gigantic values so that the original proposal $h_{gr} = nh_0$ would predict very large values for n. If p-adic and dark evolutions are independent this is not a problem since p-adic length scales need not be gigantic.

4.2.1 Evolution of the parameter β_0

Gravitational fine structure constant is given by $\alpha_{gr} = GMm/4\pi\hbar_{gr} = \beta_0/4\pi$. The basic challenge is to understand the value spectrum of β_0.

1. Kepler's law $\beta^2 = GM/r = r_S/2r$ suggests length scale evolution of form

$$\beta_N = \sqrt{\frac{r_S}{2L(N)x}} = \frac{\beta_{0,max}}{N} \quad . \tag{4.9}$$

   The coefficient x has been included in the formula because otherwise a conflict with Bohr model for planetary orbits results.

2. How to identify N?

   (a) $N = n = h_{gr}/h_0$ would give a gigantic value of N and this would give extremely small value for β_0. Actually $N = n$ for n in $h_{gr} = nh_0$ is impossible as is clear from the defining equation.

   (b) It is not clear whether N be identified as a dimension for some factor in the composition of extension to simple factors rather than as n. This would conform with the vision that there are evolutionary hierarchies of extensions of extensions of... for which the dimension is product of dimensions of the extensions involved.

   (c) The simplest option is that p-adic length scale evolution determines N as in case of the gauge interactions, and it corresponds to k in $p \simeq 2^k$. $log_2(p)$ exists also for a general prime p in real sense. In p-adic sense it exists for all primes except $p = 2$ as integer valued function. $p = 2$ could be chosen to be the exceptional prime.

   This would conform with the idea that gravitational sector and gauge interaction sector correspond to different factors in the decomposition of extension of rationals. Perhaps the gravitational part of EQ extends its gauge part. This would conform with the idea that gravitation does not differentiate between states with different gauge quantum numbers.

What can one say about the value of $\beta_{0,max}$ and its length scale evolution?

1. The value of $\beta_{0,max} = 1/2$ would give for the length scale $L = GM/\beta_{0,max} = r_S$. If one requires that the scale L is not smaller than Scwartschild radius, $\beta_{0,max} \leq 1/2$ follows. $\beta_{0,max} = 1/2$ is the first guess but it turns that number theoretical constraintss exclude it and suggest $\beta_{0,max} = \pi/6$ as the simplest guess.

2. Gravitational Bohr radius a_{gr} given by

$$a_{gr} = \frac{\hbar_{gr}}{\alpha_{gr} m} per. \tag{4.10}$$

   defines a good candidate for the minimal value of L_n as $L_1 = a_{gr}$.

3. The analogs of p-adic length scales would be equal to the radii of gravitational Bohr atom as n^2- multiples of the gravitational Bohr radius a_{gr}:

$$L_n = n^2 a_{gr} \quad , \quad a_{gr} = \frac{4\pi GM}{\beta_0^2} \quad . \tag{4.11}$$

   This expression realizes the condition $\beta_0^2 = xGM/r$ inspired by the Kepler's law with $x = 4\pi$.

4. One must fix a_{gr} as a multiple $a_{gr} = kr_S$ of r_S. Substitution to the above equation gives

$$\beta_{0,max} = \sqrt{\frac{2\pi}{k}} \ .$$

The condition $\beta_{0,max} = 1/2$ would give $k = 8\pi$ and $a_{gr} = 8\pi r_S$ as a minimal radius for a Bohr orbit. The condition $\beta_{0,max} < 1$ gives $k \geq 2\pi$ and $a_{gr} \geq 2\pi r_S$.

Just as in the case of hydrogen atom, the falling of the orbiting system to the blackhole like entity (in TGD frameworkd blackholes are replaced with what might be called flux tube spaghettis [L8, L7]) is prevented. This should have obviously consequences for the view about the dynamics around blackhole like objects. The circular orbits have as analogs s-waves and of these are realized, the falling to blackhole like entity is possible.

5. The proposed formula does not force the condition $\beta_0 < 1$ and it is not clear whether it holds true at the relativistic limit. The replacement $\beta_0 \rightarrow sinh(\eta) = \beta_0/\sqrt{1-\beta_0^2}$, where η is the hyperbolic angle, forces the condition $\beta_0 < 1$, and would give

$$\beta_0 \rightarrow \frac{\beta_0}{\sqrt{1-\beta_0^2}} = \sqrt{\frac{2\pi}{k}} \ .$$

The condition $\beta_{0,max} = 1/2$ gives $k/2\pi = 3$. This would correspond to the minimal Bohr radius $a_{gr} = 6\pi r_S \simeq 18.84 r_S$.

4.2.2 Number theoretical universality as a constraint

Also number theoretical universality could be also used as a constraint. The condition would be that only finite-dimensional extensions are allowed. π defines an infinite-D transcendental extension so that it should disappear in central formulas.

1. The appearance of 4π in the formula $a_{gr} = 4\pi G/\beta_{0,max}^2$ creates number-theoretical worries. Suppose that a_{gr} is a rational number.

2. I have proposed that G is dynamically determined and relates to the CP_2 radius via the formula $G = R^2/\hbar_{grav} = 2\pi R^2/h_{grav}$, where $h_{grav}/h_0 \sim 10^7$ holds true [K4].

This gives

$$a_{gr} = \frac{4\pi G}{\beta_{0,max}^2} = \frac{8\pi^2 R^2}{h_{grav}\beta_{0,max}^2} \tag{4.12}$$

.

3. Since $\beta_0/4\pi$ appears as coupling strength in the perturbation theory, it should also be rational. $\beta_{0,max} = \pi/6$ would realize the condition $\beta_{0,max} = 1/2$ approximately.

4. With this assumption the rationality of a_{gr} requires that h_{gr} is proportional to π so that also G would be rational. This implies that $\hbar_{eff} = h_{eff}/2\pi$ is rational. Also α_K would be rational if g_K^2 is rational. This would be true also for the other coupling constants.

5. $\beta_0 = \pi/6$ would realize the condition $\beta_0 = 1/2$ approximately. This also implies that α_{gr} is rational. The condition $k/2\pi = 1/\beta_{0,max}^2$ implies $k \propto 1/\pi$. $a_{gr} = kr_s = kGM$ is rational, and this requires $M \propto \pi$. This guarantees the rationality of GM/β_0. Gravitational fine structure constant α_{gr} would be an inverse integer multiple of $\alpha_{gr}(max) = 1/24$. It would seem that the system is consistent.

The alternative condition $\beta_0^2/(1-\beta_0^2) = 2\pi/k$ is excluded because it implies that k is a rather complex transcendental.

What makes this interesting is that 24 is one of the magic numbers of mathematics (`https://cutt.ly/RnOxOTr`) and it appears in the bosonic string model as the number of space-like dimensions.

1. Euclidian string world sheet with torus topology has a conformal equivalence class defined by the ratio ω_2/ω_1 of the complex vectors spanning the parallelogram defining torus as an analog of a unit cell. String theory must be invariant under modular group $SL(2, Z)$ leaving the periods and thus the conformal equivalence class of torus invariant. Same applies to higher genera. In TGD these surfaces correspond to partonic 2-surfaces.

2. Modular invariance raises elliptic functions (doubly periodic analytic functions in complex plane) in a special role. In particular, Weierstrass function, which satisfies the differential equation $(d\mathcal{P}/dz)^2 = 4\mathcal{P}^3 - g_2\mathcal{P} - g_3$ has a key role in the theory of elliptic functions (`https://cutt.ly/Bn0xrMS`).

   The discriminant $\Delta = g_2^3 - 3g_2^3$ of the polynomial at the r.h.s can be locally regarded as a function of the ratio of $\tau = \omega_2/\omega_1$ of the periods of $\mathcal{P}$ defining the conformal equivalence class of torus.

   $\Delta(\tau)$ is not a genuine modular invariant function of τ. Rather, Δ defines a modular form of weight 12 transforming as $\Delta(a\tau+b/(c\tau+d)) \to (c\tau+d)^{12}\Delta(\tau)$ under $SL(2, Z)$. The number 24 comes from the fact that one can express Δ as 24^{th} power of the Dedekind η function: $\Delta = (2\pi)^{12}\eta^{24}$.

3. In dimension $D = 24$ there are 24 even positive definite unimodular lattices, called the Niemeier lattices, and the so-called Leech lattice is one of them. Interestingly, in dimension 4 there exists a 24-cell analogous to Platonic solid having 24 octahedrons as its 3-D "faces".

This encourages the question whether there might be a connection between TGD and string theory based views of quantum gravitation.

4.2.3 Test cases for the proposal

Phase transitions changing β_0 are possible at $r_n/a_{gr} = n^2$ at the Bohr orbits. For instance, in the Bohr orbit model the orbit of Earth is such an orbit. It can be regarded as $n = 5$ orbital with $\beta_0 \simeq 2^{-11}$ and is nearly circular so that the phase transition with $n = 1$ orbital with $\beta_0 \to \beta_0/5$ is possible. The outer planets indeed have $\beta_0/5$.

p-Adic length scale hierarchy is replaced union of hierarchies with $\beta_0 = \beta_{0,max}/n = 1/2n$, each of which is a subset of the set of Bohr orbits for $\beta_0 = \beta_{0,max}$. One can test this hypothesis for the proposed applications [L21].

1. In the Bohr orbit model the inner planets Mercury, Venus, and Earth identifiable correspond to $n = 3, 4, 5$ orbitals for $\beta_0 \simeq 2^{-11}$. Solar radius is $R_{Sun} \simeq .7$ Gm. The orbital radius of Mercury is $R_M \simeq 58$ Gm $= 82.9 \times R_{Sun}$. This gives $a_{gr} = R_M/9 \simeq 9.2R_{sun}$. This gives $\beta_0 = \sqrt{2\pi R_S/a_{gr}} \simeq 17.1 * 10^{-4}$.

   The approximation used hitherto has been $\beta_0 = 2^{-11)} \simeq 5 \times 10^{-4}$ and is by a factor about 1/3 smaller. Using $a_{gr} = R_M$ instead of $a_{gr} = R_M/9$ would give roughly correct value.

   One could indeed regard Mercury as $n = 1$ orbit for $v_0 = v_0/3$ in which case one would have $a_{gr} = R_M$ and one would obtain $\beta_0 = .57$ which is not far from the valued used. Mercury would therefore correspond to $n = 3$ dark matter gravitationally whereas Venus must correspond to $n = 1, 2$ or $n = 4$.

2. The transition $\beta_0 \to \beta_0/5$ possible for Earth and required for outer planets could be interpreted as the increase of n having interpretation as increase of dimension of extension of rationals $n \to 5n$.

For the Earth one has $R_E = 6.371 \times 10^6$ m and $r_S = 10^{-2}$ m. The model of the superfluid fountain effect [K5] [L21] suggests $\beta_0 = 1/2$ for which one would have $GM/v_0 = 1/2$. The value of $a_{gr} = 6\pi r_S$ for the relativistic form of the Nottale condition. The principal quantum number n for the Bohr orbit of the super-fluid would be $n \simeq R_E/a_{gr} = R_E/6\pi r_S \simeq 3.4 \times 10^7$. This would correspond to the large quantum number limit. The difference of radii between nearby Bohr orbits would be $\Delta r = 2R_E/n \simeq 19$ cm, which makes sense.

The levels in the hierarchy of gravitationally dark matters are labelled by $h_{gr} = GMm/\beta_0$ with $\beta_0 = \beta_{0,max}/n$, where n is the dimension of EQ, and each level defines a hierarchy of atomic orbitals. The sets of orbital radii at various levels form a nested hierarchy and phase transitions can occur at least between the states with the same angular momentum and orbital radius.

The quantum variant of the similar picture is expected to apply in the case of the hydrogen atom and the fact that there is evidence for dark valence electrons suggests that these phase transitions indeed take place.

What about long cosmic strings thickened to flux tubes explaining galactic dark matter in the TGD framework? In this case the Kepler law gives $\beta^2 = TG$ so that the all orbiting stars would correspond to the same value of β_0 and n.

5 Appendix: Imbedding of spherically symmetric stationary symmetric metric as a guideline

There are two basic questions to be answered.

1. Is $R = l_P$ or $R = m^2 l_P$, $m = 7!$ realized?
2. Should one assume that $g_K^2 \propto \hbar_{eff}$ or $\alpha_K \propto 1/\hbar_{eff}$?

For the first option α_K is the same for dark phases but would be subject to p-adic CCE. This would conform with the notion of gravitational Planck constant predicting that the parameter. The *effective* value of α_K would be however given by α_K/n for dark phases since the Galois symmetry is n-fold multiple of the action for a "fundamental region" for the Galois group.

Second option would predict that α_K behaves like $1/n$ so that effective α_K would behave like $1/n^2$. It seems that this option is excluded and one can concentrate on the first question. The increase of g_K^2 with n is not a problem since it does not appear as a parameter of perturbative expansion since g_K is automatically absorbed to a scaling of the induced gauge potentials.

Quantum-classical correspondence suggests that classical theory theory, in particular spherically symmetric stationary imbeddings, could help to answer the first question. Even the extremal property is not absolutely necessary.

The action is a sum of Kähler action and volume term proportional to length scale dependent cosmological constant approaching zero in long length scale and in equilibrium both give contributions of the same order of magnitude. This suggests that Kähler action corresponding to $\Lambda = 0$ could serve as a guideline.

I studied the embedding of a stationary spherically symmetric metric as a space-time surface during the first 10 years of TGD and the results suggested that the $R = l_P$ option looks more realistic. p-Adic mass calculations based on the definition of the Compton length as $\hbar/M$ however led to the conclusion that the one must have $r \sim 10^{7.6} l_P$. If one replaces $\hbar$ with $\hbar_0$, $R = l_P$ is natural.

The spherically symmetric ansatz assumes that space-time surfaces has a projection to a geodesic sphere S^2 of CP_2 which can be either homologically trivial or non-trivial. Using spherical coordinates (Θ, Φ) for S^2 and spherical Minkowski (t, r, θ, ϕ) coordinates for M^4, the ansatz reads

$$\begin{aligned} s \equiv sin(\Theta) = f(r) \quad , \quad \Phi = \omega t \quad , \\ g_{tt} = 1 - k^2 s^2 \quad , \qquad k^2 = R^2 \omega^2 \quad . \end{aligned} \tag{5.1}$$

In far-away region one can approximate s as

$$s = s_0 + \tfrac{r_1}{r}, \quad s_0 = sin(\Theta_0) \quad . \tag{5.2}$$

The induced metric has component g_{tt} given by

$$g_{tt} = 1 - k^2 s_0^2 - 2k^2 s_0 \frac{r_1}{r} \quad , \tag{5.3}$$

by taking $u = t\sqrt{2 - k^2 s_0^2}$ as a new time coordinate can expresses g_{tt} in terms of the parameters of Scwartshild metric

$$\begin{aligned} g_{uu} &= 1 - 2k^2 s_0 \tfrac{r_1}{r} \equiv 1 - \tfrac{r_s}{r} \quad , \\ r_s &= 2GM = \tfrac{2k^2 s_0 r_1}{1-k^2 s_0^2} \quad , \\ r_1 &= \tfrac{1-k^2 s_0^2}{2k^2 s_0} r_s \equiv k_1 r_s \quad . \end{aligned} \tag{5.4}$$

The approximation makes sense for $s \leq 1$, which gives the condition

$$r \geq r_{min} = (1-s_0) r_1 = (1-s_0) k_1 r_s = (1-s_0)\frac{1-k^2 s_0^2}{2k^2 s_0} r_s \equiv y_1 r_s \quad . \tag{5.5}$$

Remark: The radial component of the metric goes to zero much faster than for Schwartschild metric. The shift of time coordinates depending on the radial coordinate allows to correct this problem. This is however not essential for the recent argument. Schwartchild metric however implies that $\sqrt{g}$ in the calculation of mass gives just the volume element of the flat metric since $g_{tt} g_{rr} = 1$ is true. This is assumed in the following.

One can estimate the mass of the system as Kähler electric energy. Assume that the contribution to the mass comes only from the region $r > y_1 r_s$. The Kähler electric mass $M = r_s/2G$ is given by the expression

$$\begin{aligned} M &= \tfrac{r_s}{2G} \\ &= \tfrac{\hbar_{eff}}{2\alpha_K} \tfrac{s_0^2}{1-s_0^2} r_1^2 \omega^2 \int_{r_{min}}^{\infty} \tfrac{dr}{r^2} = \tfrac{\hbar_{eff}}{2\alpha_K} \tfrac{(1-k^2 s_0^2) s_0}{2(1-s_0)} r_s \tfrac{1}{R^2} \quad . \end{aligned} \tag{5.6}$$

This gives a consistency condition relating R and l_P

$$\begin{aligned} R^2 &= \tfrac{\hbar_{eff}}{\hbar} X l_P^2 \quad , \\ X &= \tfrac{(1-k^2 s_0^2) s_0}{\alpha_K (1-s_0)} \quad . \end{aligned} \tag{5.7}$$

One can consider two cases.

1. For $\hbar_{eff} = \hbar$ the condition reduces to

$$R^2 = X l_P^2 \quad . \tag{5.8}$$

   $l_P = R$ gives $X = (1-k^2 s_0^2) s_0/\alpha_K (1-s_0) = 1$. One should have $s_0 \simeq \alpha_K$ so that the value of $1/\alpha_K$ as an analog of critical temperature would be coded to the geometry of the space-time surface.

   $R = (7!)^2) l_P$ would require $X = \hbar/\hbar_0$, one should have $1 - s_0 \sim 10^{-5}$ for $\alpha_K \sim 10^{-2}$.

2. For $\hbar_{eff} = \hbar_0$ the condition reduces to

$$R^2 = X \frac{\hbar_0}{\hbar} \times l_P^2 \quad . \tag{5.9}$$

   $l_P = R$ gives $X = \hbar/\hbar_0$. One might of course argue that α_K decreases in long scales in the discrete p-adic length scale evolution but this option does not look plausible.

To sum up, intuitively $\hbar$ option with $R = l_P$ looks the most reasonable option.

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