Does coupling constant evolution reduce to that of cosmological constant?

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Abstract

The great surprise of the last year was that twistor induction allows large number of induced twistor structures. $SO(3)$ acts as moduli space for the dimensional reductions of the 6-D Kähler action defining the twistor space of space-time surface as a 6-D surface in 12-D twistor space assignable to $M^4 \times CP_2$. This 6-D surface has space-time surface as base and sphere S^2 as fiber. The action of the twistor sphere in induced twistor structure defines running cosmological constant and one can understand the mysterious smallness of cosmological constant.

This in turn leads to the understanding of coupling constant evolution in terms of the flow changing the value of cosmological constant defined by the area of the twistor sphere of space-time surface for induced twistor structure. Cosmological constant effectively replaces cutoff of length of quantum field theories and the RG invariance reduces to the invariance of action and possibly also scattering amplitudes under small enough variations of the cosmological constant.

Zeros of Riemann zeta are believed to related to criticality. The complex integral of zeta along curve having zero of ζ as endpoint is critical with respect to the variations of the end point, which leads to the proposal that this kind of integral serves as an argument in the expressions for running coupling constants. In the case of the S^2 part of 6-D dimensionally reduced Kähler action this leads to a highly unique expression of Kähler coupling constant such that critical points assignable to the zeros of zeta. Whether the S^2 part of the 6-D action has zeros of zeta as critical points can be tested numerically.

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1 Introduction

In TGD framework coupling constants are determined by quantum criticality implying that they do not run at all in the phase assignable to given extension of rational. They are analogous to critical temperature and determined in principle by number theory. Several key principles involves has been identified and ad hoc guesses for the evolution have been made but I have not been able to propose concrete formula for the coupling constant evolution as differential equations.

The first reason has been that now the evolution - at least when restricted to adelic physics [L3, L4] - requires number theoretical universality - so that the evolution becomes discrete in adelic framework: this does not of course exclude continuous evolution in the real sector of adeles. Second problem has been that the ultraviolet cutoff playing key role in quantum field theories, is absent. The independence of scattering amplitudes on ultraviolet cutoff is what defines renormalization group equations.

The great surprise of the last year was that twistor induction allows large number of induced twistor structures. $SO(3)$ acts as moduli space for the dimensional reductions of the 6-D Kähler action defining the twistor space of space-time surface as a 6-D surface in 12-D twistor space assignable to $M^4 \times CP_2$. This 6-D surface has space-time surface as base and sphere S^2 as fiber. The action of the twistor sphere in induced twistor structure defines running cosmological constant and one can understand the mysterious smallness of cosmological constant.

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2 Some background about TGD

In this section some background about quantum TGD is discussed.

2.1 Two approaches to quantum TGD

There are two approaches to TGD: geometric and number theoretic. The "world of classical worlds" (WCW) is central notion of TGD as a geometrization of quantum physics rather than only classical physics.

1. WCW consists of 3-surfaces and by holography realized by assigning to these 3-surfaces unique 4-surfaces as preferred extremals. In zero energy ontology (ZEO) these 3-surfaces are pairs of 3-surfaces, whose members reside at opposite boundaries of causal diamond (CD) and are connected by preferred extremal analogous to Bohr orbit. The full quantum TGD would rely on real numbers and scattering amplitudes would correspond to zero energy states having as arguments these pairs of 3-surfaces. WCW integration would be involved with the definition of inner products.

2. The theory could be seen formally as a complex square root of thermodynamics with vacuum functional identified as exponent of Kähler function. Kähler geometry would allow to eliminate ill-defined Gaussian determinants and metric determinant of Kähler metric and they would simply disappear from scattering amplitudes. WCW is infinite-D space and one might argue that this kind of approach is hopeless. The point is however that the huge symmetries of WCW - super-symplectic invariance - give excellent hopes of really construction the scattering amplitudes: TGD would be integrable theory.

3. A natural interpretation would be that Kähler action as the analog of Hamiltonian defines the Kähler function of WCW and functional integral defined by it allows definition of full scattering amplitudes.

The number theoretic approach could be called adelic physics [L4, L3] providing also the physics of cognition.

1. At space-time level p-adicization as description of cognition requires discretization. Cognitive representations at space-time level consist of finite set of space-time points with preferred coordinates M^8 in extension of rationals inducing the extensions of p-adic number fields. These representations would realize the notion of finite measurement resolution. p-Adicization and adelization for given extension of rationals are possible only in this manner since these points can be interpreted as both real and p-adic numbers.

2. What about cognitive representations at the level of WCW? The discrete set of space-time points would replace the space-time surface with a finite discrete set of points serving also as its WCW coordinates and define the analog of discretization of WCW using polynomials in M^8 fixed by their values at these points [L2]. If the space-time surface is represented by a polynomial, this representation is all that is needed to code for the space-time surface since one can deduce the coefficients of a polynomial from its values at finite set of points. Now the coefficients belong to extension of rationals. If polynomials are replaced by analytic functions, polynomials provide approximation defining the cognitive representation.

During last year I realized that what I christened as micro-canonical ensemble [L6] identified as kind of complex square root of its counterpart in thermodynamics can serve as a cognitive representation of scattering amplitudes. Cognitive representations of space-time surfaces would thus give also cognitive representations of WCW and micro-canonical ensemble would realize cognitive representations for the scattering amplitudes. Cognitive representations define only a hierarhcy of approximations. The exact description would involve the full WCW, its Kähler geometry, and vacuum functional as exponent of Kähler function.

The idea of micro-canonical ensemble as a subset of space-time surfaces with the same vanishing action would select a sub-set of surfaces with the same values of coupling parameters so that the fixing the coupling parameters together with preferred extremal property selects the subset with same value of action. There are two options to consider.

1. The real part of the action vanishes and imaginary part is multiple of 2π so that the action exponential is equal to unity. For the twistor lift this actually implies the vanishing of the entire action since volume term and Kähler term have the same phase (that of $1/\alpha_K$). The role of coupling parameters would be analogous to the role of temperature and applied pressure. In principle this condition is mathematically possible. The electric part of Kähler action in Minkowskian regions has sign opposite to magnetic part and volume term (actually magnetic S^2 part of 6-D Kähler action) so that these two contributions could cancel. The problem is that Kähler function would be constant and therefore also the Kähler metric.

2. I have also proposed [L6] that the analog of micro-canonical ensemble makes sense meaning that all space-time surfaces contributing to the scattering amplitude have the same action. As a consequence, the action exponential and the usual normalization factor would cancel each other and one would obtain just a sum over space-time surfaces with same action: otherwise action exponential would not appear in the scattering amplitudes - this is the case also in perturbative QFTs. This is crucial for the p-adicization and adelization since these exponential factors belong to the extension of rationals only under very strong additional conditions.

   This option has analog also at the level of WCW since Kähler function should have for give values of zero modes only single minimum so that localization in zero modes would mean that the action exponential cancels in the normalization of the amplitudes. It seems that this option is the only possible one.

   Note that the cancellation of the metric determinant and Gaussian determinant possible for Kähler metric with the exponent of Kähler function serving as vacuum functional reduces the perturbative integrations around the minima of Kähler action to a sum over exponents, and if only single minimum contributes for given values of the zero modes, the sum contains only single term.

2.2 Number theoretic vision about coupling constant evolution

Let us return to the question about the coupling constant evolution.

1. Each extension of rationals corresponds to particular values of coupling parameters determined by the extension so that it indeed makes sense to ponder what the spectrum of values for say fine structure constant is. In standard QFT this does not make sense.

2. Coupling constant evolution as a function of momentum or length scales reduces to p-adic coupling constant evolution in TGD as function of p-adic prime. Particles are characterized by preferred p-adic primes - for instance, electron corresponds to $M_{127} = 2^{127} - 1$ - the largest Mersenne prime which does not correspond to super-astronomical Compton length - and the natural identification is as so called ramified primes of extension.

Why the interpretation of p-adic primes as ramified primes?

1. As one increases length scale resolution particle decomposes to more elementary particles.

2. Particles correspond in TGD to preferred p-adic primes. This suggests that when a prime (ideal) of given extension is looked at improved precision determined by an extension of the orignal extension it decomposes into a product of primes. This indeed happens.

   The number of primes of the larger extension appearing in the decomposition to product equals to the dimension of extension as extension of the original extension. All these primes appear and only once in the generic case. Ramified primes of ordinary extension are however odd-balls. Some primes of extension are missing and some appear as higher powers than 1 in their decomposition.

3. Ramified primes are analogous to critical systems. Polynomial with a multiple root - now prime of extension appearing as higher power - corresponds to a critical system. TGD is quantum critical so that one expects that ramified primes are preferred physically and indeed correspond to quantum critical systems.

4. Only the momenta belonging to the extension of rationals are considered and one can identify them as real-valued or p-adic valued momenta. Coupling constants do not depend on the values of the momenta for given extension of rationals and are thus analogous to critical temperature.

   This involves interesting not totally resolved technical question inspired by p-adic mass calculations for which the p-adic mass squared value is mapped to its real value by canonical identification $S\sum x_n p^n \rightarrow \sum x_n p^{-n}$. The correspondence is continuous and can be applied to Lorentz invariants appearing in scattering amplitudes [K2].

Could this correspondence be applied also to momenta rather than only mass squared values and Lorentz invariants? $M^8 - H$ correspondence [L2] selects fixed Poincare frame as moduli space for octonionic structures and at M^8 level this could make sense.

2.3 Cosmological constant and twistor lift of Kähler action

Cosmological constant Λ is one of the biggest problems of modern physics. Surprisingly, Λ turned out to provide the first convincing solution to the problem of understanding coupling constant evolution in TGD framework. In QFTs the independence of scattering amplitudes on UV cutoff length scale gives rise to renormalization group (RG) equations. In TGD there is however no natural cutoff length scale since the theory is finite. Cosmological constant should however evolve as a function of p-adic length scales and cosmological constant itself could give rise to the length scale serving in the role of cutoff length scale. Combined with the view about cosmological constant provided by twistor lift of TGD this leads to explicit RG equations for α_K and scattering amplitudes.

Cosmological constant has two meanings.

1. Einstein proposed non-vanishing value of Λ in Einstein action as a volume term at his time in order to get what could be regarded as a static Universe. It turned out that Universe expanded and Einstein concluded that this proposal was the greatest blunder of his life. For two decades ago it was observed that the expansion of the Universe acclerates and the cosmological constant emerged again. Λ must be extremely small and have correct sign in order to give accelerating rather decelerating expansion in Robertson-Walker cooordinate. Here one must however notice that the time slicing used by Einstein was different and fr this slicing the Universe looked static.

2. Λ can be however understood in an alternative sense as characterizing the dynamics in the matter sector. Λ could characterize the vacuum energy density of some scalar field, call it quintessense, proportional to 3- volume in quintessence scenario. This Λ would have sign opposite to that in the first scenario since it would appear at opposite side of Einstein's equations.

2.3.1 Cosmological constant in string models and in TGD

It has turned out that Λ could be the final nail to the coffin of superstring theory.

1. The most natural prediction of M-theory and superstring models is Λ in Einsteinian sense but with wrong sign and huge value: for instance, in AdS/CFT correspondence this would be the case. There has been however a complex argument suggesting that one could have a cosmological constant with a correct sign and even small enough size.

   This option however predicts landscape and a loss of predictivity, which has led to a total turn of the philosophical coat: the original joy about discovering the unique theory of everything has changed to that for the discovery that there are no laws of physics. Cynic would say that this is a lottery win for theoreticians since theory building reduces to mere artistic activity.

2. Now however Cumrun Vafa - one of the leading superstring theorists - has proposed that the landscape actually does not exist at all [B2] (see `http://tinyurl.com/ycz7wvng`). Λ would have wrong sign in Einsteinian sense but the hope is that quintessence scenario might save the day. Λ should also decrease with time, which as such is not a catastrophe in quintessence scenario.

3. Theorist D. Wrase et al has in turn published an article [B1] (see `http://tinyurl.com/ychrhuxk`) claiming that also the Vafa's quintessential scenario fails. It would not be consistent with Higgs Higgs mechanism. The conclusion suggesting itself is that according to the no-laws-of-physics vision something catastrophic has happened: string theory has made a prediction! Even worse, it is wrong.

   Remark: In TGD framework Higgs is present as a particle but p-adic thermodynamics rather than Higgs mechanism describes at least fermion massivation. The couplings of Higgs to fermions are naturally proportional their masses and fermionic part of Higgs mechanism is seen only as a manner to reproduce the masses at QFT limit.

4. This has led to a new kind of string war: now inside superstring hegemony and dividing it into two camps. Optimistic outsider dares to hope that this leads to a kind of auto-biopsy and the gloomy period of superstring hegemony in theoretical physics lasted now for 34 years would be finally over.

String era need not be over even now! One could propose that both variants of Λ are present, are large, and compensate each other almost totally! First I took this as a mere nasty joke but I realized that I cannot exclude something analogous to this in TGD. It turned that this is not possible. I had made a delicate error. I thought that the energy of the dimensionally reduced 6-D Kauml;hler action can be deduced from the resulting 4-D action containing volume term giving the negative contribution rather than dimensionally reducing the 6-D expression in which the volume term corresponds to 6-D magnetic energy and is positive! A lesson in non-commutativity!

The picture in which Λ in Einsteinian sense parametrizes the total action as dimensionally reduced 6-D twistor lift of Kähler action could be indeed interpreted formally as sum of genuine cosmological term identified as volume action. This picture has additional bonus: it leads to the understanding of coupling constant evolution giving rise to discrete coupling constant evolution as sub-evolution in adelic physics. This picture is summarized below.

2.3.2 The picture emerging from the twistor lift of TGD

Consider first the picture emerging from the twistor lift of TGD.

1. Twistor lift of TGD leads via the analog of dimensional reduction necessary for the induction of 8-D generalization of twistor structure in $M^4 \times CP_2$ to a 4-D action determining space-time surfaces as its preferred extremals. Space-time surface as a preferred extremal defines a unique section of the induced twistor bundle. The dimensionally reduced Kähler action is sum of two terms. Kähler action proportional to the inverse of Kähler coupling strength and volume term proportional to the cosmological constant Λ.

   Remark: The sign of the volume action is negative as the analog of the magnetic part of Maxwell action and *opposite* to the sign of the area action in string models.

   Kähler and volume actions should have opposite signs. At M^4 limit Kähler action is proportional to $E^2 - B^2$ In Minkowskian regions and to $-E^2 - B^2$ in Euclidian regions.

2. Twistor lift forces the introduction of also M^4 Kähler form so that the twistor lift of Kähler action contains M^4 contribution and gives in dimensional reduction rise to M^4 contributions to 4-D Kähler action and volume term.

   It is of crucial importance that the Cartesian decomposition $H = M^4 \times CP_2$ allows the scale of M^4 contribution to 6-D Kähler action to be different from CP_2 contribution. The size of M^4 contribution as compared to CP_2 contribution must be very small from the smallness of CP breaking [L5] [K7].

   For canonically imbedded M^4 the action density vanishes. For string like objects the electric part of this action dominates and corresponding contribution to 4-D Kähler action of flux tube extremals is positive unlike the standard contribution so that an almost cancellation of the action is in principle possible.

3. What about energy? One must consider both Minkowskian and Euclidian space-time regions and be very careful with the signs. Assume that Minkowskian and Eucidian regions have *same time orientation*.

   (a) Since a dimensionally reduced 6-D Kähler action is in question, the sign of energy density is positive Minkowskian space-time regions and of form $(E^2 + B^2)/2$. Volume energy density proportional to Λ is positive.

(b) In Euclidian regions the sign of g^{00} is negative and energy density is of form $(E^2 - B^2)/2$ and is negative when magnetic field dominates. For string like objects the M^4 contribution to Kähler action however gives a contribution in which the electric part of Kähler action dominates so that M^4 and CP_2 contributions to energy have opposite signs.

(c) 4-D volume energy corresponds to the magnetic energy for twistor sphere S^2 and is therefore positive. For some time I thought that the sign must be negative. My blunder was that I erratically deduced the volume contribution to the energy from 4-D dimensionally reduced action, which is sum of Kähler action and volume term rather than deducing it for 6-D Kähler action and then dimensionally reducing the outcome. A good example about consequences of non-commutativity!

The identification of the observed value of cosmological constant is not straightforward and I have considered several options without making explicit their differences even to myself. For Einsteinian option cosmological constant could correspond to the coefficient Λ of the volume term in analogy with Einstein's action. For what I call quintessence option cosmological constant Λ_{eff} would approximately parameterize the total action density or energy density.

1. Cosmological constant - irrespective of whether it is identified as Λ or Λ_{eff} - is extremely small in the recent cosmology. The natural looking assumption would be that as a coupling parameter Λ or Λ_{eff} depends on p-adic length scale like $1/L_p^2$ and therefore decreases in average sense as $1/a^2$, where a is cosmic time identified as light-cone proper time assignable to either tip of CD. This suggests the following rough vision.

   The increase of the thickness of magnetic flux tubes carrying monopole flux liberates energy and this energy can make possible increase of the volume so that one obtains cosmic expansion. The expansion of flux tubes stops as the string tension achieves minimum and the further increase of the volume would increase string tension. For the cosmological constant in cosmological scales the maximum radius of flux tube is about 1 mm, which is biological length scale. Further expansion becomes possible if a phase transition increasing the p-adic length scale and reducing the value of cosmological constant is reduced. This phase transition liberates volume energy and leads to an accelerated expansion. The space-time surface would expand by jerks in stepwise manner. This process would replace continuous cosmic expansion of GRT. One application is TGD variant of Expanding Earth model explaining Cambrian Explosion, which is really weird event [K1].

   One can however raise a serious objection: since the volume term is part of 6-D Kähler action, the length scale evolution of Λ should be dictated by that for $1/\alpha_K$ and be very slow: therefore cosmological constant identified as Einsteinian Λ seems to be excluded.

2. It however turns that it possible to have a large number of imbedding of the twistor sphere into the product of twistor spheres of M^4 and CP_2 defining dimensional reductions. This set is parameterized by rotations sphere. The S^2 part of 6-D Kähler action determining Λ can be arbitrarily small. This mechanism is discussed in detail in [L8, L9] and leads also to the understanding of coupling constant evolution. The cutoff scale in QFT description of coupling constant evolution is replaced with the length scale defined by cosmological constant.

2.3.3 Second manner to increase 3-volume

Besides the increase of 3-volume of M^4 projection, there is also a second manner to increase volume energy: many-sheetedness. The phase transition reducing the value of Λ could in fact force many-sheetedness.

1. In TGD the volume energy associated with Λ is analogous to the surface energy in superconductors of type I. The thin 3-surfaces in superconductors could have similar 3-surface analogs in TGD since their volume is proportional to surface area - note that TGD Universe can be said to be quantum critical.

   This is not the only possibility. The sheets of many-sheeted space-time having overlapping M^4 projections provide second mechanism. The emergence of many-sheetedness could also be caused by the increase of $n = h_{eff}/h_0$ as a number of sheets of Galois covering.

2. Could the 3-volume increase during deterministic classical time evolution? If the minimal surface property assumed for the preferred extremals as a realization of quantum criticality is true everywhere, the conservation of volume energy prevents the increase of the volume. Minimal surface property is however assumed to fail at discrete set of points due to the transfer of conserved charged between Kähler and volume degrees of freedom. Could this make possible the increase of volume during classical time evolution so that volume and Kähler energy could increase?

3. ZEO allows the increase of average 3-volume by quantum jumps. There is no reason why each "big" state function reduction changing the roles of the light-like boundaries of CD could not decrease the average volume energy of space-time surface for the time evolutions in the superposition. This can occur in all scales, and could be achieved also by the increase of $h_{eff}/h_0 = n$.

4. The geometry of CD suggests strongly an analogy with Big Bang followed by Big Crunch. The increase of the volume as increase of the volume of M^4 projection does not however seem to be consistent with Big Crunch. One must be very cautious here. The point is that the size of CD itself increases during the sequence of small state function reductions leaving the members of state pairs at passive boundary of CD unaffected. The size of 3-surface at the active boundary of CD therefore increases as also its 3-volume.

   The increase of the volume during the Big Crunch period could be also due to the emergence of the many-sheetedness, in particular due to the increase of the value of n for space-time sheets for sub-CDs. In this case, this period could be seen as a transition to quantum criticality accompanied by an emergence of complexity.

2.3.4 Is the cosmological constant really understood?

The interpretation of the coefficient of the volume term as cosmological constant has been a long-standing interpretational issue and caused many moments of despair during years. The intuitive picture has been that cosmological constant obeys p-adic length scale scale evolution meaning that Λ would behave like $1/L_p^2 = 1/p \simeq 1/2^k$ [K5].

This would solve the problems due to the huge value of Λ predicted in GRT approach: the smoothed out behavior of Λ would be $\Lambda \propto 1/a^2$, a light-cone proper time defining cosmic time, and the recent value of Λ - or rather, its value in length scale corresponding to the size scale of the observed Universe - would be extremely small. In the very early Universe - in very short length scales - Λ would be large.

A simple solution of the problem would be the p-adic length scale evolution of Λ as $\Lambda \propto 1/p$, $p \simeq 2^k$. The flux tubes would thicken until the string tension as energy density would reach stable minimum. After this a phase transition reducing the cosmological constant would allow further thickening of the flux tubes. Cosmological expansion would take place as this kind of phase transitions (for a mundane application of this picture see [K1]).

This would solve the basic problem of cosmology, which is understanding why cosmological constant manages to be so small at early times. Time evolution would be replaced with length scale evolution and cosmological constant would be indeed huge in very short scales but its recent value would be extremely small.

I have however not really understood how this evolution could be realized! Twistor lift seems to allow only a very slow (logarithmic) p-adic length scale evolution of Λ [L7]. Is there any cure to this problem?

1. The magnetic energy decreases with the area S of flux tube as $1/S \propto 1/p \simeq 1/2^k$, where $\sqrt{p}$ defines the transversal length scale of the flux tube. Volume energy (magnetic energy associated with the twistor sphere) is positive and increases like S. The sum of these has minimum for certain radius of flux tube determined by the value of Λ. Flux tubes with quantized flux would have thickness determined by the length scale defined by the density of dark energy: $L \sim \rho_{vac}^{-1/4}$, $\rho_{dark} = \Lambda/8\pi G$. $\rho_{vac} \sim 10^{-47}$ GeV^4 (see `http://tinyurl.com/k4bwlzu`) would give $L \sim 1$ mm, which would could be interpreted as a biological length scale (maybe even neuronal length scale).

2. But can Λ be very small? In the simplest picture based on dimensionally reduced 6-D Kähler action this term is not small in comparison with the Kähler action! If the twistor spheres of M^4 and CP_2 give the same contribution to the induced Kähler form at twistor sphere of X^4, this term has maximal possible value!

   The original discussions in [K6, K5] treated the volume term and Kähler term in the dimensionally reduced action as independent terms and Λ was chosen freely. This is however not the case since the coefficients of both terms are proportional to $(1/\alpha_K^2)S(S^2)$, where $S(S^2)$ is the area of the twistor sphere of 6-D induced twistor bundle having space-time surface as base space. This are is same for the twistor spaces of M^4 and CP_2 if CP_2 size defines the only fundamental length scale. I did not even recognize this mistake.

The proposed fast p-adic length scale evolution of the cosmological constant would have extremely beautiful consequences. Could the original intuitive picture be wrong, or could the desired p-adic length scale evolution for Λ be possible after all? Could non-trivial dynamics for dimensional reduction somehow give it? To see what can happen one must look in more detail the induction of twistor structure.

1. The induction of the twistor structure by dimensional reduction involves the identification of the twistor spheres S^2 of the geometric twistor spaces $T(M^4) = M^4 \times S^2(M^4)$ and of T_{CP_2} having $S^2(CP_2)$ as fiber space. What this means that one can take the coordinates of say $S^2(M^4)$ as coordinates and imbedding map maps $S^2(M^4)$ to $S^2(CP_2)$. The twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ have in the minimal scenario same radius $R(CP_2)$ (radius of the geodesic sphere of CP_2. The identification map is unique apart from $SO(3)$ rotation R of either twistor sphere possibly combined with reflection P. Could one consider the possibility that R is not trivial and that the induced Kähler forms could almost cancel each other?

2. The induced Kähler form is sum of the Kähler forms induced from $S^2(M^4)$ and $S^2(CP_2)$ and since Kähler forms are same apart from a rotation in the common S^2 coordinates, one has $J_{ind} = J + RP(J)$, where R denotes a rotation and P denotes reflection. Without reflection one cannot get arbitrary small induced Kähler form as sum of the two contributions. For mere reflection one has $J_{ind} = 0$.

   Remark: It seems that one can do with reflection if the Kähler forms of the twistor spheres are of opposite sign in standard spherical coordinates. This would mean that they have have opposite orientation.

   One can choose the rotation to act on (y, z)-plane as $(y, z) \to (cy + sz, -sz + cy)$, where s and c denote the cosines of the rotation angle. A small value of cosmological constant is obtained for small value of s. Reflection P can be chosen to correspond to $z \to -z$. Using coordinates $(u = cos(\Theta), \Phi)$ for $S^2(M^4)$ and (v, Ψ) for $S^2(CP_2)$ and by writing the reflection followed by rotation explicitly in coordinates (x, y, z) one finds $v = -cu - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = arctan[(su/\sqrt{1-u^2}cos(\Phi) + ctan(\Phi)]$. In the lowest order in s one has $v = -u - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = \Phi + scos(\Phi)(u/\sqrt{1-u^2})$.

3. Kähler form J^{tot} is sum of unrotated part $J(M^4) = du \wedge d\Phi$ and $J(CP_2) = dv \wedge d\Psi$. J' equals to the determinant $\partial(v, \Psi)/\partial(u, \Phi)$. A suitable spectrum for s could reproduce the proposal $\Lambda \propto 2^{-k}$ for Λ. The S^2 part of 6-D Kähler action equals to $(J_{\theta\phi}^{tot})^2/\sqrt{g_2}$ and in the lowest order proportional to s^2. For small values of s the integral of Kähler action for S^2 over S^2 is proportional to s^2.

   One can write the S^2 part of the dimensionally reduced action as $S(S^2) = s^2F^2(s)$. Very near to the poles the integrand has $1/[sin(\Theta) + O(s)]$ singularity and this gives rise to a logarithmic dependence of F on s and one can write: $F = F(s, log(s))$. In the lowest order one has $s \simeq 2^{-k/2}$, and in improved approximation one obtains a recursion formula $s_n(S^2, k) = 2^{-k/2}/F(s_{n-1}, \log(s_{n-1})$ giving renormalization group evolution with k replaced by anomalous dimension $k_{n,a} = k + 2log[F(s_{n-1}, log(s_{n-1})]$ differing logarithmically from k.

4. The sum $J + RP(J)$ defining the induced Kähler form in $S^2(X^4)$ is covariantly constant since both terms are covariantly constant by the rotational covariance of J.

5. The imbeddings of $S^2(X^4)$ as twistor sphere of space-time surface to both spheres are holomorphic since rotations are represented as holomorphic transformations. Also reflection as $z \rightarrow 1/z$ is holomorphic. This in turn implies that the second fundamental form in complex coordinates is a tensor having only components of type $(1,1)$ and $(-1,-1)$ whereas metric and energy momentum tensor have only components of type $(1,-1)$ and $(-1,1)$. Therefore all contractions appearing in field equations vanish identically and $S^2(X^4)$ is minimal surface and Kähler current in $S^2(X^4)$ vanishes since it involves components of the trace of second fundamental form. Field equations are indeed satisfied.

6. The solution of field equations becomes a family of space-time surfaces parameterized by the values of the cosmological constant Λ as function of S^2 coordinates satisfying $\Lambda/8\pi G = \rho_{vac} = J \wedge (*J)(S^2)$. In long length scales the variation range of Λ would become arbitrary small.

7. If the minimal surface equations solve separately field equations for the volume term and Kähler action everywhere apart from a discrete set of singular points, the cosmological constant affects the space-time dynamics only at these points. The physical interpretation of these points is as seats of fundamental fermions at partonic 2-surface at the ends of light-like 3-surfaces defining their orbits (induced metric changes signature at these 3-surfaces). Fermion orbits would be boundaries of fermionic string world sheets.

   One would have family of solutions of field equations but particular value of Λ would make itself visible only at the level of elementary fermions by affecting the values of coupling constants. p-Adic coupling constant evolution would be induced by the p-adic coupling constant evolution for the relative rotations R combined with reflection for the two twistor spheres. Therefore twistor lift would not be mere manner to reproduce cosmological term but determine the dynamics at the level of coupling constant evolution.

8. What is nice that also $\Lambda = 0$ option is possible. This would correspond to the variant of TGD involving only Kähler action regarded as TGD before the emergence of twistor lift. Therefore the nice results about cosmology [K3] obtained at this limit would not be lost.

3 Does cosmological constant replace cutoff length in coupling constant evolution?

One of the chronic problems if TGD has been the understanding of what coupling constant evolution could be defined in TGD.

3.1 Basic notions and ideas

Consider first the basic notions and ideas.

1. The notion of quantum criticality is certainly central. The continuous coupling constant evolution having no counterpart in the p-adic sectors of adele would contain as a sub-evolution discrete p-adic coupling constant evolution such that the discrete values of coupling constants allowing interpretation also in p-adic number fields are fixed points of coupling constant evolution.

   Quantum criticality is realized also in terms of zero modes, which by definition do not contribute to WCW metric. Zero modes are like control parameters of a potential function in catastrophe theory. Potential function is extremum with respect to behavior variables replaced now by WCW degrees of freedom. The graph for preferred extremals as surface in the space of zero modes is like the surface describing the catastrophe. For given zero modes there are several preferred extremals and the catastrophe corresponds to the regions of zero mode space, where some branches of co-incide. The degeneration of roots of polynomials is a concrete realization for this.

Quantum criticality would also mean that coupling parameters effectively disappear from field equations. For minimal surfaces (generalization of massless field equation allowing conformal invariance characterizing criticality) this happens since they are separately extremals of Kähler action and of volume term.

Quantum criticality is accompanied by conformal invariance in the case of 2-D systems and in TGD this symmetry extends to its 4-D analogas isometries of WCW.

2. In the case of 4-D Kähler action the natural hypothesis was that coupling constant evolution should reduce to that of Kähler coupling strength $1/\alpha_K$ inducing the evolution of other coupling parameters. Also in the case of the twistor lift $1/\alpha_K$ could have similar role. One can however ask whether the value of the 6-D Kähler action for the twistor sphere $S^2(X^4)$ defining cosmological constant could define additional parameter replacing cutoff length scale as the evolution parameter of renormalization group.

3. The hierarchy of adeles should define a hierarchy of values of coupling strengths so that the discrete coupling constant evolution could reduce to the hierarchy of extensions of rationals and be expressible in terms of parameters characterizing them.

4. I have also considered number theoretical existence conditions as a possible manner to fix the values of coupling parameters. The condition that the exponent of Kähler function should exist also for the p-adic sectors of the adele is what comes in mind as a constraint but it seems that this condition is quite too strong.

   If the functional integral is given by perturbations around single maximum of Kähler function, the exponent vanishes from the expression for the scattering amplitudes due to the presence of normalization factor. There indeed should exist only single maximum by the Euclidian signature of the WCW Kähler metric for given values of zero modes (several extrema would mean extrema with non-trivial signature) and the parameters fixing the topology of 3-surfaces at the ends of preferred extremal inside CD. This formulation as counterpart also in terms of the analog of micro-canonical ensemble (allowing only states with the same energy) allowing only discrete sum over extremals with the same Kähler action [L6].

5. I have also considered more or less ad hoc guesses for the evolution of Kähler coupling strength such as reduction of the discrete values of $1/\alpha_K$ to the spectrum of zeros of Riemann zeta or actually of its fermionic counterpart [L1]. These proposals are however highly ad hoc.

3.2 Could the area of twistor sphere replace cutoff length?

As I started once again to consider coupling constant evolution I realized that the basic problem has been the lack of explicit formula defining what coupling constant evolution really is.

1. In quantum field theories (QFTs) the presence of infinities forces the introduction of momentum cutoff. The hypothesis that scattering amplitudes do not depend on momentum cutoff forces the evolution of coupling constants. TGD is not plagued by the divergence problems of QFTs. This is fine but implies that there has been no obvious manner to define what coupling constant evolution as a continuous process making sense in the real sector of adelic physics could mean!

2. Cosmological constant is usually experienced as a terrible head ache but it could provide the helping hand now. Could the cutoff length scale be replaced with the value of the length scale defined by the cosmological constant defined by the S^2 part of 6-D Kähler action? This parameter would depend on the details of the induced twistor structure. It was shown above that if the moduli space for induced twistor structures corresponds to rotations of S^2 possibly combined with the reflection, the parameter for coupling constant restricted to that to $SO(2)$ subgroup of $SO(3)$ could be taken to be taken $s = sin(\epsilon)$.

3. RG invariance would state that the 6-D Kähler action is stationary with respect to variations with respect to s. The variation with respect to s would involve several contributions. Besides the variation of $1/\alpha_K(s)$ and the variation of the $S(^2)$ part of 6-D Kähler action defining the cosmological constant, there would be variation coming from the variations of 4-D Kähler

action plus 4-D volume term . This variation vanishes by field equations. As matter of fact, the variations of 4-D Kähler action and volume term vanish separately except at discrete set of singular points at which there is energy transfer between these terms. This condition is one manner to state quantum criticality stating that field equations involved no coupling parameters.

One obtains explicit RG equation for α_K and Λ having the standard form involving logarithmic derivatives. The form of the equation would be

$$\frac{dlog(\alpha_K)}{ds} = -\frac{S(S^2)}{(S_K(X^4)/Vol(X^4)) + S(S^2)} \frac{dlog(S(S^2))}{ds} \quad . \tag{3.1}$$

It should be noticed that the choices of the parameter s in the evolution equation is arbitrary so that the identification $s = sin(\epsilon)$ is not necessary. Note that one must use Kähler action per volume.

The equation contains the ratio $S(S^2)/(S_K(X^4) + S(S^2))$ of actions as a parameter. This does not conform with idea of micro-locality. One can however argue that this conforms with the generalization of point like particle to 3-D surface. For preferred extremal the action is indeed determined by the 3 surfaces at its ends at the boundaries of CD. This implies that the construction of quantum theory requires the solution of classical theory.

In particular, the 4-D classical theory is necessary for the construction of scattering amplitudes, and one cannot reduce TGD to string theory although strong form of holography states that the data about quantum states can be assigned with 2-D surfaces. Even more: $M^8 - H$ correspondence implies that the data determining quantum states can be assigned with discrete set of points defining cognitive representations for given adel This set of points depends on the preferred extremal!

4. How to identify quantum critical values of α_K? At these points one should have $dlog(\alpha_K)/ds = 0$. This implies $dlog(S(S^2)/ds = 0$, which in turn implies $dlog(\alpha_K)/ds = 0$ unless one has $S_K(X^4) + S(S^2) = 0$. This condition would make exponent of 6-D Kähler action trivial and the continuation to the p-adic sectors of adele would be trivial. I have considered also this possibility [L7].

   The critical values of coupling constant evolution would correspond to the critical values of S and therefore of cosmological constant. The basic nuisance of theoretical physics would determine the coupling constant evolution completely! Critical values are in principle possible. Both the numerator $J^2_{u\Phi}$ and the numerator $1/\sqrt{det(g)}$ increase with ϵ. If the rate for the variation of these quantities with s vary it is possible to have a situation in which the one has

$$\frac{dlog(J^2_{u\Phi})}{ds} = -\frac{dlog(\sqrt{det(g)})}{ds} \quad . \tag{3.2}$$

5. One can make highly non-trivial conclusions about the evolution at general level. For the extremals with vanishing action and for which α_K is critical (vanishing derivate), also the second derivative of $d^2S(S^2)/ds^2 =$ holds true at the critical point. The QFT analogs of these points are points at which beta function develops higher order zero. The tip of cusp catastrophe is second analogy.

   The points at which that the action has minimum are also interesting. For magnetic flux tubes for which one has $S_K(X^4) \propto 1/S$ and $S_{vol} \propto S$ in good approximation, one has $S_K(X^4) = S_{vol}$ at minimum (say for the flux tubes with radius about 1 mm for the cosmological constant in cosmological scales). One can write

$$\frac{dlog(\alpha_K)}{ds} = -\frac{1}{2}\frac{dlog(S(S^2))}{ds} \quad , \tag{3.3}$$

and solve the equation explicitly:

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \ . \tag{3.4}$$

A more general situation would correspond to a model with $x \neq 1/2$: the deviation from $x = 1/2$ could be interpreted as anomalous dimension. This allows to deduce numerically a formula for the value spectrum of $\alpha_{K,0}/\alpha_K$ apart from the initial values.

6. One can solve the equation also for fixed value of $S(X^4)/Vol(X^4)$ to get

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \ . \tag{3.5}$$

$$\frac{\alpha_K}{\alpha_{K,0}} = \frac{S_K(X^4)/Vol(X^4)) + S(S^2)}{S_K(X^4)/Vol(X^4))} \quad . \tag{3.6}$$

At the limit $S(S^2) = \rightarrow 0$ one obtains $\alpha_K \rightarrow \alpha_{K,0}$.

7. One should demonstrate that the critical values of s are such that the continuation to p-adic sectors of the adele makes sense. For preferred extremals cosmological constant appears as a parameter in field equations but does not affect the field equations expect at the singular points. Singular points play the same role as the poles of analytic function or point charges in electrodynamics inducing long range correlations. Therefore the extremals depend on parameter s and the dependence should be such that the continuation to the p-adic sectors is possible.

   A naive guess is that the values of s are rational numbers. Above the proposal $s = 2^{-k/2}$ motivated by p-adic length scale hypothesis was considered but also $s = p^{-k/2}$ can be considered. These guesses might be however wrong, the most important point is that there is that one can indeed calculate $\alpha_K(s)$ and identify its critical values.

8. What about scattering amplitudes and evolution of various coupling parameters? If the exponent of action disappears from scattering amplitudes, the continuation of scattering amplitudes is simple. This seems to be the only reasonable option. In the adelic approach [L4] amplitudes are determined by data at a discrete set of points of space-time surface (defining what I call cognitive representation) for which the points have M^8 coordinates belong to the extension of rationals defining the adele.

   Each point of $S^2(X^4)$ corresponds to a slightly different X^4 so that the singular points depend on the parameter s, which induces dependence of scattering amplitudes on s. Since coupling constants are identified in terms of scattering amplitudes, this induces coupling constant evolution having discrete coupling constant evolution as sub-evolution.

3.3 Could the critical values of α_K correspond to the zeros of Riemann Zeta?

Number theoretical intuitions strongly suggests that the critical values of $1/\alpha_K$ could somehow correspond to zeros of Riemann Zeta. Riemann zeta is indeed known to be involved with critical systems.

The naivest ad hoc hypothesis is that the values of $1/\alpha_K$ are actually proportional to the non-trivial zeros $s = 1/2 + iy$ of zeta [L1]. A hypothesis more in line with QFT thinking is that they correspond to the imaginary parts of the roots of zeta. In TGD framework however complex values of α_K are possible and highly suggestive. In any case, one can test the hypothesis that the values of $1/\alpha_K$ are proportional to the zeros of ζ at critical line. Problems indeed emerge.

1. The complexity of the zeros and the non-constancy of their phase implies that the RG equation can hold only for the imaginary part of $s = 1/2 + it$ and therefore only for the imaginary part of the action. This suggests that $1/\alpha_K$ is proportional to y. If $1/\alpha_K$ is complex, RG equation implies that its phase RG invariant since the real and imaginary parts would obey the same RG equation.

2. The second - and much deeper - problem is that one has no reason for why $dlog(\alpha_K)/ds$ should vanish at zeros: one should have $dy/ds = 0$ at zeros but since one can choose instead of parameter s any coordinate as evolution parameter, one can choose $s = y$ so that one has $dy/ds = 1$ and criticality condition cannot hold true. Hence it seems that this proposal is unrealistic although it worked qualitatively at numerical level.

It seems that it is better to proceed in a playful spirit by asking whether one could realize quantum criticality in terms of of the property of being zero of zeta.

1. The very fact that zero of zeta is in question should somehow guarantee quantum criticality. Zeros of ζ define the critical points of the complex analytic function defined by the integral

$$X(s_0, s) = \int_{C_{s_0 \to s}} \zeta(s) ds \quad , \tag{3.7}$$

where $C_{s_0 \to s}$ is any curve connecting zeros of ζ, a is complex valued constant. Here s does not refer to $s = sin(\epsilon)$ introduced above but to complex coordinate s of Riemann sphere.

By analyticity the integral does not depend on the curve C connecting the initial and final points and the derivative $dS_c/ds = \zeta(s)$ vanishes at the endpoints if they correspond to zeros of ζ so that would have criticality. The value of the integral for a closed contour containing the pole $s = 1$ of ζ is non-vanishing so that the integral has two values depending on which side of the pole C goes.

2. The first guess is that one can define S_c as complex analytic function $F(X)$ having interpretation as analytic continuation of the S^2 part of action identified as $Re(S_c)$:

$$\begin{aligned} & S_c(S^2) = F(X(s, s_0)) \quad , && X(s, s_0) = \textstyle\int_{C_{s_0 \to s}} \zeta(s) ds \quad , \\ & S(S^2) = Re(S_c) = Re(F(X)) \quad , && \\ & \zeta(s) = 0 \quad , && Re(s_0) = 1/2 \quad . \end{aligned} \tag{3.8}$$

$S_c(S^2) = F(X)$ would be a complexified version of the Kähler action for S^2. s_0 must be at critical line but it is not quite clear whether one should require $\zeta(s_0) = 0$.

The real valued function $S(S^2)$ would be thus extended to an analytic function $S_c = F(X)$ such that the $S(S^2) = Re(S_c)$ would depend only on the end points of the integration path $C_{s_0 \to s}$. This is geometrically natural. Different integration paths at Riemann sphere would correspond to paths in the moduli space $SO(3)$, whose action defines paths in S^2 and are indeed allowed as most general deformations. Therefore the twistor sphere could be identified Riemann sphere at which Riemann zeta is defined. The critical line and real axis would correspond to particular one parameter sub-groups of $SO(3)$ or to more general one parameter subgroups.

One would have

$$\frac{\alpha_{K,0}}{\alpha_K} = (\frac{S_c}{S_0})^{1/2} \quad . \tag{3.9}$$

The imaginary part of $1/\alpha_K$ (and in some sense also of the action $S_c(S^2)$) would determined by analyticity somewhat like the real parts of the scattering amplitudes are determined by the discontinuities of their imaginary parts.

3. What constraints can one pose on F? F must be such that the value range for $F(X)$ is in the value range of $S(S^2)$. The lower limit for $S(S^2)$ is $S(S^2) = 0$ corresponding to $J_{u\Phi} \to 0$.

   The upper limit corresponds to the maximum of $S(S^2)$. If the one Kähler forms of M^4 and S^2 have same sign, the maximum is $2 \times A$, where $A = 4\pi$ is the area of unit sphere. This is however not the physical case.

   If the Kähler forms of M^4 and S^2 have opposite signs or if one has RP option, the maximum, call it S_{max}, is smaller. Symmetry considerations strongly suggest that the upper limit corresponds to a rotation of 2π in say (y, z) plane ($s = sin(\epsilon) = 1$ using the previous notation).

   For $s \to s_0$ the value of S_c approaches zero: this limit must correspond to $S(S^2) = 0$ and $J_{u\Phi} \to 0$. For $Im(s) \to \pm\infty$ along the critical line, the behavior of $Re(\zeta)$ (see `http://tinyurl.com/y7b88gvg`) strongly suggests that $|X| \to \infty$. This requires that F is an analytic function, which approaches to a finite value at the limit $|X| \to \infty$. Perhaps the simplest elementary function satisfying the saturation constraints is

$$F(X) = S_{max} tanh(-iX) \ . \tag{3.10}$$

   One has $tanh(x + iy) \to \pm 1$ for $y \to \pm\infty$ implying $F(X) \to \pm S_{max}$ at these limits. More explicitly , one has $tanh(-i/2-y) = [-1+exp(-4y)-2exp(-2y)(cos(1)-1)]/[1+exp(-4y)-2exp(-2y)(cos(1)-1)]$. Since one has $tanh(-i/2+0) = 1-1/cos(1) < 0$ and $tanh(-i/2+\infty) = 1$, one must have some finite value $y = y_0 > 0$ for which one has

$$tanh(-\frac{i}{2} + y_0) = 0 \ . \tag{3.11}$$

   The smallest possible lower bound s_0 for the integral defining X would naturally to $s_0 = 1/2 - iy_0$ and would be below the real axis.

4. The interpretation of $S(S^2)$ as a positive definite action requires that the sign of $S(S^2) = Re(F)$ for a given choice of $s_0 = 1/2 + iy_0$ and for a propertly sign of $y - y_0$ at critical line should remain positive. One should show that the sign of $S = a \int Re(\zeta)(s = 1/2 + it)dt$ is same for all zeros of ζ. The graph representing the real and imaginary parts of Riemann zeta along critical line $s = 1/2 + it$ (see `http://tinyurl.com/y7b88gvg`) shows that both the real and imaginary part oscillate and increase in amplitude. For the first zeros real part stays in good approximation positive but the the amplitude for the negative part increase be gradually. This suggests that S identified as integral of real part oscillates but preserves its sign and gradually increases as required.

A priori there is no reason to exclude the trivial zeros of ζ at $s = -2n$, $n = 1, 2, ...$

1. The natural guess is that the function $F(X)$ is same as for the critical line. The integral defining X would be along real axis and therefore real as also $1/\alpha_K$ provided the sign of S_c is positive: for negative sign for S_c not allowed by the geometric interpretation the square root would give imaginary unit. The graph of the Riemann Zeta at real axis (real) is given in MathWorld Wolfram (see `http://tinyurl.com/55qjmj`).

2. The functional equation

$$\zeta(1-s) = \zeta(s)\frac{\Gamma(s/2)}{\Gamma((1-s)/2)} \tag{3.12}$$

   allows to deduce information about the behavior of ζ at negative real axis. $\Gamma((1-s)/2)$ is negative along negative real axis (for $Re(s) \leq 1$ actually) and poles at $n + 1/2$. Its negative maxima approach to zero for large negative values of $Re(s)$ (see `http://tinyurl.`

com/clxv4pz) whereas $\zeta(s)$ approaches value one for large positive values of s (see http://tinyurl.com/y7b88gvg). A cautious guess is that the sign of $\zeta(s)$ for $s \leq 1$ remains negative. If the integral defining the area is defined as integral contour directed from $s < 0$ to a point s_0 near origin, S_c has positive sign and has a geometric interpretation.

3. The formula for $1/\alpha_K$ would read as $\alpha_{K,0}/\alpha_K(s = -2n) = (S_c/S_0)^{1/2}$ so that α_K would remain real. This integration path could be interpreted as a rotation around z-axis leaving invariant the Kähler form J of $S^2(X^4)$ and therefore also $S = Re(S_c)$. $Im(S_c) = 0$ indeed holds true. For the non-trivial zeros this is not the case and $S = Re(S_c)$ is not invariant.

4. One can wonder whether one could distinguish between Minkowskian and Euclidian and regions in the sense that in Minkowskian regions $1/\alpha_K$ correspond to the non-trivial zeros and in Euclidian regions to trivial zeros along negative real axis. The interpretation as different kind of phases might be appropriate.

What is nice that the hypothesis about equivalence of the geometry based and number theoretic approaches can be killed by just calculating the integral S as function of parameter s. The identification of the parameter s appearing in the RG equations is no unique. The identification of the Riemann sphere and twistor sphere could even allow identify the parameter t as imaginary coordinate in complex coordinates in $SO(3)$ rotations around z-axis act as phase multiplication and in which metric has the standard form.

3.4 Some guesses to be shown to be wrong

The following argument suggests a connection between p-adic length scale hypothesis and evolution of cosmological constant but must be taken as an ad hoc guess: the above formula is enough to predict the evolution.

1. p-Adicization is possible only under very special conditions [L4], and suggests that anomalous dimension involving logarithms should vanish for $s = 2^{-k/2}$ corresponding to preferred p-adic length scales associated with $p \simeq 2^k$. Quantum criticality in turn requires that discrete p-adic coupling constant evolution allows the values of coupling parameters, which are fixed points of RG group so that radiative corrections should vanish for them. Also anomalous dimensions Δk should vanish.

2. Could one have $\Delta k_{n,a} = 0$ for $s = 2^{-k/2}$, perhaps for even values $k = 2k_1$? If so, the ratio c/s would satisfy $c/s = 2^{k_1} - 1$ at these points and Mersenne primes as values of c/s would be obtained as a special case. Could the preferred p-adic primes correspond to a prime near to but not larger than $c/s = 2^{k_1} - 1$ as p-adic length scale hypothesis states? This suggest that we are on correct track but the hypothesis could be too strong.

3. The condition $\Delta d = 0$ should correspond to the vanishing of dS/ds. Geometrically this would mean that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

4 Generalized conformal symmetry, quantum criticality, catastrophe theory, and analogies with thermodynamics and gauge theories

The notion of quantum criticality allows two realizations: as stationarity of S^2 part of the twistor lift of Kähler action and in terms of zeros of zeta are key elements in the explicit proposal for discrete coupling constant evolution reducing to that for cosmological constant.

4.1 Quantum criticality from different perspectives

Quantum criticality is however much more general notion, and one must ask how this view relates to the earlier picture.

1. At the real number side continuous coupling constant evolution makes sense. What does this mean? Can one say that quantum criticality makes possible only adelic physics together with large $h_{eff}/h_0 = n$ as dimension for extension of rationals. This hierarchy is essential for life and cognition.

   Can one conclude that living systems correspond to quantum critical values of $S(S^2)$ and therefore α_K and in-animate systems correspond to other values of α_K? But wouldn't his mean that one gives up the original vision that α_K is analogous to critical temperature. The whole point was that this would make physics unique?

   From mathematical view point also continuous α_K can make sense. α_K can be continuous if it corresponds to a higher-dimensional critical manifold at which two or more preferred extremals associated with the same parameter values co-incide - roots of polynomial $P(x, a, b)$ depending on parameters a, b serves as the canonical example. The degree of quantum criticality would vary and there would be a hierarchy of critical systems characterized by the dimension of the critical manifold. One would have a full analog of statistical physics. For mathematician this is the only convincing interpretation.

   2-D cusp catastrophe serves as a basic example helping to generalize [A1]. Cusp corresponds to the roots of $dP_4/dx = 0$ of third order polynomial $P_4(x, a, b)$, where (a, b) are control variables. The projection of region with 3 real roots to (a, b)-plane is bounded by critical lines forming a roughly V-shaped structure. d^2P_4/dx^2 vanishes at the edges of V, where two roots co-incide and d^3P_4/dx^3 vanishes at the tip of V, where 3 roots co-incide.

2. A hierarchy of quantum criticalities has been actually assumed. The hierarchy of representations for super-symplectic algebra realizing 4-D analog of super-conformal symmetries allows an infinite hierarchy of representations for which infinite-D sub-algebra isomorphic to a full algebra and its commutator with the full algebra annihilate physical states. Also classical Noether charges vanish. What is new is that conformal weights are non-negative integers. The effective dimensions of these systems are finite - at least in the sense that one one has finite-D Lie algebra (or its quantum counterpart) or corresponding Kac-Moody algebra as symmetries. This realization of quantum criticality generalize the idea that conformal symmetry accompanies 2-D criticality.

   This picture conforms also with the vision about hierarchy of hyper-finite-factors with included hyper-finite factor defining measurement resolution [K4]. Hyper-finiteness indeed means finite-dimensionality in excellent approximation.

4.2 TGD as catastrophe theory and quantum criticality as prerequisite for the Euclidian signature of WCW metric

It is good to look more precisely how the catastrophe theoretic setting generalizes to TGD.

1. The value of the twistor lift of Kähler action defining Kähler function very probably corresponds to a maximum of Kähler function since otherwise metric defined by the second derivatives could have non-Euclidian signature. One cannot however exclude the possibility that in complex WCW coordinates the (1,1) restriction of the matrix defined by the second derivatives of Kähler function could be positive definite also for other than minima.

   It would seem that one cannot accept several roots for given zero modes since one cannot have maximum of Kähler function for all of them. This would allow only the the boundary of catastrophe region in which 2 or more roots co-incide. Positive definiteness of WCW metric would force quantum criticality.

   For given values of zero modes there would be single minimum and together with the cancellation of Gaussian and metric determinants this makes perturbation theory extremely simple since exponents of vacuum functional would cancel.

2. There is an infinite number of zero modes playing the role of control variables since the value of the induce Kähler form is symplectic invariant and there are also other symplectic invariants associated with the M^4 degrees of freedom (carrying also the analog of Kähler form for the twistor lift of TGD and giving rise to CP breaking). One would have catastrophe theory with infinite number of control variables so that the number of catastrophes would be infinite so that standard catastrophe theory does not as such apply.

3. Therefore TGD would not be only a personal professional catastrophe but a catastrophe in much deeper sense. WCW would be a catastrophe surface for the functional gradient of the action defining Kähler function. WCW would consists of regions in which given zero modes would correspond to several minima. The region of zero mode space at which some roots identifiable as space-time surfaces co-incide would be analogous to the V-shaped cusp catastrophe and its higher-D generalizations. The question is whether one allows the entire catastrophe surface or whether one demands quantum criticality in the sense that only the union of singular sets at which roots co-incide is included.

4. For WCW as catastrophe surface the analog of V in the space of zero modes would correspond to a hierarchy of sub-WCWs consisting of preferred extremals satisfying the gauge conditions associated with a sub-algebra of supersymplectic algebra isomorphic to the full algebra. The sub-WCWs in the hierarchy of sub-WCWs within sub-WCWs would satisfy increasingly stronger gauge conditions and having decreasing dimension just like in the case of ordinary catastrophe. The lower the effective dimension, the higher the quantum criticality.

5. In ordinary catastrophe theory the effective number of behavior variables for given catastrophe can be reduced to some minimum number. In TGD framework this would correspond to the reduction of super-symplectic algebra to a finite-D Lie algebra or corresponding Kac-Moody (half-)algebra as modes of supersymplectic algebra with generators labelled by non-negative integer n modulo given integer m are eliminated as dynamical degrees of freedom by the gauge conditions: this would effectively leave only the modes smaller than m. The fractal hierarchy of these supersymplectic algebras would correspond to the decomposition of WCW as a catastrophe surface to pieces with varying dimension. The reduction of the effective dimension as two sheets of the catastrophe surface co-incide would mean transformation of some modes contributing to metric to zero modes.

4.3 RG invariance implies physical analogy with thermodynamics and gauge theories

One can consider coupling constant evolution and RG invariance from a new perspective based on the minimal surface property.

1. The critical values of Kähler coupling strength would correspond to quantum criticality of the S^2 part $S(S^2)$ of 6-D dimensionally reduced Kähler action for fixed values of zero modes. The relative S^2 rotation would serve as behavior variable. For its critical values the dimension of the critical manifold would be reduced, and keeping zero modes fixed a critical value of α_K would be selected from 1-D continuum.

   Quantum criticality condition might be fundamental since it implies the constancy of the value of the twistor lift of Kähler action for the space-time surfaces contributing to the scattering amplitudes. This would be crucial for number theoretical vision since the continuation of exponential to p-adic sectors is not possible in the generic case. One should however develop stronger arguments to exclude the continuous evolution of Kähler coupling strength in S^2 degrees of freedom for the real sector of the theory.

2. The extremals of twistor lift contain dependence on the rotation parameter for S^2 and this must be taken into account in coupling constant evolution along curve of S^2 connecting zeros of zeta. This gives additional non-local term to the evolution equations coming from the dependence of the imbedding space coordinates of the preferred extremal on the evolution parameter. The derivative of Kähler action with respect to the evolution parameter is by chain rule proportional to the functional derivatives of action with respect to imbedding space

coordinates, and vanish if 4-D Kähler action and volume term are *separately* stationary with respect to variations. Therefore minimal surface property as implied by holomorphy guaranteeing quantum criticality as universality of the dynamics would be crucial in simplifying the equations! It does not matter whether there is coupling between Kähler action and volume term.

Could one find interpretation for the miminal surface property which implies that field equations are separately satisfied for Kähler action and volume term?

1. Quantum TGD can be seen as a "complex" square root of thermodynamics. In thermodynamics one can define several thermodynamical functions. In particular, one can add to energy E as term pV to get enthalpy $H = E + pV$, which remains constant when entropy and pressures are kept constant. Could one do the same now?

   In TGD V replaced with volume action and p would be a coupling parameter analogous to pressure. The simplest replacement would give Kähler action as outcome. The replacement would allow RG invariance of the modified action only at critical points so that replacement would be possible only there. Furthermore, field equations must hold true separately for Kähler action and volume term everywhere.

2. It seems however that one must allow singular sets in which there is interaction between these terms. The coupling between Kähler action and volume term could be non-trivial at singular sub-manifolds, where a transfer of conserved quantities between the two degrees of freedom would take place. The transfer would be proportional to the divergence of the canonical momentum current $D_\alpha(g^{\alpha\beta}\partial_\beta h^k)$ assignable to the minimal surface and is conserved outside the singular sub-manifolds.

   Minimal surfaces provide a non-linear generalization of massless wave-equation for which the gradient of the field equals to conserved current. Therefore the interpretation could be that these singular manifolds are sources of the analogs of fields defined by M^4 and CP_2 coordinates.

   In electrodynamics these singular manifolds would represented by charged particles. The simplest interpretation would be in terms of point like charges so that one would have line singularity. The natural identification of world line singularities would be as boundaries of string world sheets at the 3-D light-like partonic orbits between Minkowskian and Euclidian regions having induced 4-metric degenerating to 3-D metric would be a natural identification. These world lines indeed appear in twistor diagrams. Also string world sheets must be assumed and they are are natural candidates for the singular manifolds serving as sources of charges in 4-D context. Induced spinor fields would serve as a representation for these sources. These strings would generalize the notion of point like particle. Particles as 3-surfaces would be connected by flux tubes to a tensor network and string world sheets would connected fermion lines at the partonic 2-surfaces to an analogous network. This would be new from the standard model perspective.

   Singularities could also correspond to a discrete set of points having an interpretation as cognitive representation for the loci of initial and final states fermions at opposite boundaries of CD and at vertices represented topologically by partonic 2-surfaces at which partonic orbits meet. This interpretation makes sense if one interprets the imbedding space coordinates as analogs of propagators having delta singularities at these points. It is quite possible that also these contributions are present: one would have a hierarchy of delta function singularities associated with string worlds sheets, their boundaries and the ends of the boundaries at boundaries of CD, where string world sheet has edges.

3. There is also an interpretation of singularities suggested by the generalization of conformal invariance. String world sheets would be co-dimension 2 singularites analogous to poles of analytic function of two complex coordinates in 4-D complex space. String world sheets would be co-dimension 2 singularities analogous to poles at light-like 3-surfaces. The ends of the world lines could be analogous of poles of analytic function at partonic 2-surfaces.

   These singularities could provide to evolution equations what might be called matter contribution. This brings in mind evolution equations for n-point functions in QFT. The resolution of the overall singularity would decompose to 2-D, 1-D and 0-D parts just like cusp catastrophe. One can ask whether the singularities might allow interpretation as catastrophes.

4. The proposal for analogs of thermodynamical functions suggests the following physical picture. More general thermodynamical functions are possible only at critical points and only if the extremals are miminal surfaces. The singularities should correspond to physical particles, fermions. Suppose that one considers entire scattering amplitude involving action exponential plus possible analog of pV term plus the terms associated with the fermions assigned with the singularities. Suppose that the coupling constant evolution from 6-D Kähler action is calculated *without* including the contribution of the singularities.

   The derivative of n-particle amplitude with respect to the evolution parameter contains a term coming from the action exponential receiving contributions only from the singularities and a term coming from the operators at singularities. RG invariance of the scattering amplitude would require that the two terms sum up to zero. In the thermodynamical picture the presence of particles in many particle scattering amplitude would force to add the analog of pressure term to the Kähler function: it would be determined by the zero energy state.

   One can of course ask how general terms can be added by requiring minimal surface property. Minimal surface property reduces to holomorphy, and can be true also for other kinds of actions such as the TGD analogs of electroweak and color actions that I considered originally as possible action candidates.

   This would have interpretation as an analog for YM equations in gauge theories. Space-time singularities as local failure of minimal surface property would correspond to sources for H coordinates as analogs of Maxwell's fields and sources currents would correspond to fermions currents.

5 Appendix: Explicit formulas for the evolution of cosmological constant

What is needed is induced Kähler form $J(S^2(X^4)) \equiv J$ at the twistor sphere $S^2(X^4) \equiv S^2$ associated with space-time surface. $J(S^2(X^4))$ is sum of Kähler forms induced from the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$.

$$J(S^2(X^4) \equiv J = P[J(S^2(M^4)) + J(S^2(CP_2))] \ , \tag{5.1}$$

where P is projection taking tensor quantity T_{kl} in $S^2(M^4) \times S^2(CP_2)$ to its projection in $S^2(X^4)$. Using coordinates y^k for $S^2(M^4)$ or $S^(CP_2)$ and x^μ forS^2, P is defined as

$$P : T_{kl} \rightarrow T_{\mu\nu} = T_{kl}\frac{\partial y^k}{\partial x^\mu}\frac{\partial y^l}{\partial x^\nu} \ . \tag{5.2}$$

For the induced metric $g(S^2(X^4)) \equiv g$ one has completely analogous formula

$$g = P[g(J(S^2(M^4)) + g(S^2(CP_2))] \ . \tag{5.3}$$

The expression for the coefficient K of the volume part of the dimensionally reduced 6-D Kähler action density is proportional to

$$L(S^2) = J^{\mu\nu}J_{\mu\nu}\sqrt{det(g)} \ . \tag{5.4}$$

(Note that $J_{\mu\nu}$ refers to S^2 part 6-D Kähler action). This quantity reduces to

$$L(S^2) = (\epsilon^{\mu\nu}J_{\mu\nu})^2\frac{1}{\sqrt{det(g)}} \ . \tag{5.5}$$

where $\epsilon^{\mu\nu}$ is antisymmetric tensor density with numerical values +,-1. The volume part of the action is obtained as an integral of K over S^2:

$$S(S^2) = \int_{S^2} L(S^2) = \int_{-1}^{1} du \int_0^{2\pi} d\Phi \frac{J_{u\Phi}^2}{\sqrt{det(g)}} \quad . \tag{5.6}$$

$(u,\Phi) \equiv (cos(\Theta,\Phi)$ are standard spherical coordinates of S^2) varying in the ranges $[-1,1]$ and $[0,2\pi]$.

This the quantity that one must estimate.

5.1 General form for the imbedding of twistor sphere

The imbedding of $S^2(X^4) \equiv S^2$ to $S^2(M^4) \times S^2(CP_2)$ must be known. Dimensional reduction requires that the imbeddings to $S^2(M^4)$ and $S^2(CP_2)$ are isometries. They can differ by a rotation possibly accompanied by reflection

One has

$$(u(S^2(M^4)), \Phi(S^2(M^4)) = (u(S^2(X^4), \Phi(S^2(X^4)) \equiv (u,\Phi) \quad ,$$

$$\left[u(S^2(CP_2)), \Phi(S^2(CP_2))\right] \equiv (v,\Psi) = RP(u,\Phi)$$

where RP denotes reflection P following by rotation R acting linearly on linear coordinates (x,y,z) of unit sphere S^2). Note that one uses same coordinates for $S^2(M^4)$ and $S^2(X^4)$. From this action one can calculate the action on coordinates u and Φ by using the definite of spherical coordinates.

The Kähler forms of $S^2(M^4)$ resp. $S^2(CP_2)$ in the coordinates $(u = cos(\Theta), \Phi)$ resp.(v,Ψ) are given by $J_{u\Phi} = \epsilon = \pm 1$ resp. $J_{v\Psi} = \epsilon = \pm 1$. The signs for $S^2(M^4)$ and $S^2(CP_2)$ are same or opposite. In order to obtain small cosmological constant one must assume either

1. $\epsilon = -1$ in which case the reflection P is absent from the above formula (RP $\rightarrow$ R).

2. $\epsilon = 1$ in which case P is present. P can be represented as reflection $(x,y,z) \rightarrow (x,y,-z)$ or equivalently $(u,\Phi) \rightarrow (-u,\Phi)$.

Rotation R can represented as a rotation in (y,z)-plane by angle ϕ which must be small to get small value of cosmological constant. When the rotation R is trivial, the sum of induced Kähler forms vanishes and cosmological constant is vanishing.

5.2 Induced Kähler form

One must calculate the component $J_{u\Phi}(S^2(X^4)) \equiv J_{u\Phi}$ of the induced Kähler form and the metric determinant $det(g)$) using the induction formula expressing them as sums of projections of M^4 and CP_2 contributions and the expressions of the components of $S^2(CP_2)$ contributions in the coordinates for $S^2(M^4)$. This amounts to the calculation of partial derivatives of the transformation R (or RP) relating the coordinates (u,Φ) of $S^2(M^4)$ and to the coordinates (v,Ψ) of $S^2(CP_2)$.

In coordinates (u,Φ) one has $J_{u\Phi}(M^4) = \pm 1$ and similar expression holds for $J(v\Psi)S^2(CP_2)$. One has

$$J_{u\Phi} = 1 + \frac{\partial(v,\Psi)}{\partial(u,\Phi)} \quad . \tag{5.7}$$

where right-hand side contains the Jacobian determinant defined by the partial derivatives given by

$$\tfrac{\partial(v,\Psi)}{\partial(u,\Phi)} = \tfrac{\partial v}{\partial u}\tfrac{\partial \Psi}{\partial \Phi} - \tfrac{\partial v}{\partial \Phi}\tfrac{\partial \Psi}{\partial u} \quad . \tag{5.8}$$

5.3 Induced metric

The components of the induced metric can be deduced from the line element

$$ds^2(S^2(X^4) \equiv ds^2 = P[ds^2(S^2(M^4)) + ds^2(S^2(CP_2))] \ .$$

where P denotes projection. One has

$$P(ds^2(S^2(M^4))) = ds^2(S^2(M^4)) = \frac{du^2}{1-u^2} + (1-u^2)d\Phi^2 \ .$$

and

$$P[ds^2(S^2(CP_2))] = P[\frac{(dv)^2}{1-v^2} + (1-v^2)d\Psi^2] \ ,$$

One can express the differentials $(dv, d\Psi)$ in terms of $(du, d\Phi)$ once the relative rotation is known and one obtains

$$P[ds^2(S^2(CP_2))] = \frac{1}{1-v^2}[\frac{\partial v}{\partial u}du + \frac{\partial v}{\partial \Phi}d\Phi]^2 + (1-v^2)[\frac{\partial \Psi}{\partial u}du + \frac{\partial \Psi}{\partial \Phi}d\Phi]^2 \ .$$

This gives

$$\begin{aligned} & P[ds^2(S^2(CP_2))] \\ & = [(\frac{\partial v}{\partial u})^2\frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2]du^2 \\ & +[(\frac{\partial v}{\partial \Phi})^2\frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2 1-v^2]d\Phi^2 \\ & +2[\frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2)]dud\Phi \ . \end{aligned}$$

From these formulas one can pick up the components of the induced metric $g(S^2(X^4)) \equiv g$ as

$$\begin{aligned} g_{uu} &= \frac{1}{1-u^2} + (\frac{\partial v}{\partial u})^2\frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2] \ , \\ g_{\Phi\Phi} &= 1-u^2 + (\frac{\partial v}{\partial \Phi})^2\frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2(1-v^2) \\ g_{u\Phi} &= g_{\Phi u} = \frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2) \ . \end{aligned} \tag{5.9}$$

The metric determinant $det(g)$ appearing in the integral defining cosmological constant is given by

$$det(g) = g_{uu}g_{\Phi\Phi} - g_{u\Phi}^2 \ . \tag{5.10}$$

5.4 Coordinates (v, Ψ) in terms of (u, Φ)

To obtain the expression determining the value of cosmological constant one must calculate explicit formulas for (v, Ψ) as functions of (u, Φ) and for partial derivations of (v, Ψ) with respect to (u, Φ).

Let us restrict the consideration to the RP option.

1. P corresponds to $z \rightarrow -z$ and to

$$u \rightarrow -u \ . \tag{5.11}$$

2. The rotation R $(x,y,z) \rightarrow (x',y',z')$ corresponds to

$$x' = x, \quad y' = sz + cy = su + c\sqrt{1-u^2}sin(\Phi) \ , \quad z' = v = cu - s\sqrt{1-u^2}sin(\Phi) \ . \quad (5.12)$$

   Here one has $(s,c) \equiv (sin(\epsilon), cos(\epsilon)$, where ϵ is rotation angle, which is extremely small for the value of cosmological constant in cosmological scales.

   From these formulas one can pick v and $\Psi = arctan(y'/x)$ as

$$v = cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \quad (5.13)$$

3. RP corresponds to

$$v = -cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[-\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \quad (5.14)$$

5.5 Various partial derivatives

Various partial derivates are given by

$$\begin{aligned}
\frac{\partial v}{\partial u} &= -1 + s\frac{u}{\sqrt{1-u^2}}sin(\Phi) \ , \\
\frac{\partial v}{\partial \Phi} &= -s\frac{u}{\sqrt{1-u^2}}cos(\Phi) \ , \\
\frac{\partial \Psi}{\partial \Phi} &= (-s\frac{u}{\sqrt{1-u^2}}sin(\Phi) + c)\frac{1}{X} \ , \\
\frac{\partial \Psi}{\partial u} &= \frac{scos(\Phi)(1+u-u^2)}{(1-u^2)^{3/2}}\frac{1}{X} \ , \\
X &= cos^2(\Phi) + [-s\frac{u}{\sqrt{1-u^2}} + csin(\Phi)]^2 \ .
\end{aligned} \quad (5.15)$$

Using these expressions one can calculate the Kähler and metric and the expression for the integral giving average value of cosmological constant. Note that the field equations contain S^2 coordinates as external parameters so that each point of S^2 corresponds to a slightly different space-time surface.

5.6 Calculation of the evolution of cosmological constant

One must calculate numerically the dependence of the action integral S over S^2 as function of the parameter $s = sin(\epsilon)$). One should also find the extrema of S as function of s.

Especially interesting values are very small values of s since for the the cosmological constant becomes small. For small values of s the integrand (see Eq. 5.6) becomes very large near poles having the behaviour $1/\sqrt{g} = 1/(sin(\Theta)+O(s))$ coming from $\sqrt{g}$ approaching that for the standard metric of S^2. The integrand remains finite for $s \neq 0$ but this behavior spoils the analytic dependence of integral on s so that one cannot do perturbation theory around $s = 0$. The expected outcome is a logarithmic dependence on s.

In the numerical calculation one must decompose the integral over S^2 to three parts.

1. There are parts coming from the small disks D^2 surrounding the poles: these give identical contributions by symmetry. One must have criterion for the radius of the disk and the natural assumption is that the disk radius is of order s.

2. Besides this one has a contribution from S^2 with disks removed and this is the regular part to which standard numerical procedures apply.

One must be careful with the expressions involving trigonometric functions which give rise to infinite if one applies the formulas in straightforward manner. These infinities are not real and cancel, when one casts the formulas in appropriate form inside the disks.

1. The limit $u \to \pm 1$ at poles involves this kind of dangerous quantities. The expression for the determinant appearing in $J_{u\Phi}$ remains however finite and $J^2_{u\phi}$ vanishes like s^2 at this limit. Also the metric determinant $1/\sqrt{g}$ remains finite expect at $s = 0$.

2. Also the expression for the quantity X in $\Psi = arctan(X)$ contains a term proportional to $1/cos(\Phi)$ approaching infinity for $\Phi \to \pi/2, 3\pi/2$. The value of $\Psi = arc(tan(X)$ remains however finite and equal to $\pm\Phi$ at this limit depending on on the sign of us.

Concerning practical calculation, the relevant formulas are given in Eqs. 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, and 5.15.

The calculation would allow to test/kill the key conjectures already discussed.

1. There indeed exist extrema satisfying $dS(S^2)/ds = 0$.

2. These extrema are in one-one correspondence with the zeros of zeta.

There are also much more specific conjctures to be killed.

1. These extrema correspond to $s = sin(\epsilon) = 2{-}k$ or more generally $s = p^{-k}$. This conjecture is inspired by p-adic length scale hypothesis but since the choice of evolution parameter is to high extent free, the conjecture is perhaps too spesific.

2. For certain integer values of integer k the integral $S(S^2)$ of Eq. 5.6 is of form $S(S^2) = xs^2$ for $s = 2^{-k}$, where x is a universal numerical constant.

   This would realize the idea that p-adic length scales realized as scales associated with cosmological constant correspond to fixed points of renormalization group evolution implying that radiative corrections otherwise present cancel. In particular, the deviation from $s = 2^{-d/2}$ would mean anomalous dimension replacing $s = 2^{-d/2}$ with $s^{-(d+\Delta d)/2}$ for $d = k$ the anomalies dimension Δd would vanish.

   The condition $\Delta d = 0$ should be equivalent with the vanishing of the dS/ds. Geometrically this means that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

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