

Is the sum of p-adic negentropies equal to real entropy?

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Abstract

It is shown that for rationals entanglement probabilities the real entropy equals to the sum of p-adic negentropies. For entanglement probabilities in an extension of rationals inducing a finite-dimensional extension of p-adic numbers this is no more true. A possible interpretation is that at the lowest level of the evolutionary hierarchy defined by the extensions of rationals the p-adic negentropy serving as a measure for conscious (trivial extension) information equals to real entanglement entropy. For algebraic extensions this is no more true and Negentropy Maximization Principle suggests that total p-adic negentropy is in general larger than real entropy. Alternative - not so attractive - interpretation is that negentropy includes also the negative real contribution so that for rational entanglement probabilities the total conscious information would vanish. Large p-adic negentropy however tends to be accompanied by large real entropy which conforms with the vision of Jeremy England.

1 Introduction

I ended almost by accident to a fascinating and almost trivial theorem. Adelic theorem for information would state that conscious information represented as sum of p-adic negentropies (entropies, which are negative) is equal to real entropy. The more conscious information, the larger the chaos in the environment as everyone can verify by just looking around.

This looks bad! Luckily, it turned out that this statement is true for rational probabilities only. For algebraic extensions it cannot be true as is easy to see. That negentropic entanglement is possible only for algebraic extensions of rationals conforms with the vision that algebraic extensions of rationals characterize evolutionary hierarchy. The rationals represent the lowest level at which there either conscious information vanishes or if equal to p-adic contribution to negentropy is accompanied by equally large real entropy.

It is not completely obvious that the notion of p-adic negentropy indeed makes sense for algebraic extensions of rationals. A possible problem is caused by the fact that the decomposition of algebraic integer to primes is not unique. Simple argument however strongly suggests that the various p-adic norms of the factors do not depend on the factorization. Also a formula for the difference of the total p-adic negentropy and real entropy is deduced.

2 Can total p-adic entanglement negentropy be larger than real entanglement entropy for algebraic entanglement probabilities?

This section is an attempt to answer the question of the title and to clarify the deep implications of positive answer to the question.

2.1 p-Adic contribution to negentropy equals to real entropy for rational probabilities but not for algebraic probabilities

The following argument shows that p-adic negentropy equals to real entropy for rational probabilities.

1. The fusion of real physics and various p-adic physics (identified as correlates for cognition, imagination, and intentionality) to single coherent whole leads to what I call adelic physics [K2]. Adeles associated with given extension of rationals are Cartesian product of real number field with all p-adic number fields extended by the extension of rationals. Besides algebraic extensions also the extension by any root of e is possible since it induces finite-dimensional p-adic extension. One obtains hierarchy of adeles and of corresponding adelic physics interpreted as an evolutionary hierarchy.

An important point is that p-adic Hilbert spaces exist only if one restricts the p-adic numbers to an algebraic extension of rationals having interpretation as numbers in any number field. This is due to the fact that sum of the p-adic valued probabilities can vanish for general p-adic numbers so that the norm of state can vanish. One can say that the Hilbert space of states is universal and is in the algebraic intersection of reality and various p-adicities.

2. Negentropy Maximization Principle (NMP) is the variational principle of consciousness in TGD framework reducing to quantum measurement theory in Zero Energy Ontology assuming adelic physics. One can define the p-adic counterparts of Shannon entropy for all finite-dimensional extensions of p-adic numbers, and the amazing fact is that these entropies can be negative and thus serve as measures for information rather than for lack of it. Furthermore, all non-vanishing p-adic negentropies are positive and the number of primes contributing to negentropy is finite since any algebraic number can be expressed using a generalization of prime number decomposition of rational number. These p-adic primes characterize given system, say elementary particle.

NMP states that the negentropy gain is maximal in the quantum jump defining state function reduction. How does one define the negentropy? As the sum of p-adic negentropies or as the sum of real negative negentropy plus the sum of p-adic negentropies? The latter option I proposed for some time ago without checking what one obtains.

3. The adelic theorem says that the norm of rational number is equal to the product of the inverses of its p-adic norms. The statement that the sum of real and p-adic negentropies is zero follows more or less as a statement that the logarithms of real norm and the product of p-adic norms for prime factors of rational sum up to zero.

The core formula is adelic formula stating that the real norm of rational number is product of its p-adic norms. This implies that the logarithm of the rational number is sum over the logarithms of its p-adic norms. Since in p-adic entropy assigned to prime p logarithms of probabilities are replaced by their p-adic norms, this implies that for rational probabilities the real entropy equals to p-adic negentropy.

It would seem that the negentropy appearing in the definition of NMP must be the sum of p-adic negentropies and real entropy should have interpretation as a measure for ignorance about the state of either entangled system. The sum of p-adic negentropies would serve as a measure for the information carried by a rule with superposed state pairs representing the instances of the rule. The information would be conscious information and carried by the negentropically entangled system.

4. What about probabilities in algebraic extensions? The probabilities are now algebraic numbers. The induced p-adic norm $N_p(x)$ for n-dimensional extension of Q is defined as the determinant $det(x)$ of the linear map defined by multiplication with x . $det(x)$ is rational number. The corresponding p-adic norm is defined as the n :th root $N_p(det(x))^{1/n}$ of the ordinary p-adic norm to guarantee that the norm co-incides with the ordinary p-adic norm for ordinary p-adic integers.

One must perform now a factorization to algebraic primes. Below an argument is given that although the factorization to primes is not always unique, the product of p-adic norms for given algebraic rational defined as ratio of algebraic integers is unique.

The p-adic norms of probabilities are however always powers of primes so that the adelic formula *cannot* be true since on the real side one has logarithms of algebraic numbers and on the p-adic side only logarithms of primes.

What could be the interpretation?

1. If conscious information corresponds to $N - P$, it accompanies the emergence of algebraic extensions of rationals at the level of Hilbert space.
2. If N corresponds to conscious information, then at the lowest level conscious information is necessary accompanied by same entropy but for algebraic extensions $N - P$ could be positive since N is maximized.

Both interpretations conform with the number theoretic vision about evolution. One expects that the value of real entropy correlates strongly with the value of negentropy. This would conform with the observation that large entropy seems to be a prerequisite for life by providing large number of states with degenerate energies providing large representative capacity. For instance, Jeremy England has made this proposal [I1]: I have commented this proposal from [L1] (see <http://tinyurl.com/zjp3bp6>).

2.2 Formula for the difference of total p-adic negentropy and real entanglement entropy

Can one write an explicit formula the difference of total p-adic entanglement negentropy and real entanglement entropy (non-negative) using prime factorization in finite dimensional algebraic extension (note that for algebraic numbers defining infinite-dimensional extension of rationals factorization does not even exist since one can write $a = \sqrt{a}\sqrt{a} = \dots$)? This requires that total p-adic entropy is uniquely defined. There is a possible problem due to the non-uniqueness of the prime factorization.

1. For Dedekind rings, in particular rings of integers, there exists by definition a unique factorization of proper ideals to prime ideals (see <http://tinyurl.com/h3oufpp>). In contrast, the prime factorization in the extensions of Q is not always unique. Already for $Q(\sqrt{-5})$ one has $6 = 2 \times 3 = (1 + \sqrt{-5})(1 - \sqrt{-5})$ and the primes involved are not related by multiplication with units.

Various factorizations are characterized by so called class group and class field theory (see <http://tinyurl.com/zdnw7j3> and <http://tinyurl.com/j9nq45d>) is the branch of number theory studying factorizations in algebraic extensions of integer rings. Factorization is by definition unique for Euclidian domains. Euclidian domains allow by definition so called Euclidian function $f(x)$ having values in R_+ with the property that for any a and b one has either $a = qb$ or $a = qb + r$ with $f(r) < f(b)$. It seems that one cannot restrict to Euclidian domains in the recent situation. Euclidian domain property is equivalent with the property that all principal ideals are generated by algebraic integer. For instant, $Q(\sqrt{-3})$ is not Euclidian domain since the integers $a + b\sqrt{-3}$, $a + b$ even, do not form a principal ideal. For instance, the integers $(2, 0)$ and $(1, 1)$ have same norm 6 but do not differ by a multiplication with unit.

- Even when the factorization in the extension is not unique, one can hope that the product of various p-adic norms for the factors is same for all factorizations. Since the p-adic norm for the extensions of primes is induced by ordinary p-adic number this requires that the p-adic prime for which the induced p-adic norm differs from unity are same for all factorizations and that the products of p-adic norms differing from unity are same. This independence on the representative for factorization would be analogous to gauge invariance in physicist's conceptualization.

The probabilities P_k belongs to a unique product of ideals labelled by primes of extension. The ideals are characterized by norms and if this norm is product of p-adic norms for any prime factorization as looks natural then the independence on the factorization follows. Number theorist can certainly immediately tell whether this is true. What is encouraging that for $Q(\sqrt{-5})$ $z = x + \sqrt{-5}y$ has determinant $\det(z) = x^2 + 5y^2$ and for $z = 1 \pm \sqrt{-5}$ one has $\det(z) = 6$ so that for the products of p-adic norms for the factorizations $6 = 2 \times 3$ and $(1 + \sqrt{-5})(1 - \sqrt{-5})$ are equal.

- If this physicist's argument is true, one can write the the difference of total p-adic negentropy N and real entanglement entropy S as

$$N - S = \sum P_k \log\left(\frac{P_k}{\prod_p N_p(P_k)}\right) . \quad (2.1)$$

Here $\prod_p N_p(P_k)$ would not depend on particular factorization. The condition $\sum P_k = 1$ poses an additional condition. It would be nice to understand whether $N - S \geq 0$ holds true generally and if not, what are the conditions guaranteeing this. The p-adic numbers of numerators of rationals involved give positive contributions to $N - S$ as the example $P_k = 1/N$ in rational case shows.

2.3 An ansatz for entanglement probabilities guaranteeing $N - S > 0$

What about entanglement probabilities in algebraic extension of rationals? In this case real number based entanglement entropy is not maximal since entanglement probabilities are different. What can one say about p-adic entanglement negentropies: are they still maximal under some reasonable conditions? The logarithms involved depend on p-adic norms of probabilities and this is in the generic case just inverse of the power of p. Number theoretical universality suggests that entanglement probabilities are of form

$$P_i = \frac{a_i}{N}$$

with $\sum_i a_i = N$ with algebraic numbers a_i not involving natural numbers and thus having unit p-adic norm.

With this assumption the p-adic norms of P_i reduce to those of $1/N$ as for maximal rational entanglement. If this is the case the p-adic negentropy equals to $\log(p^k)$ if p^k divides N . The total adelic negentropy equals to $\log(N)$ and is maximal and has the same value as for rational probabilities equal to $1/N$.

The real entanglement entropy is now in general however smaller than $\log(N)$, which would mean that p-adic negentropy is larger than the real entropy as conjectured earlier [K3] (see <http://tinyurl.com/jozwqxk>). For *rational* entanglement probabilities the generation of entanglement negentropy - conscious information during evolution - would be accompanied by a generation of equal entanglement entropy measuring the ignorance about what the negentropically entangled states representing selves are.

This conforms with the observation of Jeremy England that living matter is entropy producer [K1], [L2] (see <http://tinyurl.com/jff33xk>). For algebraic extensions of rationals this entropy could be however smaller than the total negentropy. Second law follows as a shadow of NMP if the real entanglement entropy corresponds to the thermodynamical entropy. Algebraic evolution would allow to generate conscious information faster than the environment is polluted, one might concretize! The higher the dimension of the algebraic extension rationals, the larger the difference

could be and the future of the Universe might be brighter than one might expect by just looking around! Very consolating! One should however show that the above described situation can be realized as NMP strongly suggests before opening a bottle of champagne.

2.4 Cloning of maximally negentropic states is possible: DNA replication as cloning of this kind of states?

In Facebook discussion with Bruno Marchal and Stephen King the notion of quantum cloning as copying of quantum state popped up and I ended up to ask about approximate cloning and got a nice link about which more below. From Wikipedia article (see <http://tinyurl.com/oyvklde>) one learns some interesting facts cloning. No-cloning theorem states that the cloning of *all* states by unitary time evolution of the tensor product system is not possible. It is however possible clone *orthogonal basis of states*. Does this have some deep meaning?

As a response to my question I got a link to an article of Lamourex et al (see <http://tinyurl.com/zq4kgda>) showing that the *cloning of entanglement* - to be distinguished from the cloning of quantum state - is not possible in the general case. Separability - the absence of entanglement - is not preserved. Approximate cloning generates necessarily some entanglement in this case, and the authors give a lower bound for the remaining entanglement in case of an unentangled state pair.

The cloning of maximally entangled state is however possible. What makes this so interesting is that maximally negentropic entanglement for *rational* entanglement probabilities in TGD framework corresponds to maximal entanglement - entanglement probabilities form a matrix proportional to unit matrix- and just this entanglement is favored by Negentropy Maximization Principle [K1]. Could maximal entanglement be involved with say DNA replication? Could maximal negentropic entanglement for algebraic extensions of rationals allow cloning so that DNA entanglement negentropy could be larger than entanglement entropy?

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