Neutrinos and TGD

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Abstract

Mini-Boone experiment (2018) demonstrated that the rate of charged current reaction transforming electron neutrinos $_e$ produced by $\nu_\mu - \nu_e$ mixing in ν_μ beam is so high that a conflict with what is known about neutrino mixing matrix emerges. The recent experiment of Micro-Boone collaboration however shows no evidence for sterile neutrinos. The only remaining anomaly is associated with the channel producing an electron but no hadrons in the final state.

This rather specific question forced a thorough reconsideration of the TGD view about particles and their massivation, and led to a solution of several problems of TGD.

1. The Dirac equation $D(H)\Psi = 0$ in $H = M^4 \times CP_2$ predicts that most states have mass of order CP_2 mass. To obtain massless states one must have a tachyonic ground state whose origin has remained a mystery.

   Twistor lift of TGD predicts that also M^4 has Kähler structure. Furthermore, Lorentz invariance is broken to that for M^2 and eigenstates of $D^2(M^4)$ are characterized by a conformal weight n. The covariantly constant right-handed neutrino becomes a tachyon.

These right-handed neutrinos can be used to construct massless states. They would play the role of $N = 2$ SUSY, whose possible existence has also been a long-standing source of headaches.

2. Neutrinos are massive but only left-handed neutrinos are observed. Here zero energy ontology (ZEO) provides the solution. Unitary time evolution corresponds to a scaling induced by Virasoro generator L_0. This corresponds to the square $D^2(H)$ of $D(H)$ which does not mix M^4-chiralities!

   The superpositions of right- and left-handed M^4 chiralities are possible if they are mass degenerate. The tachyon term present for only right-handed states however implies that the M^4 mass squared values of the M^4 Dirac operator are in general different. If different values of the conformal weight n allow for L-R mass degeneracy for charged leptons but not for neutrinos, a consistency with empirical facts is obtained.

This does not yet explain the conflict between Mini-Boone and Micro-Boone experiments. Because Micro-Boone observes the anomaly for single electron final states only, it seems that neutrinos must scatter from some new form of matter. TGD indeed predicts $h_{eff} > h$ phases of ordinary particles behaving like dark matter. The anomalous production of electrons by charged currents could be due to the presence of dark protons or nuclei in the detector and having large enough h_{eff}. This could scale up weak interaction Compton length by h_{eff}/h above nuclear or even atomic length scale so that weak bosons would be effectively massless particles and the scattering cross section could be of the same order of magnitude as electroweak scattering cross section.

1 Introduction

Neutrinos are problematic from the point of view of the standard model. It has become clear that neutrinos experience an analog of CKM mixing for quarks but there are anomalous findings related to the mixing. MiniBoone collaboration published 2018 findings [C2] (see `https://arxiv.org/abs/1805.12028`) related to the mixing between muon and electron neutrinos for incoming muon beam.

The transformation of electron neutrino to electron via charged current reaction was used as a signature for the electron neutrinos and the findings forced the conclusion that the number of electrons produced is too high to be consistent with the neutrino CKM matrix deduced from other experiments. The sterile neutrino was one of the many proposed explanations (see `https://cutt.ly/DRKPZYz`).

The recent experiment of Micro-Boone collaboration however shows no evidence for sterile neutrinos (`https://cutt.ly/QRKDsUA` and `https://cutt.ly/oRKS77W`). The only remaining anomaly is associated with the channel producing an electron but no hadrons in the final state. If this finding is taken seriously, it is difficult to avoid the conclusion that some new physics, which is not caught by the standard model, is involved. Could the transformation of neutrino to an electron occur in some unknown way?

As it often happens, this rather specific question led to a thorough reconsideration of the TGD view about particles and their massivation: what is really understood and what is really certain? The basic idea of the TGD based solution described at the end of the article, would not have required these considerations so that an inpatient reader can directly skip to the last section.

1.1 What is the role of right handed neutrinos in TGD?

The new view led to the conclusion that the right-handed neutrino predicted by TGD and analogous to the inert neutrino solves some long-standing problems of TGD.

1. TGD in its recent form predicts an entire tower of color excitations as modes of second quantized $H = M^4 \times CP_2$ spinor field identified as a quark field. The mass scale determined by CP_2 length scale and these give rise to bound states of 3 antiquarks having quantum numbers of leptons if TGD view about color symmetry is accepted [L13]. In particular, covariantly constant right-handed neutrino ν_R in some respects analogous to a sterile neutrino is predicted.

It is intuitively clear that ν_R must have a very special physical role. The naive proposal that ν_R and ν_R could generate the analog $N = 2$ SUSY [L6] has not led to a breakthrough. Spartners would have been created by adding zero momentum right-handed neutrinos and antineutrinos to the state: the problem is that the norm of these states vanishes if the only CP_2 Kähler form is present as in the formulation of TGD before the discovery of the twistor lift of TGD.

2. The twistor lift of TGD [K4] predicts that also M^4 has Kähler structure. This implies a breaking of Lorentz symmetry within causal diamond CD to $M^2 \subset M^4$ emerging also in the the dual M^8 picture based on number theoretical view about physics [L7, L8, L14] as a prerequisite of $M^8 - H$ duality.

   M^4 mass squared m^2 is replaced with M^2 mass squared as in the quark model of hadrons, in string models, and also in p-adic mass calculations [K2]. The M^2 mass squared spectrum for $H = M^4 \times CP_2$ spinor modes is very much like in conformal field theories and the two integers (n_1, n_2) characterizing analogs of cyclotron states are analogous to conformal weights.

   The key point is that the massless ν_R transforms to a tachyon. This is due to the presence of spin term $J^{kl}(M^4)\Sigma_{kl}$ in $D^2(H)$ vanishing for left-handed leptons. On the other hand, p-adic mass calculations [K2] require a tachyon- like ground state: otherwise massless states are impossible. The origin of tachyonicity has remained a mystery. The tachyobiuc right-handed neutrinos could provide the long sought-for mechanism allowing to reduce the conformal weight of a given many-quark state to obtain a massless state.

3. The hard problem is that neutrinos are massive but only the left-handed neutrinos are observed. The problem is that the left-handed neutrinos mix with the right-handed ones if H Dirac operator $D(H)$ determines the time evolution operator. This should be seen in neutrino mixing experiments.

   The proposed solution of the problem is based on the TGD view about time evolution in zero energy ontology (ZEO). It has become clear that the time evolution between "small" state function reductions (SSFRs) corresponds to a scaling rather than time translation, and is induced by Virasoro generator L_0 - essentially mass squared operator - rather than by Hamiltonian.

   This suggests that for the spinor modes of H, the mass squared operator, that is the square $D^2(H)$ of Dirac operator $D(H)$ - or rather, its longitudinal M^2 part - should determine the time evolution operator rather than $D(H)$. Different M^4 chiralities would *not* mix.

4. This alone does not explain why only left-handed neutrinos are observed since different M^4 chiralities for leptons can appear as superpositions if left and right M^4 chiralities have the same value of $m^2(M^2)$. However, the $J^{kl}(M^4)\Sigma_{kl}$ term in $D^2(H)$ implies L-R splitting of mass squared eigenvalues. Degeneracy is possible if different values of n_1+n_2 can compensate for this splitting.

   Empirical facts require that R-L mixing is possible for charged leptons but not for neutrino states. Right-handed neutrinos would not mix with left-handed ones and would couple only to M^4 Kähler form but not to electroweak interactions. This could explain why they are not detected but also suggests that their detection might be possible.

1.2 Mini-Boone-Micro-Boone conflict and the TGD view about dark matter

This picture looks nice but does not explain the conflict between Mini-Boone and Micro-Boone experiments. Because Micro-Boone observes the anomaly for single electron final states only, it seems that neutrinos must scatter from some new form of matter.

TGD indeed predicts $h_{eff} > h$ phases of ordinary particles behaving like dark matter. The anomalous production of electrons by charged currents could be understood by the presence of dark protons or nuclei in the detector and having large enough h_{eff}. This could scale up weak interaction Compton length by h_{eff}/h above nuclear or even atomic length scale so that weak bosons would be effectively massless particles and the scattering cross section could be of the same order of magnitude as electroweak scattering cross section.

2 Some background about TGD

Some background about TGD is necessary in order to tackle the problems related to neutrinos.

2.1 Spinor fields in TGD

Spinor fields appear in TGD at three levels. At the level of embedding space $H = M^4 \times CP_2$, at the level of space-time surface $X^4 \subset H$, and at the level of "world of classical worlds" (WCW).

2.1.1 Spinor fields in H

Consider first spinor fields and their quantization at the level of H, which actually induces the spinor structure at the level of X^4 and WCW.

1. In the TGD Universe space-times are 4-surfaces X^4 in 8-D $H = M^4 \times CP_2$. The only fundamental fermions are quarks and the TGD view about color allows us to identify leptons as composites of 3 antiquarks in the scale of CP_2: this is not possible in QCD [L6, L13]. In what follows a key assumption is that leptons behave effectively like H spinor field having a chirality opposite to that for quarks and have the same electroweak quantum numbers apart from em charge. Therefore the Dirac equation in H applies to them.

2. The quantization of spinors is carried out at the level of H and quantized quark fields in X^4 are induced, that is restricted, to X^4 so that one avoids all problems related to second quantization in curved background. One of them is the difficulty in defining what positive and negative energy solutions to the Dirac equation do really mean.

3. If the Kähler form of $J(M^4)$ of M^4 vanishes (the more general case will be discussed later on), the square $D^2(H)$ of the H Dirac operator $D(H) = D(M^4) + D(CP_2)$ allows solutions satisfying $D^2(H)\Psi = 0$ that is massless modes in 8-D sense. The solutions of $D(H)\Psi = 0$ are of form $D(M^4)\Psi_1 \otimes \Psi_2 + \Psi_1 \otimes D(CP_2)\Psi_2$. Ψ_1 is a plane wave and Ψ_2 is an eigenstate of $D^2(CP_2)$ with a quantized mass squared eigenvalue m^2. Note that chiralities are mixed in accordance with the massivation in H.

   Covariantly constant right-handed neutrino is the only massless solution of $D(H)\Psi = 0$ in the M^4 sense. Since it does not have electroweak couplings it satisfies $D(CP_2)\nu_R = 0$ and is covariantly constant in CP_2. One can say that masslessness in 4-D sense is replaced with masslessness in 8-D sense and this is crucial also for why the twistor lift of TGD applies also to massive particles.

   One can say that $D(CP_2)$ is the analog of $D(M^4) = \gamma^k p^k$ in M^4 degrees of freedom. However, it cannot be algebraized. One could also say that it acts as an analog of the Higgs field which is not a H scalar but a CP_2 vector.

2.1.2 Spinor fields in X^4

Consider next the spinor fields at the level of X^4.

1. One can define modified Dirac operator [L14] at the level of X^4 in terms of the modified Gamma matrices determined as contractions of H gamma matrices Γ^k and the canonical momentum currents T^α_k determined by the action, which for twistor lift involves volume term (length scale dependent cosmological constant) and Kähler action analogous to Maxwell action. Preferred extremals are actually minimal surfaces which are also extremals of the Kähler action in the interior of X^4 [L15].

2. Modified Dirac equation cannot be satisfied generally as an operator equation. It could be however satisfied at the boundaries of causal diamond (CD) (one might say for external free quarks there) or possibly even in the interior of X^4 for the physical states but not generally. In any case the oscillator operator algebra for quarks in H would be used to construct quantum states.

The intuitive guess is that the inverse of D canappear as a propagator. Its construction looks however a horrible problem. Fortunately, the problem disappears since $D(H)$ naturally defines a propagator between points restricted to the space-time surface.

What is remarkable is that quite generally, the propagation between points with light-like distance is essentially like massless propagation. Particle-like entities are light-like orbits of partonic 2-surfaces so that the geometric character of particles forces massive modes effectively masslessness. A more precise formulation is discussed in [L14].

The induction procedure generalizes to the level of the isometry algebra (IA) and even super-symplectic algebra (SSA) [K3] [L10, L14].

1. One can construct the representations of IA and SSA in H for the Dirac action associated with $D(H)$ and construct the Noether currents of super symplectic algebra and project the currents to the space-time surface. A natural condition would be that these currents are equal to the corresponding currents assigned to the modified Dirac action for the physical states defined at the boundaries of CD.

2. An analogous condition for classical currents was proposed in [L14] and stated that the the conserved classical current for given isometry with Killing vector j_A^k is proportional to its projection to the space-time surface.

$$\begin{array}{rcl} T_B^{A\alpha} & = & \Lambda(x) j_A^{\alpha} \ , \\ j_A^{\alpha} & = & j_A^k \partial^{\alpha} h_k \equiv j_A^k h_{kl} g^{\alpha\beta} \partial_{\beta} h^l \quad , \\ \partial_{\alpha} \Lambda j^{A\alpha} & = & 0 \ . \end{array} \tag{2.1}$$

   This condition could be true for the entire space-time surface or at the ends of X^4 at the boundaries of CD. The conserved bosonic current in H corresponds to j_A^k satisfying $D_k j_A^k = 0$. The conservation condition requires that Λ is constant along the flow lines of j_A^k.

   Quantum classical correspondence suggests that the condition can be true only for Cartan algebra. For the volume part of the action the condition is identically true and $\Lambda(x)$ corresponds to length scale dependent cosmological constant in this case. For Kähler action, the condition is non-trivial.

3. In the fermionic case, the condition would state that the conserved second quantized quark current at the level of H projected to the space-time surface is equal to the conserved fermionic current for the Dirac action in X^4. In the general case, this could hold true for the Cartan algebra and in the case of H isometries at the entire space-time surface. For the symplectic currents it could hold true at the 3-D ends of the space-time surface at boundaries of CD. The condition reads as

$$T_F^{A\alpha} = \overline{\Psi} \Gamma_k \partial_{\alpha} h^k \delta_A \Psi = k(x) \overline{\Psi} \Gamma_k T_B^{k\alpha} \delta_A \Psi \ . \tag{2.2}$$

   If the bosonic condition for $T_B^{k\alpha}$ holds true, this condition and the conservation condition are trivially satisfied for $k(x) = \Lambda(x)$ as also the conservation condition. The condition also generalizes to super-currents obtained by replacing $\overline{\Psi}$ or Ψ by a mode of H spinor field in the expression of the fermionic current.

2.1.3 WCW spinors

The third realization is at the level of the "world of classical worlds" (WCW) assigned to H consisting of 4-surfaces as preferred extremals of the action. Gamma matrices of WCW are expressible as superpositions of quark oscillator operators so that anti-commutation relations are geometrized. WCW spinors are Fock states of quarks. The conditions stating super-symplectic symmetry are a generalization of super-Kac-Moody symmetry and of super-conformal symmetry and give rise to the WCW counterpart of the Dirac operator [K3] [L10, L14] as a non-hermitian super-Virasoro generator G which however carries fermion number.

Bosonic conditions and the fermionic condition implied by them have been already discussed and would dramatically simplify the construction of the quantums states as super-symplectic representations.

WCW gamma matrices would be simply SSA super charges for the induced spinor fields obtained by integrating the 3-D SSA super currents over 3-surfaces X^3 defining the ends of X^4 at the boundaries of CD. That they are projections of 8-D conserved currents in H would make life simple.

One could construct also WCW Kähler metric and in principle all related geometric entities in terms of SSA.

1. The matrix element of the WCW Kähler metric would be obtained as anticommutators

   $$g_{\overline{A},B}=\frac{1}{2}\{Q_A^{\dagger},Q_B\} \tag{2.3}$$

   of the super symplectic charges. Super charge Q_A is obtained as a 3-D integral of super current J_A carrying quark number over the 3-surface X^3:

   $$Q_A=\int_{X^3}d^3xJ_A \quad . \tag{2.4}$$

   The anticommutators of the fermionic oscillator operators for H spinors give Kronecker deltas for both momenta and color quantum numbers.

2. The localization at 3-surface implies that $g_{\overline{A}B}$ is given by an integral of form

   $$\int_{X^3\times X^3}d^3x_1d^3x_2\sum_{p,n}\overline{T}_{A_1}(p,n,x_1)T_{A_2}(p,n,x_2) \quad . \tag{2.5}$$

   The plane waves in the product give a factor $exp(ip\cdot(m(x_1)-m(x_2)$ giving rise to interference. CP_2 spinor harmonics give a product of $\overline{\Psi}_n(s(x_1)\Psi_n(s(x_2)$. The products of factors at different points give rise to interference effects and could save from infinities.

   The replacement of point-like particles with 3-surfaces is essential since the 7-D equal-time anti-commutation relations for quark oscillator operators give a 7-D delta function in H. Indeed, for a point-like particle instead of a 3-surface, one would obtain a sum over terms $\overline{\Psi}_n(s(x_1)\Psi_n(s(x_1)$ multiplied by the volume of the corresponding mass shell.

3. More generally, the double 3-D integral over a particle like n-surfaces should compensate for the 7-D delta function divergence so that for $2n>7$ divergences would be absent. For 3-D objects one has $2n=6$, so that one cannot exclude logarithmic divergences typically present also in gauge theories. It would seem that the divergence cancellation cannot rely on mere non-locality.

4. Could the preferred extremal property be crucial? As a matter of fact, the condition guaranteeing that SSA currents for the action are equal to the projections of SSA currents for H spinors (at least at boundary CD) has been already assumed.

   Number theoretic holography fixes the space-time region in terms of roots of a polynomial with rational coefficients and is an extremely powerful condition also on 3-surfaces at the boundary of CD.

   Also the geometry of $\delta CD=\delta cd\times CP_2$ might be relevant as also the precise definition of the integral. One has a 6-D integral over $\delta cd\times\delta cd$. It seems that this is the correct intuition.

The following argument indeed shows that the geometry of CD (and thus ZEO) is highly relevant.

1. For $m_1 - m_2 = 0$, the CP_2 anticommutator gives a 4-D delta function in CP_2 as a singularity for $s(m_1) = s(m_2)$. For $m_1 = m_2$, one also has a 3-D delta function corresponding to equal time anticommutation relations. This would give 7-D delta function and the integral would diverge and be ill-defined. This is the source of troubles and raises the question whether one should one define the integral as a limit in which the ill-defined 7-D delta function contribution is avoided.

2. Denote by D the diagonal set $Diag(\delta cd \times \delta cd)$ of points $m_1 = m2$ of $\delta cd \times \delta cd$. Assign to D a thin 3-D layer $D \times L$ with L having a thickness l and define the integral over the volume $cd \times cd \setminus D \times L$ and take the limit $l \to 0$. This removes the problematic 7-D delta function singularity and leaves only the 1-D light-ray singularity at δcd [L12, L11] under consideration so that the anticommutator is well-defined and finite.

3. Irrespective of mass, fermion anticommutator has 1-D delta function type singurity as a 1-D delta function $\delta(a)$, $a^2 = (m_1 - m_2)^2$. Now both m_1 and m_2 are points at δcd, and the delta function defines light-like geodesic rays from origin connecting m_1 and m_2. This delta function eliminates 1 integration variable from 6 integration variables in the integration measure $dV = d^3m_1 d^3m_2$ associated with $\delta cd \times \delta cd$.

   d^3m is determined by the the determinant of the induced metric and if the CP_2 coordinates are not constant, the determinant is manifestly non-trivial even if one uses radial light-like coordinate r and angle coordinates Ω of $R_+ \times S^2$ as coordinates. This leaves a 5-D integration volume $X^5 \subset \delta cd \times \delta cd$. Note that for canonically embedded M^4 as a minimal surface extremal the integration measure is trivial so that the 3-surfaces do not belong to WCW.

4. The geometry of δcd would be highly relevant. If one had E^3 as time= constant slice instead of M^4, the same definition of the integral would give a vanishing result since light-like radial rays as singularities would be lost. This picture supports the importance of light-cone boundary as a basic notion but strictly speaking does not force CD.

One could worry for the somewhat ad hoc elimination of 7-D delta function singularity and perhaps take it as a signal telling that something important is still missing. There indeed exists a variant of gamma matrices with which I ended up from the cancellation of fermionic divergences in ZEO. This option is inspired by the multi-locality of the Yangian variants of the super symplectic algebra and isometry algebra for H.

1. The fermionic creation and annihilation operators appearing as building bricks of super symplectic (SSA) charges defining the gamma matrices would be at the opposite boundaries of CD and 3-D states at the opposite boundaries would relate like bras and kets. Annihilation operators would act like creation operators at the opposite boundary of CD.

   The conserved isometry currents in H would be replaced by bilocals with Ψ and $\overline{\Psi}$ and opposite boundaries of CD and remain conserved currents thanks to the (covariant) constancy of M^4 gamma matrices. Note that although SSA currents are not conserved, the Noether charges at the boundaries of CD are well-defined.

2. Can one apply this recipe to the WCW gamma matrices as bi-local entities having 3-surfaces at opposite boundaries as arguments? For supersymmetry generators associated with H isometries, the conservation laws hold and one can calculate the anticommutators. They are non-vanishing and the dominating contributions come from pairs of points with light-like separations. One can use the same CP_2 and S^2 coordinates at both light-like boundaries and only the radial light-like coordinates are different. The 3-D delta function singularity does not appear at all. This would justify the notion of CD rather than only light-cone boundary.

3. The commutators of SSA charges associated with 3-surfaces at different boundaries of CDs or even at boundaries of different CDs generate a poly-local algebra, which could have an interpretation as the Yangian algebra of SSA acting as isometries for WCW.

2.2 Twistor lift predicts M^4 Kähler force

The twistor lift of TGD suggests also a modification of the neutral weak forces.

1. The twistor lift of TGD requires that there is a covariantly constant self-dual Kähler form also in M^4. This would contribute to the electromagnetic and Z^0 fields an additional coupling analogous to that of electroweak hypercharge to U(1) gauge potential.

2. M^4 Kähler form contributes to the Kähler action an additional term. The M^4 contribution is fixed by the condition that the M^4 metric is the square of the Kähler form. Also H-spinors couple to M^4 Kähler gauge potential defining a self-dual Abelian field: essentially constant electric and magnetic fields, which are orthogonal and have the same strength, is in question.

   The scale of the M^4 metric defines the normalization of $J(M^4)$. Here one however encounters a problem since M^4 does not have any inherent scale in its geometry. The size scale L causal diamond ($CD = cd \times CP_2$), where cd is the intersection of light-cones with opposite direction, serves as a natural scale allowing to identify dimensionless coordinates for M^4 in such a manner that the range of variation for the dimensionless coordinates does not depend on the size of CD.

   In these coordinates the self-dual Kähler form scales $E = B = k/L^2$, k a constant near unity. At the limit of long length scales $E = B$ would approach zero. The identification of L as a length scale determined by the cosmological constant is attractive. The breaking of Lorentz symmetry to that of M^4 for the Dirac operator $D(H)$ would be small in long length scales. In very short length scales associated with quarks, the breaking would be large.

   Remark: One cannot completely exclude the alternative option $E = B = k/R^2$, where R is CP_2 scale for which the breaking of Lorentz invariance would be large in all scales.

The presence of M^4 Kähler structure has non-trivial implications also at the level of particle physics.

1. In particular, M^4 Kähler gauge potential $A(M^4)$ couples also to neutrinos unlike $A(CP_2)$, where the net coupling vanishes. The effects are expected to be small in the TGD view about space-time sheets at particle level.

2. The prediction is that all particles have an additional M^4 contribution in their Z^0 and em force and also right-handed neutrinos couple to M^4 Kähler gauge potential.

   Remark: The Kähler gauge potential A does not correspond to a genuine gauge invariance and each choice defines a different physics. The proposal is that the so-called Hamilton-Jacobi structures could correspond to different choices of A.

3. At the level of H the square $D^2(H)$ of the modified Dirac operator would allow spinors to be eigen states of energy and single momentum component. Self duality and covariant constancy imply that $D^2(H)$ contains a term proportional to charge matrix $J^{kl}(M^4)\Sigma_{kl} \propto (\sigma_{03} + \Sigma_{12})$, which vanishes for the second M^4 chirality.

4. 2 components of the 3-momentum would correspond to harmonic oscillator states so that the states would be confined to a finite transversal volume to a harmonic oscillator state characterized by transversal momenta of order magnetic length $\sqrt{B_K}$.

   Suppose that for the transversal degrees of freedom in E^2 with signature (-1,-1), Kähler gauge potential can be chosen to be $A_x = B_K y$. For an eigenstate of p_x, one obtains for the square of the E^2 part of the square D^2 of the Dirac operator,

   $$D^2(E^2) = -(\partial_x - B_K y)^2 - \partial_y^2 = p_x^2 + \partial_y^2 - B_K y^2 - 2ip_x B_K y \ .$$

   The sign of the harmonic oscillator term is correct and the complex shift does not produce problems if the notion of hermiticity is generalized so that PT replaces complex conjugation. Eigenvalues of $p_y^2 + ..$ are essentially the eigenvalues of energy in harmonic oscillator potential and proportional to $2nB_K$ with $n = 1$ assignable to the ground state.

5. In the longitudinal degrees of freedom M^2, the signature of the metric is (1,-1). If A is given by $A_t = B_K z$, the M^2 part of the square of the Dirac operator for an energy eigenstate reduces to $D^2(M^2) = (iE - iB_K z)^2 - \partial_z^2 = -E^2 - \partial_z^2 - B_K^2 z^2 - 2EB_K z$. One obtains a harmonic oscillator potential with a wrong sign and has suffered a complex shift by $z \rightarrow z + iE/B_K$. Harmonic oscillator Gaussian would be replaced with an imaginary exponential - this is of course familiar from free quantum field theories based on path integral defined by Gaussian. The size scale of CD would bring to the theory an arbitrarily long p-adic length scale as a fundamental level scale but expressible in terms of CP_2 radius.

Some physics inspired comments are in order.

1. This picture brings strongly in mind the parton model of hadrons. If cosmological constant Λ characterizes the size scale L, it must correspond to the scale which is essentially geometric mean of Planck length and the p-adic length scaled defined defining the length scale dependent cosmological constant Λ (of order Hubble scale). In the TGD framework, cosmological constant is length scale dependent, and the value of Λ assignable to cosmology would correspond to length L of order 10^{-4} meters assignable to a large neuron.

2. The spectrum of the M^2 mass squared operator is integer valued using B as a unit. The mass squared spectrum is similar to the spectrum in string models. This picture also conforms with the idea that the transversal Kac-Moody modes in $M^2 \times E^2$ are dynamical. Also transversality of polarizations in gauge theories conforms with this picture. Also the properties of "massless extermals" support this picture.

3. What comes to mind is that the values of integers n_i characterizing harmonic oscillator states are analogous to fermionic conformal weights. One has conformal weight for both the light-like radial coordinate of super symplectic representations and for the Kac-Moody type representations associated with light-like orbits of partons: the light-likeness of the partonic 2-surfaces and of light-cone boundary make them metrically 2-D and implies a generalization of conformal invariance.

   This conforms with the notion of induction. The fermion super symplectic charges should be constructible in terms of the quark oscillator operators for the second quantized quark fields of H.

2.3 How can massless particles exist at all and how do they become massive?

One must understand why there are light particles at all and what makes them massive.

1. The mass scale for CP_2 is about 10^{-4} Planck masses and the only massless particle is a right-handed neutrino of only $J(CP_2)$ is present. Also the color quantum numbers depend on the em charge. Therefore physical elementary particles cannot correspond to the quarks as such. The situation remains essentially the same if $J(M^4)$ is present.

   The proposal has been that H spinor modes define ground states for super-symplectic representations and operators carrying conformal weight contribute to mass squared additively create the physical states. The lowest states have vanishing mass squared. The introduction of $J(M^4)$ suggests that the quark oscillator operators labelled by two integers could actually be interpreted as conformal weights and that M^2 momentum would take the role of M^4 momentum. The number of ground states of super-symplectic representations could be much smaller.

2. p-Adic thermodynamics however mixes these states with states of higher conformal weight and this gives rise to the mass of the light particles. One must assume that there is a negative tachyonic contribution to the ground state conformal weight since only the right-handed neutrino is massless in 4-D sense. The origin of this negative conformal weight has remained a mystery.

3. M^8-H duality provides a possible insight to the mystery of the tachyonic conformal weight. The map of 4-surfaces in M_c^8 (complexified octonions) by M^8-H duality involves selection of M^4 as a 4-D linear subspace in M^8. This choice is not unique. Momenta and color quantum numbers in H correspond to 8-momenta in M^8 such that 8-D mass squared vanishes at both sides and M^4 momenta are identical. For a suitable choice of $M^4 \subset M^8$, the 8-momentum is parallel to M^4 and the state is massless!

   Could the introduction of negative tachyonic conformal weight provide an alternative description of this choice? This choice can be made only for a single, naturally dominant contribution of the state, and the remaining contributions to mass squared coming from higher conformal weights give rise to massivation described by p-adic thermodynamics.

4. Here the twistor lift comes to rescue. Twistor lift of TGD requires that also M^4 has Kähler structure defined by a self-dual Kähler form $J_{kl}(M^4)$ (constant E and B with $vertB| = |E|$ orthogonal to each other). Depending on the selected correlation between M^4 and CP_2 chiralities guaranteeing that quarks correspond to a fixed H chirality, $D^2(H)$ contains for either left- or right-handed M^4 modes a nonvanishing spin term $J^{kl}(M^4)\Sigma_{kl}$. The reason is that for left-/right-handed mode the eigenvalues of Σ_{03} and Σ_{12} have the same/opposite sign or vice versa.

   This would give a mass splitting between left-and right-handed modes and also spin splitting for left- or right-handed modes. The spin-splitting could give rise to a negative contribution to the mass squared in the case of right-handed neutrinos. Could the tachyonic state of the right-handed neutrino give rise to the mysterious tachyonic ground states required by p-adic mass calculations? Could a suitable number of tachyonic right-handed neutrinos allow to nullify arbitrarily high conformal weight of ground state?

2.4 How to describe the unitary time evolution of quantum states in the TGD Universe?

The first question is how to describe the time evolution of quantum states in general. The time evolution at the single particle level is involved with the mixing of neutrinos.

Remark: One must remember that physical particles are multiquark composites: even leptons are local composites of 3 antiquarks). Therefore the description in terms of H-spinors applied in the sequel can be criticized.

1. In the TGD framework the standard 4-D approach based on the Hamiltonian picture can be only an approximate description since it neglects masslessness in the 8-D sense and is not relativistically invariant.

2. The empirical fact is that neutrinos are massive but always left- handed. The trivial explanation could be that right-handed neutrinos have only gravitational interaction so that their detection is not possible. The mixing of left-handed neutrinos with right-handed ones should however be visible in neutrino mixing experiments.

   In the TGD framework Dirac equation in H forces the mixing of quark chiralities for the modes of H-spinors. The covariantly constant right-handed neutrino is an exception. Induction as a mere restriction to the space-time surface respects this property! This implies that left-handed neutrino modes mix with right-handed ones and this could make itself visible in the neutrino beam experiments like Mini-Boone and Micro-Boone.

   The problem can be avoided if it is possible to have massive neutrinos with well-defined M^4 chirality and a time evolution which does not mix the chiralities. Could this kind of time evolution allow a realization?

3. Certainly, if the Dirac operator in H, or equivalently, the modified Dirac operator in X^4 defines the phenomenological Hamiltonian operator, the chirality mixing seems unavoidable. There is however no deep reason why $D(H)$ or $D(X^4)$ should define the propagation.

4. To get some guidance, one can also consider the level of "world of classical worlds" (WCW). The gamma matrices of WCW are constructed in terms of anticommuting oscillator operators

of H-spinors and at tat that level the analog of the Dirac operator is a generator G of superconformal algebra whereas the scaling generator L_0 is essentially $GG^\dagger$. However, G carries a quark number and therefore it does not make sense to talk about a propagator defined by G or an analog of Hamiltonian.

The only reasonable unitary time evolution operator at WCW level is defined by the exponent of L_0, which is essentially mass squared operator obtained as "square" of WCW Dirac operator and has at the level of H counterpart of mass squared operator $D^2(H)$.

In fact, in superstring models, the time evolution operator for the string world sheet is defined by L_0 so that this idea is not new. Also p-adic thermodynamics is defined by the exponent of L_0, at this time real, and its existence in the p-adic sense is responsible for the predictive power of p-adic thermodynamics.

Here one must be more precise. Entire L_0 cannot be in question if it annihilates the physical states. In p-adic mass calculations L_0 is identified as the vibrational part $L_{0,vib}$ and for physical states in the string model satisfy $L_0\Psi = (p^2 - kL_{0,vib})\Psi = 0$. One could say that one has thermodynamics for states with different values of mass squared but satisfying the Virasoro condition. p^2 could also correspond to the longitudinal M^2 momentum and transversal momentum would be absorbed to $L_{0,vib}$. Both p-adic mass calculations and M^4 Kähler form favor this option and this picture conforms also with the stringy picture with M^2 effectively replacing the string world sheet.

Also the TGD based quantum measurement theory [L5] [K5] leads to the conclusion that the unitary time evolutions between "small" state function reductions (SSFRs) correspond to the exponential of L_0. Unitary time evolution as a time translation is replaced with a scaling which is a Lorenz invariant notion and better suited for relativistic purposes.

5. L_0 does not mix chiralities! If the initial state of a neutrino is left-handed, it remains left-handed. But how can the initial state of a neutrino be left-handed if spinor modes at the level of H are mixtures of left and right-handed modes as $D(H)\Psi = 0$ demands?

   Massless Dirac equation cannot be satisfied at the level of X^4 and at the level of WCW it does not make sense. Could one consider the radical possibility of giving it up altogether so that at the level of H one would require only that $D^2(H)\Psi = 0$ is satisfied and $D^2(H)$ would define counterpart of fermionic L_0 and time evolution.

   If so, the number of modes is doubled except for the right-handed neutrino. This implies mirror neutrinos. Could left and right-handed charged leptons and quarks be interpreted in terms of the mirror modes? Mirror neutrino hypothesis does not however have empirical support at available energies. One explanation is that the right-handed neutrino modes are very massive or somehow special.

6. If $J(M^4)$ is present, the masses of the left-handed mode and corresponding right-handed mode differ by the $S = J^{kl}(M^4)\Sigma_{kl}$ whose eigenvalues define the vacuum conformal weight $\pm h_{vac}$. Assume that S is non-vanishing for the right-handed mode. The number of right-handed modes with tachyonic mass squared would be the number of CP_2 modes with mass squared smaller than h_{vac}. Covariantly constant neutrino 0 would certainly define this kind of state.

   If the mass is identified as the longitudinal M^2 mass, it might be possible to select the values of the conformal weights n_1 and n_2 for the modes in such a manner that the masses are identical for the left- and right-handed modes and they can superpose. This should happen for charged modes. If this is not possible for neutrinos, the mixing of chiralities could not occur. This does not work.

   The masses of modes related by multiplication with Dirac operator have always identical mass squared values as follows from the commutativity of D and D^2. However, the covariantly constant right-handed neutrino does not have a left-handed companion. Both mixed states as modes of D and unmixed states satisfy $D^2\Psi = 0$. Why would neutrinos always have a definite handedness? Does the absence of standard model interactions for ν_R imply that the state preparation and reduction involving weak interactions creates only purely left-handed neutrinos?

In the TGD Universe, even covariantly constant right-handed neutrino mode couples to M^4 Kähler form. Could this make it possible to project from mostly left-handed neutrino the non-covariantly constant right-handed part? Could their large mass make their creation impossible?

3 Problems related to neutrinos

In what follows, the problem of missing right-handed neutrinos and the problem created by apparently contradictory findings of Mini-Boone and Micro-Boone about neutrino mixing are discussed. Also the topological model for neutrino and D-quark CKM mixing is briefly considered.

3.1 Why only left-handed neutrinos are observed?

A basic theoretical motivation for the sterile neutrinos is the difficulty posed by the fact that the neutrinos behave like massive particles. This is not consistent with their left-handedness, which is an experimental fact.

As a matter of fact, the sterile neutrinos would be analogous to the covariantly constant right-handed neutrinos in TGD if only $J(cP_2)$ would be present.

Remark: As already stated, in the sequel it is assumed that leptons as bound states of 3 antiquarks can be described using spinors of H with chirality opposite to that for quarks. They have colored modes and the action of super-symplectic algebra is assumed to neutralize the color and also give rise to a massless state getting its small mass by p-adic thermodynamics.

How could one understand the fact that only left-handed neutrinos are observed although neutrinos are massive? One can consider two approaches leading to the same conclusion.

Is it possible to have time evolution respecting M^4 chirality and neutrinos with fixed chirality possible despite their mass?

1. All spinor modes in CP_2 are of the form Φ_L or $D(CP_2)\Phi_L$ and therefore generated from left-handed spinors Φ_L.

   If one assumes $D(H)\Psi = 0$, the spinor modes of H are of the form $D(M^4)\Psi_R \otimes \Phi_L + \Psi_R \otimes D(CP_2)\Phi_L$. The modes of form $D(M^4)\Psi_L \otimes \Phi_R + \Psi_L \otimes D(CP_2)\Phi_R$ are therefore of the form $D(M^4)\Psi_L \otimes D\Phi_L + \Psi_L \otimes D^2(CP_2)\Phi_L$. The mixing of chiralities is unavoidable.

2. However, if one assumes only the condition $D^2(H)\Psi = 0$, one can obtain both left- and right-handed modes without mixing of M^4 chiralities and M^4 Kähler structure could make the lowest mass second right-handed neutrino (covariantly constant in CP_2) tachyonic. The time evolution generated by the exponent of L_0 would respect M^4 chirality.

   This does not prevent superpositions of right- and left-handed fermions if their masses are the same. If only charged leptons can satisfy this condition, one can understand why right-handed neutrinos are not observed.

An alternative approach would rely on quantum measurement theory but leads to the same conclusion.

1. Suppose that neutrinos can appear as superpositions of both right- and left-handed components. To detect a right-handed neutrino, one must have a measurement interaction, which entangles both length and right-handed components of the neutrino with the states of the measuring system. Measurement would project out the right-handed neutrino. If only the $J(CP_2)$ form is present, the right-handed neutrino has only gravitational interactions, and this kind of measurement interaction does not seem to be realizable.

2. Putting it more explicitly, the reduction probability should be determined by a matrix element of a neutral (charged) weak current between a massive neutrino (charged lepton) spinor with a massless right-handed neutrino spinor. This matrix element should have the form $\overline{\Psi}_R O \Psi_L$, where O transforms like a Dirac operator. If it is proportional to $D(H)$, the matrix element vanishes by the properties of the massless right-handed neutrino.

3. There is however a loophole: the transformation of left- to right-handed neutrinos analogous to the transformation to sterile neutrino in the neutrino beam experiments could demonstrate the existence of ν_R just like it was thought to demonstrate the existence of the inert neutrino in Mini-Boone experiment. Time evolution should thus respect M^4 chirality.

If $J(M^4)$ is present, one might understand why right- and left-handed neutrinos have different masses.

1. Also the right-handed neutrino interacts with Kähler gaug potential $A(M^4)$ and one can consider an entanglement distinguishing between right- and left-handed components and the measurement would project out the right-handed component. How could this proposal fail?

   Could it be that right- and left-handed neutrinos cannot have modes with the same mass so that these superpositions are not possible as mass eigen states? Why charged modes could have the same mass squared but not the neutral ones?

2. The modes with right-handed CP_2 chirality are constructed from the left-handed ones by applying the CP_2 Dirac operator to them and they have the same CP_2 contribution to mass squared. However, for the right-handed modes the $J^{kl}(M^4)\Sigma_{kl}$ term splits the masses. Could it be that for right- and left-handed charged leptons the same value of mass is possible.

   The presence of $J(M^4)$ breaks the Poincare symmetry to that for M^2 which corresponds to a Lagrangian manifold. This suggests that the physical mass is actually M^2 mass and the QCD picture is consistent with this. Also the p-adic mass calculations strongly support this view. The E^2 degrees of freedom would be analogous to Kac-Moody vibrational degrees of freedom of string. This would allow right- and left-handed modes to have different values of "cyclotron" quantum numbers n_1 and n_2 analogous to conformal weights. This could allow identical masses for left- and right-handed modes. For a Lagrangian manifold M^2, one would have $n_1 = n_2 = 0$, which could correspond to ground states of super-symplectic representation.

3. Why identical masses would be impossible for right- and left-handed neutrinos? Something distinguishing between right- and left-handed neutrinos should explain this. Could the reason be that Z^0 couples to left-handed neutrinos only? Could the fact that charged leptons and neutrinos correspond to different representations of color group explain why only charged states can have right and left chiralities with the same mass?

   Perhaps it is of interest to notice that the presence of $J^{kl}(M^4)\Sigma_{kl}$ for right-handed modes makes possible the existence of a mode for which mass can vanish for a suitable selection of B.

3.2 Mini-Boone and Micro-Boone anomalies and TGD

After these preliminaries we are ready to tackle the anomalies associated with the neutrino mixing experiments. The incoming beam consists of muonic neutrinos mixing with electron neutrinos. The neutrinos are detected as they transform to electrons by an exchange of W boson with nuclei of the target and the photon shower generated by the electron serves as the experimental signature.

The basic findings are as follows.

1. Mini-Boone collaboration reported 2018 [C2] an anomalously large number of electrons generated in the charged weak interaction assumed to occur between neutrino and a nucleus in the detector. "Anomalous" meant that the fit of the analog of the CKM matrix of neutrinos could not explain the finding. Various explanations including also inert neutrinos were proposed. Muonic inert neutrino would transform to inert neutrino and then to electron neutrino increasing the electro neutrino excess in the beam.

2. The recently published findings of Micro-Boone experiment [C1] studied several channels denoted by $1eNpM\pi$ where $N = 0, 1$ is the number of protons and $M = 0, 1$ is the number of pions. Also the channel 1eX, where "X" denotes all possible final states was studied.

> It turned out that the rate for the production of electrons is below or consistent with the predictions for channels 1e1p, 1eNp0π and 1eX. Only one channel was an exception and corresponds to 1e0p0π.
>
> If one takes the finding seriously, it seems that a neutrino might be able to transform to an electron by exchanging the W boson with a nucleus or hadron, which does not belong to the target.

In TGD, the only imaginable candidate for this interaction could be charged current interaction with a dark nucleus or with a nucleon with $h_{eff} > h$. This could explain the absence of ordinary hadrons in the final state for 1e events.

1. Dark particles are identified as $h_{eff} > h$ phases of the ordinary matter because they are relatively dark with respect to phases with a different value of h_{eff}. Dark protons and ions play a key role in the TGD inspired quantum biology [L16] and even in the chemistry of valence bonds [L2]. Dark nuclei play a key role in the model for "cold fusion" [L1, L9] and also in the description of nuclear reactions with nuclear tunnelling interpreted as a formation of dark intermediate state [L4].

2. I have proposed that dark protons are also involved with the lifetime anomaly of the neutron [L3] [L3]. The explanation relies on the transformation of some protons produced in the decay of neutrons to dark protons so that the measured life time would appear to be longer than real lifetime. In this case, roughly 1 percent of protons from the decay of n had to transform to dark protons.

3. If dark protons have a high enough value of h_{eff} and weak bosons interacting with them have also the same value of h_{eff}, their Compton length is scaled up and dark W bosons behave effectively like massless particles below this length scale. The minimum scale seems to be nuclear or atomic scale. This would dramatically enhance the dark rate for $\nu p \rightarrow e + n$ so that it would have the same order of magnitude as the rates for electromagnetic interactions. Even a small fraction of dark nucleons or nuclei could explain the effect.

3.3 CKM mixing as topological mixing and unitary time evolution as a scaling

The scaling generator L_0 describes basically the unitary time evolution between SSFRs [L5] [K5] involving also the deterministic time evolutions of space-time surfaces as analogs of Bohr orbits appearing in the superposition defining the zero energy state. How can one understand the neutrino mixing and more generally quark and lepton mixing in this picture?

1. In the TGD framework, quarks are associated with partonic 2-surfaces as boundaries of wormhole contacts, which connect two Minkowskian space-time sheets and have an Euclidean signature of induced metric and light-like projection to M^4 [K1, K2].

2. For some space-time surfaces in their superposition defining a zero energy state, the topology of the partonic 2-surfaces can change in these time evolutions. The mixing of boundary topologies would explain the mixing of quarks and leptons. The CKM matrix would describe the difference of the mixings for U and D type quarks and for charged and neutral leptons. The topology of a partonic 2-surface is characterized by the genus g as the number of handles attached to a sphere to obtain the topology.

   The 3 lowest genera with $g \leq 2$ have the special property that they always allow Z_2 as a conformal symmetry [K1, K2]. The proposal is that handles behave like particles and thanks to Z_2 symmetry $g = 2$ the handles form a bound state. For $g > 2$ one expects a quasi-continuous spectrum of mass eigenvalues. These states could correspond to so-called unparticles introduced by Howard Georgi (`https://cutt.ly/sRZKSFm`).

3. The time evolution operator defined by L_0 induces mixing of the partonic topologies and in a reasonable idealization one can say that L_0 has matrix elements between different genera. The dependence of the time evolution operator on mass squared differences is natural in this

framework. In standard description it follows from the approximation of relativistic energies as $p_0 \simeq p+m^2/2p$. Also the model of hadronic CKM relies on mass squared as a basic notion and involves therefore L_0 rather than Hamiltonian.

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