Dance of the honeybee and New Physics

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Abstract

For more than two decades ago mathematician Barbara Shipman made rather surprising finding while working with her thesis. The 2-D projections of certain curves in flag manifold $F = SU(3)/U(1) \times U(1)$ defined by the so called momentum map look like the waggle part of the dance of the honey bee. Shipman found that one could reproduce in this framework both waggle dance and circle dance (special case of waggle dance) and the transition between these occurring as the distance of the food source from the nest reduces below some critical distance. Shipman introduced a parameter, which she called α, and found that the variation of α allows to integrate various forms of the honeybee dance to a bigger picture. Since $SU(3)$ is the gauge group of color interactions, this unexpected finding led Shipman to as whether there might be a profound connection between quantum physics at quark level and macroscopic physics at the level of honeybee dance.

The average colleague of course regards this kind of proposal as crackpottery: the argument is that there simply cannot be any interaction between degrees of freedom in so vastly different length scales. Personally I however found this finding fascinating and wrote about the interpretation of this finding in the framework of TGD and TGD inspired consciousness. During more than two decades a lot of progress has taken place in TGD, in particular I have learned that the flag manifold F has interpretation as twistor space of CP_2 and plays a fundamental role in twistor lift of TGD. Hence it is interesting to look what this could allow to say about honeybee dance.

It turned out that one could understand the waggle parts of the honeybee dance at space-time level in terms of the intersection of the space-time surface with the image of the Cartan sub-algebra of SU(3) represented in CP_2 using exponential map. This allows to code the positional data about the food source. The frequencies assignable to the wing vibrations and waggling turn could have interpretation as cyclotron frequencies as expected if the magnetic body of the bee controls the waggle dance utilizing resonance mechanism. They could also correspond to the momenta (frequencies) defining constants of motion for geodesic in $U(1) \times U(1)$ defining one particular point of flag manifold F. Also a connection with the Chladni effect emerges: the waggle motion is along time-like curve at which Kähler force vanishes. Also the transition from waggle dance do circle dance involving also a short waggle period can be understood.

Contents

1 Introduction

For more than two decades ago mathematician Barbara Shipman made rather surprising finding while working with her thesis [A3, A2]. The 2-D projections of certain curves in flag manifold $F = SU(3)/U(1) \times U(1)$ defined by the so called momentum map look like the waggle part of the dance of the honey bee (see `http://tinyurl.com/c7pljpw`). Shipman found [A4, A5, A1] that one could reproduce in this framework both waggle dance and circle dance (special case of waggle dance) and the transition between these occurring as the distance of the food source from the nest reduces below some critical distance of about 10-20 meters. Shipman introduced a parameter, which she called α, and found that the variation of α allows to integrate various forms of the honeybee dance to a bigger picture. Since $SU(3)$ is the gauge group of color interactions, this unexpected finding led Shipman to as whether there might be a profound connection between quantum physics at quark level and macroscopic physics at the level of honeybee dance.

The average colleague of course regards this kind of proposal as crackpottery: the argument is that there simply cannot be any interaction between degrees of freedom in so vastly different length scales. This argument actually resembles the argument of nuclear physicists against "cold fusion" and is based on the dogma of length scale reductionism. Personally I however found this finding fascinating and wrote about the interpretation of this finding in the framework of TGD and TGD inspired consciousness [K3, K1].

During more than two decades a lot of progress has taken place in TGD, in particular I have learned that flag manifold F has interpretation as twistor space of CP_2 and plays a fundamental role in twistor lift of TGD [K10, K16, K17, K15]. Hence, when Johan Frisch contacted and asked whether I could help him to get material about the work of Shipman, I got interested in honeybee dance and realized that the earlier picture could be made much more detailed. I am grateful for Jerry Decker for finding links and references to the work of Shipman from web.

It is appropriate to begin by summarizing the new elements of TGD relevant for the honeybee dance.

1. In TGD framework an entire hierarchy of scaled variants of QCD like physics is possible by p-adic length scale hypothesis stating that preferred p-adic length scales $L_p \propto \sqrt{p}$ correspond to primes $p \simeq 2^k$. This hypothesis was inspired by the success of p-adic mass calculations [K12, K4]. In particular, in biologically especially relevant length scale range from 10 nm (cell membrane thickness) to 2-5 μ (size of cell nucleus) as many as 4 candidates for scaled variants of QCDs could exist: they would correspond to Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$, $k = 151, 157, 163, 167$. The existence of so many Gaussian Mersennes in so narrow a length scale range is a number theoretical miracle. The interaction of honeybee could be with scaled up variant of QCD like physics and the quarks could have the size of cell nucleus!

2. The flag manifold $F = SU(3)/U(1) \times U(1)$ has an interpretation as the space for the choices for the quantization axes of color quantum numbers (color hypercharge and isospin). Few years ago it turned out that F is the twistor space of CP_2 and possesses Kähler structure [K10, K15, K17]. As a matter of fact, S^4 and CP_2 are the only compact spaces with twistor space possessing Kähler structure. Also M^4 and E^4 - kind of non-compact variants of S^4 - allow twistor space with Kähler structure (M^4 in generalized sense). Hence the existence of twistor lift of TGD implies that TGD is completely unique.

TGD inspired theory of consciousness [K5] leads to a proposal concerning the identification of qualia [K3]. One can distinguish between non-geometric qualia - colors, tastes, and odours - and geometric qualia representing geometric information such as angles and distances. Flag manifold qualia would be universal "general purpose" geometric qualia representing geometric information. In the model for the honeybee dance the point of 6-D flag manifold F would represent positional information about the food source and waggle dance would represent this point of F as a dynamical pattern very much like the point of momentum space is represented as orbit in configuration space.

3. TGD predicts hierarchy of Planck constants $h_{eff}/h = n$ labelling the levels of a dark matter hierarchy identified as phases of ordinary matter residing at flux tubes of magnetic bodies (MBs) assignable to ordinary physical systems. In the adelic vision h n corresponds to the dimension of the extension of rationals defining particular adele in the hierarchy of adeles having interpretation in terms of an evolutionary hierarchy [L10] [L11]. The scaling of Planck constant by n means similar scaling of Compton lengths implying zooming up of the microscopic physics. These scaled up variants of particles at the MB of the living system play a crucial role in TGD inspired quantum biology, and even suggests new physics associated with the notion of valence bond highly relevant to metabolism [L8] (see `http://tinyurl.com/ycg94xpl`).

This background gives good motivations for looking whether Shipman's findings could make sense in TGD Universe. It however turned out difficult to find any material relating to Shipman's work in web and the popular articles do not tell the details. There are several questions to be answered.

What do momentum map and 2-dimensional projection really mean? What the curves studied by Shipman really are?

1. Momentum map μ is a standard notion and actually familiar for physics albeit being represented using totally different language. In the case of general Lie group G acting as symmetries of symplectic manifold M, μ maps the elements of g ($su(3)$ now) represented as vector fields of M or the images of corresponding one-parameter groups (flows) to the elements of the co-adjoint algebra g^* of g having Poisson structure. One-parameter groups associated with the elements of g are mapped to conserved Hamiltonians associated with them. Mathematician speaks of co-adjoint orbits as images of orbits in M.

2. Physicist would see the situation either at the level of configuration space ("q-space") or momentum space ("p-space"). Exponential map takes each element X of Cartan algebra $h \subset g$ to an image of corresponding one-parameter group by exponential map, the orbit of the flow defined by X.

   Since M allows symplectic structure and G acts as symmetries, each orbit is characterized by conserved Hamiltonians associated with elements of g. Only the Hamiltonians assignable to h commute with respect to Poisson bracket.

   The image of H in M is spanned by 1-parameter subgroups associated with H. In momentum space picture given orbit corresponds to single point in co-adjoint algebra g^* defined by the two conserved Hamiltonians defining the momentum of the particle.

3. The choices of H are labelled by flag manifold F and each point of F defines a 2-momentum in h^*. The projection of F to $h*$ defines so called momentum polytope, which is hexagon. This notion makes sense completely generally.

What could be the TGD counterpart of this general picture? Consider first the general dynamics.

1. In TGD framework the new element is that the 2-D image Y of $U(1) \times U(1) \subset SU(3)$ in CP_2 is further projected to the space-time surface $X^4 \subset M^4 \times CP_2$: one simply forms the intersection $X = Y \cap X^4$. X (as already Y) carries vanishing induced Kähler form being thus analogous to Lagrangian sub-manifold. X is also analogous to the so called Chladni surface at which electric field vanishes: the physical meaning of these surfaces is discussed in [L4].

The dynamics of the twistor lift of Kähler action [K10, K17, K15] reducing to a 4-D generalization of a dynamics coupling geodesic motion of point particle to induced Kähler field (analogous to Maxwell field) would fix space-time surfaces and therefore also the surfaces X. This dynamics could be also seen as a generalization of Chladni mechanism. Asymptotic self-organization patterns indeed correspond to the vanishing of the induced Kähler force inside given space-time sheet. These space-time regions correspond to external particles entering CD in ZEO based view about scattering. At point-like limit the external particles would be geodesic lines and in interaction regions move under Kähler force.

2. In the general case the dimension $D(X)$ of X satisfies $D(X) \leq 2$. One can have $D(X) = 2$ if space-time surface carries vanishing induced Kähler form: these surfaces are special case of minimal surface extremals for the twistor lift of Kähler action [K15] [L2].

   There are also other kinds of preferred extremals. Cosmic string solutions are of form $X^2 \times S^2 \subset M^4 \times CP_2$, where X^2 is minimal surface - string world sheet and S^2 is geodesic sphere of CP_2. CP_2 has two non-equivalent geodesic spheres. The first one has vanishing induced Kähler form and second is homologically non-trivial (non-contractible) and carries Kähler magnetic flux.

   One has also more general preferred extremals $X^2 \times Y^2$, where Y^2 is complex sub-manifold of CP_2 obtained by replacing S^2 with a sphere with $g \geq 1$ handles. These flux tubes are infinitely thin but one can deform them in M^4 directions to get magnetic flux tubes of finite thickness, which are key players in TGD inspired quantum biology.

3. The proposal is that simple modifications of these extremals exist as preferred extremals. One can "kick" Y^2 in rotational rigid body motion in CP_2 such that there are separate rotations in temporal and spatial directions of X^2. The surface X would be 2-D projection of $U(1) \times U(1)$ to X^2. Symmetry breaking can occur and reduce the projection esentially to that for $U(1) \subset U(1) \times U(1)$, and one obtains a unique waggle run along flux tube. Note that this ansatz works also for the M^4 deformations of cosmic strings.

   Surfaces X at string world sheets X^2 or equal to them would thus serve as representations for the points of F.

What about the details of the waggle dance?

1. Additional information such as waggling can be coded by the dynamics of the modified $X^2 \times Y^2$ with rotating Y^2 deformed in M^4 degrees of freedom. X has one time-like direction so that the two waggle runs must correspond to two distinct points of F related by a symmetry realized as a reflection with respect to the line connecting the hive to the food source (the two waggle runs give rise to a V shape with edges representing the horizontal projection of the line to the food source).

   The necessity of the crucial phase transition from waggle dance to circle dance (special case of waggle dance) follows actually without any assumption about the model for the proposed coding of position information about food source.

2. The temporal duration assignable to X defines naturally the duration of the waggle dance in turn coding for the distance of the food source and identifiable as TGD counterpart for the parameter α of Shipman.

3. Waggle run involves two important frequencies: vibration frequency f_v of wings and waggle frequency f_w: these frequencies could correspond to the two conserved Hamiltonians - essentially frequencies (f_1, f_2) associated with the waggle run.

In the sequel I will summarize some basic facts about honeybee dance, sum up what I understand from the work of Shipman, and discuss the TGD based model and compare it with Shipman's work. The TGD inspired model is inspired by the twistor lift of TGD giving special status for the twistor space of CP_2 as flag manifold $F = SU(3)/U(1) \times U(1)$, by the general vision provided by TGD about living matter, by the TGD based model for qualia, by the basic knowledge about honeybee dance - in particular the intriguing observation the two basic frequencies associated with dance correspond to cyclotron frequencies - , and by the interpretation of the findings of Shipman.

Before continuing it is appropriate to list references to Shipman's work. Work related to Toda lattices can be found at [A3, A2]. There are also articles in arXiv (see `http://tinyurl.com/y998f9v6`, `http://tinyurl.com/yapgjprt`, and `http://tinyurl.com/y7a47f39`). The work related to honeybee dance can be found in the articles [A5, A1].

2 Some empirical facts

The background for TGD based proposal concerning the dance of honeybee relies on some key empirical facts and the attempt to understand the intriguing findings of Barbara Shipman in TGD framework.

There are articles in ScienceDirect discussing waggle dance from the perspective of neuroscience (see `http://tinyurl.com/ycuhjybt`). For instance, "Dance Language" by Dyer and "Learning Theory and Behaviour" by Marco and Menzel are warmly recommended.

The beginning of the article of Dyer gives some idea about the importance of the decoding of waggle dance by Frisch.

> *Anyone who has watched bees dance for food, and is aware of the function of this behavior, cannot help but be amazed. Karl von Frischs decoding of the dance language is certainly one of the great discoveries in modern biology. This is not only because of the inherent fascination that the dance holds for curious human observers. Even more important is the extent to which von Frischs discovery laid the foundation for the study of deep questions about animal behavior. When we consider the role that the dance language has played in the study of vision, olfaction, audition, learning, circadian rhythms, decision making, social organization, and behavioral evolution, it is easy to see why von Frisch regarded the dance language as a magic well of discovery. Furthermore, with advances in neuroscience, genomics, and evolutionary theory, it seems clear that the value of the dance as a model system will continue for many years to come.*

From this it is clear that waggle dance involves multi-sensory communications including vision, hearing, and olfaction. There are several questions to be answered. What information does the dance convey? How the dancer gathers this information? How the audience extracts this information from the dance? The basic puzzle is how an insect with so small cognitive capacity (no cortex nor limbic brain) is able to carry out this feat?

Swarm intelligence is proposed as an answer: this would not involve consciousness. I would be surprised if deep learning were not proposed as a solution. But is this enough? Should one consider bee as part of larger conscious entity - the hive - just like one regards single neuron as part of brain? And what about cognition: should one have genuine theory of consciousness describing also cognition: the formulation of TGD as adelic physics indeed provides a theory of cognition [L11] [L10].

2.1 Basic facts about the dance

The dance of the honeybee occurs at the vertical face of the honeycomb and codes the information about the distance and direction of the food source. Von Frisch discovered the choreographic syntax and interpretation of the dance and published the results of his work in his 1967 book "*Dance language and Orientation of Bees*" [J2].

The pattern of the dance is that of figure eight above certain critical distance to the food source and that of a circle below this distance.

1. The angle of the figure eight pattern with respect to the vertical codes the angle between the direction of the food source and the horizontal projection of Sun. For instance, when the food source is in the direction of Sun, figure eight pattern is vertical. The dancer waggles and produces buzzing sound during the first phase of the dance and then walks to the original position along the other circle of the figure eight. After that the dancer waggles again but now along the second circle of the figure eight so that the waggling phases of the dance form the pattern of a figure V in the middle of the figure 8. The buzzing sound produced by the wings of the dancer makes it possible for the audience to locate the dancer (dance occurs

in darkness). The opening angle of the figure V codes the distance to the food source for distances above some critical distance.

2. Below the critical distance the pattern changes to a circle. Now the waggling parts of the dance correspond to two disjoint straight line portions located at the opposite sides of the hexagon.

Besides waggle dance and circle dance as special case of waggle dance there are also other forms of honeybee dance such as tremble dance and grooming dance.

One can find summary about more quantitative empirical facts related to the waggle dance in Wikipedia (see `http://tinyurl.com/c7pljpw`). The following represents some facts relevant to the updated TGD inspired model are listed.

1. Waggle dance involves 100 or more circuits. The duration of the straight portion, waggle run, depends on distance: according to Wikipedia 1 second corresponds to 1 km. This might make sense at long enough distances but for short distances in the range 10-20 m this would mean millisecond time scale and this cannot make sense.

2. The dance language codes positional information about the food source (this need not be the only information: also information about the quality of the nectar might be represented). The direction of the food source relative to Sun is coded to the direction of relative to the direction of gravity (vertical direction). The polar angle θ_S with respect to the direction of Sun is mapped to polar angle $\theta_{gr} = \theta_S$ with respect to the vertical direction defined by gravitational force. The atzimutal angle ϕ with respect to the projection of the solar direction to the horizontal plane is represented as such.

   The distance d of the food source is coded to the duration, call it T - of the waggle period and also to the number of wagtail movements during single waggle run (this is trivially true if the wagtail movements occur with same frequency f_w always). Also the vibration frequency f_v of wings correlates with d. One has $f_w \simeq 13$ Hz and $f_v \in [200, 300]$ Hz.

   Remark: These frequencies are not too far from cyclotron frequencies $f(Ca^{2+}) = 15$ Hz and $f(p) = 300$ Hz in endogenous magnetic field $B_{end} = 0.2$ Gauss explaining the quantal effects of ELF radiation on vertebrate brain [J1] [K6, K7].

3. What is remarkable that the orientation information - that is the values of θ and ϕ - is automatically updated to take into account the motion of the Sun. In brain as a computer paradigm this would require quite impressive computation taking into account the size of the brain of honeybee (no cortex and no limbic brain). Even humans are unable to perform this feat. Deep learning paradigm might help but personally I remain skeptic.

2.2 How forager bee could collect the position information?

Honeybee dance mediates information about both the direction and distance of the food source. In neuroscience approach identifying brain as a computer this information would be stored by computation. The proposal is that the foraging bee utilizes solar compass. The information about the direction in the plane of Earth would be stored by using the information extracted from the polarization of the sunlight. The cloud free regions can provide this information even in cloudy day.

It has been proposed that the information about the distance of food source is coded by the net motion of the visual features of environment along retina during the flight. Experimentation has shown that it is the projected distance to the food source (rather than absolute distance) which is coded in this manner.

During flight bee develops positive electromagnetic charge, call it Q, to its abdomen. The charge is due to moving and rubbing together of the body parts. Also Q serves as a measure for the distance of the food source (see `http://tinyurl.com/y8vcqc7n` and `http://tinyurl.com/ycn32wrk`).

Interestingly, the pollen in flowers is negatively charged relative to environment and sticks to positively charged bees. The electric field of flow changes for 100 seconds after the visit of bee to prevent from futile visits. Bees also detect the electric field created by flower possessing negative charge relative to environment. Bees also detect electric oscillations.

2.3 Communications in other sensory modalities

Dance language is not purely visual. There are also communications in sensory modalities other than vision. Dancing bee produces and releases hydrocarbons: two alkanes, tricosane and pentacosane, and two alkenes, (Z)-9-tricosene and (Z)-9-pentacosene, onto their abdomens and into the air. This makes possible communication by olfaction.

Also acoustic and electromagnetic communications are involved.

1. During dance sounds with frequencies f_v in the range 250-300 Hz are emitted from the vibrations of the wings. Flight sounds are in the same frequency range. It is reported (see `http://tinyurl.com/y8qklrhx`) that there is a preferred value of f_v around 265 Hz. Honeybees also prefer rhythmic sounds. These preferences allow to detect the sounds produced by honeybee dance in dark and noisy environment.

2. The value of f_v correlates with the distance of the food source decreasing with the distance (see `http://tinyurl.com/ya4kq8b8`). In the example discussed in the article f_v decreases from 315 Hz to 207 Hz at interval 50-1600 m. Also the duration of the wag run and the number of wagtail movements during the run increase with the distance.

   This vibration mediates auditory information. Acoustic oscillations can be however transformed to electromagnetic vibrations in the body of honeybee since living systems are piezo electrets. The antennae of bees are sensitive to em radiation at ELF frequencies.

3. The frequency f_w of lateral swinging of the body of dancer is reported to be 13 Hz (see `http://tinyurl.com/ycsmlxt7`). This frequency is below the consciously audible range in the case of humans but also now the transformation to electromagnetic oscillations is possible. During waggle run electric fields are emitted and a natural expectation is that the frequencies f_v and f_w define important electromagnetic frequencies.

What is intriguing is that f_w is in EEG range. As already observed, f_v and f_w could correspond to cyclotron frequencies of proton and Ca^{2+} in a magnetic field near to the endogenous magnetic field $B_{end} = .2$ Gauss needed to explain the quantal effects of ELF em fields in vertebrate brain.

Could radiation with frequencies in EEG range be important also in beehive? For years ago I asked the crazy question whether beehive could have the analog of EEG communicating information from beehive to its MB and making possible the control of beehive by MB using cyclotron radiation. The condition that cyclotron frequencies for MB are identical with some relevant frequencies of biological body is essential for resonance making possible communication and control.

3 The findings of Shipman

A popular article describing the findings of Barbara Shipman [A4, A5, A1] related to honeybee dance can be found at web (see `http://tinyurl.com/96kzbw`). These are however difficult to find. There are also articles about Toda lattices [A3, A2], which she studied in her thesis (see `http://tinyurl.com/yde7h6q4`).

The basic notions used by Shipman relate to the theory of symplectic manifolds M with symmetry group G. The notion of flag manifold is in an essential role. Moment(um) map to the co-adjoint g^* of the Lie algebra g having symplectic structure is involved. Also projections to the Cartan algebra $h \subset g$ and its image in M and to the co-adjoint h^* of the Cartan algebra of G are involved. These notions are standard.

What about the particle dynamics having curves as orbits? This question one cannot be answered firmly without access to the work of Shipman but I failed to find the work of Shipman in web anymore. The natural guess is however that the orbits correspond to actions inf M of one-parameter subgroups of G parameterized by g, which for symmetric spaces such as CP_2 correspond to geodesic lines. The subset of M spanned by the orbits assignable to the elements of $h \subset g$ is therefore a natural object.

Shipman studies in her thesis "Ĉonvex polytopes and duality in the geometry of the full Kostant-Toda lattice") (see `http://tinyurl.com/yde7h6q4`) the dynamics of Toda lattices. These systems are completely integrable systems and Shipman uses generalized flag manifolds for this purpose.

The groups involved are non-compact and have non-trivial Borel sub-groups (consisting of matrices with vanishing lower diagonal). I have briefly summarized the ideas related to Toda lattice in [K3].

It however seems that in the applications to honeybee dance one can study $SU(3)$, which is compact. For $SU(3)$ the Borel sub-group would formally reduce to $U(1) \times U(1)$ consisting of diagonal matrices.

Momentum map seems to be very general and allows very general dynamics. What is needed is that one can assign to each point of the orbit values of Hamiltonians $H_i = \mu(X_i)$ defined as contractions of vector fields X^i with the 1-form μ. The Hamiltonians of Cartan algebra commute with respect to Poisson bracket and therefore it is natural to consider the orbits for which these Hamiltonians are constant in Hamiltonian dynamics having G as symmetries. It would however seem that co-adjoint orbit (it would not reduce to a point for non-Hamiltonian dynamics) and its Cartan projection are always well-defined: even when the dynamics itself is not Hamiltonian and Hamiltonians are not conserved.

3.1 Dance of the honeybee

The following piece of text is summary of Shipman's findings that I wrote as I proposed g the TGD inspired model for honeybee dance in [K3]. It must be emphasized that the model to be discussed differs from this model introduced for more than twenty years ago. I cannot guarantee that Shipman would agree with all what I claim.

What Barbara Shipman found [A4] was that the images of certain curves of 6-dimensional flag manifold under the so called momentum map reproduce the dancing pattern of the honeybee if the six initial values determining the curve are chosen suitably. Only two of these parameters code the information about the food source. The article about the model of honeybee dance is not published yet but on the basis of short abstract [A4] it seems that the curves in question are solution curves associated with a completely integrable system known as a full Kostant-Toda lattice studied by Barbara Shipman [A3, A2].

The solutions of the $2(n-1)$ equations of motion associated with this model can be mapped to the solutions of certain completely integrable Hamiltonian system in the flag manifold $F_n = SL(n,C)/B$, where $SL(n,C)$ is the space of complex matrices with unit determinant and B is the space of upper triangular matrices with unit determinant. F_n is in turn isomorphic with $SU(n)/U(1)^{n-1}$ and this implies a connection with the quantum measurement theory of color charges in $n = 3$ case.

The dance of honeybee should somehow map the some curve of the flag manifold to a planar curve representing the dancing pattern. $SU(n)$ acts as Hamiltonian transformations of the flag manifold but not as symmetries of Kostant-Toda lattice (see `http://tinyurl.com/ybds7us2`): in particular, the Cartan algebra generators define Hamiltonians $H_I(x)$ and $H_Y(x)$ in F_3. The so called momentum map associating to the point x of the flag manifold F_3 the point $(H_I(x), H_Y(x))$ characterizing the values of the isospin and hypercharge Hamiltonians at the point x. The image of F_3 under this map is hexagonal region of plane and the image of Kostant-Toda orbit under this map is identified as the dancing pattern of the honeybee. It is obvious that $SU(3)$ cannot act as symmetries of the Kostant-Toda system since in this case Hamiltonians would be constant along the solution curves and momentum map would map every orbit to single point.

To summarize the result concisely:

1. If the orbit of 3-surface in the flag manifold is characterized by Hamiltonian equations related to the so called Kostant-Toda lattice, which is a completely integrable system,

2. if the hexagonal planar region defined by the image of the momentum map corresponds to the "dance floor" and

3. if the orbit of the bee corresponds to the image of the orbit of flag manifold under the momentum momentum map,

one can understand the basic aspects of the waggle dance.

One can indeed understand the dance of honeybee as a representation for the information content of thought of the honeybee. What forces one to take the model seriously is that it reproduces also the dependence of the dancing pattern on bee community and predicts correctly the spectacular change of the V shaped dancing pattern to a union two disjoint lines on the opposite boundaries of the hexagon like region.

Remark: The recent TGD inspired model to be discussed deviates from this picture since the intersection X of Shipman's projection with space-time surface defines the parquette for $D(X) = 2$ and also the dance for $D(X) = 1$.

3.2 Basic mathematical notions

It is appropriate to introduce the basic mathematical notions used by Shipman although the TGD based model is formulated without using these notions explicitly: the dance parquette is identified as the surface $X \subset X^4$ defined as the intersection of X^4 with the orbit of $U(1) \times U(1)$ in CP_2. For $D(X) = 1$ dance parquette reduces to dance pattern. For given orbit $U(1) \times U(1)$ the Hamiltonians associated with $u(1) \times u(1)$ Lie-algebra generators are constant.

1. The definition of the moment(um) map can be found from Wikipedia. One considers manifold M with symplectic structure and allowing group G as isometries. Any element of Lie-algebra g of G can be represented as a vector field X of M giving rise to orbits by exponential map. If X is symplectic transformation, the one parameter group associated with X is represented as orbit in M obeying Hamiltonian dynamics defined by the conserved Hamiltonian $H(X)$ assignable to X.

   At any point of M can map $X(x)$ to the dual g^* of g (co-adjoint of g) by contracting it with symplectic form J defining the symplectic structure. Momentum map gives just the Hamiltonian $H(X)$ associated with X. One starts from the formula

   $$d\langle \mu, X \rangle = dH(X)$$

   for the contraction between 1-form μ and vector field X. Clearly, the Hamiltonians are defined only modulo additive constant. Along the orbit of the flow defined by X $H(X)$ is constant since one has Hamiltonian flow

   $$\frac{dY}{dt} = \{H, Y\}$$

   applied to $Y = X$. Also the Hamiltonians associated with Lie-algebra generators commuting with X are constant along the orbits of X.

   One can define momentum map as one-form μ by requiring that the value of μ at point x of M for any X equals to Hamiltonian $H(X)$ at this point:

   $$H(X(x)) = \langle \mu(x), X(x) \rangle \ .$$

   Since the number of components of μ is the dimension $D(M)$ of M and g is $D(g)$-dimensional, this gives $D(M)$ equations for $D(N) > D(M)$ variables so that solutions exist. The condition that the Poisson brackets of Hamiltonians represent the Lie-algebra gives additional conditions allowing to fix μ.

2. Momentum map allows to assign to the orbits of the dynamical system obeying Hamiltonian dynamics conserved Hamiltonians and for completely integrable systems such as Toda lattice these conserved Hamiltonian fix the solution completely.

3. The Cartan sub-algebra h of g determines maximal number of commuting conserved quantities quantum mechanically and one can assign to the image of the classical system in g^* its projection to h^*. In TGD framework one can however argue that this does not provide an interesting representation of the waggle dance since a stationary position of the bee at dancing parquette would code for the position information.

   Rather, it would seem that the dual of this representation in which point of h^* determines the direction and magnitude of the momentum/velocity of the bee is more appropriate. One can also indeed h as a union of orbits of generators of h in M. Waggle run would correspond to one particular point of h. The Hamiltonians associated with vector fields of h would be constant at this surface.

4. The projection of flag manifold to the image of h in flag manifold $F = SU(3)/U(1) \times U(1)$ or any manifold M, say CP_2 with symplectic $SU(3)$ action would determine the 2-D dance parquette for $G = SU(3)$. At these 2-surfaces orbits would be parameterized by constant values of Hamiltonians defining color hypercharge and isospin. The choices of the subgroup $U(1) \times U(1)$ are parameterized by F and at each surface. As already noticed, one must consider the intersection of this set with space-time surface in TGD framework.

   One could say that the points of F representing the choices of quantization axes for color quantum numbers are represented by 2-D Lagrangian surface in CP_2 in TGD framework. This would realize quantum classical correspondence realizing the quantization axis as a dynamical pattern. As noticed, the projection to space-time surface need not be 2-D.

5. The projection map of $F = G/H$ to h^* defines so called momentum polytope having dimension of h (see `http://tinyurl.com/ycxddqz5`). In the case of $SU(3)$ polytope is 2-D hexagon. The fact that beehive has the structure of hexagonal lattice is taken by Shipman as an accident but one can ask whether this is really so.

4 TGD based model

The purpose of honeybee dance is to represent symbolically a behavioral pattern leading to a desired goal, a kind of a program. In ZEO behavioral patterns are fundamental whereas time=constant snapshots of dynamics are fundamental in standard positive energy ontology (PEO). ZEO is extremely restrictive: the preferred extremals of the action principle satisfy infinite number of additional gauge conditions reducing the effective number of space-time dimensions to 2 corresponding to the strong form of holography.

Number theoretic approach [L11, L10] forces even stronger form of holography: in which finite measurement resolution is a key aspect of dynamics reduces the locus of initial values to a set of discrete space-time points providing a cognitive representation for the system at space-time level [L7]. This picture conforms with the computationalistic idea that that finite number of numbers fixes the time evolution as an analog of computer program [L6].

The idea that forager bee would perform complex neuronal computations to store the data about the path to the food source looks to me somewhat questionable. At least these computations involved cannot be conscious. AI enthusiast would propose deep learning as a formation of associations leading to the miraculous ability of the bee to remember the path and represent it by dance pattern. This option looks more promising.

To me a more plausible view to consider is that the positional information is stored automatically to the MB of honeybee. This brings in the radical possibility that the forager bee actually generates temporary flux tube connections with the food source and has a permanent contact with Sun and Earth via gravitational flux tubes. This would store the information to the MB of the bee and the updating would be automatic.

4.1 Some ideas of TGD and TGD inspired neuroscience and quantum biology

One should be able to model honeybee dance without introducing any adhoc assumptions. In particular, the dance itself should emerge at space-time level from the fundamental dynamics of TGD. Central notions are ZEO and magnetic body (MB) carrying dark matter as $h_{eff} = n \times h$ phases of ordinary matter. This hierarchy has first principle description in terms of adelic physics [L11, L10].

4.1.1 Zero energy ontology

TGD inspired theory of consciousness and quantum biology rely on few key ideas and notions. Zero energy ontology (ZEO) is of them. ZEO leads to an extension of quantum measurement theory to a theory of consciousness [L12]. The notion of causal diamond (CD) plays a key role in ZEO. ZEO implies that time=constant snapshots as counterparts of physical states are replaced by preferred time evolutions as 3-surfaces (analogs of Bohr orbits) connecting the 3-surfaces at the opposite light-like boundaries of CD analogous. Zero energy states states can be regarded as events with initial and final states at opposite boundaries of CD and classically represented as 3-surfaces.

Field equations in the twistor lift of TGD [K10, K17, K15] can be regarded as a generalization of the dynamics of geodesic motion coupled to Kähler force obtained by replacing 1-D curve with 4-D orbit X^4 of 3-surface. The preferred extremals can be divided to two kinds of regions. Regions of first kind represent external particles for which Kähler 4-force vanishes and which are minimal surfaces as analogs of light-like geodesics. Regions of second kind are interaction regions inside CDs where the Kähler 4-force is non-vanishing. Following biologists and neuroscientists one could speak about a generalization of the notion of behavioral pattern or biological function. Computer scientist would talk about programs.

In ZEO the act of free will would be analogous to a replacement of a deterministic program with a new one [L6]. ZEO is actually forced by the acceptance of the fact that we have free will, which must be consistent with the determinism of field equations. At quantum level, classical program as preferred extremal is replaced with a quantum superposition of classical programs, which in some resolution cannot be distinguished from each other.

4.1.2 The notion of magnetic body

The basic distinction between TGD and Maxwell's electrodynamics and gauge theories is that in TGD Universe any system has a field identity as separate space-time sheets, topological field quanta. They correspond to magnetic flux sheets or tubes and also to electric field has topological quanta. This follows from the notion of the induced gauge field. In Maxwell's theory fields of different systems interfere, in TGD they correspond to separate space-time sheets but particle experiences the sum of the forces caused by them since it touches these space-time sheets.

This modification forces the replacement

organism + environment → MB + organism + environment.

MB receives sensory input from biological body (BB) and controls it. Sensory input to MB can be in terms of generalized Josephson radiation from cell membrane acting as generalized Josephson junction and coding nerve pulse patters to frequency modulations. The control by MB can be realized in terms of cyclotron radiation to DNA (accompanied by what I call dark DNA [L3]).

4.1.3 Hierarchy of Planck constants

The hierarchy $h_{eff} = n \times h$, $n = 1, 2, 3, ..$ of Planck constants gives rise to a hierarchy of dark matters. $h_{eff} = n \times h$ labels the onion like layers of MB. The size scale of give layer is by uncertainty principle of order of cyclotron wavelength $\lambda \propto m/eB$ and thus proportional to particle mass m. The value of Planck constant determines the hierarchy level: n can be identified as the dimension of the algebraic extension of rationals defining the adele [L11], and measures the complexity of the algebraic extension associated with the dynamics at the basic level, and therefore serves as a kind of IQ. Evolution corresponds to a gradual and unavoidable increase of $h_{eff}/h = n$ in statistical sense.

1. At the atomic level the value of n seems to be $n = 6$ rather than $n = 1$ [L8, L5]. For valence bonds the value of n is already larger and increases along the rows of the periodic table being largest for the molecules containing atoms towards the right end of the period: biologically important atoms C, N, O, S, P are examples associated with valence bonds with large n.

2. For protons at hydrogen bonds the value of n is much higher than for electrons of valence bonds and the generation of hydrogen bonds could be seen as a crucial aspect of bio-chemistry. Metabolic energy is measured as the difference of the energy of bond for ordinary value of h_{eff} from the real one and one can say that metabolic energy provides for the system ability to increase its negentropy. Metabolic energy increases h_{eff} resources: this is why we must eat.

An important additional hypothesis generalizes the notion of gravitational Planck constant due to Nottale [E1].

1. The hypothesis [K13, K14] states that at the flux tubes mediating gravitational interactions (propagation of gravitons) one has

$$\hbar_{eff} = n\hbar = \hbar_{gr} = \frac{GMm}{v_0} \ ,$$

where M and m are the masses associated with the ends of the flux tube and $v_0 < c$ has dimensions of velocity. This formula holds true if Mm/v_0 exceeds Planck mass squared and implies that the coupling parameter GMm in perturbation series is replaced with $v_0/c < 1$ so that one achieves convergence.

2. For large values of M the value of h_{gr} can be very large, which means that long range gravitational interaction can give rise to systems with very high cognitive resources. This hypothesis generalizes also to other interactions in rather obvious manner and the phase transition increasing the value of h_{eff} leads to dark phase in which perturbation theory converges (the value of the coupling strength $\alpha \propto 1/\hbar_{eff}$ is reduced).

3. The value of M depends on the state of the network defined by the flux tubes mediating gravitational interaction. At the limit of ordinary quantum gravity M would be mass of elementary particle. There is however entire dynamical fractal hierarchy of gravitational flux tubes completely analogous to those postulated flux tube hierarchies in neural system and in endocrine system. For instance, the fountain effect of superfluidity could correspond to a situation involving large value of h_{gr}. In living matter the mass of large neuron is of order Planck mass and defines kind of critical mass in the sense that gravitational interaction between two large neurons could correspond to h_{gr}.

4. $h_{eff} = h_{gr}$ hypothesis implies that cyclotron energies do not depend on the mass m of the charged particle and are therefore universal. The proposal is that the energy scale of bio-photons, which is in visible and UV appropriate for molecular transitions, corresponds to the energies of dark cyclotron photons, which can transform to bio-photons [K11]. The spectrum of the values of "endogenous" magnetic field B_{end} with nominal value $B_{end} = .2$ Gauss would corresponds to the energy range of bio-photons. Cyclotron photons would play central role in the control of biological body by MB based on resonance mechanism. Also the communications from biological body to MB would involve resonance mechanism.

4.1.4 Flag manifold qualia

TGD inspired theory of consciousness leads to a proposal concerning the identification of qualia [K3]. The original proposal was based on standard ontology and the sensory qualia were identified in terms of changes of quantum numbers in state function reduction: the problem of the interpretation is that the outcome of the reduction is random and qualia could be defined only in statistical sense.

The recent view is based on the vision about self as a generalized Zeno effect [L12]. In ZEO qualia would correspond to quantum numbers measured repeatedly during the Zeno period having also interpretation as so called weak measurement.

1. One can distinguish between non-geometric qualia like colors, tastes, and odours, and geometric qualia representing geometric information such as angles and distances. Flag manifold qualia would be universal geometric qualia. In the model for the honeybee dance [K3] the point of 6-D flag manifold F would represent positional information about the food source and waggle dance would represent a point f of F (or an orbit inside the 2-surface of CP_2 representing f) as a dynamical pattern.

2. F has symplectic structure and this encourages the question whether flag manifold qualia could be divided to position type qualia and momentum type qualia. The symplectic structure of F forces to ask whether only degrees of freedom which correspond to mutually commuting Hamiltonians are representable. If so then the representations of qualia at space-time

level could correspond to 2-surfaces for which Hamiltonians assignable to $U(1) \times U(1)$ are constant. Motion in this plane dictated by the values of these Hamiltonians as momenta would provide the representation of the geometric qualia at the level of CP_2.

3. The natural proposal is that the surface $X \subset X^4$ obtained as intersection of space-time surface and the orbit of $U(1) \times U(1)$ in CP_2 and depending on the dimension $D(X)$ analogous to string world sheet, curve, or even point corresponds to a kind of dance parquettes or dance itself.

4. In the case of M^4 twistor lift forces to introduce the geometric variant of twistor space as $M^4 \times CP_2$ and also generalization of Kähler structure and symplectic structure. The counterpart of $U(1) \times U(1)$ consists of translations in time-like plane and the point of the twistor space correspond to a choice of time axis (energy quantization axis) and quantization axis of spin.

   In fact, octonionic approach to TGD reducing the dynamics of TGD to algebraic geometry forces to introduce preferred time axis and spatial axis: they correspond to octonionic real unit and preferred imaginary unit [L7]. The 6-D twistor space $M^4 \times S^2$ labelling the choice of these axes would code for geometric information, and also now one would have a representation in terms of the intersection of space-time surface with this plane.

These arguments suggest that flag manifold qualia are something very fundamental and gives support for the discovery of Shipman. Honeybee dance would provide also support for the coherence of long range classical color gauge fields predicted by TGD.

4.2 Waggle and vibration frequencies as clues

The basic vision is that MB uses biological body (BB) as a motor instrument and sensory receptor. Control and communication mechanisms are based on resonance mechanism requiring that the changes of energies for some transitions are same at the level of MB and BB: this givesvery powerful constraints on prebiotic scenarios and allows to understand why just certain molecules were chosen as bio-molecules [L9, L13]. Cyclotron frequencies are in a special role and one expects that the resonant frequencies at the level of biological body correspond to cyclotron frequencies. Large value of h_{eff} guarantees that low frequency quanta have energies about thermal energy and therefore effective.

The fundamental dynamics would be that of magnetic flux tubes. Bee could simply move along a flux tube carrying dark ions. A more ore detailed model will be discussed later. The orbits at the image Y of $U(1) \times U(1)$ in CP_2 are labelled by two momenta, essentially frequencies since angle variables are in question. Could the frequencies (f_1, f_2) have counterparts in honeybee dance? There are indeed two key frequencies involved: waggle frequency f_w and vibration frequency f_v for the wings of the bee: could the identification $(f_w, f_v) = (f_1, f_2)$ make sense?

Some of the cyclotron frequencies involved should correspond to f_w and f_v.

1. The vibration frequency f_v for the wings of the bee varies in the range 200-300 Hz roughly. For $B_{end} = .2$ Gauss, which explains Blackman's findings about the quantal effects of ELF radiation [J1], the cyclotron frequency of Ca^{2+} would be $f(Ca^{2+}) = 15$ Hz (or its multiple corresponding to higher cyclotron transitions).

   300 Hz would correspond to protons cyclotron frequency for B_{end}. For $f_c(p) = 200$ Hz the value of B would be $B = 2B_{end}/3$. f_v could correspond also to electromagnetic frequency since acoustic signals are transformed to electric signals in living matter, which consists of piezo electrets.

2. The observed waggle frequency f_w is around 13 Hz and suggests that B_{end} is scaled down by factor 13/15 in this case. This scaling down reduces f_v to 250 Hz. The preferred value of f_v is reported to be around 265 Hz (see `http://tinyurl.com/y8qklrhx`) .

3. The average value of f_v is reported to decrease with the distance from 315 Hz at 50 m to 207 Hz at 1600 m (see `http://tinyurl.com/ya4kq8b8`). Therefore also the value of B_{end} should decrease with the distance. Interestingly, the lower bound $f_v = 200$ Hz corresponds

to lower bound $f_w = 10$ Hz in alpha band and in the case of humans defines the lowest frequencies correlating directly with conscious experience. Alpha band indeed dominates in the transition from awake state to sleep.

4. These observations support the view that f_w and f_v allow interpretation as cyclotron frequencies, and force to ask whether proton and Ca^{2+} cyclotron frequencies are in key role in the communications between the dancer and the audience. It is indeed known that the dancer generates electric oscillations and the bees can detect them by their antennae. Proton and Ca^{2+} are also in a key role in the function of cells and neurons.

One cannot avoid the question whether beehive could have EEG or at least alpha band. Bees should not have EEG if the usual neuroscience interpretation for EEG frequencies as being produced by cortex is correct but in TGD one cannot be certain about this.

4.3 What should one understand?

One can try to understand the basic topology of the dance by starting from the interpretation for the information coded by it. This does require introduction any specific model for how the information is represented.

1. Why the waggle pattern transforms from two parallel lines for large distances to V shape at shorter distances and finally to two disjoint pieces of circle dance? A possible answer is that the angle between the edge of V and its diagonal represents the angle between the direction of the food source and its projection in horizontal direction. For long distances the angle is small so that the lines are nearly parallel.

   For short distances the angle becomes large. At criticality the upper edge of V becomes vertical and the dancing pattern must change since other wise the direction to the source is interpreted to opposite from the real one. Waggle periods must be in a direction parallel to the direction of food sources to code for the direction of the food source. Waggle period becomes short since it codes for a short distance.

2. Why two waggle runs - left and right run - as mirror images of each other with respect to the diagonal of V are needed. If waggle direction is actually the direction along the surface of Earth to the food source, waggle run and its mirror images are necessary for coding the information about the diagonal of V defining the direction to the food source.

Suppose that the position information about the source is represented by a point of F.

1. How the coordinates of the point of F characterizing choices of $U(1) \times U(1)$ code for the position information? The intersections X of the orbits of $U(1)\times U(1)$ in CP_2 with the space-time surface have in the generic case $D(X) \leq 2$ and should code the position information. X can (and must) have one time-like direction. For $D(X) = 1$ this gives just single waggle run. The temporal length T of X codes defining the duration of the waggle codes for the distance of the food source.

2. The parameter α introduced by Shipman correlates with distance and could code it. Since the duration of the waggle run correlates with the distance, a possible interpretation of α as temporal duration T assignable to X. Also the value of the charge Q generated during the flight and proportional to the duration of the flight is roughly proportional to the distance and thus T and α.

3. Waggle frequency f_w is additional dynamical parameter related to the motion of the dancer. According to Wikipedia the higher the value of f_w is, the more excited the bee is. This would suggests that f_w varies. Waggling and f_w would relate to the dynamics of space-time surface involved (flux tube perhaps) in M^4 degrees of freedom rather than to the rather simple dynamics of geodesic motion in CP_2. Oscillating string is what comes in mind as approximation to the dynamics of the flux tube.

4. Vibration frequency f_v for wings is a further additional parameter. Also f_v would to the dynamics of X if the flux tube controls the motion of the bee. Cyclotron frequency hypothesis implies that the ratio f_v/f_w of waggle frequency and vibration frequency is constant equal to the mass number of Ca divided by two $f_v/f_w = A/2 = 20$.

   If $(f_w, f_v) = (f_1, f_2)$ identification makes sense then also (f_w, f_v) would be coded by the point of F.

   Remark: Amusingly, the same number 20 appears in the model for life like properties of a simple system of plastic balls in Argon gas: now the ratio of atomic weight of Argon and proton ($A(Ar) = 20$) gives it [L9] (see `http://tinyurl.com/y8wexfqo`). An explanation for the decrease of the f_v and (possibly of f_w too) with the distance of the food source would be needed. Could the long distance to the food source imply that the dancer is less excited? This would require the decrease of the value of B_{end} to which f_w and f_v are proportional with distance. The value of B_{end} could correspond to the magnetic field at the flux tube.

5. It seems that one can understand what happens to the position information at the criticality. One should also understand how the change of the dancing pattern is represented at the level of F and X. Why the intersections of $U(1) \times U(1)$ with space-time surface get short? Is this simply due to the fact that the temporal length of X is determined directly by the length of the path to the source.

How the angle between Sun and target could be updated automatically?

1. In neuroscience approach identifying brain as a computer this information would be stored by computation. Deep learning algorithms would be proposed by AI people. Standard physics mechanism for storing the information about the direction angles are proposed. Foraging bee would utilize solar compass. The information about directions in plane of Earth would be stored by using the information coming the polarization of the sunlight.

2. Automatic updating of the direction S of Sun and the direction L of the food source relative to it should be understood. This requires computation and learning in neuroscience approach. I do not know enough about deep learning to articulate precisely why I do not believe this option.

   A more radical option is that MB of bee stores this information into its own geometry? The proposal has been that MB explains the third person perspective of consciousness: this would explain also OBEs [K8, K9]. Could the MB provide a representation for the dynamics of the bee and its environment including the Sun? If MB contains flux tube S in the direction of Sun defining a pointer of sundial so to say, a temporary flux tube L in the direction of food source, and a temporary flux tube H along projection of L parallel to Earth, this is guaranteed and updating takes place automatically.

3. How the temporary flux tubes would generated? In TGD inspired theory of consciousness flux tubes serve as a correlate for attention. Dancer has directed its attention to the food source and has become connected bya flux tube it. Could this bond be preserved so that the bee would be connected by flux tubes to the target? To be precise, these temporary flux tubes would be actually by pairs of flux tubes generated as flux tube loops from bee and food source reconnect. One could also imagine kind of miniature variant of this representation if this sounds too non-local.

4. The flux tubes S in the direction of Sun would be naturally gravitational flux tubes possibly carrying dark matter with $h_{eff} = h_{gr}$. The mass M appearing in h_{gr} could be some fraction of solar mass. The angle between L and H a would code the information needed to realize the V shape. Same would apply to the gravitational flux tubes E of Earth. Earlier work suggests that a fraction 10^{-4} of Earths mass of the gravitational flux of Earth is at dark flux tubes $h_{eff} = h_{gr}$.

   These flux tubes would provide a cognitive representation for the direction S of Sun, for the line L connecting the hive to the food source, and for the projection H of Lalong the surface of Earth. The MB of bee should have also gravitational flux tubes of Earth and since they are orthogonal to H: could these two kinds of gravitational flux tubes make possible the representation of H as edge of V? H and L would be behave like rigid body whereas S would be like a pointer of sundial.

5. Does the rotation of the reference direction from L to E mean rigid body rotation for the MB (and body) of the dancer? Do solar flux tubes become flux tubes inside the vertical flux tubes? If so, the first part of waggle run would take place along the flux tube to target along H turned by $\pi/2-\theta$. The second part of waggle run would take place along its mirror image with respect to rotated L.

Consider now the role of $h_{eff}/h = n$ having two widely different ranges of values.

1. The cyclotron energies of p and Ca^{2+} are extremely small for the ordinary value of Planck constant. This was one of the reasons motivating to the introduction of the hierarchy of Planck constants $h_{eff}/h = n$ [K2, K13]. The hypothesis $h_{eff} = h_{gr}$ implies that the cyclotron energies do not depend on the mass m of the particle and are in the range of energies of bio-photons (visible and UV). Also the gravitational Compton lengths of particles are independent of m. Also this encourages the consideration of the possibility that the MB of bee has flux tubes carrying gravitational flux tubes from Sun. The values are roughly of the order of 10^{14} if EEG photons have energies in visible and UV.

   The directions of gravitational flux tubes to Sun and Earth define two preferred stationary directions. It seems natural to assign to them gravitational Planck constants $h_{E,gr}$ and $h_{S,gr}$.

2. Relatively small values of n are assignable to electrons of valence bonds and of aromatic cycles [L8]. $n < n_{max} = 100$ is a rough estimate. Thus they are much smaller than the values of $n = h_{gr}/h$ assignable to dark protons at magnetic flux tubes of say hydrogen bonds and assignable to gravitational fields. The model for valence bonds based on TGD predicts that n increases along the row of the periodic table and the molecules appearing as nutrients have the highest values of n associated with their valence bonds.

   $h_{eff}/h = n$ serves as a kind of measure for IQ of the system. More precisely, the recent interpretation is that n expresses the ability of the system to generate negentropy. I have proposed that chemical senses might detect the value of n. For instance, the higher the value of n, the more pleasant the odour. Aromatic compounds with aromatic cycles would have dark electrons at the flux tubes assignable to the cycles and therefore would have value of n larger than the usual.

   Could the average value $\langle n\rangle$ measure the quality of the nectar? Could the dancer communicate the value of $\langle n\rangle$. Could be be more excited if the average value of n is large. This should be reflected in the value of f_w via the value of B_{end} if f_w really measures how excited the bee is.

4.4 Concrete model for the coding of the information about waggle dance at MB

Magnetic flux tubes forming part of MB serve as controllers of BB in TGD inspired quantum biology. This suggests that it could be possible to build a concrete model for the control of waggle dance in terms of magnetic flux tubes.

1. The simplest flux tubes are infinitely thin and thus their orbits have 2-D M^4 projection. I call them cosmic strings. They are space-time surface of form $X^4 = X^2 \times S^2 \subset M^4 \times CP_2$, where X^2 is minimal surface - string world sheet - and S^2 is geodesic sphere of CP_2. CP_2 has two non-equivalent geodesic spheres. The first one has vanishing induced Kähler form and second is homologically non-trivial (non-contractible) and carries Kähler magnetic flux.

2. One has also more general preferred extremals $X^2 \times Y^2$, where Y^2 is complex sub-manifold of CP_2 obtained by replacing S^2 with a sphere with $g \geq 0$ handles.

3. One can deform these extremals in M^4 directions to get magnetic flux tubes, which are key players in TGD inspired quantum biology.

All geodesic circles of $Y^2 = S^2$ give rise to X with $D(X) = 1$ but the space-time projection is space-like and corresponds to single point in X^2. How could one get time-like X as a projection of $U(1) \times U(1)$ orbits? The idea comes from the dynamics of rigid body generalized to that for the complex surface $Y^2 \subset CP_2$.

1. Think Y^2 as a rigid body in CP_2 and "kick" it into a rotational motion. This extremely simple motion might produce a preferred extremal. The idea can be illustrated for a geodesic circle S^1 of ordinary sphere. One can "kick" S^1 to a rotational motion around any axis defined by a line from origin to a point of S^2. This motion describes geodesic circle as the image of a Cartan group $SO(2) \subset SO(3)$.

2. For infinitely thin flux tubes the space-time is effectively the string world sheet $X^2 \subset M^4$. X should define surface at X^2. For a rotating Y^2 the orientation of Y^2 in CP_2 depends on the time coordinate t of of X^2. One would have a geodesic motion corresponding to $U(1) \subset U(1) \times U(1) \subset SU(3)$.

3. One can also imagine the dependence of the orientation of Y^2 on the space-like coordinate x of Y^2: there would be "rotation" also in the x direction! The orientation of rotating Y^2 would depend on two string coordinates. Given $(t, x) \in X^2$ would correspond to a point $(\Phi, \Psi) \in U(1) \times U(1)$ and string world sheet itself to $U(1) \times U(1) \in F$! String world sheet would represent flag manifold qualia.

4. This picture is not yet realistic enough. One must have magnetic flux tubes with M^4 projection, which is not infinitely thin. They are obtained for the deformations $X^2 \times S^2$ solutions in M^4 directions increasing the dimension of M^4-projection so that it is 4-D. $D(X) \geq 1$ is however needed to explain honeybee dance.

5. Also for the realistic flux tubes one obtains the rotation by allowing rotation in the additional two directions. A reasonable first guess is that the rotation is everywhere in fixed $U(1)\times U(1)$: this would correspond to a global choice of quantization axes for color quantum numbers.

   Can one identify unique string world sheet X^2 now? What I call fermionic string world sheets are fundamental in TGD. They connect the orbits of partonic 2-surfaces carrying fermion numbers at their ends are indeed realized at the orbits of magnetic flux tubes. This brings in mind strong form of holography (SH) implied by strong form of general coordinate invariance in TGD. Maybe honeybee dance is in certain sense a holographic representation?

6. The temporal size scale T of X would correspond to the duration of the dance and would thus code for the distance to the food source. Hence T must be more or less equivalent with the parameter α of Shipman. The electromagnetic charge Q generated during the flight of forager correlates also with the distance (1 second corresponds to 1 km) and also corresponds to T.

   T would naturally correspond to a finite size scale for CD assignable to the conscious self assignable to the honeybee dance.

7. It is quite possible that the full symmetry breaks down, and the intersection with X^4 gives only single geodesic in the torus $U(1)\times U(1)$. It is characterized by winding numbers (m, n). Waggle run involves two important frequencies: vibration frequency f_v of wings and waggle frequency f_w: these frequencies could correspond to the two conserved Hamiltonians assigning two-momentum to the waggle orbit. These momenta would be equivalent with frequencies. If (f_w, f_v) corresponds to the pair (f_1, f_2) of rotation frequencies at torus $U(1)\times U(1)$ for the rigid body motion one would have $m/n = 20 = f_v/f_w$. The same frequency ratio appears in the system of plastic balls exhibiting life like properties [L9]. Could the dynamics of preferred extremals favor this value of m/n and give Ca^{2+} its unique role in biology and neuroscience?

4.5 Summary

The basic vision behind TGD view is that flag manifold coordinates represent geometric qualia and honeybee dance represents them. The choice of the subgroup $U(1) \times U(1)$ representing point of flag manifold is represented at space-time level. In TGD framework geodesic dynamics coupled to Kähler force is the physically attractive first guess since it would be 1-D idealization of the dynamics of classical TGD, which is obtained from this dynamics by replacing point-like particle with 3-surfaces. This dynamics follows from the twistor lift of TGD by dimensional reduction occurring dynamically. At the point-like limit it gives geodesic motion coupled to Kähler force and allowing $SU(3)$ charges as conserved charged. If one requires that also action $\int j \cdot A$ is invariant the symmetries reduce to $U(1) \times U(1)$ characterizing particular choice of Kähler function of CP_2 (it does not depend on coordinates (Φ, Ψ) assignable to $U(1) \times U(1)$).

The classical dynamics of TGD could explain how the map from the dual of $u(1) \times u(1)$ algebra to the space-time level - the beehive - is realized.

1. The orbits for completely integrable systems are parameterized by the conditions that maximal number of commuting Hamiltonians are constant. TGD is integrable theory and what suggests itself is that the Hamiltonians (P_Φ, P_Ψ) assignable to the phase angle coordinates (Φ, Ψ) parameterizing $U(1) \times U(1)$ orbit in CP_2 are constants at the projection X of the $U(1) \times U(1)$ orbit to X^4 the dimension $D(X)$ satisfies $D(X) \leq 2$. For $D(X) > 0$ the representation is dynamica.

   Space-time sheet could allows $D(X) > 0$ only for very few values of the momentum (P_1, P_2): the projections of other points of F would be discrete. One parameter subgroup of $U(1) \times (1)$ (torus orbit) would define the line H and its mirror image along magnetic flux tube. The edges of V would be obtained by π rotation from each other.

   Waggle run would correspond to time-like line and its spatial projection would represent the orientation angles (θ, ϕ) of the food source as those associated with with the diagonal of V. The temporal size T of X would determine the duration of waggle run and therefore the distance to the source. The electromagnetic charge Q generated to the abdomen of the bee during the flight is proportional to T of the waggle run and codes for the length of the path.

2. The natural parameterization of the situation is in terms of Darboux coordinates for CP_2 for which Kähler potential is given by $A = P_k dQ^k$. Using standard complex coordinates ξ_i for CP_2 [L1], one can choose Q_i to be phase angles of ξ^i: $Q_1 = \Phi$, $Q_2 = \Psi$. These coordinates are cyclic coordinates not appearing in Kähler function of CP_2 and they correspond to $U(1) \times U(1)$ isometries of CP_2. They are constants of motion also for the geodesic dynamics coupled to Kähler form. Conservation laws would correspond to the constancy of the corresponding Hamiltonians P_i. The orbit at X (or equal to it) would be therefore surfaces $P_i = constant$.

   The ratio $v = P_1/P_2$ would define the velocity $v = d\Psi/d\Phi$. The interpretation of P_i as frequencies is natural - hence the notation $P_i = \omega_i$ is more appropriate and an interesting possibility is that these frequencies could serve as a measure for the eagerness of the bee. An interesting possibility is that the frequencies f_v and f_w correspond to f_i. This would give $v \simeq 1/20$.

   X is intersection of HY with X^4. In so called Chladni mechanism [L4] for which self-organization patters for charged particles correspond to the nodes of electromagnetic field so that the force vanishes. The situation would be exactly the same now but em field would be replaced by the induced Kähler form.

3. This picture would allow to understand why the two waggle runs become parallel at large distances and why they form V shape at smaller distances. Also the criticality could be understood. At criticality the second branch of V would become vertical and the geometry of the dance orbit would change so that waggle periods would be parallel to Earth and at opposite sides of the circle to code the information about the direction of the food source.

4. The waggle pattern characterized by the frequency f_w represents information related to the dynamics in M^4 degrees of freedom allowing perhaps only very limited number of continuous orbits. Also vibrational frequency f_v would represent additional information. The interpretation as cyclotron frequencies for Ca^{2+} and proton makes sense. These frequencies could correspond to the conserved momenta or equivalently frequencies associated with Ψ and Φ.

5. The rotation of the frame defined by the direction of Sun to that defined by the direction of local gravitation would correspond to a rotation of the MB of the bee. Here the permanent dark gravitational flux tubes would play a key role in defining the frame.

Needless to say, the proposed representation is very general and perhaps provide a universal manner to represent geometric information. Flag manifold qualia might be universal manner to represent geometric information. In the case of M^4 twistor lift forces to introduce the geometric variant of twistor space as $M^4 \times CP_2$ and also generalization of Kähler structure and symplectic structure. Now the counterpart of $U(1) \times U(1)$ consists of time translations and translations in some spatial direction and point of the twistor space correspond to a choice of time axis (energy quantization axis) and quantization axes of spin.

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