# Variation of Newton's constant and of length of day

M. Pitkänen
Email: matpitka@luukku.com.
http://tgdtheory.com/public_html/.

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**Abstract**

J. D. Anderson et al have published an article discussing the observations suggesting a periodic variation of the measured value of Newton constant and variation of length of day (LOD). This article represents TGD based explanation of the observations in terms of a variation of Earth radius. The variation would be due to the pulsations of Earth coupling via gravitational interaction to a dark matter shell with mass about $1.3 \times 10^{-4} M_E$ introduced to explain Flyby anomaly: the model would predict $\Delta G/G = 2\Delta R/R$ and $\Delta LOD/LOD = 2\Delta R_E/R_E$ with the variations pf G and length of day in opposite phases. The expermental finding $\Delta R_E/R_E = M_D/M_E$ is natural in this framework but should be deduced from first principles.

The gravitational coupling would be in radial scaling degree of freedom and rigid body rotational degrees of freedom. In rotational degrees of freedom the model is in the lowest order approximation mathematically equivalent with Kepler model. The model for the formation of planets around Sun suggests that the dark matter shell has radius equal to that of Moon's orbit. This leads to a prediction for the oscillation period of Earth radius: the prediction is consistent with the observed 5.9 years period. The dark matter shell would correspond to $n = 1$ Bohr orbit in the earlier model for quantum gravitational bound states based on large value of Planck constant if the velocity parameter $v_0$equals to the rotation velocity of Moon. Also $n > 1$ orbits are suggestive and their existence would provide additional support for TGD view about quantum gravitation. Further amazing co-incidences: the gravitational Compton length of particle is very near to to the Earth's radius in case Earth if central mass is Earth mass. For the mass of dark matter shell it is the variation $\Delta R_E$. This strongly suggest that quantum coherence in astrophysical scales has been and perhaps still is present.

# 1 Inroduction

J. D. Anderson et al [E1] have published an article discussing the observations suggesting a periodic variation of the measured value of Newton constant and variation of length of day.

According to the article, about a dozen measurements of Newton's gravitational constant, G, since 1962 have yielded values that differ by far more than their reported random plus systematic errors. Authors find that these values for G are oscillatory in nature, with a period of $P = 5.899 \pm 0.062$ yr , an amplitude of , $S = 1.619 \pm 0.103 \times 10^{-14}$ m$^3$kg$^{-1}$ s$^{-2}$ and mean-value crossings in 1994 and 1997. The relative variation $\Delta G/G \sim 2.4 \times 10^{-4}$. Authors suggest that the actual values of $G$ does not vary but some unidentified factor in the measurement process is responsible for an apparent variations.

According to the article, of other recently reported results, the only measurement with the same period and phase is the Length of Day (LOD defined as a frequency measurement such that a positive increase in LOD values means slower Earth rotation rates and therefore longer days). The period is also about half of a solar activity cycle, but the correlation is far less convincing. The 5.9 year periodic signal in LOD has previously been interpreted as due to fluid core motions and inner-core coupling. We report the G/LOD correlation, whose statistical significance is 0.99764 assuming no difference in phase, without claiming to have any satisfactory explanation for it. Least unlikely, perhaps, are currents in the Earth's fluid core that change both its moment of inertia (affecting LOD) and the circumstances in which the Earth-based experiments measure G. In this case, there might be correlations with terrestrial-magnetic-field measurements.

In the popular article "Why do measurements of the gravitational constant vary so much?" (`http://phys.org/news/2015-04-gravitational-constant-vary.html`) Anderson states that there is also a possible connection with Flyby anomaly [E2], which also shows periodic variation.

In the following TGD inspired model for the findings is developed. The gravitational coupling would be in radial scaling degree of freedom and rigid body rotational degrees of freedom. In rotational degrees of freedom the model is in the lowest order approximation mathematically equivalent with Kepler model. The model for the formation of planets around Sun suggests that the dark matter shell has radius equal to that of Moon's orbit. This leads to a prediction for the oscillation period of Earth radius: the prediction is consistent with the observed 5.9 years period. The dark matter shell would correspond to $n = 1$ Bohr orbit in the earlier model for quantum gravitational bound states based on large value of Planck constant if the velocity parameter $v_0$appearing in $\hbar_{gr} = GM_EM_D/v_0$ equals to the rotation velocity of Moon. Also $n > 1$ orbits are suggestive and their existence would provide additional support for TGD view about quantum gravitation. There are further amazing co-incidences. The gravitational Compton length $GM/v_0$ of particle is very near to to the Earth's radius in case Earth if central mass is Earth mass. For the mass of dark matter shell it is the variation $\Delta R_E$. This strongly suggest that quantum coherence in astrophysical scales has been and perhaps still is present.

# 2 Coupled oscillations of radii of Earth and dark matter shell as an explanation for the variations

A possible TGD explanation for the variation emerges from the following arguments.

1. By angular momentum conservation requiring $I\omega = L = constant$ the oscillation of the length of day (LOD) can be explained by the variation of the radius $R_E$ of Earth since the moment of inertia is proportional to $R_E^2$. This gives $\Delta LOD/LOD = 2\Delta R/R$. This explains also the apparent variation of $G$ since the gravitational acceleration at the surface of Earth is $g = GM/R_E^2$ so that one has $\Delta g/g = 2\Delta R/R$. Note that the variations have opposite phase.

2. Flyby and Pioneer anomalies [K1] relies on the existience of dark matter shell with a universal surface mass density, whose value is such that in the case of Earth the total mass in the shell would be $M_D \sim 10^{-4} M_E$. The value $M_D/M_E \simeq 1.3 \times 10^{-4}$ suggested by TGD is of the same order of magnitude as $\Delta R/R$. Even galactic dark matter around galactic core could correspond to a shell with this surfaces density of mass [K1]. This plus the claim that also Flyby anomaly has oscillatory character suggest a connection. Earth and dark mass shell are in a collective pulsation with a frequency of Earth pulsation about 6 years and the interaction is gravitational attraction. Note that the frequencies need not be the same. Momentum conservation in radial direction indeed requires that both of them participate in oscillation.

# 3 A detailed model

One can construct a model for the situation.

1. Earth and dark matter shell are modelled as rigid bodies with spatially constant density except that their radii can change. Earth and dark matter shell are characterized by moments of inertia $I_E = (3/5) \times M_E r_E^3$ and $I_D = (2/3) \times M_D r_D^2$. If one restricts the consideration to a rigid body rotation around fixed axis (call it z-axis), one has effective point masses $M_1 = 3M_E/5$ and $M_2 = 2M_D/3$ and the problem is mathematically very similar to a motion point like particles with these effective masses in plane subject to the mutual gravitational force obtained by averaging the gravitational $1/r$ potential over the volumes of the two mass distributions. In the lowest order the problem is very similar to a central force problem with $1/r$-potential plus corrections coming as series in $r_E/r_D$. This problem can be solved by using angular momentum conservation and energy conservation.

2. In the lowest order approximation $r_E/r_D = 0$ one has just Kepler problem in $1/r_D$ force between masses $M_1$ and $M_2$ for $M_D$ and one obtains the analogs of elliptic orbit in the analog of plane defined by $r_D$ and $\phi$. Kepler's law $T_D^2 \propto r_D^3$ fixes the average value of $r_D$, call this value $R_D$.

3. In the next approximation one feeds this solution to the equations for $r_E$ by replacing $r_D$ with its average value $R_D$ to obtain the interaction potential

depending on the radius $r_E$. It must be harmonic oscillator potential and the elastic constant determines the oscillation period of $r_E$. The value of this period should be about 6 yr.

The Lagrangian is sum of kinetic terms plus potential term

$$
\begin{aligned}
& L = T_E + T_D + V_{gr} \ , \\
& T_E = \frac{1}{2}M_E(\frac{dR_E}{dt})^2 + \frac{1}{2}I_E(\frac{d\Phi_E}{dt})^2 \ , \quad T_D = \frac{1}{2}M_D(\frac{dR_D}{dt})^2 + \frac{1}{2}I_D(\frac{d\Phi_D}{dt})^2 \ .
\end{aligned} \tag{3.1}
$$

One could criticize the choice of the coefficients of the kinetic terms for radial coordinates $R_E$ and $R_D$ as masses and one could indeed consider a more general choices. One can also argue, that the rigid bodies cannot be completely spherically since in this case it would not be possible to talk about rotation - at least in quantum mechanical sense.

Gravitational interaction potential is given by

$$
\begin{aligned}
& V_{gr} = -G\int dV_E \int dA_D \rho_E \sigma_D \frac{1}{r_{D,E}} \ , && r_{D,E} = |\overline{r}_D - \overline{r_E}| \ , \\
& dA_D = r_D^2 d\Omega_D && dV_E = r_E^2 dr_E d\Omega_E \ , \\
& \rho_E = \frac{3M_E}{4\pi R_E^3} \ , && \sigma_D = \frac{M_D}{4\pi R_D^2} \ .
\end{aligned} \tag{3.2}
$$

The integration measures are the standard integration measures in spherical coordinates.

One can extract the $r_D$ factor from $r_{D,E}$ (completely standard step) to get

$$
\begin{aligned}
& \frac{1}{r_{D,E}} = \frac{1}{r_D}X \ , \\
& X = \frac{1}{|\overline{n}_D - x\overline{n}_E|} = \frac{1}{[1+x^2-2xcos(\theta)]^{1/2}} = \frac{1}{(1+x^2)^{1/2}}\frac{1}{(1-2xcos(\theta)/(1+x^2))^{1/2}} \ , \\
& x = \frac{r_E}{r_D} \ , \ cos(\theta) = \overline{n}_D \cdot \overline{n}_E \ .
\end{aligned} \tag{3.3}
$$

Angular integration over $\theta$ is trivial and only the integration over $r_E$ remains.

$$
\begin{aligned}
& V_{gr} = -GM_D M_E \frac{3r_D^2}{r_E^3} \int_0^{r_E/r_D} F(\epsilon(x)) \frac{x^2}{(1-x^2)^{1/2}} dx \ , \\
& F(\epsilon) = \frac{(1+\epsilon)^{1/2} - (1-\epsilon)^{1/2})}{\epsilon} \simeq 1 - \frac{\epsilon}{8} \ , \\
& \epsilon = \frac{2x}{1+x^2} \ , \ x = \frac{r_E}{r_D} \ .
\end{aligned} \tag{3.4}
$$

In the approximation $F(\epsilon) = 1$ introducing error of few per cent the outcome is

$$V_{gr} = -\frac{3GM_DM_E}{r_D} \times \left[arcsin(x) - x\sqrt{1-x^2}\right] \;\; = \frac{3GM_DM_E}{r_D}\left[\frac{2}{3} + \frac{x^2}{5} + O(x^3) + ...\right] \;\; , \tag{3.5}$$

$$x = \frac{r_E}{r_D} \;\; .$$

The physical interpretation of the outcome is clear.

1. The first term in the series gives the gravitational potential between point like particles depending on $r_D$ only giving rise to the Kepler problem. The orbit is closed - an ellipse whose eccentricity determines the amplitude of $\Delta R_D/R_D$. In higher orders one expects that the strict periodicity is lost in the general case. From the central force condition $M_2\omega_d^2 r_D = GM_DM_E/r_D^2$ one has

$$T_D = \sqrt{\frac{2}{3}} \times \sqrt{\frac{R_D}{r_{S,E}}\frac{2\pi R_D}{c}} \;\; , \;\; r_{S,E} = 2GM_E \;\; . \tag{3.6}$$

   $r_{S,E} \simeq 8.87$ mm is the Earth's Schwartschild radius. The first guess is that the dark matter shell has the radius of Moon orbit $R_{Moon} \simeq 60.33 \times R_E$, $R_E = 6.731 \times 10^6$ m. This would give $T_D = T_{Moon} \simeq 30$ days.

2. Second term gives harmonic oscillator potential $k_ER_E^2/2$, $k_E = 6GM_DM_E/5R_D^3$ in the approximation that $r_D$ is constant. Oscillator frequency is

$$\omega_E^2 = \frac{k_E}{M_E} \times \frac{6GM_D}{5R_D^3} \;\; .$$

   The oscillator period is given by

$$T_E \;\; = \;\; 2\pi \times \sqrt{\frac{5R_D^3}{6GM_D}} = 2\pi \times \sqrt{5}3 \times \sqrt{\frac{R_D}{R_{S,D}}} \times \frac{R_D}{c} \;\; . \tag{3.7}$$

   In this approximation the amplitude of oscillation cannot be fixed but the non-linearity relates the amplitude to the amplitude of $r_D$.

3. One can estimate the period of oscillation by feeding in the basic numbers. One has $R_D \sim R_{Moon} = 60.34R_E$, $R_E = 6.371 \times 10^6$ m. A rough earlier estimate for $M_D$ is given by $M_D/M_E \simeq 1.3 \times 10^{-4}$. The relative amplitude of the oscillation is $\Delta G/G = 2\Delta R/R \simeq 2.4 \times 10^{-4}$, which suggests $\Delta R/R \simeq M_D/M_E$.

   The outcome is $T_E \simeq 6.1$ yr whereas the observed period is $T_E \simeq 5.9$ yr. The discrepancy could be due to non-linear effects making the frequency continuous classically.

An interesting question is whether macroscopic quantal effects might be involved.

1. The applicability of Bohr rules to the planetary motion [K2] first proposed by Nottale [E3] encourages to ask whether one could apply also to the effective Kepler problem Bohr rules with gravitational Planck constant $\hbar_{gr} = GM_E M_D/v_0$, where $v_0$ is a parameter with dimensions of velocity. The rotation velocity of Moon $v_0/c = 10^{-5}/3$ is the first order of magnitude guess. Also one can ask whether also $n > 1$ other dark matter layers are possible at Bohr orbits so that one would have the analog of atomic spectroscopy.

2. From angular momentum quantization requires $L = m\omega^2 R = n\hbar_{gr}$ and from central force condition one obtains the standard formula for the radius of Bohr orbit $r_n = n^2 GM_E/v_0^2$. For $n = 1$ the radius of the orbit would be radius of the orbit of Moon with accuracy of 3 per cent. Note that the mass of Moon is about 1 per cent of the Earth's mass and thus roughly by a factor 100 higher than the mass of the spherical dark matter shell.

Clearly, the model might have caught something essential about the situation. What remains to be understood is the amplitude $\Delta R/R$. It seems that $\Delta R/R \simeq M_D/M_E$ holds true. This is not too surprising but one should understand how this follows from the basic equations.

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