New Aspects of $M^8 - H$ Duality

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Abstract

$M^8 - H$ duality is now a central part of TGD and leads to new findings. $M^8 - H$ duality can be formulated both at the level of space-time surfaces and light-like 8-momenta. Since the choice of M^4 in the decomposition of momentum space $M^8 = M^4 \times E^4$ is rather free, it is always possible to find a choice for which light-like 8-momentum reduces to light-like 4-momentum in M^4 - the notion of 4-D mass is relative. This leads to what might be called $SO(4)-SU(3)$ duality corresponding to the hadronic and partonic views about hadron physics. Particles, which are eigenstates of mass squared are massless in $M^4 \times CP_2$ picture and massive in M^8 picture. The massivation in this picture is a universal mechanism having nothing to do with dynamics and results in zero energy ontology automatically if the zero energy states are superpositions of states with different masses. p-Adic thermodynamics describes this massivation. Also a proposal for the realization of ADE hierarchy emerges.

4-D space-time surfaces correspond to roots of octonionic polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$. These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t = r_n, r_M \leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball. These special values of M^4 time lead to a deeper understanding of ZEO based quantum measurement theory and consciousness theory.

Can one imagine modifications of $M^8 - H$ duality? Such modification emerged when I became critical about the notion of twistor space of M^4.

I have assumed that what I call geometric twistor space of M^4 is simply $M^4 \times S^2$. It however turned out that one can consider standard twistor space CP_3 with metric signature (3,-3) as an alternative. This option reproduces the nice results of the earlier approach but the philosophy is different: there is no fundamental length scale but the hierarchy of causal diamonds (CDs) predicted by zero energy ontology (ZEO) gives rise to the breaking of the exact scaling invariance of M^8 picture.

M^4 in H would not be replaced with conformally compactified 4-D causal diamond cd (cd_{conf}) for which a natural identification is as CP_2 with second complex coordinate replaced with hypercomplex coordinate. The sizes of twistor spaces of cd_{conf} using CP_2 size as unit would reflect the hierarchy of size scales for CDs. $M^8 - H$ duality would map the points of M^4 to point of $CP_{3,h}$ and project it to a point of $CP_{2,h}$, where "h" tells that hyperbolic variant of CP_n is in question. $CP_{n,h}$ can be indeed defined as projective space. Note that also the original form of $M^8 - H$ duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.

Contents

1 Introduction

$M^8 - H$ duality ($H = M^4 \times CP_2$) [L1] has become one of central elements of TGD. $M^8 - H$ duality implies two descriptons for the states.

1. $M^8 - H$ duality assumes that space-time surfaces in M^8 have associative tangent- or normal space M^4 and that these spaces share a common sub-space $M^2 \subset M^4$, which corresponds to complex subspace of octonions (also integrable distribution of $M^2(x)$ can be considered). This makes possible the mapping of space-time surfaces $X^4 \subset M^8$ to $X^4 \subset H = M^4 \times CP_2$) giving rise to $M^8 - H$ duality.

2. $M^8 - H$ duality makes sense also at the level of 8-D momentum space in one-one correspondence with light-like octonions. In $M^8 = M^4 \times E^4$ picture light-like 8-momenta are projected to a fixed quaternionic $M^4_T \subset M^8$. The projections to $M^4_T \supset M^2$ momenta are in general massive. The group of symmetries is for E^4 parts of momenta is $Spin(SO(4)) = SU(2)_L \times SU(2)_R$ and identified as the symmetries of low energy hadron physics.

   $M^4 \supset M^2$ can be also chosen so that the light-like 8-momentum is parallel to $M^4_L \subset M^8$. Now CP_2 codes for the E^4 parts of 8-momenta and the choice of M^4_L and color group $SU(3)$ as a subgroup of automorphism group of octonions acts as symmetries. This correspond to the usual description of quarks and other elementary particles. This leads to an improved understanding of $SO(4) - SU(3)$ duality. A weaker form of this duality $S^3 - CP_2$ duality: the 3-spheres S^3 with various radii parameterizing the E^4 parts of 8-momenta with various lengths correspond to discrete set of 3-spheres S^3 of CP_2 having discrete subgroup of $U(2)$ isometries.

3. The key challenge is to understand why the MacKay graphs in McKay correspondence and principal diagrams for the inclusions of HFFs correspond to ADE Lie groups or their affine variants. It turns out that a possible concrete interpretation for the hierarchy of finite subgroups of $SU(2)$ appears as discretizations of 3-sphere S^3 appearing naturally at M^8 side of $M^8 - H$ duality. Second interpretation is as covering of quaternionic Galois group. Also the coordinate patches of CP_2 can be regarded as piles of 3-spheres and finite measurement resolution. The discrete groups of $SU(2)$ define in a natural manner a hierarchy of measurement resolutions realized as the set of light-like M^8 momenta. Also a concrete interpretation for Jones inclusions as inclusions for these discretizations emerges.

4. A radically new view is that descriptions in terms of massive and massless states are alternative options leads to the interpretation of p-adic thermodynamics as a completely universal massivation mechanism having nothing to do with dynamics. The problem is the paradoxical looking fact that particles are massive in H picture although they should be massless by definition. The massivation is unavoidable if zero energy states are superposition of massive states with varying masses. The M^4_L in this case most naturally corresponds to that associated with the dominating part of the state so that higher mass contributions can be described by using p-adic thermodynamics and mass squared can be regarded as thermal mass squared calculable by p-adic thermodynamics.

5. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory. 4-D space-time surfaces correspond to roots of octonionic

polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$.

These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t=r_n, r_M\leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball.

What suggests itself is following "braney" picture. 4-D space-time surfaces intersect the 6-spheres at 2-D surfaces identifiable as partonic 2-surfaces serving as generalized vertices at which 4-D space-time surfaces representing particle orbits meet along their ends. Partonic 2-surfacew would define the space-time regions at which one can pose analogs of boundary values fixing the space-time surface by preferred extremal property. This would realize strong form of holography (SH): 3-D holography is implied already by ZEO.

This picture forces to consider a modification of the recent view about ZEO based theory of consciousness. Should one replace causal diamond (CD) with light-cone, which can be however either future or past directed. "Big" state function reductions (BSR) meaning the death and re-incarnation of self with opposite arrow of time could be still present. An attractive interpretation for the moments $t=r_n$ would be as moments assignable to "small" state function reductions (SSR) identifiable as "weak" measurements giving rise to to sensory input of conscious entity in ZEO based theory of consciousness. One might say that conscious entity becomes gradually conscious about its roots in increasing order. The famous question "What it feels to be a bat" would reduce to "What it feels to be a polynomial?"! One must be however very cautious here.

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1. I have assumed that what I call geometric twistor space of M^4 is simply $M^4\times S^2$. It however turned out that one can consider standard twistor space CP_3 with metric signature (3,-3) as an alternative. This option reproduces the nice results of the earlier approach but the philosophy is different: there is no fundamental length scale but the hierarchy of causal diamonds (CDs) predicted by zero energy ontology (ZEO) gives rise to the breaking of the exact scaling invariance of M^8 picture.

2. M^4 in H would not be replaced with conformally compactified M^4 (M^4_{conf}) but conformally compactified 4-D causal diamond cd (cd_{conf}) for which a natural identification is as CP_2 with second complex coordinate replaced with hypercomplex coordinate. The sizes of twistor spaces of cd_{conf} using CP_2 size as unit would reflect the hierarchy of size scales for CDs. M^8-H duality would map the points of M^4 to point of $CP_{3,h}$ and project it to a point of $CP_{2,h}$, where "h" tells that hyperbolic variant of CP_n is in question. $CP_{n,h}$ can be indeed defined as projective space. Note that also the original form of M^8-H duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.

2 M^8-H duality

The generalization of the standard twistor Grassmannian approach to TGD remains a challenge for TGD and one can imagine several approaches. M^8-H duality is essential for these approaches and will be discussed in the sequel. The original form of M^8-H duality assumed $H=M^4\times CP_2$ but quite recently it turned out that if one replaces the twistor space of M^4 identified as $M^4\times S^2$ with $CP_{3,h}$, which is hyperbolic variant of CP_3 one must replace H with $H=CP_{2,h}\times CP_2$.The symmetry between two factors is amazing!

2.1 M^8-H duality at the level of space-time surfaces

M^8-H duality [L1] relates two views about space-time surfaces X^4: as algebraic surfaces in complexified octonionic M^8 and as minimal surfaces with singularities in $H=M^4\times CP_2$.

1. Octonion structure at the level of M^8 means a selection of a suitable decomposition $M^8 = M^4 \times E^4$ in turn determining $H = M^4 \times CP_2$. Choices of M^4 share a preferred 2-plane $M^2 \subset M^4$ belonging to the tangent space of allowed space-time surfaces $X^4 \subset M^8$ at various points. One can parameterize the tangent space of $X^4 \subset M^8$ with this property by a point of CP_2. Therefore X^4 can be mapped to a surface in $H = M^4 \times CP_2$: one M^8-duality. One can consider also the possibility that the choice of M^2 is local but that the distribution of $M^2(x)$ is integrable and defines string world sheet in M^4. In this case $M^2(x)$ is mapped to same $M^2 \subset H$.

2. Since 8-momenta p_8 are light-like one can always find a choice of $M^4_L \subset M^8$ such that p_8 belongs to M^4_L and is thus light-like. The momentum has in the general case a component orthogonal to M^2 so that M^4_L is unique by quaternionicity: quaternionic cross product for tangent space quaternions gives the third imaginary quaternionic unit. For a fixed M^4, call it M^4_T, the M^4 projections of momenta are time-like. When momentum belongs to M^2 the choices is non-unique and any $M^4 \subset M^2$ is allowed.

3. Space-time surfaces $X^4 \subset M^8$ have either quaternionic tangent- or normal spaces. Quantum classical correspondence (QCC) requires that charges in Cartan algebra co-incide with their classical counters parts determined as Noether charges by the action principle determining X^4 as preferred extremal. Parallelity of 8-momentum currents with tangent space of X^4 would conform with the naive view about QCC. It does not however hold true for the contributions to four-momentum coming from string world sheet singularities (string world sheet boundaries are identified as carriers of quantum numbers), where minimal surface property fails.

An important aspect of $M^8 - H$ duality is the description of space-time surfaces $X^4 \subset M^8$ as roots for the real or imaginary part (real and imaginary parts of octonion are quaternions) of octonionic polynomial with real coefficients: these options correspond to quaternionic tangent or normal spaces.

One cannot exclude rational functions and or even real analytic functions in the sense that Taylor coefficients are octonionically real (propotional to octonionic real unit). Number theoretical vision - adelic physics [L2], suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM^8_+ of M^8 with tip at the origin of coordinates is an exception [L1]. At δM^8_+ the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM^8_+ as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary

Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal property realizing the huge symplectic symmetries and making $M^8 - H$ duality possible as also classical twistor lift.

   I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4 \cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

   Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i = \pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

2.2 $M^8 - H$ duality at the level of momentum space

$M^8 - H$ duality occurs also at the level of momentum space and has different meaning now.

1. At M^8 level 8-momenta are quaternionic and light-like. The choices of $M^4_L \supset M^2$ for which 8-momentum in M^4_L, are parameterized by CP_2 parameterizing also the choices of tangent or normal spaces of $X^4 \subset M^8$ at space-time level. This maps M^8 light-like momenta to light-like M^4_L momenta and to CP_2 point characterizing the M^4 and depending on 8-momentum. One can introduce CP_2 wave functions expressible in terms of spinor harmonics and generators of of a tensor product of Super-Virasoro algebras.

2. For a fixed choice M^4_T momenta in general time-like and the E^4 component of 8-momentum has value equal to mass squared. E^4 momenta are points of 3-sphere so that $SO(3)$ harmonics with $SO(4)$ symmetry could parametrize the states. The quantum numbers are $M^4_T \supset M^2$ momenta with fixed mass and the two angular momenta with identical values for S^3 harmonics, which correspond to the quantum states of a spherical quantum mechanical rigid body, and are given by the matrix elements $D^j_{m,n}$ $SU(2)$ group elements ($SO(4)$ decomposes to $SU(2)_L) \times SU(2)_R$ acting from left and right).

   This picture suggests what one might call $SO(4) - SU(3)$ duality at the level of momentum space. There would be two descriptions of states: as massless states with $SU(3)$ symmetry and massive states with $SO(4)$ symmetry.

3. What about the space formed by the choices of the space of the light-like 8-momenta? This space is the space for the choices of preferred M^2 and parameterized by the 6-D (symmetric space $G_2/SU(3)$, where $SU(3) \subset G_2$ leaving complex plane M^2 invariant is subgroup of quaternionic automorphic group $G(2)$ leaving octonionic real unit defining the rest system invariant. This space is moduli space for octonionic structures each of which defines family of space-time surfaces. 8-D Lorent transformations produce even more general octonionic structures. The space for the choices of color quantization axes is $SU(3)/U(1) \times U(1)$, the twistor space of CP_2.

2.2.1 Do M^4_L and M^4_T have analogs at the space-time level?

As found, the solutions of octonionic polynomials consisting of 4-D roots and special 6-D roots coming as 6-sphere S^6 s at 7-D light-cone of M^8. The roots at $t = r$ light-cone boundary are given by the roots $r = r_N$ of the polynomial $P(t)$ and correspond to M^4 slices $t_M = r_N, r_M \leq r_N$. At point r_M S^3 fiber as radius $r(S^3) = \sqrt{r_N^2 - r_M^2}$ and contracts to a point at its boundaries.

Could M^4_L and M_T have analogies at the space-time level?

1. The sphere S^3 associated M^4_T could have counterpart at the level of space-time description. The momenta in M^4_T would naturally be mapped to momenta in the section $t = r_n$ in this case the S^3:s of different mass squared values would naturally correspond to S^3:s assignable to the points of the balls $t = r_n$ and code for mass squared value.

   The counterpart of M^4_L should correspond to light-cone boundary but what does CP_2 correspond? Could the pile of S^3 associated with $t = r_n$ correspond also to CP_2. Could this be the case if there is wormhole contact carrying monopole flux at the origin so that the analog for the replacement of 3-sphere at $r_{CP_2} = \infty$ with homologically non-trivial 2-sphere would be realized?

2. Does the 6-sphere as a root polynomial have counterpart in H? The image should be consistent with $M^8 - H$ duality and correspond to a fixed structure depending on the root r_n only. Since S^3 associated with the E^4 momenta reduces to a point for M^4_L, the natural guess is that S^6 reduces to $t = r_n, 0 \leq r_M \leq r_n$ surface in H.

2.2.2 $S^3 - CP_2$ duality

$S^3 - CP_2$ duality at the level of quantum numbers suggest strongly itself. What does this require? One can approach the problem from two different perspectives.

1. The first approach would be that the representations of $SU(3)$ and $SO(4)$ groups somehow correspond to each other: one could speak of $SU(3) - SO(4)$ duality [K2, K3]. The original form of this duality was this. The color symmetries of quark physics at high energies would be dual to the $SO(4) = SU(2)_L \times SU(2)_R$ symmetries of the low energy hadron physics. Since the physical objects are partons and hadrons formed from the one cannot expect the duality to hold true at the level of details for the representations, and the comparison of the representations makes this clear.

2. The second approach relies on the notion of cognitive representation meaning discretization of CP_2 and S^3 and counting of points of cognitive representations providing discretization in terms of M^8 or H points belonging to the extension of rationals considered. In this case it is more natural to talk about $S^3 - CP_2$ duality.

   The basic observation is that the open region $0 \leq r < \infty$ of CP_2 in Eguchi-Hanson coordinates with r labeling 3-spheres $S^3(r)$ with finite radius can be regarded as pile of $S^3(r)$. In discretization one would have discrete pile of these 3-spheres with finite number of points in the extension of rationals. They points of given S^3 could be related by isometries in special cases.

How $S^3 - CP_2$ duality at the level of light-like M^8 momenta could emerge?

1. Consider first the situation in which one chooses $M^4 \supset M^2$ sub-spaces so that momentum projection to it is light-like. For cognitive representation the choices of $M^4 \supset M^2$ correspond to ad discrete set of points of CP_2 and thus points in the pile of S^3 with discrete radii since all E^4 parts of momenta with fixed length squared to to zero in this choice and each E^4 momentum with fixed lengthand thus identifiable as discrete point of S^3 would correspond to one choice.

   All these choices would give rise to a pile of S^3:s identifiable as set $0 \leq r < \infty$ of CP_2. The number of CP_2 points would be same as total number of points in the pile of discrete S^3s. This is what $S^3 - CP_2$ duality would say.

   Remark: The volumes of CP_2 and S^3 with unit radius are $8\pi^2$ and $2\pi^2$ so that ration is rational number.

2. Consider now the situation for M^4_T so that one has non-vanishing M^4 mass squared equal to E^4 mass squared, having discretized values. The E^4 would momenta correspond to points for a pile of discretized S^3 and thus to the points of CP_2 by above argument. One would have $S^3 - CP_2$ correspondence also now as one indeed expects since the two manners to see the situation should be equivalent.

3. In the space of light-like M^8 momenta E^8 momenta could naturally organize into representations of finite discrete subgroups of $SU(2)$ appearing in McKay correspondence with ADE groups: the groups are cyclic groups, dihedral groups, and the isometry groups associated with tetrahedron, octahedron (cube) and icosahedron (dodecahedron) (see `http://tinyurl.com/yyyn9p95`).

4. Could a concrete connection with the inclusion hierarchy of HFFs be based on increasing momentum resolution realized in terms of these groups at sphere S^3. Notice however that for cyclic and dihedral groups the orbits are circles and pairs of circles for dihedral groups so that the discretization looks too simple and is rotationally asymmetric. Discretization should improve as n increases.

   One can of course ask why C_n and D_n with single direction of rotation axes would appear? Could it be that the directions of rotation axis correspond to the directions defined by the vertices of the 5 Platonic solids. Or could the orbits of fixed axis under the 5 Platonic orbits be allowed. Also this looks still too simple.

   Could the discretization labelled by n_{max} be determined by the product of the groups up to n_{max} and define a group with infinite order. One can consider also groups defined by subsets $\{n_1, n_2...n_3\}$ and these a pair of sequences with larger sequence containing the smaller one could perhaps define an inclusion. The groups C_n and D_n allow prime decomposition in obvious manner and it seems enough to include to the product only the groups C_p and D_p, where p is prime as generators so that one would have set $\{p_1, ...p_n\}$ of primes labelling these groups besides the Platonic groups. The extension of rationals used poses a cutoff on the number of groups involved and on the group elements representable since since too high roots of unity resulting in the multiplication of C_{p_i} and D_{p_j} do not belong to the extension.

   At the level of momentum space the hierarchy of finite discrete groups of $SU(2)$ would define the notion measurement resolution. The discrete orbits of $SU(2) \times U(1)$ at S^3 would be analogous to tesselations of sphere S^2 known as Platonic solids at sphere S^2 and appearing in the ADE correspondence assignable to Jones inclusions as description of measurement resolution. This would also explain also why Z_2 coverings of the subgroups of $SO(3)$ appear in ADE sequence.

   This picture is probably not enough for the needs of adelic physics [L2] allowing all extensions of rationals. Besides roots of unity only the roots of small integers $2, 3, 5$ associated with the geometry of Platonic solids would be included in these discretizations. One could interpret these discretizations in terms of subgroups of discrete automorphism groups of quaternions. Also the extensions of rationals are probably needed.

Could $S^3 - CP_2$ duality make sense at space-time level? Consider the space-time analog for the projection of M^8 momenta to fixed M^4_T.

1. Suppose that the 3-surfaces determining the space-time surfaces as algebraic surfaces in $X^4 \subset M^8$ are given at the surfaces $t = r_N, r_M \leq r_N$ and have a 3-D fiber which should be surface in CP_2. On can assign to each point of this ball $S^3(r_M)$ with radius going to zero at $r_M = r_N$. One has pile of $S^3(r_M)$ which corresponds to the region $0 \leq r < \infty$ of CP_2. This set is discretized. Suppose that the discretization is like momentum discretization. If so, the points would correspond to points of CP_2. It is not however clear why the discretization should be so symmetric as in momentum discretization.

2. The initial values could be chosen by choosing discrete set of points in this pile of S^3:s and this would give rise to a discrete set of points of CP_2 fixing tangent or normal plane of X^4 at these points. One should show that the selection of a point of S^6 at each point indeed determines quaternionic tangent or normal plane plane for a given polynomial $P(o)$ in M^8.

It would seem that this correspondence need not hold true.

2.3 $M^8 - H$ duality and the two manners to describe particles

The isometry groups for $M^4 \times CP_2$ is $P \times SU(3)$ (P for Poincare group). The isometry group for $M^8 = M^4 \times E^4$ with a fixed choice of M^4 breaks down to $P \times SO(4)$. A further breaking by selection $M^4 \subset M^2$ of preferred octonionic complex plane M^2 necessary in the algebraic approach to space-time surfaces $X^4 \subset M^8$ brings in preferred rest system and reduces the Poincare symmetry further. At the space-time level the assumption that the tangent space of X^4 contains fixed M^2 or at least integral distribution of $M^2(x) \subset M^4$ is necessary for $M^8 - H$ duality [L1].

The representations $SO(4)$ and $SU(3)$ could provide alternative description of physics so that one would have what I have called $SO(4) - SU(3)$ duality [K2]. This duality could manifest in the description of strong interaction physics in terms of hadrons and quarks respectively (conserved vector current hypothesis and partially conserved axial current hypothesis based on $Spin(SO(4)) = SU(2) \times SU(2)_R$. The challenge is to understand in more detail this duality. This could allow also to understand better how the two twistor descriptions might relate.

$SO(4) - SU(3)$ duality implies two descriptions for the states and scattering amplitudes.

Option I: One uses projection of 8-momenta to a fixed $M^4_T \supset M^2$.

Option II: One assumes that $M^4_L \supset M^2$ is defines the frame in which quaternionic octonion momentum is parallel to M^4_L: this M^4_L depends on particle state and describes this dependence in terms of wave function in CP_2.

2.3.1 Option I: fixed $M^4_T \supset M^2$

For Option I the description would be in terms of a *fixed* $M^4_T \subset M^8 = M^4_T \times E^4$ and $M^2 \subset M^4_T$ fixed for both options. For given quaternionic light-like M^8 momentum one would have projection to M^4_T, which is in general massive. E^4 momentum would have same the length squared by light-likeness.

De-localization M^4_T mass squared equal to $p^2(M^4_T) = m^2$ in E^4 can be described in terms of $SO(4)$ harmonics at sphere having $p^2(E^4) = m^2$. This would be the description applied to hadrons and leptons and particles treated as massive particles. Particle mass would be due to the fixed choice of M^4_T. What dictates this choice is an interesting question. An interesting question is how these descriptions relate to QFT Higgs mechanism as (in principle) alternative descriptions: the choice of fixed M^4_T could be seen as analog for the generation of vacuum expectation of Higgs selecting preferred direction in the space of Higgs fields.

2.3.2 Option II: varying $M^4_L \supset M^2$

For Option II the description would use $M^4_L \supset M^2$, which is *not fixed* but chosen so that it contains light-like M^8 momentum. This would give light-like momentum in M^4_L identifiable as quaternionic sub-space of complexified octonions.

1. One could assign to the state wave function function for the choices of M^4 and by quaternionicity of 8-momenta this would correspond to a state in super-conformal representation with vanishing M^4_L mass: CP_2 point would code the information about E^4 component light-like 8-momentum. This description would apply to the partonic description of hadrons in terms of massless quarks and gluons.

2. For this option one could use the product of ordinary M^4 twistors and CP_2 twistors. One challenge would be the generalization of the twistor description to the case of CP_2 twistors.

2.3.3 p-Adic particle massivation and ZEO

The two pictures about description of light-like M^8 momenta do not seem to be quite consistent with the recent view about TGD in which H-harmonics describe massivation of massless particles. What looks like a problem is following.

1. The resulting particles are massive in M^4. But they should be massless in $M^4 \times CP_2$ description. The non-vanishing mass would suggest the correct description in terms of Option I for which the description in terms of E^4 momenta with length equal to mass and thus

identifiable as points of S^3. Momentum space wave functions at S^3 - essentially rigid body wave functions given by representation matrices of $SU(2)$ could characterize the states rather than CP_2 harmonic.

2. The description based on CP_2 color partial waves however works and this would favor Option II with vanishing M^4 mass. What goes wrong?

To understand what might be involved, consider p-adic mass calculations.

1. The massivation of physical fermion states includes also the action of super-conformal generators changing the mass. The particles are originally massless and p-adic mass squared is generated thermally and mapped to real mass squared by canonical identification map.

   For CP_2 spinor harmonics mass squared is of order CP_2 mass squared and thus gigantic. Therefore the mass squared is assumed to contain negative tachyonic ground state contribution due to the negative half-odd integer valued conformal weight $h_{vac} < 0$ of vacuum. The origin of this contribution has remained a mystery in p-adic thermodynamics but it makes possible to construct massless states. h_{vac} cancels the spinorial contributions and the contribution from positive conformal weights of super-conformal generators so that the particle states are massless before thermalization. This would conform with the idea of using varying M^4_L and thus CP_2 description.

2. What does the choice of M^4 mean in terms of super-conformal representations? Could the mysterious vacuum conformal weight h_{vac} provide a description for the effect of the needed $SU(3)$ rotation of M^4 from standard orientation on super-conformal representation. The effect would be very simple and in certain sense reversal to the effect of Higgs vacuum expectation value in that it would cancel mass rather than generate it.

   An important prediction is that heavy states should be absent from the spectrum in the sense that mass squared would be p-adically of order $O(p)$ or $O(p^2)$ (in real sense of order $O(1/p)$ or $O(1p^2)$). The trick would be that the generation of h_0 as a representation of SU(3) rotation of M^4 makes always the dominating contribution to the mass of the state vanishing.

   Remark: If the canonical identification I mapping the p-adic mass integers to their real numbers is of the simplest form $m = \sum_n x_n p^n \rightarrow I(m) = \sum_n x_n p^{-n}$, it can happen that the image of rational m/n with p-adic norm not larger than 1 represented as p-adic integer by expanding it in powers of p, can be near to the maximal value of p and the mass of the state can be of order CP_2 mass - about 10^{-4} Planck masses. If the canonical identification is defined as $m/n \rightarrow I/(m)/I(n)$ the image of the mass is small for small values of m and n.

Why p-adic massivation should occur at all? Here ZEO comes in rescue.

1. In ZEO one can have superposition of states with different 4-momenta, mass values and also other charges: this does not break conservation laws. How to fix M^4 in this case? One cannot do it separately for the states in superposition since they have different masses. The most natural choices is as the M^4 associated with the dominating contribution to the zero energy state. The outcome would be thermal massivation described excellently by p-adic thermodynamics [K1]. Recently a considerable increase in the understanding of hadron and weak boson masses took place [L7].

2. In ZEO quantum theory is square root of thermodynamics in a well-defined formal sense, and one can indeed assign to p-adic partition function a complex square root as a genuine zero energy state. Since mass varies, one must describe the presence of higher mass excitations in zero energy state as particles in M^4 assigned with the dominating part of the state so that the observed particle mass squared is essentially p-adic thermal expectation value over thermal excitations. p-Adic thermodynamics would thus describe the fact that the choice of M^4_L cannot not ideal in ZEO and massivation would be possible only in ZEO.

3. Current quarks and constituent quarks are basic notions of hadron physics. Constituent quarks with rather large masses appear in the low energy description of hadrons and current quarks in high energy description of hadronic reactions. That both notions work looks rather

paradoxical. Could massive quarks correspond to M_T picture and current quarks to M_L^4 picture but with p-adic thermodynamics forced by the superposition of mass eigenstates with different masses.

The massivation of ordinary massless fermion involves mixing of fermion chiralities. This means that the $SU(3)$ rotation determined by the dominating component in zero energy state must induce this mixing. This should be understood.

2.4 $M^8 - H$ duality and consciousness

$M^8 - H$ duality is one of the key ideas of TGD and one can ask whether it has implications for TGD inspired theory of consciousness and it indeed forces to challenge the recent ZEO based view about consciousness [L3] .

2.4.1 Objections against ZEO based theory of consciousness

Consider first objections against ZEO based view about consciousness.

1. ZEO (zero energy ontology) based view about conscious entity can be regarded as a sequence of "small" state function reductions (SSRs) identifiable as analogs of so called weak measurements at the active boundary of causal diamond (CD) receding reduction by reduction farther away from the passive boundary, which is unchanged as also the members of state pairs at it. One can say that weak measurements commute with the observables, whose eigenstates the states at passive boundary are. This asymmetry assigns arrow of time to the self having CD as imbedding space correlate. "Big" state function reductions (BSRs) would change the roles of boundaries of CD and the arrow of time. The interpretation is as death and re-incarnation of the conscious entity with opposite arrow of time.

   The question is whether quantum classical correspondence (QCC) could allow to say something about the time intervals between subsequent values of temporal distance between weak state function reductions.

2. The questionable aspect of this view is that $t_M = constant$ sections look intuitively more natural as seats of quantum states than light-cone boundaries forming part of CD boundaries. The boundaries of CD are however favoured by the huge symplectic symmetries assignable to the boundary of M^4 light-cone with points replaced with CP_2 at level of H. These symmetries are crucial or the existence of the geometry of WCW ("world of classical worlds").

3. Second objection is that the size of CD increases steadily: this nice from the point of view of cosmology but the idea that CD as correlate for a conscious entity increases from CP_2 size to cosmological scales looks rather weird. For instance, the average energy of the state assignable to either boundary of CD would increase. Since zero energy state is a superposition of states with different energies classical conservation law for energy does not prevent this [L6]: essentially quantal effect due to the fact that the zero energy states are not exact eigenstates of energy could be in question. In BSRs the energy would gradually increase. Admittedly this looks strange and one must be keen for finding more conventional options.

4. Third objection is that re-incarnated self would not have any "childhood" since CD would increase all the time.

One can ask whether $M^8 - H$ duality and this braney picture has implications for ZEO based theory of consciousness. Certain aspects of $M^8 - H$ duality indeed challenge the recent view about consciousness based on ZEO (zero energy ontology) and ZEO itself.

1. The moments $t = r_n$ defining the 6-branes correspond classically to special moments for which phase transition like phenomena occur. Could $t = r_n$ have a special role in consciousness theory?

   (a) For some SSRs the increase of the size of CD reveals new $t = r_n$ plane inside CD. One can argue that these SSRS define very special events in the life of self. This would not modify the original ZEO considerably but could give a classical signature for how many ver special moments of consciousness have occurred: the number of the roots of P would be a measure for the lifetime of self and there would be the largest root after which BSR would occur.

(b) Second possibility is more radical. One could one think of replacing CD with single truncated future- or past-directed light-cone containing the 6-D universal roots of P up to some r_n defining the upper boundary of the truncated cone? Could $t = r_n$ define a sequence of moments of consciousness? To me it looks more natural to assume that they are associated with very special moments of consciousness.

2. For both options SSRs increase the number of roots r_n inside CD/truncated light-one gradually and thus its size? When all roots of $P(o)$ would have been measured - meaning that the largest value r_{max} of r_n is reached -, BSR would be unavoidable.

   BSR could replace $P(o)$ with $P_1(r_1 - o)$: r_1 must be real and one should have $r_1 > r_{max}$. The new CD/truncated light-cone would be in opposite direction and time evolution would be reversed. Note that the new CD could have much smaller size size if it contains only the smallest root r_0. One important modification of ZEO becomes indeed possible. The size of CD after BSR could be much smaller than before it. This would mean that the re-incarnated self would have "childhood" rather than beginning its life at the age of previous self - kind of fresh start wiping the slate clean.

   One can consider also a less radical BSR preserving the arrow of time and replacing the polynomial with a new one, say a polynomial having higher degree (certainly in statistical sense so that algebraic complexity would increase).

2.4.2 Could one give up the notion of CD?

A possible alternative view could be that one the boundaries of CD are replaced by a pair of two $t = r_N$ snapshots $t = r_0$ and $t = r_N$. Or at least that these surfaces somehow serve as correlates for mental images. The theory might allow reformulation also in this case, and I have actually used this formulation in popular lectures since it is easier to understand by laymen.

1. Single truncated light-cone, whose size would increase in each SSR would be present now since the spheres correspond to balls of radius r_n at times r_n. If $r_0 = 0$, which is the case for $P(o) \propto o$, the tip of the light-cone boundary is one root. One cannot avoid association with big bang cosmology. For $P(0) \neq r_0$ the first conscious moment of the cosmology corresponds to $t = r_0$. One can wonder whether the emergence of consciousness in various scales could be described in terms of the varying value of the smallest root r_0 of $P(o)$.

   If one allows BSR:s this picture differs from the earlier one in that CDs are replaced with alternation of light-cones with opposite directions and their intersections would define CD.

2. For this option the preferred values of t for SSRs would naturally correspond to the roots of the polynomial defining $X^4 \subset M^8$. Moments of consciousness as state function reductions would be due to collisions of 4-D space-time surfaces X^4 with 6-D branes! They would replace the sequence of scaled CD sizes. CD could be replaced with light-one and with the increasing sequence $(r_0, ...r_n)$ of roots defining the ticks of clock and having positive and negative energy states at the boundaries r_0 and r_n.

3. What could be the interpretation for BSRs representing death of a conscious entity in the new variant of ZEO? Why the arrow of time would change? Could it be because there are no further roots of $P(o)$? The number of roots of $P(o)$ would give the number of small state function reductions?

   What would happen to $P(o)$ in BSR? The vision about algebraic evolution as increase of the dimension for the extension of rationals would suggest that the degree of $P(o)$ increases as also the number of roots if all complex roots are allowed. Could the evolution continue in the same direction or would it start to shift the part of boundary corresponding to the lowest root in opposite direction of time. Now one would have more roots and more algebraic complexity so that evolutionary step would occur.

In the time reversal one would have naturally $t_{max} \geq r_{n_{max}}$ for the new polynomial $P(t-t_{max})$ having $r_{n_{max}}$ as its smallest root. The light-cone in M^8 with tip at $t = t_{max}$ would be in opposite direction now and also the slices $t - t_{max} = r'_n$ would increase in opposite direction! One would have two light-cones with opposite directions and the $t = r_n$ sections would replace boundaries of CDs. The reborn conscious entity would start from the lowest root so that also it would experience childhood.

This option could solve the argued problems of the previous scenario and give concrete connection with the classical physics in accordance with QCC. On the other hand, a minimal modification of original scenario combined with $M^8 - H$ duality with moments $t = r_n$ as special moments in the life of conscious entity allows also to solve these problems if the active boundary of CD is interpreted as boundary beyond which classical signals cannot contribute to perceptions.

2.4.3 What could be the minimal modification of ZEO based view about consciousness?

What would be the minimal modification of the earlier picture? Could one *assume* that CDs serve as imbedding space correlates for the perceptive field?

1. Zero energy states would be defined as before that is in terms of 3-surfaces at boundaries of CD: this would allow a realization of huge symmetries of WCW and the active boundary A of CD would define the boundary of the region from which self can receive classical information about environment. The passive boundary P of CD would define the boundary of the region providing classical information about the state of self. Also now BSR would mean death and reincarnation with an opposite arrow of time. Now however CD would shrink in BSR before starting to grow in opposite time direction. Conscious entity would have "childhood".

2. If the geometry of CD were fixed, the size scale of the $t = r_n$ balls of M^4 would first increase and then start to decrease and contract to a point eventually at the tip of CD. One must however remember that the size of $t = r_n$ planes increases all the time as also the size of CD in the sequences of SSRs. Moments $t = r_n$ could represent special moments in the life of conscious entity taking place in SSRs in which $t = r_n$ hyperplane emerges inside CD with increased size. The recent surprising findings challenging the Bohrian view about quantum jumps [L4] can be understood in this picture [L4].

3. $t = r_n$ planes could also serve as correlates for memories. As CD increases at active boundary new events as $t = r_n$ planes would take place and give rise to memories. The states at $t = r_n$ planes are analogous to seats of boundary conditions in strong holography and the states at these planes might change in state function reductions - this would conform with the observations that our memories are not absolute.

To sum up, the original view about ZEO seems to be essentially correct. The introduction of moments $t = r_n$ as special moments in the life of self looks highly attractive as also the possibility of wiping the slate clear by reduction of the size of CD in BSR.

2.5 Challenging the identification $H = M^4 \times CP_2$

One can challenge the identification $H = M^4 \times CP_2$. Poincare invariance is realized at level of the moduli space of the octonionic structures of M^8: given octonion structure breaks Poincare invariance to that for $T \times SO(2)$, which corresponds to a choice of rest frame and spin quantization axis. Therefore one can consider the replacement of M^4 with a space without Poincare symmetries. There is also a breaking of scaling invariance characterized by a hierarchy of 8-D causal diamonds (CD_8) inducing 4-D hierarchy of causal diamonds (cds).

The proposed identification of twistor space of M^4 as $M^4 \times S^2$ is different from the standard identification as hyperbolic variant $CP_{3,h}$ of CP_3. What if the twistor space could be $CP_{3,h}$ after all?

The key idea is that the twistor space and its base space represents CD so that one obtains scale hierarchy of twistor spaces with varying sizes as a realization of broken scale invariance giving rise to the p-adic length scale hierarchy.

1. I have identified the twistor space of M^4 simply as $T(M^4) = M^4 \times S^2$. The interpretation would be at the level of octonions as a product of M^4 and choices of M^2 as preferred complex sub-space of octonions with S^2 parameterizing the directions of spin quantization axes. Real octonion axis would correspond to time coordinate. One could talk about the space of of light-like directions. Light-like vector indeed defines M^2. This view could be defended by the breaking of both translation and Lorentz invariance in the octonionic approach due to the choice of M^2 and by the fact that it seems to work.

   Remark: $M^8 = M^4 \times E^4$ is complexified to M^8_c by adding a commuting imaginary unit i appearing in the extensions of rationals and ordinary M^8 represents its particular sub-space. Also in twistor approach one uses often complexified M^4.

2. The objection is that it is ordinary twistor space identifiable as CP_3 with (3,-3) signature of metric is what works in the construction of twistorial amplitudes. CP_3 has metric as compact space and coset space. Could this choice of twistor space make sense after all as geometric twistor space?

   Here one must pause and recall that the original key idea was that Poincare invariance is symmetry of TGD for $X^4 \subset M^4 \times CP_2$. Now Poincare symmetry has been transformed to a symmetry acting at the level of M^8 in the moduli space of octonion structures defined by the choice of the direction of octonionic real axis reducing Poincare group to $T \times SO(3)$ consisting of time translations and rotations. Fixing of M^2 reducrs the group to $T \times SO(2)$ and twistor space can be seen as the space for selections of quantization axis of energy and spin.

3. But what about the space H? The first guess is $H = M^4_{conf} \times CP_2$. According to [B1] (see `http://tinyurl.com/y35k5wwo`) one has $M^4_{conf} = U(2)$ such that $U(1)$ factor is time- like and $SU(2)$ factor is space-like. One could understand $M^4_{conf} = U(2)$ as resulting by addition and identification of metrically 2-D light-cone boundaries at $t = \pm\infty$. This is topologically like compactifying E^3 to S^3 and gluing the ends of cylinder $S^3 \times D^1$ together to the $S^3 \times S^1$.

   The conformally compactified Minkowski space M^4_{conf} should be analogous to a base space of CP_3 regarded as bundle with fiber S^2. The problem is that one cannot imagine an analog of fiber bundle structure in CP_3 having $U(2)$ as base. The identification $H = M^4_{conf} \times CP_2$ does not make sense.

4. In ZEO based breaking of scaling symmetry it is CD that should be mapped to the analog of M^4_{conf} - call it cd_{conf}. The only candidate is $cd_{conf} = CP_2$ with one hypercomplex coordinate. To understand why one can start from the following picture. The light-like boundaries of CD are metrically equivalent to spheres. The light-like boundaries at $t = \pm\infty$ are identified as in the case of M^4_{conf}. In the case of CP_2 one has 3 homologically trivial spheres defining coordinate patches. This suggests that cd_{conf} is simply $CP_{2,h}$: CP_2 with second complex coordinate made hypercomplex. M^4 and E^4 differ only by the signature and so would do $cd_{conf} = CP_{2,h}$ and CP_2.

   The twistor spheres of CP_3 associated with points of M^4 intersect at point if the points differ by light-like vector so that one has and singular bundle structure. This structure should have analog for the compactification of CD. CP_3 has also bundle structure $CP_3 \to CP_2$. The S^2 fibers and base are homologically non-trivial and complex analogs of mutually orthogonal line and plane and intersect at single point. This defines the desired singular bundle structure via the assignment of S^2 to each point of CP_2.

   The M^4 points must belong to the interior of *cd* and this poses constraints on the distance of M^4 points from the tips of *cd*. One expects similar hierarchy of *cd*s at the level of momentum space.

5. In this picture $M^4_{conf} = U(2)$ could be interpreted as a base space for the space of CDs with fixed direction of time axis identified as direction of octonionic real axis associated with various points of M^4 and therefore of M^4_{conf}. For Euclidian signature one would have base and fiber of the automorphism sub-group $SU(3)$ regarded as $U(2)$ bundle over CP_2: now one would have CP_2 bundle over $U(2)$. This is perhaps not an accident, and one can ask whether

these spaces could be interpreted as representing local trivialization of $SU(3)$ as $U(2)\times CP_2$. This would give to metric cross terms between $U(2)$ and CP_2.

The outcome of these considerations is surprising.

1. For modified M^8-Hduality one would have $T(H)=CP_3\times F$ and $H=CP_{2,H}\times CP_2$, where $CP_{2,H}$ has hyperbolic metric with metric signature $(1,-3)$ having M^4 as tangent space so that the earlier picture could be understood as an approximation. This would reduce the construction of preferred extremals of 6-D Kähler action in $T(H)$ to a construction of polynomial holomorphic surfaces and also the minimal surfaces with singularities at string world sheets should result as bundle projection. Since M^8-H duality must respect algebraic dynamics the maximal degree of the polynomials involved must be same as the degree of the octonionic polynomial in M^8.

2. The hyperbolic variant Kähler form and also spinor connection of $CP_{2,h}$ brings in new physics beyond standard model. This Kähler form would serve as the analog of Kähler form assigned to M^4 earlier, and suggested to explain the observed CP breaking effects and matter anti-matter asymmetry for which there are two explanations [L5].

Note that also the original form of M^8-H duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$. Therefore one cannot say that $H=M^4\times CP_2$ identification with CDs realizing the scale hierarchy in M^4 is wrong.

REFERENCES

Theoretical Physics

[B1] Jadczyk A. Conformally compactified minkowski space: myths and facts. arXiv:1105.3948 [math-ph]. Available at: https://arxiv.org/abs/1803.00545, 2011.

[B2] Minev ZK et al. To catch and reverse a quantum jump mid-flight. arXiv:1803.00545 [quant-ph]. Available at: https://arxiv.org/abs/1803.00545, 2019.

Books related to TGD

[K1] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#mless, 2006.

[K2] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#visionb, 2006.

[K3] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#numbervision, 2014.

[K4] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry? In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#ratpoints, 2017.

Articles about TGD

[L1] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry? Available at: http://tgdtheory.fi/public_html/articles/ratpoints.pdf, 2017.

[L2] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.

[L3] Pitkänen M. Re-examination of the basic notions of TGD inspired theory of consciousness. Available at: http://tgdtheory.fi/public_html/articles/conscrit.pdf, 2017.

[L4] Pitkänen M. Copenhagen interpretation dead: long live ZEO based quantum measurement theory! Available at: http://tgdtheory.fi/public_html/articles/Bohrdead.pdf, 2019.

[L5] Pitkänen M. SUSY in TGD Universe. Available at: http://tgdtheory.fi/public_html/articles/susyTGD.pdf, 2019.

[L6] Pitkänen M. Tesla still inspires. Available at: http://tgdtheory.fi/public_html/articles/teslastill.pdf, 2019.

[L7] Pitkänen M. Two anomalies of hadron physics from TGD perspective. Available at: http://tgdtheory.fi/public_html/articles/twoanomalies.pdf, 2019.