About $h_{eff}/h = n$ as the number of sheets of Galois covering

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/.

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Abstract

The precise view about how space-time surface can be regarded as Galois covering for an extension of rationals in TGD framework is discussed. The so called $M^8-M^4\times CP_2$-correspondence reducing the dynamics at the level of M^8 to algebraic equations for algebraic surfaces and at the level of $M^4\times CP_2$ to the dynamics of preferred extremals satisfying besides variational principle also infinite number of gauge conditions allows to understand how Galois coverings emerge. The polynomials determining space-time surface as algebraic surface determine also the extension of rationals used.

The following considerations were motivated by the observation of a very stupid mistake that I have made repeatedly in some articles about TGD. Planck constant $h_{eff}/h = n$ corresponds naturally to the number of sheets of the covering space defined by the space-time surface.

I have however claimed that one has $n = ord(G)$, where $ord(G)$ is the order of the Galois group G associated with the extension of rationals assignable to the sector of "world of classical worlds" (WCW) and the dynamics of the space-time surface (what this means will be considered below).

This claim of course cannot be true since the generic point of extension G has some subgroup H leaving it invariant and one has $n = ord(G)/ord(H)$ dividing $ord(G)$. Equality holds true only for Abelian extensions with cyclic G. For singular points isotropy group is $H_1 \sup H$ so that $ord(H_1)/ord(H)$ sheets of the covering touch each other. I do not know how I have ended up to a conclusion, which is so obviously wrong, and how I have managed for so long to not notice my blunder.

This observation forced me to consider more precisely what the idea about Galois group acting as a number theoretic symmetry group really means at space-time level and it turned out that M^8-H correspondence [L2] (see http://tinyurl.com/yd43o2n2) gives a precise meaning for this idea.

Consider first the action of Galois group (see http://tinyurl.com/y8grabt2 and http://tinyurl.com/ydze5psx).

1. The action of Galois group leaves invariant the number theoretic norm characterizing the extension. The generic orbit of Galois group can be regarded as a discrete coset space G/H, $H\subset G$. The action of Galois group is transitive for irreducible polynomials so that any two points at the orbit are G-related. For the singular points the isotropy group is larger than for generic points and the orbit is G/H_1, $H_1 \sup H$ so that the number of points of the orbit divides n. Since rationals remain invariant under G, the orbit of any rational point contains only single point. The orbit of a point in the complement of rationals under G is analogous to an orbit of a point of sphere under discrete subgroup of $SO(3)$.

   $n = ord(G)/ord(H)$ divides the order $ord(G)$ of Galois group G. The largest possible Galois group for n-D algebraic extension is permutation group S_n. A theorem of Frobenius states that this can be achieved for $n = p$, p prime if there is only single pair of complex roots (see http://tinyurl.com/y8grabt2). Prime-dimensional extensions with $h_{eff}/h = p$ would have maximal number theoretical symmetries and could be very special physically: p-adic physics again!

2. The action of G on a point of space-time surface with imbedding space coordinates in n-D extension of rationals gives rise to an orbit containing n points except when the isotropy

group leaving the point is larger than for a generic point. One therefore obtains singular covering with the sheets of the covering touching each other at singular points. Rational points are maximally singular points at which all sheets of the covering touch each other.

3. At QFT limit of TGD the n dynamically identical sheets of covering are effectively replaced with single one and this effectively replaces h with $h_{eff} = n \times h$ in the exponent of action (Planck constant is still the familiar h at the fundamental level). n is naturally the dimension of the extension and thus satisfies $n \leq ord(G)$. $n = ord(G)$ is satisfied only if G is cyclic group.

The challenge is to define what space-time surface as Galois covering does really mean!

1. The surface considered can be partonic 2-surface, string world sheet, space-like 3-surface at the boundary of CD, light-like orbit of partonic 2-surface, or space-time surface. What one actually has is only the data given by these discrete points having imbedding space coordinates in a given extension of rationals. One considers an extension of rationals determined by irreducible polynomial P but in p-adic context also roots of P determine finite-D extensions since e^p is ordinary p-adic number.

2. Somehow this data should give rise to possibly unique continuous surface. At the level of $H = M^4 \times CP_2$ this is impossible unless the dynamics satisfies besides the action principle also a huge number of additional conditions reducing the initial value data ans/or boundary data to a condition that the surface contains a discrete set of algebraic points.

   This condition is horribly strong, much more stringent than holography and even strong holography (SH) implied by the general coordinate invariance (GCI) in TGD framework. However, preferred extremal property at level of $M^4 \times CP_2$ following basically from GCI in TGD context might be equivalent with the reduction of boundary data to discrete data if $M^8 - H$ correspondence [L2] (see `http://tinyurl.com/yd43o2n2`) is accepted. These data would be analogous to discrete data characterizing computer program so that an analog of computationalism would emerge [L1] (see `http://tinyurl.com/y75246rk`).

One can argue that somehow the action of discrete Galois group must have a lift to a continuous flow.

1. The linear superposition of the extension in the field of rationals does not extend uniquely to a linear superposition in the field reals since the expression of real number as sum of units of extension with real coefficients is highly non-unique. Therefore the naive extension of the extension of Galois group to all points of space-time surface fails.

2. The old idea already due to Riemann is that Galois group is represented as the first homotopy group of the space. The space with homotopy group π_1 has coverings for which points remain invariant under subgroup H of the homotopy group. For the universal covering the number of sheets equals to the order of π_1. For the other coverings there is subgroup $H \subset \pi_1$ leaving the points invariant. For instance, for homotopy group $\pi_1(S^1) = Z$ the subgroup is nZ and one has $Z/nZ = Z_p$ as the group of n-sheeted covering. For physical reasons its seems reasonable to restrict to finite-D Galois extensions and thus to finite homotopy groups.

   $\pi_1 - G$ correspondence would allow to lift the action of Galois group to a flow determined only up to homotopy so that this condition is far from being sufficient.

3. A stronger condition would be that π_1 and therefore also G can be realized as a discrete subgroup of the isometry group of $H = M^4 \times CP_2$ or of M^8 ($M^8 - H$ correspondence) and can be lifted to continuous flow. Also this condition looks too weak to realize the required miracle. This lift is however strongly suggested by Langlands correspondence [**?**, K3] (see `http://tinyurl.com/y9x5vkeo`).

The physically natural condition is that the preferred extremal property fixes the surface or at least space-time surface from a very small amount of data. The discrete set of algebraic points in given extension should serve as an analog of boundary data or initial value data.

1. M^8-H correspondence [L2] (see http://tinyurl.com/yd43o2n2) could indeed realize this idea. At the level of M^8 space-time surfaces would be algebraic varieties whereas at the level of H they would be preferred extremals of an action principle which is sum of Kähler action and minimal surface term.

   They would thus satisfy partial differential equations implied by the variational principle and infinite number of gauge conditions stating that classical Noether charges vanish for a subgroup of symplectic group of $\delta M^4_{\pm} \times CP_2$. For twistor lift the condition that the induced twistor structure for the 6-D surface represented as a surface in the 12-D Cartesian product of twistor spaces of M^4 and CP_2 reduces to twistor space of the space-time surface and is thus S^2 bundle over 4-D space-time surface.

   The direct map $M^8 \rightarrow H$ is possible in the associative space-time regions of $X^4 \subset M^8$ with quaternionic tangent or normal space. These regions correspond to external particles arriving into causal diamond (CD). As surfaces in H they are minimal surfaces and also extremals of Kähler action and do not depend at all on coupling parameters (universality of quantum criticality realized as associativity). In non-associative regions identified as interaction regions inside CDs the dynamics depends on coupling parameters and the direct map $M^8 \rightarrow CP_2$ is not possible but preferred extremal property would fix the image in the interior of CD from the boundary data at the boundaries of CD.

2. At the level of M^8 the situation is very simple since space-time surfaces would correspond to zero loci for $RE(P)$ or $IM(P)$ (RE and IM are defined in quaternionic sense) of an octonionic polynomial P obtained from a real polynomial with coefficients having values in the field of rationals or in an extension of rationals. The extension of rationals would correspond to the extension defined by the roots of the polynomial P.

   If the coefficients are not rational but belong to an extension of rationals with Galois group G_0, the Galois group of the extension defined by the polynomial has G_0 as normal subgroup and one can argue that the relative Galois group $G_{rel} = G/G_0$ takes the role of Galois group.

   It seems that M^8-H correspondence could allow to realize the lift of discrete data to obtain continuous space-time surfaces. The data fixing the real polynomial P and therefore also its octonionic variant are indeed discrete and correspond essentially to the roots of P.

3. One of the elegant features of this picture is that the at the level of M^8 there are highly unique linear coordinates of M^8 consistent with the octonionic structure so that the notion of a M^8 point belonging to extension of rationals does not lead to conflict with GCI. Linear coordinate changes of M^8 coordinates not respecting the property of being a number in extension of rationals would define moduli space so that GCI would be achieved.

Does this option imply the lift of G to π_1 or to even a discrete subgroup of isometries is not clear. Galois group should have a representation as a discrete subgroup of isometry group in order to realize the latter condition and Langlands correspondence supports this as already noticed. Note that only a rather restricted set of Galois groups can be lifted to subgroups of $SU(2)$ appearing in McKay correspondence and hierarchy of inclusions of hyper-finite factors of type II_1 labelled by these subgroups forming so called ADE hierarchy in 1-1 correspondence with ADE type Lie groups [K2, K1] (see http://tinyurl.com/ybavqvvr). One must notice that there are additional complexities due to the possibility of quaternionic structure which bring in the Galois group SO(3) of quaternions.

Remark: After writing this article a considerable progress in understanding of $h_{eff}/h = n$ as number of sheets of Galois covering emerged. By M^8-duality space-time surface can be seen as zero locus for real or imaginary part (regarding octonions as sums of quaternionic real and imaginary parts) allows a nice understanding of space-time surface as an $h_{eff}/h = n$-fold Galois covering. M^8 is complexified by adding an imaginary unit i commuting with octonionic imaginary units. Also space-time surface is complexified to 8-D surface in complexified M^8. One can say that ordinary space-time surface is the "real part" of this complexified space-time surface just like x is the real part of a complex number $x + iy$. Space-time surface can be also seen as a root of n:th order polynomial with n complex branches [L2] and the projections of complex roots to "real part" of M^8 define space-time surface as an n-fold covering space in which Galois group acts.

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