

Can TGD predict the value of Newton's constant?

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Abstract

Newton's constant G cannot be a fundamental constant in TGD framework, where CP_2 radius R and Kähler coupling strength as the analog of fine structure constant are the fundamental constants. Dimensionally G corresponds to R^2/\hbar . This gives guidelines for predicting G . TGD predicts a hierarchy of effective Planck constants $\hbar_{eff}/\hbar_0 = n$, where n is the order of Galois group of Galois extension defining extension of rationals. Dimension n factorizes to a product $n = n_1 n_2 \dots$ for extension E_1 of extension E_2 of ... rationals. $M^8 - H$ correspondence allows to associate the Galois group with an irreducible polynomial characterizing space-time surface as an algebraic surface in M^8 . The gradual increase of extension by forming a functional composite of a new polynomial with the already existing one ($P \rightarrow P_{new} \circ P$) would be analogous to the evolution of genome: earlier extensions would be analogous to conserved genes.

The proposal modifying the earlier proposal is $G = R^2/n_{gr}\hbar_0$, where n_{gr} is the order of Galois group G_{gr} "at the bottom" of the hierarchy of extensions, and one has $\hbar = 6\hbar_0$. One would have $n = n_1 n_2 \dots n_{gr}$. G_{gr} "at the bottom" is proposed to represent number theoretically geometric information about the imbedding space by providing a discretization for the product of maximal finite discrete sub-group of isometries and tangent space rotations of imbedding space. By $M^8 - H$ duality these sub-groups should be identical for H and M^8 . The prediction is that maximal G_{gr} is product of icosahedral group I with 3 copies of coverings \bar{I} . Rather remarkably, the prediction for G is correct if one assumes that the value of R is what p-adic mass calculation for electron mass gives.

Since the hierarchy of Planck constants relates to number theoretical physics proposed to describe the correlates of cognition, the connection with cognition strongly suggests itself. Icosahedral and tetrahedral geometries occur also in the TGD based model of genetic code in terms of bio-harmony, which suggests that genetic code represents geometric information about imbedding space symmetries. These connections are discussed in detail.

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1 Introduction

Newton's constant G cannot be a fundamental constant in TGD framework. G has dimensions of length squared divided by Planck constant and CP_2 length R is the only fundamental length in TGD Universe. The analog of Newton's constant $G = R^2/\hbar$ is too larger by factor of order $10^7 - 10^8$: the estimate of [K2] gives this factor the value $2^{24} = 16,777,216 = 1.6777216 \times 10^7$.

The first guess was that one must modify the formula by replacing \hbar with h_{eff} with $h_{eff} = nh_0$, $h = 6h_0$: $G = R^2/h_{eff}$ (see [L5, L13, L19]).

n has however has arbitrarily values and the proposal cannot be correct as such if one accepts the notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0 = h_{eff} = nh_0$: here M and m are masses of systems having gravitational interaction and $v_0 < c$ is velocity parameter having value $v_0/c \simeq 2^{-11}$ for inner planets [E1] [K7, K5, K6]. \hbar_{gr} is assigned with the flux tubes mediating gravitational interaction.

One can assign also to other interactions corresponding effective Planck constants - for instance, $h_{em} = Z_1 Z_2 e^2/\beta_0$, $\beta = v_0/c < 1$ to electromagnetic interactions. The general idea is that when the value of coupling strength $Q_1 Q_2 g^2/\hbar$ for a two particle system becomes so large that perturbation theory fails, Planck constant is replaced with h_{eff} and perturbation theory works again. Topologically this means a phase transition replacing space-time sheets with their n -fold coverings.

1.1 More general formula for G

The more general proposal is that h_{eff} in the formula for G must be replaced with $\hbar_{gr,1} = n_{gr} h_0$, where n_{gr} is closely related to the $n = h_{eff}/h_0$ but not equal to it. The estimate $G\hbar/R^2 \simeq 1.6 \times 10^7$ and $\hbar = 6\hbar_0$ gives the estimate $n_{gr} = 6 \times 2^{24} \simeq 1.00663296 \times 10^8$.

To make the continuation easier, it is good to express the idea in more detail.

1. CP_2 "radius" R identified in terms of geodesic length $l = 2\pi R$ is the fundamental geometrically realized unit of length measurement, and takes the role of Planck length $l_P^2 = G\hbar$ having only dimensional analytic justification. G is now the prediction and the first guess is $G = R^2/\hbar_{gr,1}$, where $\hbar_{gr,1} = n_{gr} \times \hbar_0$ is effective Planck constant with n_{gr} identified as dimension of "gravitational" extension of rationals.

$n = h_{eff}/h_0$ is the number of sheets of covering of space-time surface transformed to each other by Galois group. Since l_P/R is in the range $10^{-7} - 10^{-8}$, one must have $\hbar_{gr,1}/\hbar$ in the range $10^7 - 10^8$.

2. In principle $n_{gr}/6$ could have values $10^{-7} - 10^{-8}$ times smaller than the value associated with G . If so, G could be up to factor $10^7 - 10^8$ times larger than the standard value $G = G_N$. The

downwards fluctuations of \hbar_{eff} strengthen the gravitational attraction. One cannot exclude even large fluctuations of G .

1.2 The first attempt to identify n_{gr} fails

The motivation for this article came from an attempt to understand the value of gravitational constant G as a prediction of TGD - I have already earlier developed a model in which gravitational constant is predicted in terms of CP_2 radius R and a number related to effective Planck constant $\hbar_{eff} = n\hbar_0$ [K2].

1. The first proposal was that one can write $\hbar_{eff}/\hbar_0 = n$ as $n = n_1 \times n_2$, where n_1 is the number of sheets of space-time surface as a covering M^4 (space-time points with same CP_2 coordinates) and n_2 is the number of sheets as covering CP_2 (space-time points with same M^4 coordinates). There would be n_1 different space-time sheets for given M^4 projection - this corresponds to the idea about many-sheeted space-time. There would be n_2 different regions of space-time for given region of CP_2 projection. One can imagine n_2 parallel flux tubes in M^4 forming a coherent structure. This intuitive picture could but need not survive in more precise formulation.
2. The improved formula would be $G = R^2/n_{gr}\hbar_0$, where one as either *a*): $n_{gr} = n_1$ or *b*): $n_{gr} = n_2$. Which option - if either of them - is the correct one? Note that the option $n_{gr} = n$ is not possible since b can have huge values and G would approach to zero for dark matter in long length scales: with the recent understanding of physics this does not look plausible.

The limit $n_{gr} < n_{max} \sim 10^8$ means a bound on the number of space-time sheets over M^4 or CP_2 .

1. For option *a*) with $n_{gr} = n_1 < n_{max} \sim 10^8$ one can imagine that the Galois group corresponds to a discrete finite sub-group of $SU(3)$, analogous to the isometry groups of Platonic solids. In the case of $SO(3)$ the order of this group is bounded to the order 60 of isosahedral group unless the group is Abelian. The largest discrete sub-group of $SU(3)$ analogous to icosahedral group has order 1080 and is too small by several orders of magnitude.

Remark: The number parallel flux tubes could be arbitrarily large for this option - a possible interpretation would be that gravitational quantum coherence is true in very long length scales.

2. For option *b*) with $n_{gr} = n_2 < n_{max}$ would state that the number of parallel flux tubes forming a coherent structure is bounded. The number of space-time sheets over M^4 could be arbitrarily large. The only natural symmetry group for M^4 is discrete sub-group of $SO(3)$. For the icosahedral group the order is 60 and quite too small.

Both options fail.

1.3 A modified formula for G

The failure forces to consider a more general formula for G . The outcome is the following argument.

1. TGD predicts a hierarchy of effective Planck constants $\hbar_{eff}/\hbar_0 = n$, where n is the order of Galois group of Galois extension defining extension of rationals [L10, L11] [K3, K6]. Dimension n of extension factorizes to a product $n = n_1 n_2 \dots$ for extension E_1 of extension E_2 of rationals. $M^8 - H$ correspondence allows to associate the Galois group with an irreducible polynomial characterizing space-time surface as an algebraic surface in M^8 . The gradual increase of extension by forming a functional composite of a new polynomial with the already existing one ($P \rightarrow P_{new} \circ P$) would be analogous to the evolution of genome: earlier extensions would be analogous to conserved genes.
2. The proposal modifying the earlier proposal is $G = R^2/n_{gr}\hbar_0$, where n_{gr} is the order of Galois group G_{gr} "at the bottom" of the hierarchy of extensions, and one has $\hbar = 6\hbar_0$. One would have $n = n_1 n_2 \dots n_{gr}$. G_{gr} "at the bottom" is proposed to represent number

theoretically geometric information about the imbedding space by providing a discretization for the product of maximal finite discrete sub-group of isometries and tangent space rotations of imbedding space.

3. By $M^8 - H$ duality these sub-groups should be identical for H and M^8 . The prediction is that maximal G_{gr} is product of icosahedral group I with 3 copies of coverings \bar{I} . Rather remarkably, the prediction for G is correct if one assumes that the value of R is what p-adic mass calculation for electron mass gives.

Since the hierarchy of Planck constants relates to number theoretical physics proposed to describe the correlates of cognition, the connection with cognition strongly suggests itself. Icosahedral and tetrahedral geometries occur also in the TGD based model of genetic code in terms of bio-harmony [L1], which suggests that genetic code represents geometric information about imbedding space symmetries. These connections are discussed in detail.

2 A formula for G in terms of order of gravitational Galois group and implications

In the sequel the formula $G = R^2/n_{gr}\hbar_0$ will be deduced from number theoretical vision based on adelic physics [L10, L11] and $M^8 - H$ duality [L7, L8, L9, L17]. The prediction allows variation of G - G is indeed known to vary more than expected [K2]. These “small” variations and also possible large variations are discussed (there is a detailed discussion also in Appendix). The successful prediction forces to consider seriously the connections between quantum gravitation, cognition, and quantum biology, in particular genetic code.

2.1 An improved attempt to identify n_{gr}

The original proposal for a formula of $G = R^2/n_{gr}\hbar_0$ failed and one must try something more general.

1. The Galois group of Galois extension has a decomposition in terms of a hierarchy of normal sub-groups. G can be represented as product of maximal normal sub-group H and group G/H . H in turn has similar decomposition and the process can be continued to get a hierarchical decomposition. This leads to a concrete model for “small” state function reduction (SSFR) as a cascade of cognitive measurements. Some special normal sub-group in the hierarchy relevant for gravitation is a good candidate for “gravitational” Galois group G_{gr} , whose order is n_{gr} .

An attractive assumption is that the Galois group assignable to gravitational interactions is fundamental in the sense that it corresponds to the lowest step of the Galois ladder. The vision about evolution inspired by $M^8 - H$ duality is as an increasing hierarchy of polynomials P with rational coefficients defining space-time surfaces as algebraic surfaces in complexified M^8 : their real projections would define 4-D space-time surfaces mapped to $H = M^4 \times CP_2$ by $M^8 - H$ duality [L7, L8, L9].

Polynomials P would be functional composites as a generalization of abstraction process as statements about statements and evolution would proceed as sequence of abstraction steps $P \rightarrow P_{new} \circ P$. This step preserve the roots of P if the polynomials involved vanish at origin: $P(0) = 0$. Besides Galois groups the roots associated with earlier steps would be evolutionary invariants analogous to conserved genes. If one has decomposition $P = P_1 \circ P_2 \dots \circ P_{gr}$, one could understand why n_{gr} is almost universal constant.

2. Gravitation relates to space-time geometry and a good guess is that G_{gr} provides a representation for a discrete finite sub-group of the isometry group of imbedding space and perhaps also for the sub-group $SO(3) \subset SO(3,1)$ acting in $E^3 \subset M^4$ or its lift to $SU(2)$. Octonionic structure in M^8 indeed selects unique rest system and even the spatial origin of the linear coordinate system is fixed. This would reduce the attempt to identify n_{gr} to a study of finite discrete sub-groups of imbedding space isometries and spin covering of its tribein rotation group.

It must be made clear that G_{gr} would be associated with the space-time sheets mediating gravitational interactions: this would include gravitational flux tubes with $\hbar_{gr} = GMm/v_0$. For flux tubes mediating - say - electromagnetic interaction the counterpart of G_{gr} could be much smaller, it would however include the group $Z_2 \times Z_3$, which is center for $SU(2) \times SU(3)$ predicting $h = 6h_0$ suggested by some empirical findings [L5, L13, L19].

3. By $M^8 - H$ duality one must consider the isometry groups and 3-D tangent space-groups of both M^8 and H to see whether n_{gr} could find a natural identification. $M^8 - H$ duality requires that the gravitational sub-group is same for M^8 and H options.

The group $SO(3) \times U(2)$ is shared by $SO(3) \times SU(3)$ for H and $SO(3) \times SO(4)$. Tangent space group of $E^3 \subset M^4$ is $SU(2)$ and tangent space group of CP_2 is $U(2)$ in the two cases and if only maximally non-Abelian groups are accepted $U(2)$ effectively reduces to $SU(2)$, which can however correspond to a non-trivial sub-group of $U(2)$. This would mean that the maximal finite discrete sub-group of isometries and vielbein groups is direct product of 4 groups, which are icosahedral groups I or their coverings \bar{I} .

The orders of icosahedral group I without reflection *resp.* its covering \bar{I} is 60 *resp.* 120. $SU(2)$ for the tangent space groups is natural hypothesis since one has also fermions. For $I \times \bar{I}^3$ one would have $n_{gr} = 8 \times 6^4 \times 10^4 \simeq 32^{11} \times 15^4 = 1.0368 \times 10^8$. This is to be compared with the rough estimate $n_{gr} = 6 \times 2^{24} \simeq 1.00663296 \times 10^8$. The proposal works amazingly well!

4. Also other Platonic groups assignable to Platonic solids (tetrahedron, cube and octahedron, icosahedron and dodecahedron) are in principle possible: actually all discrete and finite sub-groups of $SU(2)$ can be considered. The non-Platonic groups however act on plane polygons, and might be more naturally assignable other than gravitational interactions. They are also associated with Mac-Kay correspondence [K9] assigning to these finite groups ADE Lie groups/Kac-Moody algebras. This hierarchy is also associated with inclusions of hyperfinite factors of type II_1 (HFFs) proposed in TGD framework to provide a representation for finite measurement resolution [K10, K4].

2.2 Could Newton's constant vary and what about formulas for other coupling strengths?

The proposed formula for G forces to consider the possibility of large variations of G due to the variation of n_{gr} as the order of gravitational Galois group G_{gr} . This group is fixed by $M^8 - H$ duality to be a product of finite discrete sub-groups of $SO(3)$ with 3 discrete sub-groups of $SU(2)$. By loosening the conditions one can however think also the possibility of other choices

1. The allowance of only Platonic solids would make possible to understand possible large increases of G but not its reduction. What is interesting is that the increase of G implies increase of gravitational Compton length $\Lambda_{gr} = G(M + m)/v_0$ unless G/v_0 is constant.
2. If one accepts also the non-Platonic finite sub-groups of $SU(2)$ with representations are realized as 2-D polygons, the range of values of G is much larger and both large and small variations of G from the preferred value become possible as variations of n_{gr} .
3. If one wants to explain the reported small but theoretically too large variations of G allowing only Platonic solids, one must allow superpositions of space-time surfaces with different values of n_{gr} . In general $\langle n_{gr} \rangle$ would be smaller than the maximal value and $\langle G \rangle$ would increase. Large variations decreasing G cannot be explained in terms of Platonic solids or their superpositions.
4. If one gives up $M^8 - H$ duality larger variations of G downwards become possible. For instance, \bar{I} in the case of $SU(3)$ isometries could be replaced with $\Sigma(1080)$ with 1080 elements (<http://tinyurl.com/uq3nxxko>). This would reduce G by factor $1080/120 = 9$. More generally, in $SU(3)$ there are following analogs of Platonic groups labelled as $\Sigma(n)$, $n \in \{60, 168, 36 \times 3, 72 \times 3, 21 \times 3, 72 \times 3 = 216, 216 \times 3 = 648, 360 \times 3 = 1080\}$. Also the semi-direct products $\Sigma(60) \times Z_3$ and $\Sigma(168) \times Z_3$ belong to the list.

The counterpart of ADE hierarchy for $SU(3)$ is obvious interest from the point of view of color interactions if one allows the breaking of $M8 - H$ duality. There is an article by Ludl in arXiv [B1] (<http://tinyurl.com/uq3nrxko>) about the finite discrete sub-groups of $SU(3)$. Table 1 of the article provides a summary of the discrete sub-groups.

1. There are 3 series parameterized by several integers with no general formula for the order. There are however infinite series of groups which belong to these series and have unbounded order. These groups are semi-direct products, which makes their representability as Galois groups of Galois extensions possible.

Could these groups be associated with the flux tubes mediating color interactions? Could colour coupling strength be expressible as $\alpha_s = g_s^2/4\pi\hbar_s$, where $\hbar_s = n_s\hbar_0$? Could the value of g_s^2 be equal to the square g_K^2 of Kähler coupling defining fundamental constant. Could similar expression hold true also for electroweak coupling strengths. Could the breaking of gauge and gravitational symmetries be coded by different values of $n_s, n_{SU(2)_{ew}}, n_{U(1)}$, and n_{gr} .

2. There are also following exceptional groups analogous to Platonic groups for $SU(3)$ and labelled as $\Sigma(n)$, $n \in \{60, 168, 36 \times 3, 72 \times 3, 21 \times 3, 72 \times 3 = 216, 216 \times 3 = 648, 360 \times 3 = 1080\}$. Also the semi-direct products $\Sigma(60) \times Z_3$ and $\Sigma(168) \times Z_3$ belong to the list. The largest order for this series is 1080. The smallest order is 60 and corresponds to icosahedral group.

The discrete sub-groups of $SO(4)$ are interesting in M^8 picture and could contain also semi-direct products as sub-groups for products of sub-groups of $SO(3)$ and $SU(2)$. These sub-groups are listed in the appendix of the article by de Medeiros and Figueroa-O'Farrill (<http://tinyurl.com/tyagn3c>).

2.3 What do experiments say?

What do experiments say? Various experiments are discussed in [K2].

1. Several experiments suggests small variations of G , which are however too large theoretically. There are experiments in millimeter scales and also Podkletnov's experiment [H2, H1] [L2].
2. Could the fountain effect of super-fluidity be understood as a large reduction the value of G . It seems that a more elegant explanation is in terms of macroscopic quantum coherence due to the large value of $h_{gr} = GMm/v_0$ for space-time sheets mediating gravitation in the case of super-fluid [K3] [K2].
3. The findings reported by Martin Grusenick [K8] - *if true* - would suggests a huge increase of G by a factor of order 10^5 if the increase of spatial lengths in the direction of the Earth's magnetic field causes the effect. The variation is too large to have an explanation allowing only Platonic solids alone. The effect could be due to the contraction of the measurement apparatus under its own weight.

Perhaps a more elegant explanation for Grusenick's claim would be in terms of warping of space-time surface possible even in absence of gravitational field predicted by TGD. Warping means that the space-time surface has metric isometric with Minkowski metric but when the M^4 coordinates of $M^4 \subset M^4 \times CP_2$ are used, there is a scaling of the metric in various directions since CP_2 projection of the imbedding is not a point but geodesic circle. This would modify the propagation velocity in radial direction.

4. One can also ask whether the unexpected mass for the blackhole candidates observed by LIGO could be due to anomalously large value of G . In TGD framework the view about blackhole like entities is much more detailed than in GRT and one could understand them also without variation of G .

Since consciousness, cognition, and gravitation are closely related in TGD Universe, one cannot avoid association with the claims made by meditators about levitation. Could the experience about levitation mean a genuine levitation of dark matter at the level of magnetic body (MB), which corresponds to a higher level cognitive consciousness and naturally gravitational consciousness by huge values of h_{gr} .

1. Could G be reduced producing anti-gravitational effect at MB? If one allows only $M^8 - H$ duality and Platonic solids G is smallest possible and cannot be reduced. Allowing also polygons would allow arbitrary small values of G . This option does not look however plausible since one can argue that the experience would reduce from 3-D for Platonic solids to 2-D for regular polygons.
2. Perhaps a more elegant explanation is that levitation experiences and out-of-body experiences [K1] (OBEs, which I have had also myself), are due to the delocalization of particles of “personal” MB due to the large value of h_{gr} . One could perhaps say that the active flux tubes of MB correspond to those mediating gravitational interaction and having $h_{gr} = GMm/v_0$. Ironically, gravitational consciousness would be experience of no having no weight.

2.4 A connection gravitation and genetic code?

A deep connection between gravitation and genetic code suggests itself.

1. TGD suggests at least two fundamental representations of genetic code besides the usual chemical representation. The first representation is in terms of dark nuclei consisting of sequences dark proton triplets representing codons [L3, L12]. Both DNA, RNA, tRNA and amino-acids have analogs as dark proton sequences. Second representation is in terms of dark photon triplets defining what I call bio-harmony [L1]. Basic objection against emission of 3-dark photons simultaneously is that the process is extremely probable. If one has however Galois confinement in the sense that only Galois singlets appear as asymptotic states, the assumption that dark photons are Z_3 triplets allows only the emission of triplets [L19].
2. What is fascinating that both icosahedral and tetrahedral groups appear in the model for the genetic code in terms of bio-harmony [L1, L15, L16]. Could genes and associated molecules DNA, RNA, tRNA, and amino-acids code for information about the geometry of imbedding space in some sense? DNA codons correspond to 20 triangular faces of icosahedron (3 Hamiltonian cycles are used obtain $20+20+20=60$ codons) and 4 triangular faces of tetrahedron to get the remaining 4 codons. By icosahedral-dodecahedral duality gene as a sequence of these faces defines a path at dodecahedron - two subsequent codon of gene would not however map to nearest points at dodecahedron. What could this mean if anything?
3. Genes code for information and therefore could relate to cognition, and the proposed representations of genetic code would mean that genes emerge already at the fundamental level: chemical representation would be only mimicry of the dark nuclear code at higher, chemical level. The hierarchy of Planck constants relates also directly to information and h_{eff} can be seen as a kind of “IQ”.

The dependence of G on n_{gr} suggest that also gravitation relates to cognition. This would not be surprising since the long-ranged non-screened character of gravitation could make possible quantum coherence in astrophysical scales: the value $h_{eff}/h_0 = n = h_{gr}/h_0$ is indeed a direct measure of the evolutionary level.

The connection with cognition could also explain why ancient mathematicians managed to discover the mathematical structures encountered two millenia later in theories trying to unify fundamental interactions.

2.5 Could Newton’s constant relate to cognition?

After having discovered the above argument fixing n_{gr} from $M-H$ duality, I could have written conclusions of the paper. The emphasis however shifted to TGD based view about evolution and cognition and its connection with gravitation. h_{eff} indeed closely relates to an evolutionary hierarchy of cognition via the idea that gravitational/geometric part of Galois group is fundamental and “at the bottom” of the hierarchy of Galois extensions of rationals. Extensions of rationals would define cognitive representations representing discretizations of spaces of various dimensions as subsets of reals or complex numbers, and also allow to represent discrete sub-groups approximating continuous groups as Galois groups.

The most fundamental physics related groups to be approximated as Galois groups would relate to the isometries and vielbein rotations of imbedding space. The maximally compact sub-group would be in question both cases. The important point would be that these groups would act on extension of rationals providing cognitive representation as subset of reals/complex numbers rather than in imbedding space. This kind of representation would be analogous to a linguistic, linear representation of geometric object as opposed to concrete geometric representation in imbedding space.

3 Could gravitation and geometric cognition relate?

It has been already demonstrated how one can predict the value G correctly as in TGD framework. The emphasis of this section is on geometric cognition and the possibility that the value of G directly reflects this connection.

3.1 Hierarchy of effective Planck constants and Galois extensions of rationals

In adelic vision [L10, L11, L7, L8, L8] about TGD $n = h_{eff}/h_0$ corresponds to the dimension of extension of rationals characterizing space-time surface. n is also the order of Galois group of extension for Galois extensions. Recall that Galois extension has the nice property that the order of Galois group equals to the dimension of the extension. Galois extension can be regarded as extension of extension of...rationals and there is a hierarchy of Galois group such that the included sub-groups are normal sub-groups. One can express n as a product $n = n_1 n_2 \dots$ of the dimensions of these extensions.

This leads to the vision about the reduction of evolution to a hierarchy of Galois extensions such that evolution means increase of the extension and therefore number theoretical complexity and of h_{eff} meaning increase of quantum coherence scale.

If the extensions tend to emerge as further extensions preserving the earlier extensions - as is natural to think -, the extension "at the bottom" of the hierarchy of extensions is rather stable. Since the geometric cognitive consciousness can be argued to be fundamental, the dimension of Galois group corresponds to n_{gr} in $n = n_1 n_2 \dots n_{gr} = m \times n_{gr}$. n_{gr} would be rather stable factor of n .

n_{gr} would be analogous to the conserved genes of primary life form from which evolution started. The change of genome at this level would induce dramatic changes making the survival of the new life form implausible. This alone would not predict unique value for n_{gr} but only that its value is dynamically rather stable. One must of course understand why this particular value of n_{gr} would be selected. What distinguishes this extension from a general extension? The groups in question allow infinite number of finite discrete sub-groups but $M^8 - H$ duality would select highly unique sub-groups as common to both. Only groups, which are products of 4 isometry groups of Platonic solids or their double coverings and maximal order for the group minimizing G would leave only the icosahedral group I and its coverings into consideration.

3.2 $M^8 - H$ duality and representation of space-time surfaces in M^8 as algebraic surfaces assignable to polynomials with rational coefficients

$M^8 - H$ duality [L17] provides a concrete realization of the number theoretic vision in terms of space-time surfaces, and also allows to realize the view about number theoretical evolution in terms of a hierarchy of polynomials obtained by functional composition of polynomials.

The articles [L7, L8, L9] contain a detailed description of $M^8 - H$ duality. The article [L18] described a possible connection with chaos theory and Mandelbrot/Julia fractals based on the possibility that time evolution by "small" state function reductions (SSFRs) correspond in good approximation iteration of polynomial. The article [L21] describes a model of SSFR as a cognitive measurement identified as a reduction cascade in the group algebra of Galois group having a decomposition in terms of normal sub-groups.

3.2.1 Basic vision

Consider first what TGD space-time is.

1. In TGD framework space-times can be regarded 4-surfaces in $H = M^4 \times CP_2$ or in complexification of octonionic M^8 . Linear Minkowski coordinates or Robertson-Walker coordinates for light-cone (used in TGD based cosmology) provide highly unique coordinate choice and this problem disappears.
2. The solutions of field equations are preferred extremals satisfying extremely powerful additional conditions giving rise to a huge generalization of the ordinary 2-D conformal symmetry to 4-D context. In fact, twistor twist of TGD predicts that one has minimal surfaces, which are also extremals of 4-D Kähler action apart from 2-D singularities identifiable as string world sheets and partonic 2-surfaces having a number theoretical interpretation. The huge symmetries act as maximal isometry group of “world of classical worlds” (WCW) consisting of preferred extremals connecting pair of 3-surfaces, whose members are located at boundaries of causal diamond (CD). These symmetries strongly suggest that TGD represents completely integrable system and thus non-chaotic and diametrical opposite of a chaotic system. Therefore the chaos - if present - would be something different.

$M^8 - H$ duality suggests an analogous picture at the level of M^8 . $M^8 - H$ duality in its most restrictive form states that space-time surfaces are characterized by “roots” of rational polynomials extended to complexified octonionic ones by replacing the real coordinate by octonionic coordinate o [L7, L8, L9].

1. One can define the imaginary and real parts $IM(P)$ and $RE(P)$ of $P(o)$ in octonionic sense by using the decomposition of octonions $o = q_1 + I_4 q_2$ to two quaternions so that $IM(P)$ and $RE(P)$ are quaternion valued. For 4-D space-time surfaces one has either $IM(P) = 0$ or $RE(P) = 0$ in the generic case. The curve defined by the vanishing of imaginary or real part of complex function serves as the analog.
2. If the condition $P(0) = 0$ is satisfied, the boundary of δM_+^8 of M^8 light-cone is special. By the light-likeness of δM_+^8 points the polynomial $P(o)$ at δM_+^8 reduces to ordinary real polynomial $P(r)$ of the radial M^4 coordinate r identifiable as linear M^4 time coordinate t : $r = t$.

Octonionic roots $P(o) = 0$ at M^8 light-cone reduce to roots $t = r_n$ of the real polynomial $P(r)$ and give rise to 6-D exceptional solutions with $IM(P) = RE(P) = 0$ vanish. The solutions are located to δM_+^8 and have topology of 6-sphere S^6 having 3-balls B^3 with $t = r_n$ as of M_+^4 projections. The “fiber” at point of B^3 with radial M^4 coordinate $r_M \leq r_n$ is 3-sphere $S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ contracting to point at the δM_+^4 .

These 6-D objects are analogous to 5-branes in string theory and define “special moments in the life of self”. At these surfaces the 4-D “roots” for $IM(P)$ or $RE(P)$ intersect and intersection is 2-D partonic surface having interpretation as a generalization of vertex for particles generalized to 3-D surfaces (instead of strings). In string theory string world sheets have boundaries at branes. Strings are replaced with space-time surfaces and branes with “special moments in the life of self”.

Quite generally, one can consider gluing 4-D “roots” for different polynomials P_1 and P_2 at surface $t = r_n$ when r_n is common root. For instance, P and its iterates $P^{\circ N}$ having r_n and the lower inverse iterates as common roots can be glued in this manner.

3. It is possible complexify M^8 and thus also r . Complexification is natural since the roots of P are in general complex. Also 4- space-time surface is complexified to 8-D surface and real space-time surface can be identified as its real projection.

To sum up, space-time surfaces would be coded a polynomial with rational or at most algebraic coefficients. Essentially the discrete data provided by the roots r_n of P would dictate the space-time surface so that one would have extremely powerful form of holography.

3.2.2 Should one allow also transcendental extensions?

One can consider generalizations of the simplest picture.

1. One can also consider a generalization of polynomials to general analytic functions F of octonions obtained as octonionic continuation of a real function with rational Taylor coefficients: the identification of space-time surfaces as “roots” of $IM(F)$ or $RE(F)$ makes sense.
2. What is intriguing that for space-time surfaces for which $IM(F_1) = 0$ and $IM(F_2) = 0$, one has $IM(F_1F_2) = RE(F_1)IM(F_2) + IM(F_1)RE(F_2) = 0$. One can multiply space-time surfaces by multiplying the polynomials. Multiplication is possible also when one has $RE(F_1) = 0$ and $IM(F_2) = 0$ or $RE(F_2) = 0$ or $IM(F_1) = 0$ since one has $RE(F_1F_2) = RE(F_1)RE(F_2) - IM(F_1)IM(F_2) = 0$.

For $IM(F) = 0$ type space-time surfaces one can even define polynomials analytic functions of the space-time surface with rational Taylor coefficients. One could speak of functions having space-time surface as argument, space-time surface itself would behave like number.

3. One can also form functional composites $P \circ Q$ (also for analytic functions with complex coefficients). Since $P \circ Q$ at $IM(Q) = 0$ surface is quaternionic, its image by P is quaternionic and satisfies $IM(P \circ Q) = 0$ so that one obtains a new solution. One can iterate space-time surfaces defined by $Im(P) = 0$ condition by iterating these polynomials to give $P, P^{circ2}, \dots, P^{\circ N} \dots$. From $IM(P) = 0$ solutions one obtains a solutions with $RE(Q) = 0$ by multiplying the M^8 coordinates with I_4 appearing in $o = q_1 + I_4q_2$.

The $Im(P) = 0$ solutions can be iterated to give $P \rightarrow P \circ P \rightarrow \dots$, which suggests that the sequence of SSFRs could at least approximately correspond to the dynamics of iterations and generalizations of Mandelbrot and Julia sets and other complex fractals and also their space-time counterparts. Chaos (or rather, complexity theory) including also these fractals could be naturally part of TGD!

3.3 Evolution of cognition

Polynomials in M^8 obtained as continuation of real polynomials with rational (or perhaps even algebraic) coefficients and vanishing at origin define a concrete representation for the extensions of rationals. There is infinite number of polynomials realizing the same extension. The interpretation is as an evolutionary hierarchy.

Since the number of extensions larger than given extension is larger than those smaller than it, the sequence of BSFRs changing the extension leads unavoidably to evolution as a statistical increase of the dimension of extension. The functional composition of polynomials which vanish at origin gives rise to evolutionary hierarchies for which the number theoretical complexity increases as one climbs up in the hierarchy. Extensions in these hierarchies are analogous to conserved genes if the replacement of extension F in BSFR can only extend F to larger extension E . This might be true in statistical sense.

Extensions could increase statistically also in SSFRs. In [L18] I considered the possibility that the sequence of SSFRs could correspond in reasonable approximation to an iteration of polynomial P . This would give direct connection with the Mandelbrot and Julia fractals.

The basic question is whether the number theoretical vision based on M^8 and adelic physics could be seen as exact dual of the geometric vision based on $H = M^4 \times CP_2$ and the notion of “WCW” (WCW) or does number theoretical view describe cognitive representations as approximate mimicry of actual physics so that the duality would be many-to-1.

The latter option seems to more plausible. Evolution leads to an improved representations but 1-1 correspondence is not reached even at the level of algebraic numbers allowing cognitive representations dense at space-time surface, but might be reached by accepting transcendental extensions replacing polynomials with analytic functions with rational (or even algebraic) coefficients to guarantee the continuation to p-adic number fields. One argument in favor of transcendentals is that exponential functions and trigonometric functions should be possible. Exponential functions would force e which however defines finite-D extension of p-adic numbers. The roots of trigonometric functions would bring in π and its powers.

3.3.1 General ideas about cognition and cognitive representations

Consider first cognitive representations at space-time level.

1. Cognitive representations at the level space-time surfaces would be provided by the points of space-time surface with imbedding space coordinates in extension of rationals considered. One the coordinates of imbedding space are fixed, these discretization are unique. The selection of coordinates is in the octonionic case highly unique. Only time translation in the rest system defined by the linear octonion coordinates is allowed. Also in case H the coordinates are unique apart from color rotations. Also vielbein/spin rotation group of 3-surface could have representation as a Galois group.
2. Galois group would act on the cognitive representation at space-time level and in general would not leave it invariant so that one would obtain new space-time surface. The wave functions in the space of space-time surfaces would correspond to wave functions in the space of cognitive representations which would correspond to elements of Galois group or factor space if sub-group of Galois group leaves the representation invariant. Wave functions would be elements of the group algebra of Galois group with possible conditions corresponding of invariance with respect to sub-group restricting the function to coset space effectively. This picture leads to a vision about “small” state function reductions (SSFRs) as cascades of measurements leading to a tensor product of states in the hierarchy of normal sub-groups of Galois group [L21]. The interpretation would be as cognitive measurements.
3. What about fermions? Fermionic Fock states have in TGD framework interpretation in terms of quantum variant Boolean algebra realized in terms of multi-qubits. One can say that the spinor structure of space is kind of square root of metric and describes correlates of logic [L20]. This would apply even at the level of WCW.

What could finite measurement and cognitive resolution for fermions mean? The natural hypothesis is that the group algebras of Galois groups generated by wave functions in Galois group and having dimension n equal that for extension of rationals describe bosonic degrees of freedom and that fermionic state correspond to the spinors in this algebra- possible restrictions come from chirality restrictions. The dimension of the spinor space would be at most 2^n .

Cognitive representations at space-time level would be rather concrete. But is it possible to realize mathematical imagination, is it possible to imagine higher-D spaces?

1. Cognitive representations would indeed occur already at the level of number system. The extension of rationals can be regarded as n-D space over rationals instead of reals and would be mapped to a dense subset of real variant of n-D space. One can say that subset of real (or complex) numbers represents cognitively the higher-D space. The Galois group would represent discretization for the symmetries of these n-D space and from this one can say something about the possible isometry group of the corresponding real or complex space.

This ability to imagine real and complex spaces of arbitrary dimension and might be fundamental aspect of mathematical consciousness.

2. If one takes seriously the idea about the connection with Newton’s constant G , one can ask whether the evolution of the mathematical cognition proceeded via the gradual increase of the order of G_{gr} and meant gradual reduction of G in rather dramatic steps if only Platonic groups are allowed.

Remark: Nottale’s proposal for h_{gr} implies that gravitational Compton length for two particle system is $G(M + m)/v_0$ and increase with G since h_{gr} increases. If the velocity parameter v_0 and G do not correlate, larger value of G and therefore smaller value of n_{gr} and lower level of space-time consciousness would mean longer gravitational Compton length as a measure for quantum coherence and higher level of consciousness. This looks somewhat strange. Should one conclude that v_0 and G correlate: for instance, could G/v_0 be independent of G_{gr} ?

How could mathematical physics as correlation between cognitive/imagined and sensory worlds have emerged?

1. Somehow the idea that we live in Euclidian 3-space emerged and later emerged special relativity, general relativity and its followers. It seems essential that the cognitive representations at the level of number field found counterparts at the level of sensory world represented as 3-space and eventually space-time and imbedding space.

Quaternions and octonions are naturally assignable to M^8 , M^4 and H . Quaternions have $SO(3)$ as the analog of Galois group with concrete geometric interpretation. The discovery would be that this group acts on the object of sensory world. Could it be that these two equivalent choices of imbedding space are the only ones for which this consciousness about this sensory-cognitive correspondence can evolve? The essential point would be that the symmetry groups of physics would be sub-groups of automorphism groups for octonions and quaternions.

Remark: The extension allowing discrete sub-group of $SO(3)$ as Galois group must be distinguished from much smaller extension needed to represent this sub-group as 3×3 orthogonal matrices.

2. Could the emergence of the idea of Platonic solids - say in mathematics of ancient Greece - correspond to a step in evolution in which this sensory-cognitive correspondence emerged. Cognitive and sensory started to resonate, as one might say.

3.3.2 Could Galois groups provide a representation for the discrete sub-groups of isometries and tangent space rotations of imbedding space?

I have already earlier considered the possibility that Galois groups could provide representations for the finite sub-groups of isometry groups of $H = M^4 \times CP_2$ and $M^8 = M^4 \times E^4 = M^2 \times E^2 \times E^4$, see for instance [L21].

1. A natural looking assumption is that only finite discrete sub-groups having a hierarchical decomposition in terms of normal sub-groups characterizing Galois extensions and having thus order equal to dimension of extension would be allowed.

In case of sub-groups of the rotation group, one can of course consider also sub-group generated as products of discrete sub-groups but they have infinite number of elements, which does not conform with the idea about finiteness of cognition. For instance, one can take Platonic groups and groups C_n and D_{2n} such that there rotation axis does not go through a point of Platonic solid and generate the product group. This group would have the product of Galois groups as Galois group. One could think that also these are allowed if one has finite measurement resolution and cognitive resolution. This brings in the notion of approximation, which might have emerged in cognitive evolution too.

2. In terms of polynomials defining the space-time surface in M^8 as algebraic surface, one would have $P = P_1 \circ \dots \circ P_N \circ P_{gr}$. The Galois group associated with gravitational polynomial P_{gr} of degree n_{gr} would be normal sub-group of the entire Galois group and the Galois group of $P_1 \circ \dots \circ P_N$ would be factor group. This polynomial would correspond to higher evolutionary level and perhaps consciousness not directly related to imbedding space geometry.

G_{gr} would be sub-group of imbedding space isometries and vielbein rotations and therefore have the characteristic decomposition to a direct product. Direct product decomposition could be replaced with sub-direct product decomposition for sub-groups of direct product. Product- or semi-direct product decomposition would correspond to that assumed for the original proposal and interpreted in terms of many-sheetedness over M^4 resp. CP_2 (flux tube bundles in M^4).

3. $M^8 - H$ duality forces the identification of the direct product as four-fold product of discrete sub-groups of $SU(2)$ appearing in McKay correspondence and to the special role of icosahedral group and its covering. As found in the introduction, the condition that the Gal_{gr} is discrete finite sub-group of product of M^8 and H isometries leads to a unique identification for this

group as $I \times \bar{I} \times \bar{I} \times \bar{I}$, where I is icosahedral group and \bar{I} its covering, and predicts correctly the value of G .

The assumption that the product of discrete isometry groups of the factors of imbedding space is representable as Galois group of Galois extension representable in terms of a polynomial can be criticized. Can the Galois group for Galois extension of rationals defined by irreducible polynomial be a direct product of Galois groups for extensions?

1. The answer to the question can be found from web (<http://tinyurl.com/sj26xrc>): it is found that this is possible for Galois extensions if the product of extensions is the extension and the intersection of extensions consists of rationals. This question is physically highly relevant since Z_6 should have representation as Galois group having interpretation as direct product of centers of $SU(2)$ and $SU(3)$.
2. If this were not the case, one would be in trouble since this would exclude representations of the products $G_1 \times G_2$ of discrete sub-groups associated with isometries H and M^8 as Galois groups. One can of course think of having discrete sub-groups of $G_1 \times G_2$ having a lower order with direct products of sub-groups of G_i excluded. These are possible. $Z_2 \times Z_2$ allows the sub-groups $\{(0, 0), (1, 0)\}$, $\{(0, 0), (0, 1)\}$, and $\{(0, 0), (1, 1)\}$ and these are not products.
3. More generally, one could have a semi-direct product of normal sub-groups of $H_1 \subset G_1$ and $H_2 \subset G_2$ (<http://tinyurl.com/zhx5xpz>). This implies a correlation between the discrete isometries of the factors of imbedding space, which would have physical interpretation. Semi-direct product allows surjective projections to $p_i : G_i \rightarrow H_i$ with normal sub-groups N_i as kernels. The product group $G_1/N_1 \times G_2/N_2$ is the graph of isomorphism $G_1/N_1 \cong G_2/N_2$. This obviously poses strong conditions on the groups. For $G_1 = G_2$ one can would have $N_1 = N_2$. Since Z_2 is always normal sub-group, one would obtain an acceptable group in this manner if both factors have even order, and the order would be reduced by factor 1/4. The orders of the acceptable sub-groups are factors of $ord(G_1) \times ord(G_2)$.

Remark: One should be of course be very cautious in considering the isometry groups. For instance, could the discrete sub-groups automorphism group G_2 of octonions be relevant in M^8 picture? One can also ask whether the finite discrete sub-groups of $SO(7)$ as maximal compact subgroup of $SO(1, 7)$ might be relevant.

3.3.3 Genetic code and geometric consciousness

TGD predict at least two representations of genetic code. The first representation is in terms of dark photon triplets and second representation in terms of dark proton triplets.

TGD based model for genetic code based on bio-harmony realizes genetic code as a code for communications by dark photons. Triplet of dark photons having interpretation as 3-chord of bio-harmony is the basic idea. Icosahedral and tetrahedral geometries connect bio-harmony with geometry [L1, L16].

1. 12-note scale is represented as Hamiltonian cycle at icosahedron having 12 vertices. By assigning to edge of the Hamiltonian cycle quint (scaling of frequency by factor 3/2), the Hamiltonian cycle defines a harmony with 20 3-chords assignable to the triangular faces of the icosahedron. Hamiltonian cycles are characterized by their symmetry group S , which is Z_6 , Z_4 and Z_2 (here one has two variants one depending on whether Z_2 represents reflection or rotation by π) or Z_1 (no symmetry, disharmony). By combining 3 Hamiltonian cycles with symmetries Z_6 , Z_4 , and Z_2 one obtains 60 3-chords.
2. One can assign to given 3-chord DNA codon and the analog amino-acid as the orbit of this chord under the symmetry group of the cycle. One almost obtains vertebrate genetic code with correct number of DNA codons associated with given amino-acid as number of faces at the orbit associated with it. Only 4 amino-acids and 4 DNA codons are missing. Tetrahedral harmony defined by unique Hamilton cycle gives the remaining 4 chords assignable to the triangular faces of tetrahedron. The outcome is vertebrate genetic code.

3. Icosahedron is in a unique position. Icosahedron has 17 Hamiltonian cycles whereas tetrahedron cube and dodecahedron have only 1 and octahedron 2. In case of dodecahedron the Hamiltonian cycle divides the dodecahedron to two identical parts with 6 pentagons suggesting that the the symmetry group is Z_6 and the number of amino-acids is 2.
4. There is large number of bioharmonies obtained by combining unique Z_6 harmony with pairs of Z_4 and Z_2 harmonies. Since music expresses and induces emotions, the identification would be as correlates for fundamental emotion/moods appearing already at molecular level, and perhaps even at deeper levels [L14]. The interpretation of codon as 6-bit would correspond to the standard reductionistic view about information represented as bit sequences. Harmony would code for the holistic aspects of information. These two views would correspond to intelligence in the usual sense and emotional intelligence.

Second representation of genetic code is in terms of dark nuclei consisting of sequences of dark protons triplets [L3, L19]. Codon corresponds to an entangled state of 3 dark protons forming a linear or circular structure with ordering of protons. The dark protons sequences associated with flux tubes parallel to ordinary DNA double strands would provide pairing of dark and ordinary DNA. Also RNA, tRNA, and amino-acids would be represented as dark proton triplets and DNA-amino-acid correspondence has a natural description.

One can raise questions about the interpretation of these two representations of the genetic code (and also about chemical representation).

1. Could genetic code be represented in terms of bio-harmony provide a quantum representation for two Platonic solids: icosahedron and tetrahedron, perhaps their product in $M^4 \times CP_2$. This would answer the question why both icosahedron and tetrahedron. An alternative interpretation is that one has product of isometries and tangent space rotations for M^4 (or CP_2).

Could genes somehow represent concretely information about imbedding space geometry and its symmetries - could one even imagine that genes are kind of statements? Could also dark proton representation have interpretation as a concrete representation in sensory realm.

2. One can raise questions about the bio-harmony. Why just 3 Hamiltonian cycles at icosahedron plus tetrahedral cycle? Could these 4 factors correspond to the 2+2 factors due to the $M^4 \times CP_2$ isometries and tangent space rotations. One would have representation for all these factors. But why one of them would be tetrahedron rather than icosahedron in which case one would have 80 codons? Why the symmetry groups S of Hamiltonian cycles would be Z_6 , Z_4 and Z_2 ?

Remark: Tetrahedral symmetries and orientation preserving octahedral symmetries are sub-groups of icosahedral symmetries (<http://tinyurl.com/vav2n2r>).

3. What about representation of color symmetries of CP_2 Platonic solid in terms of dark codons? Could one assign to dark codon formed by protons a representation in $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ to get colored variants of genetic code. Genes would have vanishing total color. Can one consider representation of color as a subgroup of Galois group. Also more general Galois groups can be considered and genes as units would be defined as Galois singlets [L19].

Could the notion of genetic code generalize to the level of more general Galois groups.

1. Could one consider a generalization of the genetic code to cognitive representations based on Galois group and its coset groups. Restrict first the consideration to any finite discrete subgroup of isometries of H or M^8 . Represent it physically in M^4 or CP_2 as a discrete structure analogous to Platonic solid. Form all Hamiltonian paths in the discretization and identify the n-D basic cells of this n-D structure as basic entities - analogs of DNA codons/chords. Identify the orbits of these entities under symmetry group of the cycle as analogs of amino-acids. Define the analog of genetic code as in the case of ordinary genetic code.
2. Could one imagine cognitive representation of arbitrary Galois group in terms of wave functions in group or its coset space. Could one consider generalization of bio-harmony in terms

of Hamiltonian cycles in this coset space. Could one assign analogs of DNA codons to the faces of the polyhedron and could amino-acids correspond to the orbits of the faces under symmetries of the Hamiltonian cycle? Amino-acid wave functions would be constant at the orbits of the symmetry group of the cycle.

3. The relation to the model of “small” state function reductions (SSFs) [L21] is interesting. SSFs would have an interpretation as cognitive measurements in Galois group of extension. Let E be the extension of rationals and F the largest sub-field of E : let the corresponding Galois groups be G and H . The reduction would be a cascade starting with a reduction of the wave function in Galois group of E/F to a product of wave functions in G/H and H . At the next step same would take place for H and after finite number of steps one would have full reduction [L21].

These reduction cascades provide a model for cognitive processing as cognitive quantum measurements. This process brings in mind the translation of DNA to amino-acids. Could map to amino-acid involving transition from I to sub-group I/S , S the symmetry group of bio-harmony, be analogous to a state function reduction.

4 Appendix: What do experiments say about variation of Newton's constant?

In the sequel some experiments suggesting both small and large variations of G are discussed in the sequel. These experiments have been discussed earlier in [K2]. The experiment of Martin Grusenick is discussed in [K8].

4.1 Experiments suggesting small variations of G

The experiments measuring G use typically torsion pendulum: this method was introduced by Henry Cavendish in 1798.

Remark: A remark about terminology is in order. Torque $\tau = F \times r$ on particle has dimensions Nm. Torsion (see <http://tinyurl.com/q8esymu>) in solid is essentially the density of torque per volume and has dimensions N/m². Twist angle is induced by torsion in equilibrium. The situation is governed by the theory of elasticity.

Basically one has torsion balance in which the gravitational torque produced by two source masses on masses associated with a torsion pendulum - dumbbell shaped system having identical masses at the ends of a bar and hanging from a thread at the middle point of the bar. As the source masses are rotated a twist of the thread emerges and twist angle corresponds to an equilibrium in which the torsion of the thread compensates the torque produced by gravitational interaction with source masses. Cavendish achieved 1 per cent accuracy in his measurements.

Refined variations of these measurements have been developed during years and the current precision is 47 parts per million (ppm). In some individual experiments the precision is 13.7 ppm. Disagreements larger than 500 ppm are reported, which suggests that new physics might be involved.

The latest experiments were made by the above mentioned research group. Two methods are used. TOS (Time Of Swing) and AAF (Angular Acceleration Feedback). AAF results deviates from the accepted value whereas TOS agrees. The accuracies were 11.64 ppm and 11.61 ppm in TOS and AAF respectively. AAF however gave by 45 ppm larger value of G .

In TOS technique the pendulum oscillates. The frequency of oscillation is determined by the positions of the external masses and G can be deduced by comparing frequencies for two different mass configurations. There are two equilibrium positions. The pendulum is either parallel to the line connecting masses relatively near to each other (“near” position). The pendulum orthogonal to the line connecting masses in “far” position. By measuring the different oscillation frequencies one can deduce the value of G .

Angular-acceleration feedback (AAF) method involves rotating the external masses and the pendulum on two separate turn tables. Twist angle is kept zero by changing the angular velocity of the other turn table: thus feedback is involved. If I have understood correctly, the torsion induced by gravitational torque compensates the torsion created by twisting of the thread around

its axis in opposite direction and from the value of torsion for zero twist angle one deduces G . One could perhaps say that in AAF torsion is applied actively whereas in TOS it appears as reaction.

Why the measured value obtained for G would be larger for AAF? Could the active torsion inducing compensating twisting of the torsion pendulum actually increase G ?

4.1.1 Fluctuations of Newton's constant in sub-millimeter scales

Sabine Hossenfelder had a post with link to an article "*Hints of Modified Gravity in Cosmos and in the Lab?*" [E3] (see <http://tinyurl.com/y6j8sntw>). Here is the part of abstract that I find the most interesting.

On sub-millimeter scales we show an analysis of the data of the Washington experiment (Kapner et al. (2007) searching for modifications of Newton's Law on sub-millimeter scales and demonstrate that a spatially oscillating signal is hidden in this dataset. We show that even though this signal cannot be explained in the context of standard modified theories (viable scalar tensor and $f(R)$ theories), it is a rather generic prediction of nonlocal gravity theories.

What is interesting from TGD point of view that the effect - if it is indeed real - appears in scale of .085 mm about 10^{-4} μm , which is the scale defined by the density of dark energy in recent universe and thus by cosmological constant. This is also size scale of large neuron.

Washington group studied gravitational torque on torque pendulum for sub-millimeter distances of masses involved [E2] (see <http://tinyurl.com/y2un6686>). Figure 19 of [E3] (see <http://tinyurl.com/y6j8sntw>) illustrates data points representing the deviation of the gravitational torque from the Newtonian prediction as a function of distance in the range .05-10 mm.

The deviation can be parameterized in terms of effective scaling $G \rightarrow kG$ of Newton's constant, which is assumed to be predictable rather than due to fluctuations and depend on the distance only

$$k = 1 + x \cos\left(\frac{2\pi r}{\lambda} + \frac{3\pi}{4}\right) .$$

x is a numerical parameter. The highly non-trivial assumption is that Newton's potential is modified by an oscillating term, which must go to zero at large distances: its amplitude could approach to zero like $1/r$. The model predicts an anomalous gravitational torque $\Delta\tau$ proportional to $k - 1$ and having the form

$$\Delta\tau = a \cos\left(\frac{2\pi r}{\lambda} + \frac{3\pi}{4}\right) ,$$

where r is the distance between the masses. The parameter $\lambda = \hbar/m$ is formally analogous to Compton length for imaginary mass m .

The finding is that the statistical significance for the best fit to the data is $(a, \lambda) = (0.004 \text{ fNm}, 65 \text{ mm}^{-1})$ is more than 3σ , where a is the amplitude of the deviation. The highly non-trivial problem is however that one obtains also other minima of χ^2 measuring the goodness of the fit with different values of the parameter λ .

I am not specialist but while looking at the data, I cannot avoid the feeling that the fit does not make much sense and reflects theoretical prejudices (belief in modified gravity of some kind) rather than reality. My first impression that fluctuations in the value of Newton's constant G are in question. The value of G is indeed known to vary from experiment to experiment and the variation is too large to be explained in terms of measurement inaccuracies [E4] (see <http://tinyurl.com/yanvzxj6>).

Could it be that the value of G fluctuates, and for some reason in the length scale range around .1 mm the fluctuations are especially large meaning different values of G are large? Could some kind of criticality enhanced rather dramatically below .1 mm be involved?

4.1.2 Does Podkletnov effect involve non-standard value of G ?

Podkletnov observed [H2] at eighties a few percent reduction of gravity: he immediately lost his job in Tampere University in Finland. It was regarded as a scandalous event. Something new might have been discovered in Finnish laboratory!

I have considered a possible mechanism explaining the finding of Podkletnov [L2]. One could however ask whether the presence of superconductor involving also presence of phase with non-standard value of Planck constant could also affect the value of h_{eff} assignable to the flux tubes of the Kähler magnetic field mediating Earth's gravitational flux? The mechanism would be same as in fountain effect. The change $\Delta g_{eff}/g$ from the normal value would have been few per cent in this case.

4.2 Does fountain effect involve large deviation from non-standard value of G ?

Deviations in the value of G are not new, and I have written about several gravitational anomalies. This could mean also anti-gravity effects in a well-defined sense which is however not the same as often thought (negative gravitational masses or repulsive gravitational force).

In particular, there is well-known fountain effect (<http://tinyurl.com/kx3t52r>) in superfluidity in which superfluid seems to defy gravitation. I have proposed that $h_{eff}/h = n$ increases at superfluid flux tubes to h_{gr} and this gives to the effect as a de-localization in much longer scale [K3]. If also G is reduced so that the effect would be possible also classically? Since in superfluidity one has h_{eff} larger than usually, this might happen if gravitons travel also along flux tubes at which super fluid flows. This would change the earlier quantum estimates: in Schrödinger equation kinetic term scales up like $(h_{eff}/h)^2$ as before but gravitational potential of Earth would now scaled down like h/h_{eff} .

A simple model for the situation discussed in [K3] would rely on Schrödinger equation at the flux quantum which is locally a thin hollow cylinder turning around at the top of the wall of the container. In the following a slightly modified discussing replacing the gravitational acceleration g at surface of Earth with g_{eff}

1. One obtains 1-dimensional Schrödinger equation

$$\left(-\frac{\hbar_{eff}^2 \partial_z^2}{2m} + mg_{eff}z\right)\Psi = E\Psi \quad , \quad h_{eff} = nh_0 = \frac{nh}{6} \quad . \quad (4.1)$$

It is easy to see that the energy spectrum is invariant under the scaling $h \rightarrow h_{eff} = xh$ and $z \rightarrow z/x$. One has $\Psi_{xh, g_{eff}=g/x}(z) = \Psi_{h,g}(z/x)$ so that simple scaling of the argument z in question. The energy of the solution is same. If the ordinary solution has size scale L , the scaled up solution has size scale xL .

The height for a trajectory in gravitational field of Earth is scaled up for a given initial vertical velocity v_i is scaled as $h \rightarrow xh$ so quantum behavior corresponds to the classical behavior and de-localization scale is scaled up. Could this happen at various layers of magnetic body for dark particles so that they would be naturally at much higher heights. Cell scale would be scaled to Earth size scale of even larger sizes for the values of $h_{eff}/h = n$ involved.

For classical solution with initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g = 5$ cm. Quantum classical correspondence would be given by $E = mv_i^2/2 = E$ and this allows to look the delocalization scale of a solution.

2. One can introduce the dimensionless variable

$$u = \frac{z - \frac{E}{mg_{eff}}}{z_0} \quad , \quad z_0 = \left[\frac{2m^2 g_{eff}}{\hbar_{eff}^2}\right]^{-1/3} = \frac{n}{6} \left(\frac{h_{eff}(gr)}{h_{eff}}\right)^{1/3} \left(\frac{m}{m_p}\right)^{1/3} x \times \frac{\hbar c}{m_p} \quad , \quad (4.2)$$

$$\frac{\hbar c}{m_p} = \frac{L_p}{2\pi} \simeq .38 \times 10^{-12} \text{ m} \quad , \quad x = \left(\frac{m_p c^3}{\hbar g}\right)^{1/3} \simeq 1.5 \times 10^{10} \quad .$$

Here m_p denotes proton mass and L_p proton Compton length. z_0 scales as h_{eff} as one might expect. z_0 characterizes roughly the scale of the solution.

This allows to cast the equation to the standard form of the equation for Airy functions encountered in WKB approximation

$$-\frac{d^2\Psi}{du^2} + u\Psi = 0 . \quad (4.3)$$

Remark: Note that the classical solution depends on m . In central force problem with $1/r$ and $h_{eff} = GMm/v_0$ the binding energy spectrum $E = E_0/n^2$ has scale $E_0 = v_0^2 m$ and is universal.

3. The interesting solutions correspond to Airy functions $Ai(u)$ which approach rapidly zero for the values of $u > 1$ and oscillate for negative values of u . These functions $Ai(u + u_1)$ are orthogonal for different values of u_1 . The values of u_1 correspond to different initial kinetic energies for the motion in vertical direction. In the recent situation these energies correspond to the initial vertical velocities of the super-fluid in the film. $u = u_0 = 1$ defines a convenient estimate for the value of z coordinate above which wave function approaches rapidly to zero.

For classical solution with initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2 = E$ and this allows to look the delocalization scale of a solution.

The Airy function $Ai(u)$ approaches rapidly to zero (see the graph of <http://tinyurl.com/zrf7djo>) and one can say that above $u_0 = 3$ the function vanishes. Already at $u_0 = 1$ wave function is rather small as compared with its value at $u = 0$. This condition translates to a condition for z as

$$z_0 = z_{cl} + u_0 z_0 , \quad z_{cl} = \frac{E}{mg_{eff}} , \quad z_0 = \frac{h_{eff}}{h} \left[\frac{\hbar^2}{2m^2g} \right]^{1/3} . \quad (4.4)$$

The condition is consistent with the classical picture and the classical height z_{cl} scales like h_{eff}/h . The parameter $u_0 z_0$ defines the de-localization scale consistent with the expectations. Below z_{cl} the wave function oscillates which intuitively corresponds to the sum of waves in upwards and downwards directions.

This picture however leads to an objection.

1. If one has $\hbar_{eff}(gr)/\hbar \simeq 2^{24}$ at the flux tubes mediating gravitational interaction for the ordinary value of g (the estimates for $R^2\hbar/G$ are within range $10^6 - 10^7$), one can argue that one must use this value of \hbar in the Schrödinger equation for a particle in the gravitational field of Earth. One would have $z_0 \simeq 2^{-8} \times 250$ m. This is much larger than the value $z_0 \simeq 5.7$ mm for \hbar and the high value might be excluded already by the existing data for neutron's behavior in Earth's gravitational field. This value is also higher than the de-localization scale of order 1 meter in fountain effect.
2. If one assumes $h_{eff} = h$ and scaled up value of g corresponding to $G_{eff} = R^2/\hbar$, one obtains scaling of z_0 by $(h/\hbar_{eff}(gr))^{1/3} \simeq 2^{-8}$ giving $z_0 \simeq .2$ mm from the previous equation - note however the dependence on E . This could correspond to the ordinary situation. At electromagnetic flux tubes h_{eff} would be smaller and also G_{eff} considerably smaller as the radially symmetric stationary extremals studied during eighties indeed suggested. The increased gravitation would be masked by much stronger electromagnetic interaction so that the testing of this prediction is difficult. At gravitational flux tubes one would have a spectrum of values and h_{gr} might represent the upper bound at quantum criticality for which the dependence of scattering amplitudes on masses disappears. $\hbar_{eff}(gr)$ would correspond to the measured value of $G_{eff} = G$.

Remark: One can of course ask whether $h_0 = h/6$ indeed represents the minimal value of h_{eff} . In principle one can also consider smaller values \hbar/k and this would give rise to $G_{eff} = kR^2$ with a shorter de-localization scale.

Can one say something about the spectrum of h_{eff} ? If one assumes that number theoretical evolution corresponds to the increasing order of the Galois group such that the new Galois group

contains earlier Galois group as sub-group (this would serve as an analogy for conserved genes in biological evolution). Larger Galois groups would naturally contain the “standard” Galois group associated with N as a sub-group. From number theoretic point of view the proposal $\hbar_{eff}/\hbar = N = 2^{24}$ is perhaps the simplest one since all Galois groups appearing as its sub-groups would have order with is 6×2^k for $h = 6h_0$. \hbar_{eff}/\hbar should have N as a factor.

It seems that one must assume accept G_{eff} is indeed different for different flux tubes.

1. For proton mass $m = m_p$, $\hbar_{eff} = h$, and $g_{eff} = 2^{24}g$ one would have $z_0 \simeq .2$ mm as one finds from the previous equation. $\hbar_{eff}/\hbar_{eff}(gr) = 2^{12}$ would give $z_0 \simeq 89.1$ cm, which makes sense for fountain effect. The value $\hbar_{eff}/\hbar = 2^{36}$ looks quite conceivable at flux tubes mediating electromagnetic interaction and carrying suprafluid flow. I have considered years ago the hypothesis that \hbar_{eff}/\hbar could come as powers of 2^{11} . Note that the estimate $v_0 \simeq 2^{-11}$ is also power of 2 so that powers of 2 are suggestive.
2. $\hbar_{eff} = \hbar_{gr} = GM_D m/v_0$ corresponds to a large value of \hbar_{eff} and might be assignable to flux tubes mediating dark part of gravitational interaction

$$z_0 = \frac{c}{v_0} \frac{r_S}{2} \frac{M_D}{M_E} \left(\frac{m}{m_p}\right)^{1/3}, \quad x = \left(\frac{m_p c^3}{\hbar g}\right)^{1/3} \simeq 1.5 \times 10^{10}, \quad r_S = 2GM_E \simeq 9 \text{ mm} .$$

More concretely:

$$z_0 \simeq \frac{M_D}{M_E} \times 6 \times 10^7 \text{ km} .$$

The estimate for M_D/M_E is $M_D/M_E \sim 10^{-4}$. An open question is whether M_D should be interpreted as dark mass or whether one should interpret M_D/M_E as a mere parameterization for $\hbar_{eff} = (n/6)\hbar$ as $\hbar_{eff} = (M_D/M_E)\hbar_{gr}$. z_0 characterizes the de-localization scale for the solutions. It is clear that this scale is many orders of magnitudes larger than the de-localization scale about 1 m for superfluids.

4.3 Did LIGO observe non-standard value of G and are galactic blackholes really supermassive?

Also smaller values of G than the G_N are possible and in fact, in condensed matter scales it is quite possible that $n = R^2/G$ is rather small. Gravitation would be stronger but very difficult to detect in these scales. Neutron in the gravitational field of Earth might provide a possible test. The general rule would be that the smaller the scale of dark matter dynamics, the larger the value of G and maximum value would be $G_{max} = R^2/h_0$, $h = 6h_0$.

4.3.1 Are the blackholes detected by LIGO really so massive?

LIGO (see <http://tinyurl.com/bszfs29>) has hitherto observed 3 fusions of black holes giving rise to gravitational waves. For TGD view about the findings of LIGO see [L6, L4] (see <http://tinyurl.com/y79yqw6q> and <http://tinyurl.com/ya8ctxgc>). The colliding blackholes were deduced to have unexpectedly larger large masses: something like 10-40 solar masses, which is regarded as something rather strange.

Could it be that the masses were actually of the order of solar mass and G was actually larger by this factor and \hbar_{eff} smaller by this factor? The mass of the colliding blackholes could be of order solar mass and G would larger than its normal value - say by a factor in the range (10,50). If so, LIGO observations would represent the first evidence for TGD view about quantum gravitation, which is very different from superstring based view. The fourth fusion was for neutron stars rather than black holes and stars had mass of order solar mass.

This idea works if the physics of gravitating system depends only on $G(M + m)$. That classical dynamics depends on $G(M + m)$ only, follows from Equivalence Principle. But is this true also for gravitational radiation? If the power of gravitational radiation distinguishes between different values of M when GM is kept constant, the idea is dead.

1. If the power of gravitational radiation distinguishes between different values of $M+m$, when $G(M+m)$ is kept constant, the idea is dead. This seems to be the case. The dependence on $G(M+m)$ only leads to contradiction at the limit when $M+m$ approaches zero and $G(M+m)$ is fixed. The reason is that the energy emitted per single period of rotation would be larger than $M+m$. The natural expectation is that the radiated power per cycle and per mass $M+m$ depends on $G(M+m)$ only as a dimensionless quantity.
2. From arXiv one can find an article (see <http://tinyurl.com/y99j3fpr>) in which the energy per unit solid angled and frequency radiated in collision of blackholes is estimated. The outcome is proportional to $E^2 G(M+m)^2$, where E is the energy of the colliding blackhole. The result is proportional mass squared measured in units of Planck mass squared as one might indeed naively expect since $G(M+m)^2$ is analogous to the total gravitational charge squared measured using Planck mass. The proportionality to E^2 comes from the condition that dimensions come out correctly. Therefore the scaling of G upwards would reduce mass and the power of gravitational radiation would be reduced down like $M+m$. The power per unit mass depends on $G(M+m)$ only. Gravitational radiation allows to distinguish between two systems with the same Schwarzschild radius, although the classical dynamics does not allow this.
3. One can express the classical gravitational energy E as gravitational potential energy proportional to GM/R . This gives only dependence on GM as also Equivalence Principle for classical dynamics requires and for the collisions of blackholes R is measured by using $G(M+m)$ as a natural unit.

Remark: The calculation uses the notion of energym which in general relativity is precisely defined only for stationary solutions. Radiation spoils the stationarity. The calculations of the radiation power in GRT is to some degree artwork feeding in the classical conservation laws in post-Newtonian approximation lost in GRT. In TGD framework the conservation laws are not lost and hold true at the level of $M^4 \times CP_2$.

4.3.2 What about supermassive galactic blackholes?

What about supermassive galactic black holes in the centers of galaxies: are they really super-massive or is G super-large! The mass of Milky Way super-massive blackhole is in the range $10^5 - 10^9$ solar masses. Geometric mean is $n = 10^7$ solar masses and of the order of the standard value of $R^2/G_N = n \sim 10^7$. Could one think that this blackhole has actually mass in the range 1-100 solar masses and assignable to an intersection of galactic cosmic string with itself! How galactic blackholes are formed is not well understood. Now this problem would disappear. Galactic blackholes would be there from the beginning!

The general conclusion is that only gravitational radiation allows to distinguish between different masses $M+m$ for given $G(M+m)$ in a system consisting of two masses so that classically scaling the opposite scalings of G and $M+m$ is a symmetry.

4.4 Grusenick's experiments

Martin Grusenick performed years ago a variant of Mickelson-Morley experiment. I commented this experiment in [K8]. Grusenick reported that the effective velocity of light depends on direction such that the velocity is smallest in vertical direction. Aether interpretation is however excluded since the velocity would be in direction radial to Earth. The effect could be due to an experimental error due to not taking into account the contraction of the system under its own weight.

If one takes the finding seriously, the gravitational field of Earth is the first explanation to come into mind. Could the interference pattern for the two signals arriving from orthogonal directions be understood as being due to different spatial distances $s = \int ds_3$, $ds_3^2 = g_{ij} dx^i/dt \times dx_j/dt$ travelled along light-like geodesics. One can say, that the propagation velocities along vertical and horizontal line directions differ in GRT by the ratio $\sqrt{g_{tt}/g_{rr}}$, which leads to phase difference and interference pattern. The effect caused by gravitation is proportional to G but is by a factor 10^{-5} too small.

In TGD framework the explanation based on large variation of G is in principle allowed if one accepts the general formula for G . Could G increase by a factor of say order 10^5 in some circumstances? Certainly this increase cannot occur at ordinary space-time sheets since large G would have been observed. Photons in the experiments should propagate along gravitational space-time sheets with much larger value of G . If one assumes Platonic solid, the replacement of icosahedron with tetrahedron provides largest increase of G given by $8 \times 60^4/12^4 = 5,000$, which is too small. This option does not look plausible to me but cannot be excluded.

One can consider also a second explanation in TGD framework.

1. M^4 allows besides the canonical imbedding to $H = M^4 \times CP_2$ obtained by putting CP_2 coordinates constant also warped imbeddings for which CP_2 projection corresponds to a geodesic circle S^1 with angle coordinate Φ given by a linear function $\Phi = k \cdot m$ of linear Minkowski coordinate m : m and wave vector k are 4-vectors.

This gives rise to flat surfaces, which are warped meaning that induced metric differs by scalings from the metric of M^4 . The velocity of photon in M^4 coordinates is scaled by the ratio r_t/r_s of the scaling factors in time direction and propagation direction: $c \rightarrow (r_t/r_s)c$, $r_s = \sqrt{1 + R^2 k_s^2}$, where k_s is the projection of k to the direction of propagation and $r_t = \sqrt{1 - R^2 \omega^2}$ is the projection to the time direction. Also the density of the volume energy due to length scale dependent cosmological constant differs from that for standard imbedding so that the effect is physical. Note however that warping occurs only for flat M^4 (in 2-D E^2 there is a concrete representation for the warping in terms of paper sheet).

2. There are two manners to represent photon. As “massless extremal” (ME) in long scales or as CP_2 type extremal. CP_2 type extremal has light-like geodesic as M^4 projection and can be said to move with light-velocity.

It does not seem plausible that the notion of warping generalizes to “massless extremals” (MEs). One can however consider many-sheeted structures. Test particle feels the superposition of the effects from space-time sheets since it touches all of them. The QFT limit of TGD is based on this picture. Gauge potentials and deviations from M^4 metric effectively sum up to standard model gauge potentials and GRT metric. If photons can be regarded as test particles, this picture applies also to photon. At QFT limit the deviations of the induced metric from M^4 metric would sum up to the counterpart of GRT metric and photon would move along light-like geodesic of this metric with velocity smaller than the maximal signal velocity in M^4 .

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