Condensed matter simulation of 4-D quantum Hall effect from TGD point of view

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January 13, 2018

Abstract

There is an interesting experimental work related to the condensed matter simulation of physics in space-times with $D = 4$ spatial dimensions meaning that one would have $D = 1 + 4 = 5$-dimensional space-time. What is simulated is 4-D quantum Hall effect (QHE). In M-theory $D = 1 + 4$-dimensional branes would have 4 spatial dimensions and also 4-D QH would be possible so that the simulation allows to study this speculative higher-D physics but of course does not prove that 4 spatial dimensions are there. In this article I try to understand the simulation, discuss the question whether 4 spatial dimensions and even 4+1 dimensions are possible in TGD framework in some sense, and also consider the general idea of the simulation higher-D physics using 4-D physics. This possibility is suggested by the fact that it is possible to imagine higher-dimensional spaces and physics: maybe this ability requires simulation of high-D physics using 4-D physics.

1 Introduction

I learned about an interesting experimental work related to the condensed matter simulation of physics in space-times with D=4 spatial dimensions meaning that one would have D=1+4=5-dimensional space-time. The simulation was discussed in popular article "*Leaving Flatland Quantum Hall Physics in 4D*" (see http://tinyurl.com/ycoxr48s).

What was simulated is 4-D quantum Hall effect (QHE). In M-theory D= 1+4 dimensional branes would have 4 spatial dimensions and also 4-D QH would be possible so that the simulation to study this speculative higher-D physics. To avoid misunderstandings it must be emphasized that it has not been demonstrated that 4:th spatial dimension exists as layman might think first.

A condensed matter simulation of a 4-D QHE possible in 1+4-dimensional space-time [D1] (see http://tinyurl.com/y7nxd5k3) is in question. Professors Immanuel Bloch (LMU/MPQ) and Oded Zilberberg (ETH Zürich) are the leaders of the team behind the work. Using ultracold atoms trapped in a periodically modulated two-dimensional superlattice potential, the scientists could observe a dynamical version of a novel type of QHE that is predicted to occur in four-dimensional systems.

The theory of the 4-D QHE is discussed in [D3] (see http://tinyurl.com/y8nk5jp3). This model assumes that spatial dimensions correspond to 4-sphere but also more general topologies are possible. In the simulation the topology was that of 4-torus.

4-D QH conductivity is proportional to a topological invariant known has second Chern number [D2]- gauge theorists talk about instanton number. This invariant is space-time integral of a quantity quadratic in gauge field so that the effect is non-linear.

2-D QH conductivity is proportional to the first Chern number which is essentially magnetic charge and non-vanishing if the second homology group is non-trivial (space has a non-contractible 2-D surface) and can be identified in the experiment considered as an analog of magnetic flux over torus but in momentum space rather than space-time. In the case of 2-D QHE in the real world the spatial topology is that of a 2-disk, which is compact only if boundary is included: one can define the first Chern class as Gauss-Bonnet invariant in this case. My interpretation is however that one considers Chern number in momentum space for the boundary of Fermi surface and that the effective monopole magnetic field corresponds to the area form of this surface: certainly this should be the case for the simulation.

2 The ideas of the simulation of 4-D QHE

The basic idea is that one tries to find an ordinary 1+3-D system having a dynamics mathematically equivalent to that of QHE in 4+1-D spacetime. Fig 1 of [D1] (see `http://tinyurl.com/y7nxd5k3`) illustrates the basic idea.

1. One wants to simulate the topology $(S^1 \times S^1) \times (S^1 \times S^1)$. 2-D QHE would take place at tori $S^1 \times S^1$. The basic observation is that the union $S^1 \times S^1 \cup S^1 \times S^1$ of two tori as 2-D surfaces in 3-space is Cartesian product $(S^1 \times S^1) \times (S^1 \times S^1)$ as far as degrees of freedom are counted. Therefore it might be possible to simulate physics of this system by using two 2-D tori plus suitable coupling between them. This idea is familiar from elementary quantum mechanism where the physics of N-particle system in 3-D space as physics of single particle system in 3N-D space.

   One cannot realize these tori as 2-D surfaces in 3-space. The problem is that magnetic field should be orthogonal to the torus. This would require monopole charge distribution along circle at the center of torus. This is not realizable at space-time level using the known physics. It can be however realized as effective magnetic field in momentum space at the boundary of Fermi surface, where one can define effective magnetic monopole field using the area form.

   I understand that the idea is to get effective torus topology in momentum space by using lattice like structure. The momenta differing by lattice momenta are equivalent: physically this means that wave lengths scale smaller than lattice constant are not detectable. This identification is standard manner to define torus topology. Even the lattice structure is realized in a rather exotic manner - as a photon lattice.

2. From the figure 1 one learns that for the first torus $S^1 \times S^1$ is obtained from a lattice-structure in z- and x-directions by the proposed identifications. The Fourier transform of the electric field E_z of 2-D QHE is in the z-direction and the transversal velocity component to Lorentz force is in x-direction. E_z is created by time varying real magnetic flux in x-direction of ordinary space-time by Faraday's law. Lorentz force in momentum space is caused by fictive circular monopole distribution in momentum space generating magnetic flux Φ_{xz}.

   The plane defined by the center circle of the second second torus is orthogonal to that of the first one. One has $(z, x) \rightarrow (w, y)$. x- and y-axis of the cyclinders are thus orthogonal as also induce orthogonal velocities v_x and v_y in 2-D QHE for these systems.

3. In order to get the analog of 4-D QHE one adds a coupling between the two systems modellable using real magnetic field B_{xw} orthogonal to the fictive magnetic flux Φ_{xz}. This implies additional Lorentz force F_w in the direction of E_w in momentum space. Φ_{yw} induces therefore an additional velocity component parallel to v_y and proportional to both Φ_{xz} and Φ_{yw}. This gives rise to additional 4-D QHE proportional to the second Chern number as the integral of the instanton density in momentum space, which is essentially the product of Φ_{xz} and Φ_{yw} so the second Chern number is product of first Chern numbers (I must admit that I do not understand the details of the argument). This gives rise to QHE conductivity bi-linear in the effective magnetic fluxes and proportional to the second Chern number.

The actual realization of the situation involves quite refined condensed matter physics. The simulation of 2-D QH lattices is in terms of photon crystals creating 2-D periodic potentials to which a gas of ultracold boson atoms is added. As already confessed, I do not understand how the mathematical model for the situation leads to 4-D QHE. "*By implementing a 2D topological charge pump with ultracold bosonic atoms in an angled optical superlattice, we realize a dynamical version of the 4D integer quantum Hall effect*" does not tell much to a non-specialist. One can only admire the abstractness of the theory and skills of experimentalists.

3 TGD inspired comments about the simulation

The simulation raises several questions. Can one imagine 4 space-like dimensions or even 4+1 dimensions in TGD? Can one emerging a general simulation of imagined higher-D physics in terms of 4-D physics in TGD framework.

3.1 Are 4-D space-like regions possible in TGD?

In braneology of M-theory 4-D QHE is in principle possible and it might serve as a signature for the existence of fourth spatial dimension if branes really are there. There are however objections against large fourth space-like dimension.

1. Additional large spatial dimensions would have been probably detected if there are everywhere: for instance, additional conserved component of momentum is implied. This implies that the additional dimension must be small enough. One cannot however exclude regions of space-time, where the additional dimension is large.

2. The dimension 3 for hydrogen atom is very special. In fact, the $1/\hbar^2$ proportionality of the binding energies is crucial in TGD inspired biology, where Planck constant has spectrum: $\hbar_{eff}/\hbar = n$. At the level of chemistry one ends up with valence bond theory in which n characterizes the bonds [L5].

   The binding energy spectrum changes dramatically in other dimensions. In particlar, in dimension $D = 4$ the dependence of binding energies on Planck constant is not a power law as it is in other dimensions [L2] (see `http://tinyurl.com/yam7rbk6`). The energies of the hydrogen atom depend on $\hbar_{eff} = n \times h$ as $\hbar_{eff}^m$, $m = -2 < 0$. Hydrogen atoms in dimension D have Coulomb potential behaving as $1/r^{D-2}$ from Gauss law and the Schrödinger equation predicts for $D \neq 4$ that the energies satisfy $E_n \propto (h_{eff}/h)^m$, $m = 2 + 4/(D-4)$. For $D = 4$ the formula breaks since in this case the dependence on $\hbar$ is not given by power law. m is negative only for $D = 3$ and one has $m = -2$. There $D = 3$ would be unique dimension in allowing the hydrino-like states [L3]. The temporary reduction of n makes possible biocatalysis and life in the proposed scenario.

Are 4-D space-like regions possible in TGD?

1. In TGD space-times are 4-D surfaces in $H = M^4 \times CP_2$ picture. Space-time regions with Euclidian signature of metric (time is like fourth spatial coordinate) are predicted and could accompany any system as space-time sheet having same size as the system.

2. $M^8 - H$ duality is now a key piece of TGD and states that one can regard space-times as surfaces in either $H = M^4 \times CP_2$ or M^8 [?]see `http://tinyurl.com/yd43o2n2`). In M^8-picture space-time surfaces are zero loci for $RE(P)$ or $IM(P)$, where P is octonionic polynomial obtained as a continuation of real polynomial. In this picture one obtains also 1+4-D 1+5-D space-time surfaces as singular solutions but it is unclear whether they have any physical meaning since they do not have $M^4 \times CP_2$ counterpart. If the two descriptions are equivalent, 4-D QH effect is not possible.

3.2 Is 4-D QHE possible in TGD?

Is 4-D QHE possible in TGD? One can consider the question in two different pictures: $M^8 - M^4 \times CP_2$ duality [L4] states that the descriptions of space-time surfaces as algebraic surfaces in M^8 on one hand, and as surfaces satisfying field equations in $H = M^4 \times CP_2$ are physically equivalent.

1. As noticed, space-time regions with Euclidian signature of metric are predicted but since one has only 4-D space rather than 1+4-D space-time, 4-D QHE is not possible.

   One could however consider the possibility that ZEO makes 4+1-D situation effectively possible. The size of CD increases in each "small" reduction identifiable as an analog of weak measurement since one can say that the active boundary of CD shifts farther away from the stationary passive boundary where the members of state pairs are unaffected [L6] (see http://tinyurl.com/ycxm2tpd).

The proper time parameter telling the distance between the tips of CD corresponds to clock time correlating with experienced time. Clock time is discrete since the increments are discrete for it but one can ask whether it could give rise to effective additional space-time coordinate and for space-like regions of space-time realized as surface inside CD this could make possible 4-D QHE. Perhaps a better manner to see this clock time is as the size scale of space-time surface which changes. One could also consider 4-D QHE in which time is replaced by a size scale.

2. In M^8-picture space-time surfaces are zero loci for real and imaginary parts $RE(P)$ or $IM(P)$ (in quaternionic sense using he decomposition of octonion to two quaternions) of octonionic polynomials P obtained as a continuation of real polynomials. Rather surprisingly, one obtains as singular solutions also 1+4-D and 1+5-D space-time surfaces but it is unclear whether they have any physical meaning since they do not have $M^4 \times CP_2$ counterpart. If the two descriptions are equivalent 4-D QH effect does not seem to be possible.

3.3 Other effects involving instanton number

One can of course imagine that there could be other effects involving 4-D instanton number (second Chern number). But can one have non-vanishing instanton number in TGD?

1. The induced color gauge field is proportional to induced Kähler gauge field and the counterpart of color action reduces to Kähler action. So that it seems to enough to consider the situation for the Kähler form (of CP_2) induced to space-time surface.

2. Instanton number is winding number for the map $X^4 \to CP_2$ and requires that the CP_2 projection of the space-time surface is 4-D. Therefore one can locally represent the instanton as a map $CP_2 \to M^4$. The asymptotic regions of M^4 and the boundary of CD are however exceptions. Call these regions just S. Here CP_2 coordinates are constant and M^4 coordinates are the appropriate coordinates near S. The map $M^4 \to CP_2$ can be however multiple-valued such that the branches co-indice in S.

Consider first Minkowskian signature for the induced metric, that is maps representable as graphs $M^4 \to CP_2$ (note that locally also the representation as map $CP_2 \to M^4$ are possible at points where the instanton density is non-vanishing).

1. One must can allow multiple-valued maps $M^4 \to CP_2$. One could see M^4- or CD coordinates as coordinates for CP_2, and CP_2 require at least 3 coordinate patches, which strongly suggests at least 3-fold covering and 3-valuedness except at singular regions in which some sheets coincide.

   The effective dynamical compactification of the space-time surface requires that the CP_2 coordinates are constant in S. All gauge field components therefore vanish at S. Instanton number is divergence of a topological current and reduces to a sum of surface integrals. The contribution from S vanishes.

   The topological current is proportional to Kähler gauge potential and since Kähler field is monopole field one must take into account the gauge discontinuities at coordinate patches coming from the gauge transformation associated with the transitions between patches. If one has instanton number n, there are $3n$ patches giving a non-vanising contribution and their sum could give a non-trivial instanton number.

2. There are good reasons to expect that the induced gauge fields have $n = 0$ in space-time regions with Minkowskian signature of the induced metric. At least this would be the case for the induced Kähler form. For the twistor lift of Kähler action reducing to a sum of Kähler action and volume term, preferred extremals representing a map of $M^4 \to CP_2$ or $CD \to CP_2$ with winding number n very probably do not exist [L1] (see http://tinyurl.com/yboog5sr).

3. At QFT limit one consider only Minkowkian regions so that there would be no instantons in TGD Universe. Note that one would avoid the strong CP problem of QCD, which is due to instantons.

Consider next Euclidian signature of the induced metric.

1. For a non-vanishing value of n the representation as a map $CP_2 \to M^4$ is possible except at the intersections with S unless they are not discrete points. If the intersection with the boundary of CD discrete point or empty, one can have instanton number $n = 1$. One can represent CP_2 as a surface in $H = M^4 \times CP_2$ obtained by putting M^4 coordinates to constant. This solution is however not consistent with the assumption that space-time surfaces have ends at the opposite boundaries of CD.

   Elementary particles have wormhole contacts identifiable as deformed pieces of CP_2 as building bricks. CP_2 type extremal can be indeed deformed so that M^4 projection is a light-like geodesic. The resulting surface has two holes and they should reduce to points at the boundaries of CD. One can of course imagine also more holes. What could the instanton number of CP_2 with punctures be?

2. One could try to use 2-D analogy. Sphere CP_1 with punctures looks like a good analogy for CP_2 with punctures. The first Chern number for sphere with punctures is proportional to Gauss-Bonnet invariant expressible in terms of curvature scalar and corrections from the punctures. The first Chern number becomes proportional to $1 - n/2$, where n is the number of punctures. For two holes, one has vanishing Gauss-Bonnet invariant since one has topologically cylinder allowing flat metric.

   If an analogous formula holds also for CP_2, the second Chern number becomes fractional. CP_2 differs from sphere CP_1 in that it has 3 poles instead of 2. The removal of poles of CP_1 gives a vanishing first Chern number (cylinder). The removal of 3 poles from CP_2 should give vanishing second Chern number. Thus second Chern number would be proportional to $1 - n/3$.

   If CP_2 as surface in $H = M^4 \times CP_2$ allows n-fold coverings, they have instanton number n for the Abelian gauge field defined by the induced Kähler form. Is this possible? Could one have M^4 projection consisting of n light-like geodesics? One can argue that the sheets of n-fold covering defined by the light-like geodesics must be transformable continuously to each other so that the light-like geodesics must co-incide, and one can argue that one has 1-fold covering.

One can say that instanton number for Kähler form plays a fundamental role at the level of particle physics and has highly nontrivial physical implications and that they are directly seen in the scales of elementary particles if they have wormhole contacts as basic building bricks. This physics is however not seen at QFT limit of TGD.

3.4 Is the simulation of higher-dimensional physics/mathematic possible in TGD?

The idea of simulation of higher-D physics using 4-D physics is especially natural in TGD using N disjoint space-time surfaces. Time coordinate would be common to all N space-time surfaces, say proper time coordinate for either light-cone associated with CD so that the number of degrees of freedom would be $D = 3N + 1$. For light-line 3-D light-like partonic orbits defining the boundaries between Minkowskian and Euclidian regions the dimension would be $D = 2N + 1$ and for string world sheets it would be $D = N + 1$ so that multi-string states would allow the simulation of physics in any dimension $D \geq 2$.

At the level of imbedding space this would correspond to a simulation of physics for surfaces $H^N = (CD \times CP_2)^N$, such that time coordinate is same for all 3-D surfaces and one has effectively $(H_7)^N \times T = (E^3 \times CP_2)^N \times T$ where T denote time axis and E^3 to time= constant section. One can replace E^3 with the hyperbolic space H^3 and M^4 time t with the proper time a future or past directed light-cone.

I have proposed this possibility as a reaction to an objection against TGD. If space-time dimension is $D = 4$, how it is possible for a mathematician to imagine higher dimensions? Doesn't mathematical cognition of higher dimensions require a physical simulation of the higher-D dynamics? The proposed dynamics would indeed allow the physical simulation of the higher-D mathematics.

The simulation is trivial unless there is a non-trivial interaction between the separate space-time surfaces. This could be achieved by coupling them using flux tubes. If the surfaces are space-time sheets on top of each other with respect to CP_2 degrees of freedom, wormhole contacts define this interaction. What is interesting that homologically non-trivial wormhole contacts are basic building bricks of elementary particles. For homologically trivial wormhole contacts the contact is unstable against splitting.

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