About TGD counterparts of twistor amplitudes

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Contents

Abstract

The twistor lift of TGD, in which $H = M^4 \times CP_2$ is replaced with the product of twistor spaces $T(M^4)$ and $T(CP_2)$, and space-time surface $X^4 \subset H$ with its 6-D twistor space as 6-surface $X^6 \subset T(M^4) \times T(CP_2)$, is now a rather well-established notion and $M^8 - H$ duality predicts it at the level of M^8.

Number theoretical vision involves $M^8 - H$ duality. At the level of H the quark mass spectrum is determined by the Dirac equation in H. In M^8 mass squared spectrum is determined by the roots of the polynomial P determining space-time surface and are in general complex. By Galois confinement the momenta are integer valued when p-adic mass is used as a unit and mass squared spectrum is also integer valued. This raises hope about a generalization of the twistorial construction of scattering amplitudes to TGD context.

It is always best to start from a problem and the basic problem of the twistor approach is that physical particles are not massless.

1. The intuitive TGD based proposal has been that since quark spinors are massless in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes. However, no obvious mechanism has been identified. One step in this direction was the realization that in H quarks propagate with well-defined chiralities and only the square of Dirac equation is satisfied. For a quark of given helicity the spinor can be identified as helicity spinor.
2. M^8 quark momenta are in general complex as algebraic integers. They are the counterparts of the area momenta x_i of momentum twistor space whereas H momenta are identified as ordinary momenta. Total momenta of Galois confined states have as components ordinary integers.
3. The M^8 counterpart of the 8-D massless condition in H is the restriction of momenta to mass shells $m^2 = r_n$ determined as roots of P. The M^8 counterpart of Dirac equation in H is octonionic Dirac equation, which is algebraic as everything in M^8 and analogous to massless Dirac equation. The solution is a helicity spinor $\tilde{\lambda}$ associated with the massive momentum.

The outcome is an extremely simple proposal for the scattering amplitudes.

1. Vertices correspond to trilinears of Galois confined many-quark states as states of super symplectic algebra acting as isometries of the "world of classical worlds" (WCW). Quarks are on-shell with H momentum p and M^8 momenta x_i, x_{i+1}, $p_i = x_{i+1} - x_i$. Dirac operator $x_i^k \gamma_k$ restricted to fixed helicity L, R appears as a vertex factor and has an interpretation as a residue of a pole from an on-mass-shell propagator D so that a correspondence with twistorial construction becomes obvious. D is expressible in terms of the helicity spinors of given chirality and gives two independent holomorphic factors as in case of massless theories.
2. The scattering amplitudes would be rational functions in accordance with the number theoretic vision. The absence of logarithmic radiative corrections is not a problem: the coupling constant evolution would be discrete and defined by the hierarchy of extensions of rationals.
3. The scattering amplitudes for a single 4-surface X^4 characterixzing interaction region are determined by a polynomial P. External particles are Galois singlets consisting of off-mass shell quarks with mass squared values coming as roots of the polynomial P characterizing the interaction region. External particles are characterized by polynomials P_i satisfying $P_i(0) = 0$. P is identified as the functional composite of P_i since it inherits the masses of incoming particles as their roots. This allows only cyclic permutations of P_i. The scattering event is essentially a re-combination of incoming Galois singlets to new Galois singlets and quarks propagate freely: hence OZI rule generalizes. Also a connection with the dual resonance models emerges.
4. The integration over WCW is replaced with a summation of polynomials characterized by rational coefficients. Monic polynomials are highly suggestive. A connection with p-adicization emerges via the identification of the p-adic prime as one of the ramified primes of P. Only (monic) polynomials having a common p-adic prime are allowed in the sum. The counterpart of the vacuum functional $exp(-K)$ is naturally identified as the discriminant D of the extension associated with P and p-adic coupling constant evolution emerges from the identification of $exp(-K)$ with D.

Unitary, locality, and the failure to find the twistorial counterparts of non-planar Feynman diagrams are the basic problems of the twistor Grassmannian approach. Also the non-existence of twistor spaces for most Riemannian manifolds is a problem in GRT framework but in TGD the existence of twistor spaces for M^4 and CP_2 solves this problem. In the TGD framework, the replacement of point-like particles with 3-surfaces leads to the loss of locality at the fundamental level. The analogs of non-planar diagrams are eliminated since only cyclic permutations of P_i are allowed.

This leaves only the problem with unitarity. The TGD counterpart of unitarity realized in terms of Kähler geometry of fermionic state space is very natural in the geometrization of quantum physics. Scattering probabilities are identified as products of covariant and contravariant matrix elements of the metric, and unitary conditions are replaced by the definition of the contravariant metric. Probabilities are complex but real and imaginary parts are separately conserved. The interpretation in terms of Fisher information is possible. Due to the infinite-D character of the state space, the Kähler geometry exists only if it has a maximal group of isometries and is a unique constant curvature geometry. Also the interpretation of this approach in zero energy ontology is discussed.

There are physical motivations for considering the number theoretic generalizations of the amplitudes. For an iterate of fixed P (say large number of gravitons), the roots of the iterate of P defined virtual mass squared values, approach to the Julia set of P. The construction of scattering amplitudes thus leads to chaos theory at the limit of an infinite number of identical particles.

The construction generalizes also to the surfaces defined by real analytic functions and the fermionic variant of Riemann zeta and elliptic functions are discussed as examples.

1 Introduction

The twistor program was originally introduced by Penrose [B27]. The application of twistors to gauge theories, in particular $\mathcal{N} = 4$ SUSY, led to a dramatic progress in the mathematical understanding of these theories. For beginners like me (still), the article of Elvang and Huang [B12] is an extremely helpful introduction to twistor scattering amplitudes.

I am not a specialist in the field. Therefore the following list of works that have had effect in my attempts to understand how twistors might relate to TGD, must look rather random in the eyes of a professional. It however gives some idea about the timeline of ideas.

- Witten's work (2003) [B10] on perturbative string theory in twistor space.
- The proof of Britto, Cachazo, Feng and Witten (2005) [B6] for tree level recursion relation (BCFW recursion) in Yang-Mills theory.
- The work of Hodges (2005) [B2] about twistor diagram recursion for gauge-theory amplitudes.
- The works of Mason and Skinner (2009) on scattering amplitudes and BCFW recursion in twistor space [B25] and on dual superconformal invariance, momentum twistors and Grassmannians (2009) [B26]. There is also the work of Bullimore, Mason and Skinner (2009) on twistor strings, Grassmannians and leading singularities [B7].
- The work of Drummond, Henn and Plefka (2009) [B9] on Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ SUSY.
- The work of Goncharov *et al* (2010) [B20] on classical polylogarithms for amplitudes and Wilson loops.
- Nima Arkani-Hamed and colleagues have made impressive contributions. There is a work by Arkani-Hamed *et al* on S-Matrix in twistor space (2009) [B15, B14]; a work about unification of residues and Grassmannian dualities (2010) [B16]; a proposal for all-loop integrand for scattering amplitudes for planar $\mathcal{N} = 4$ SUSY (2011) [?] a work on scattering amplitudes and positive Grassmannian (2012) [B13]; the proposal of amplituhedron (2013) [B5] and work about positive amplitudes in amplituhedron [B4] (2014); a proposal of MHV on-shell amplitudes beyond the planar limit (2014) [B18] ; the notion of associahedron (2017) [B3].

The TGD approach to twistors [L2, L7] [L9, L17] has developed gradually during the last decade. The evolution of ideas began with the attempt to geometrize twistors in the same manner as standard model gauge fields are geometrized in TGD. Only quite recently, the number theoretic approach to twistors has started to evolve.

The twistor lift of TGD geometrizes the notion of twistor by replacing the twistor field configurations with 6-D surfaces assigning to space-time surfaces analog of its twistor space obtained by inducing the twistor structure of the product $T(M^4) \times T(CP_2)$ of the twistor spaces of M^4 and CP_2. The construction requires that these twistor spaces have a Kähler structure. M^4 and CP_2 are unique in that only their twistor spaces allow a Kähler structure [A4]. Therefore TGD is mathematically unique: the same conclusion is forced by standar model symmetries and M^8-H duality. This gives strong motivation for an attempt to construct the TGD counterparts of the twistor scattering amplitudes.

The number theoretic view about twistors based on M^8-H duality [L20, L21, L35] has developed during this year (2021) and this article tries to articulate this vision and leads to a proposal for how to construct twistor scattering amplitudes in the TGD framework.

1.1 Some background

In the following, the basic facts related to twistors are described. I cannot say anything about the technicalities of the twistorial computations and my basic aim is to clarify myself the contents of the notions involved and understand how the twistors diagrammatics might generalize to the TGD context.

1.1.1 Basic facts about twistors and bi-spinors

It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a\tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality (see `http://tinyurl.com/y6bnznyn`).

1. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle\lambda,\mu\rangle &= \epsilon_{ab}\lambda^a\mu^b \ , \\ \left[\tilde{\lambda},\tilde{\mu}\right] &= \epsilon_{a'b'}\tilde{\lambda}^{a'}\tilde{\mu}^{b'} \ , \\ p\cdot q &= \langle\lambda,\mu\rangle\left[\tilde{\lambda},\tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a\tilde{\mu}_{a'}) \ . \end{aligned} \tag{1.1}$$

2. Spinor indices are lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$. If the particle has spin one can assign it a positive or negative helicity $h=\pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a\tilde{\lambda}_{a'}}{\langle\mu,\lambda\rangle} \ , \ \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a\tilde{\mu}_{a'}}{\left[\tilde{\mu},\tilde{\lambda}\right]} \ , \ \text{negative helicity} \ . \end{aligned} \tag{1.2}$$

   In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

3. What makes 4-D twistors unique is the existence of the index raising and lifting operations using antisymmetric ϵ tensors. In higher dimensions they do not exist and this causes difficulties. For octonionic twistors with quaternionic components possibly only in $D=8$ the situation changes.

Also massive momenta and any point of M^4 can be expressed in terms of helicity spinors but momenta different by a light-like momenta on some light-like geodesic give rise to the same twistor.

1. One has $p_{a\dot{b}} = \mu_a \tilde{\lambda}_{\dot{b}}$. The spinors μ and λ are determined only modulo opposite complex scalings. One can say that the twistor line (sphere CP_1) determines a point of M^4. A possible interpretation is that the points of CP_1 correspond to the choices of spin quantization axis for momentum p and the scaling changes its direction.

2. The incidence relation $\mu^a = p^{a\dot{b}}\lambda_{\dot{b}}$ is also true for $p^{a\dot{b}}+k\lambda^a\lambda\dot{b}$, for any k, so that the points of a light-like line in M^4 are mapped to a point of the twistor space and therefore would correspond to the same direction of spin quantization axis. Physically this could be interpreted by saying that this is the case because the points with a light-like separation are not causally independent.

Twistors allow an elegant formulation of the kinematics and the Mandelstam variable $s_{ij} = (p_i - p_j)^2 = m_i^2 + m_j^2 - 2p_i \cdot p_j$ can be expressed in terms of twistors by expressing p as

$$p = |\mu\rangle[\tilde{\lambda}| + |\tilde{\mu}]\langle\lambda|$$

Since the states are massive, the inner product $p_1 \cdot 2$ can be expressed as

$$p_1 \cdot p_2 = \langle\lambda_1\mu_2\rangle[\tilde{\lambda}_1\tilde{\mu}_2] \ ,$$

Since <> and [] are not complex conjugates of each other and can be regarded as independent complex variables. For massless case this is not case that the expression for $p_1 \cdot p_2$ reduces to modulus squared=

The notion of momentum twistor is nicely explained by Claude Durr in the slides of a talk "Momentum twistors, special functions and symbols" (`https://cutt.ly/AY7QYv3`). Momentum twistors are essential in the twistorial construction of the scattering amplitudes.

1. The notion makes sense for planar diagrams for which the momenta can be ordered. For non-planar diagrams this is not the case. Whether the embedding of non-planar diagrams to a surface with some minimal genus could allow the ordering (if two lines which cross in plane, the other line could go along the handle), is not clear to me.

2. One ends up with the momentum twistors Z_i, as opposed to ordinary twistors denoted by W_i, by performing a Fourier transform of a massless twistor amplitude, which is holomorphic in variables $\langle\lambda_i\lambda_j\rangle$ so that the relation of the helicity spinor μ to λ is essentially that of wave vector to a position vector. The helicity spinor pair $Z = (\omega, \lambda)$, where ω is essentially the complex conjugate of λ in massless case is replaced with (ω, μ). This transform makes sense also in the massive case.

   Momentum twistors correspond to what are called dual or area momenta. The ordinary momenta p_i can be expressed as their differences $p_i = x_{i+1} - x_i$ and area momenta in turn as as $x_i = \sum_{1\leq k\leq i} x_k$. The term area momentum comes from the observation that the planar diagrams divide the plane into disjoint regions and the area momenta can be assigned to these regions.

3. At the level of symmetries the possibility of momentum twistors means extension of the algebra of conformal symmetries of M^4 to a Yangian algebra whose generators are labeled by non-negative integers and which are poly-local so that the corresponding charges contain multilocal contributions (note that potential energy is bilocal and somewhat analogous notion). The generators generating conformal symmetries in the space of area momenta correspond to generators of conformal weight $h = 1$ and whereas ordinary conformal generators have conformal weight $h = 0$.

Remark: TGD suggests the interpretation of two kinds of twistors in terms of $M^8 - H$ duality. Area momenta and momentum twistors could correspond to M^8 level and ordinary momenta and twistors to H level. M^8 indeed has interpretation as analog of momentum space and $M^8 - H$ duality as the TGD counterpart of momentum-position duality having no generalization in quantum field theories where momentum and position are not dynamical variables.

1.1.2 MHV amplitudes as basic amplitudes

The following comments about MHV amplitudes sketch only the main points as I see them from my limited TGD perspective. One reason for this, besides my very limited practical experience with these amplitudes, is that it seems that The TGD approach in its recent form does not force their introduction.

The article of Elvang and Huang [B12] provides an excellent summary about the construction of twistor amplitudes explaining the important details (see also the slides by Claude Durr at https://cutt.ly/AY7QYv3). Maximally helicity violating (MHV) amplitudes with $k = 2$ negative helicity gluons are defined as tree amplitudes of say $\mathcal{N} = 4$ SUSY and involve gluons and their superpartners. It is convenient to drop the group theory factor $Tr(T_1T_2 \cdots T_n)$ related to gluons.

NMHV amplitudes have $k > 2$ and can be classified by the number of loops as also $k = 2$ diagrams. NMHV diagrams are constructible in terms of MHV diagrams and the construction is known as BSFW construction which by recursion reduces these diagrams to $k = 2$ diagrams, about which 3-gluon vertices is the simplest example. To my amateurish understanding, it is not yet clear whether also the planar Feynman diagrams allow twistorialization. The basic problem is that the area moment x_i with $p_i = x_{i+1} - x_i$ must be ordered and this is not possible for non-planar diagrams.

The construction gives a recursion formula allowing to express the amplitudes in terms of MHV tree amplitudes. Rather remarkably, all loop amplitudes are proportional to the tree level MHV amplitudes so that the singularity structure of the amplitudes is completely determined by the MHV amplitudes. A holography at the level of momentum space is realized in the sense that the singularities dictate the amplitudes completely.

1. The starting point is the observation that tree amplitude with $k = 0$ or $k = 1$ vanishes. The simplest MHV amplitudes have exactly $k = n - 2$ gluons of same helicity- taken by a convention to be negative - have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle \lambda_x, \lambda_y \rangle^4}{\prod_{i=1}^{n} \langle \lambda_i, \lambda_{i+1} \rangle} \tag{1.3}$$

   When the sign of the helicities is changed $\langle .. \rangle$ is replaced with [..].

2. It is essential that the amplitudes are expressible in terms of the antisymmetric bi-linears $\langle \lambda_i, \lambda_j \rangle = \epsilon^{ab} \lambda_{i,a} \lambda_{j,b}$. This implies holomorphy and homogeneity with respect to $\langle \lambda_i, \lambda_j \rangle$ follows for massless field theories by the cancellation of the []:s or ¡¿:s of spinor inner products with [] or ¡¿ appearing in $p_i \cdot p_j$ appearing in the massless propagator.

3. $k = 2$ MHV amplitudes take the role of vertices in the construction of amplitudes with $k > 2$ negative helicity gluons. These amplitudes are connected together by off-shell propagator factors $1/P^2$. MHV diagrams allow to develop expressions for the planar on tree amplitudes and also of loop amplitudes using recursion.

4. The treatment of off-mass shell gluons forces to introduce an arbitrary fixed spinor η such that η is not a complex conjugate of λ. η is not the helicity spinor μ assignable uniquely to a massive particle (now a virtual particle). This assumption makes sense for momentum twistors assignable to internal lines of the MHV diagrams since area momenta are in general off-mass-shell.

1.1.3 Yangian symmetry, Grassmannians, positive Grassmannians, and amplituhedron

The work by Nima Arkani Hamed [B14, B17, B13, B1, B5, B28] and other pioneers has led to a very beautiful vision in which the twistorial scattering amplitudes $A_{k,n}$ for $\mathcal{N} = 4$ SUSY are expressible as residue integrals over Grassmannians $Gr_{k,n}$ of integrands which depend on the twistors characterizing the external only via delta functions forcing the integration to surfaces of $Gr(k,n)$. BCFW diagrams and therefore the Grassmannian integrals as their representations are Yangian invariants.

> The amplitudes are defined as residue integrals over $Gr(k,n)$ and contain data about momenta coded by twistors in the arguments of delta functions. The counterparts of the $< ij >$ or $ij]$ determining the integrand are the Plücker coordinates defined as the k-minors, that is determinants of the $k \times n$ matrices, characterizing the point of $Gr(k,n)$. The included minors are taken in cyclic order and contain subsequent columns [B12] (`https://cutt.ly/yY7QzQg`). One integrates over the k-planes, or equivalently, over $n-k$-planes, of C^n and the integral is residue integral. $Gr(n,k) = U(n)/U(k) \times U(n-k)$ has also an interpretation as a flag-manifold. The residues are located in the positive Grassmannian $Gr^{\geq 0}_{n,k}$. The integral reduces to a mere residue selecting a special k-plane of Grassmannian (note that a gauge fixing eliminating gauge degrees of freedom due to the $Gr(k)$ and $Gr(n)$ symmetries is performed). In the massless case, the delta function constraints state that the n-helicity spinors are orthogonal to $k-D$ and $n-k$-D planes of $GR_{k,n}$ and the conditions imply momentum conservation. In the massive case, the momentum conservation constraint states $\sum p_i = |\mu_i > [\tilde{\lambda}_i| + |\tilde{\mu}]_i < \lambda|_i = 0$. Also now, the interpretation as the inner product of n-helicity spinors is suggestive. A technically important detail is that the quadratic momentum delta function $\delta(\sum_i \lambda_i la\tilde{m}bda_i)$ is forced by a product of linear delta function constraints associated with part of $Gr_(k,n)$ to two parts corresponding to k and $n-k$ gluons with opposite helicities. The gauge invariance of these parts with respect to $Gl(k)$ and $Gl(n-k)$ allows a coordinate choice in $Gr(k,n)$ simplifying the calculation drastically.

This work has led to the notions of positive Grassmannian $Gr^{\geq 0}_{k,n}$ [B12] (`https://arxiv.org/abs/2110.10856`) defined as a sub-space of Grassmannian in which all Plücker coordinates defined by the $k \times k$ minors appearing in the expression of the twistor amplitude are non-negative. Any $n \times (k+m)$, whose minors are positive induces a map from $Gr^{\geq 0}_{k,n}$ whose image is the amplituhedron $\mathcal{A}_{\backslash,\Vert,\updownarrow\Updownarrow}$ (`https://arxiv.org/pdf/1912.06125.pdf` and `https://en.wikipedia.org/wiki/Amplituhedron`) introduced by Arkani-Hamed and Trnka. For $m = 4$ the BSFW recurrence relations for the scattering amplitudes can be used to produce collections of $4k$-dimensional cells in $Gr^{\geq 0}_{k,n}$, whose images are conjectured to sub-divide the amplituhedron. $\mathcal{A}_{\backslash,\Vert,\updownarrow\Updownarrow}$ generalizes the positive Grassmannian.

Tree-level amplituhedron can be regarded as a generalization of convex hull of external data and the scattering amplitudes can be extracted from a unique differential form having poles at the boundaries of the amplituhedron.

1.2 How to generalize twistor amplitudes in the TGD framework?

Twistor approach works so beautifully in massless case such as $calN = 4$ SUSY because the scattering amplitudes for massless gluons can be written as holomorphic homogeneous functions of arguments constructed from the helicity spinors characterizing the momenta of the external massless particles.

It is always best to start from a problem and the basic problem of the twistor approach is that physical particles are not massless. In the massive QFT, one cannot write a simple twistorial expression of the amplitudes, which would be holomorphic homogenous polynomials in the twistor components and involve only the twistor bilinears $< ij >$ or $[ij]$. The reason is that the external and internal particles are massive. For massive particles, the Mandelstam variables $s_{ij} = (p_i - p_j)^2$ do not factorize as $s_{ij} =< ij > [ij]$.

> The intuitive TGD based proposal has been that since quark spinors are massless in 8-D sense in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes. However, no obvious mechanism has been identified. One step in this direction was however the realization that in H quarks propagate with well-defined M^4 chiralities and only the $D^2(H)$ of Dirac operator annihilates the spinors. M^8 quark momenta are in general complex as algebraic integers. They are identifiable as the counterparts of the area momenta x_i of the momentum twistor

space whereas H momenta can be identified as ordinary momenta. The total momenta of Galois confined states have as components ordinary integers and the momentum spectra in H and M^8 are identical by M^8-H duality. The mass squared spectrum is quantized as integers for Galois confined states in accordance with supersymplectic invariance implying "stringy" mass spectrum. The natural first guess is that in H the free quarks satisfy the Dirac equation $D(H)\Psi=0$. There are however excellent reasons to ask whether H spinors satisfy $D(M^4)\Psi=0$. If so, the M^8 spinors as octonionic spinors would correspond to off-mass shell states with mass squared values given by the roots $m^2=r_n$ of P, which in general are complex. This conforms with an idea that the super-symplectic conformal weights have an imaginary part and conformal confinement forces total conformal weights to be integers. This would give rise to twistor holomorphy.

The outcome is an extremely simple proposal for the scattering amplitudes.

1. Vertices correspond to trilinears of Galois confined many-quark states as states of super symplectic algebra acting as isometries of the "world of classical worlds" (WCW).

2. Both M^8 and H quarks are on-shell with H momentum p_i and M^8 momenta x_i, x_{i+1}, $p_i = x_{i+1}-x_i$. Dirac operator $x^k\gamma_k$ restricted to a fixed helicity L, R appears as a vertex factor and has an interpretation as a residue of a pole from an on-mass-shell propagator D so that a correspondence with twistorial construction becomes obvious. M^8 quarks are effectively massless but off-shell but the helicity spinors μ and λ are independent unlike for massless particles.

3. The solutions of the octonionic Dirac operator $D(X^4)$ is expressible in terms of helicity spinors of given chirality and this gives two independent holomorphic factors: in the case of massless theories they would be complex conjugates and the other one must cancel by a spinor contraction.

4. The scattering amplitudes would be rational functions in accordance with the number theoretic vision.

5. In the TGD framework the construction of the scattering amplitudes for a single space-time surface is not enough. One must also understand what the WCW integration could mean at the level of scattering amplitudes based on cognitive representations. WCW integration would be naturally replaced by a summation over polynomials such that the corresponding 4-surface correspond at the level of H maxima of the Kähler function. Monic polynomials are highly suggestive.

   A connection with the p-adicization emerges via the identification of the p-adic prime as one of the ramified primes of P. Only (monic) polynomials having a common ramified prime are allowed in the sum. The counterpart of the vacuum functional $exp(-K)$ is naturally identified as the discriminant D of the extension associated with P and p-adic coupling constant evolution emerges from the identification of $exp(-K)$ with D. This leads to the proposal that discriminant equals the exponent of Kähler function. This forces the identification of p-adic prime as ramified prime and fixes coupling constant evolution to a high degree.

1.3 Scattering as recombination of quarks to Galois singlets

The view about scattering event is as follows.

1. External particles are Galois singlets consisting of off-mass shell massless quarks with mass squared values coming as roots of the polynomial P characterizing the interaction region. External particles are characterized by polynomials P_i satisfying $P_i(0)=0$. P is identified as the functional composite of P_i since it inherits the roots (mass squared values) of the incoming particles. The TGD view about cognitive state function reduction [L26] allows only cyclic permutations of P_i in the superposition.

2. The scattering event is essentially a re-combination of incoming Galois singlets to new Galois singlets and quarks propagate freely: hence OZI rule generalizes. Also a connection with the dual resonance models emerges. Finiteness is manifest since the integration of virtual moments is restricted to a summation over a finite number of mass shells.

1.4 Comparison with the gauge theory picture

There are several differences between the standard twistor approach applied in gauge theories and the TGD based vision.

1. Vertices involve external H line and two internal N^8 lines. If it indeed does not make sense to speak about internal on-mass-shell quark lines in H, the BCFW construction using MHV amplitudes as building bricks and utilizing now also internal H quark lines, is not needed. One can of course ask, whether the M^8 quark lines could be regarded as analogs of lines connecting different MHV diagrams replaced with Galois singlets. It seems that also Grassmannians, positive Grassmannians, and amplituhedron are unnecessary.

2. The identification of the twistor amplitudes as Yangian invariants is extremely attractive. The proposal has been that the super-symplectic algebra (SSA) and the extended half-Kac Moody algebra of isometries acting as symmetries of WCW extend to Yangians and that the higher charges of Grassmannisans with conformal weight $h > 0$ correspond to multiparticle contributions to conserved charges with potential energy as a very familiar 2-particle example.

   Hence the TGD based construction should produce the scattering amplitudes as Yangian invariants. One cannot of course exclude the possibility that the integration over the "world of classical worlds", which is not considered in this article, could produce analogs BCFW diagrams and their Grassmannian representations.

   Since ordinary particles correspond basically to massless Galois singlests with mass resulting from p-adic thermodynamics, it is very natural to expect that the QFT limit of TGD is a massless QFT. At this limit, the twistor Grassmannian approach would be very natural.

3. Another difference relates to the M^4 conformal invariance of the twistor approach. M^4 conformal invariance is not a symmetry of TGD and the fact that quarks in M^8 are massive in the M^4 sense, reflects this. Massivation forces to extend the twistor holomorphy to both bi-spinors defining the twistor for massive momenta. By the properties of M^8 mass, the masses do not appear explicitly in the amplitudes so effectively the M^8 quarks are massless off-mass shell states. The Yangians would be therefore associated with various super-symplectic algebras rather than with the M^4 conformal group.

4. In the TGD framework, the loop corrections are predicted to vanish and the scattering amplitudes for a given space-time surface would therefore be rational functions in accordance with the number theoretic vision. The absence of logarithmic radiative corrections is not a problem: the coupling constant evolution would be discrete and defined by the hierarchy of extensions of rationals. Also this supports the view that Grassmannians are not needed.

1.5 What about unitarity?

Unitary, locality, and the failure to find the twistorial counterparts of non-planar Feynman diagrams are the basic problems of the twistor Grassmannian approach. Also the non-existence of twistor spaces for most Riemannian manifolds is a problem in GRT framework but in TGD the existence of twistor spaces for M^4 and CP_2 solves this problem. In the TGD framework, the replacement of point-like particles with 3-surfaces leads to the loss of locality at the fundamental level. The analogs of non-planar diagrams are eliminated since only cyclic permutations of P_i are allowed.

This leaves only the problem with unitarity. Unitary is essentially a non-relativistic concept and unitary time evolution is a completely ad hoc notion. My feeling is that this problem reflects a lack of some deep principle. In the spirit of Einstein's program for the geometrization of physics, I have proposed in [L27] a geometrization of the state space. Replace the unitary S-matrix with the Kähler metric of Hilbert space. If this metric is non-trivial it is by infinite dimension highly unique. The unitarity conditions are replaced with the conditions $g^{A\overline{B}}g^{\overline{B}C} = \delta^A_C$. The twistorial scattering amplitudes as zero energy states define the Kähler metric $g_{A\overline{B}}$ of quark state space, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric.

Scattering probabilities are identified as products of covariant and contravariant matrix elements of the metric and are complex but real and imaginary parts are separately conserved. The interpretation in terms of Fisher information is possible. Due to the infinite-D character of the state space, the Kähler geometry exists only if it has a maximal group of isometries and is a unique constant curvature geometry. Also the interpretation of this approach in zero energy ontology is discussed.

1.6 Objections and critical questions

Objections and critical questions are the best way to make progress by making the picture more precise, and allowing us to see which assumptions might not be final. For instance, twistor holomorphy, M^4 conformal symmetry number theoretically, and many other arguments strongly suggest that free quark spinors do not satisfy $D(H)\Psi = 0$ but $D(M^4)\Psi = 0$ and are therefore massless. The propagation of any massive particle along a light-like geodesic is however effectively massless and CP_2 type extremals have light-like M^4 projection so that one must leave this issue open.

1.7 Number theoretical generalizations of scattering amplitudes

Last section discusses the number theoretical generalizations of the scattering amplitudes. For an iterate of fixed P (say large number of gravitons), the roots of the iterate of P defined virtual mass squared values, approach to the Julia set of P. The construction of scattering amplitudes thus leads to chaos theory at the limit of an infinite number of identical particles.

The construction generalizes also to the surfaces defined by real analytic functions and the fermionic variant of Riemann zeta and elliptic functions are discussed as examples.

2 TGD related considerations and ideas

The goal is to generalize twistorial construction of scattering amplitudes in the simplest possible manner to the TGD framework. One of the key challenges is the twistorial description of massivation. In this section I summarize briefly the ideas of TGD which seem to be relevant for the construction of the twistor amplitudes.

2.1 The basic view about ZEO and causal diamonds

In the following are listed the ideas and concepts behind ZEO [K22] that seem to be rather stable.

1. General Coordinate Invarince (GCI) plays a crucial role in the construction of the Kähler geometry of WCW and implies holography, Bohr orbitology and zero energy ontology (ZEO) [L15, L35] [K22].

2. X^3 is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. A finite failure of determinism is however possible and is discussed in [L38]. Preferred extremals would be simultaneous extremals of both volume action and Kähler action outside singularities and thus minimal surfaces analogous to soap films spanned by frames. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD=$cd\times CP_2$) defined as intersection of future and past directed light cones (cds) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. ?? Fig. ??**).

4. The opposite light-like boundaries of CD correspond for fermions dual vacuums (bra and ket) annihilated by fermion annihilation - *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in the fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD. This seems to be the correct view in H. At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing would be by mass shells (hyperboloids) at the level of M^8 and by CDs with same center point at the level of H.

5. Zeno effect can be understood if the states at either cone of CD do not change in "small" state function reductions (SSFRs). SSFRs are analogs of weak measurements (`https://cutt.ly/nURW3QE`). One could call this half-cone call as a passive half-cone. I have also talked about passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate identifiable as geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change. The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like deterministic time evolution leading to the final state [L12] as observed by Minev *et al* [L12].

7. h_{eff} hierarchy [K11, K12, K13, K14] implied by the number theoretic vision [L20, L21] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

8. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. Provides also a mechanism of self-organization [L13]: dissipation with reversed arrow of time looks like self-organization [L46].

2.2 Galois confinement

The notion of Galois confinement emerged originally in TGD inspired quantum biology [L46, L24, L28, L32]. Galois group for the extension of rationals determined by the polynomial defining the space-time surface $X^4 \subset M^8$ acts as a number theoretical symmetry group and therefore also as a physical symmetry group.

1. The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L24, L32, L31]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units.

2. The idea generalizes also to nuclear physics and suggests an interpretation for the findings claimed by Eric Reiter [L36] in terms of dark N-gamma rays analogous to BECs and forming Galois singlets. They would be emitted by N-nuclei - also Galois singlets - quantum coherently [L36]. Note that the findings of Reiter are not taken seriously because he makes certain unrealistic claims concerning quantum theory.

It seems that Galois confinement might define a notion, which is much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Why not also hadrons could be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L14, L23, L4].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states? The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action inside the space-time surface is not well-defined. At the level of M^8 octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H. In contrast to this, weak gauge transformations acting as holonomies act in the tangent space of H.

   One can argue that the symmetries of H and even of WCW should/could have some kind of reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (momenta) of M^8_c with coordinates, which are algebraic integers for the extension.

   Momenta as points of M^8_c would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of (in general complex) momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are algebraic, possibly complex, integers.

   Also other quantum numbers, in particular color, could correspond to representations of the Galois group. In the case of angular momentum, Galois confinement would allow algebraic fractional angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L, the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential phase factors, which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers, the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

2.3 No loops in TGD

There are several arguments suggesting that there is no counterpart for loops of quantum field theories (QFTs) in TGD. Purely rational scattering amplitudes are required by number theoretic vision but the logarithmic corrections from loops would spoil the number theoretic beauty.

Loops however give rise to coupling constant evolution, which is a physical fact. What could be the TGD counterpart of coupling constant evolution?

1. The number theoretic and p-adic coupling constant evolutions, which are discrete rather than continuous, look natural. The effective coupling constant should be renormalized because the allowed momentum exchanges depend on the roots of a polynomial P or at least on their number. If the p-adic prime p corresponds to a ramified prime of extension, the dependence of the effective coupling parameters on the extension of rationals defined by P implies dependence on the prime p characterizing the p-adic length scale. The emerging picture will be described in more detail in the next section.

   In the scatterin amplitudes, a power of coupling g identifiable as Kähler coupling constant g_K appears. Also the factors from Galois singlets appear as well as the states, which correspond to the super-symplectic representations.

It seems that for given external momenta a sum of several terms appear. If the number of momenta is small, a higher dimension of extension gives a larger number of diagrams and this could lead to number theoretic coupling constant evolution. If a given extension of rationals prefers some p-adic primes, mot naturally the ramified primes of the extension, number theoretic coupling constant evolution translates to a p-adic coupling constant evolution.

2. Does the integration over the WCW give Kähler coupling strength and various couplings or is Kähler coupling present at vertices from the beginning? The latter option would look natural. $M^8 - H$ duality strongly suggests that the exponent $exp(-K)$ of Kähler function K defining vacuum functional has a number theoretic counterpart. The unique counterpart would be the discriminant of the polynomial P and suggests that the value of $exp(-K)$ is equal to discriminant for maxima of K, which would naturally correspond to the space-time surface defining the cognitive representation.

2.4 Twistor lift of TGD

One could end up with the twistor lift of TGD from problems of the twistor Grassmannian approach originally due to Penrose [B27] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B11, B6, B18, B3]. For a very readable representation see [B12].

Twistor lift of TGD [L3, L18, L19] generalizes the ordinary twistor approach [L10, L11]. The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense. This gives hopes about massive twistorialization.

The basic recipe is simple: replace fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

he pleasant surprise was that the wistor space has the necessary Kähler structure only for M^4 and CP_2 [A4]: this had been discovered already when started to develop TGD! Since the Kähler structure is necessary for the twistor lift of TGD (the action principle is 6-D variant of Kähler action), TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory [L7]. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

2.5 Yangian of supersymplectic algebra

The notion of Yangian for conformal symmetry group of Minkowski space plays a key role in the construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY as Yangian invariants. There are excellent reasons to expect that also in TGD the scattering amplitudes are Yangian invariants.

2.5.1 Yangian symmetry

The notion equivalent to that of Yangian [A5] [B8, B9, B22] was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras.

The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [L2]. Besides ordinary product in the enveloping algebra there is co-product Δ, which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B8]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with the discrete index n being replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N}=4$ SUSY). One of the conditions conditions is that the tensor product $R\otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n=0$ and $n=1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n=1$ generators with themselves are however something different for a non-vanishing deformation parameter h.

Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h=0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n>0$ are $n+1$-local in the sense that they involve $n+1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2.5.2 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, there is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N}=4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A1] and Virasoro algebras [A2] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras.

2. In the twistor approach conformal symmetries of M^4 are crucial. The isometries of H do not include scalings and inversions. The massless states of the super-symplectic representation would allow conformal invariance of M^4 as dynamical symmetries.

   There are however several alternatives.

   (a) The spectrum of the Dirac operator $D(H)$ contains only right-handed neutrino ν_R as a massless state and if M^4 Kähler structure is assumed it becomes tachyon.

   (b) The second option is that $D(M^4)$ annilates spinor modes. Dirac propagator would reduce to a delta function in CP_2 degrees of freedom. This option is favored by M^8-H duality and also by the associativity of the octonionic spinors implying that M^8 momenta reduce to M^4 momenta. This is actually achieved by a suitable choice of $M^4\subset M^8$ always.

   (c) If $D(M^4)$ contains no coupling to M^4 Kähler gauge potential $A(M^4)$, on-mass-shell quarks are massless and realize M^4 conformal invariance. The appearance of roots polynomials as mass squared values in quark propagators would realize number theoretic breaking of M^4 conformal invariance at the level scattering amplitudes and allow twistor holomorphy.

If $A(M^4)$ coupling is present, all quarks appear as spin doublets with positive and negative mass squared. M^4 conformal symmetry at the quark level is achieved only at long length scales when the spin term vanishes. The quark propagator in the scattering amplitudes would contain the coupling to $A(M^4)$ so that twistor holomorphy seems to be lost. M^4 gauge potential could explain small CP breaking, and one can imagine that the induced M^4 gauge potential appears only in the modified Dirac equation for the induced spinors.

3. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($cd \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones.

   The polygon with light-like momenta would be naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

4. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $cd \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups.

   This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

2.5.3 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

2.5.4 How could the Yangian structure of the super-symplectic algebra emerge?

The isometries of WCW should generalize conformal symmetries of string models and supersymplectic transformations of the light-like boundary of CD are a highly natural candidate in this respect.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [K3].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

Consider next the general structure of the super-symplectic algebra (SSA).

1. The SSA and the TGD analogs of Kac-Moody algebras assignable to light-like partonic 3-surfaces have the property that the conformal weights assigned to the light-like coordinate r are non-negative integers. One can say that they are analogs of "half"-Kac-Moody algebras. Same holds true for the Yangian algebras, which suggests that these algebras could extend to Yangian algebras.

2. SCA (and also the Kac-Moody analogs) has fractal hierarchies of sub-algebras isomorphic to the algebra SSA itself at the lowest level. The conformal weights of sub-algebra SSA_n an n-multiplets of those of SSA: one obtains hierarchies of sub-algebras $SSA \supset SCA_{n_1} \supset SSA_{n_2 n_1}, ...$

3. This leads to the proposal that there is a hierarchy of analogs of "gauge symmetry" breakings. For the maximal "gauge symmetry", the entire SSA annihilates the states and classical Noether charges vanish. For SSA_n, only SSA_n and the commutator $[SSA_n, SSA]$ annihilate the physical states.

   One can ask whether these hierarchies could correspond to the hierarchies of extensions for rationals defined by the composition of polynomials defining 4-surfaces in M^8 and by M^8-H duality in H.

Cognitive representations play a key role and correspond to many quarks states.

1. Cognitive representations consist of the points of $X^4 \subset M^8$ with $M^4 \subset M^8$ coordinates belonging to an extension of rationals defined by a polynomial P defining X^4. It has become clear that here only the mass shells corresponding to the roots r_n of P need to be considered and that only algebraic integers defining the components of M^4 momenta need to be considered.

2. Cognitive representations consist of only those points which are "active", i.e. contain quark or antiquark. $M^8 - H$ duality maps the cognitive representations to H. The points of a given mass shell to the light-like boundary of CD. Momentum p as a point of $M^4 \subset M^8$ is mapped to a geodesic line starting from the center of CD and yields the image point as its intersection with the boundary of CD. The momenta at a given mass shell are actually mapped to the boundaries of all CDs forming a Russian doll hierarchy with common center points.

3. The cognitive representation codes for the physical states in quark degrees of freedom and should reflect themselves in the properties of the SSA state construction. The natural condition is that the Hamiltonians of SSA generate transformations leaving invariant the image points of cognitive representation at the boundaries of CD. This requires that the Hamiltonians vanish at the points of the cognitive representation. This is achieved if the Hamiltonians are obtained by multiplying the usual Hamiltonians, which can be chosen to define irreducible representations of $SU(2) \times SU(3)$, by a Hamiltonian H_{cogn}, which vanishes at the points of the cognitive representation.

   The condition that also the super-generators vanish at the points of cognitive representation implies that also the corresponding Hamiltonian vector field j vanishes so that at the points of cognitive representation all Hamiltonians vanish and are extrema. One would have a modification of the hierarchy of SSA_n but the gauge conditions would remain as such. These conditions could be regarded as a realization of quantum criticality.

4. The cognitive representation defined by the multi-quark states in M^8 would modify the SSA in H by multiplying its Hamiltonians with H_{cogn}. The level of WCW the role of the subalgebra SSA_{cogn} defined by cognitive representation would be similar to the algebra of isotropy group $SO(3)$ of particle momentum as a subgroup of $SO(3,1)$.

   This suggests that the induction procedure generating the irreducible representations for finite-dimensional Lie groups generalizes. The representations of $SO(3)$ have as an analog the representations of SSA_{cogn}. From these representations one would obtain by general symplectic transformations states analogous to the Lorentz boosts of a particle at rest. Note that for cognitive representations the Galois group acts non-trivially but one would have Galois singlet. One could have it in geometric sense so that the momenta would simply add up as vectors or in quantum sense as a many-quark state, with quarks at different points of the mass shell or at different mass shells.

How could one understand the generalization of the duality between momenta and area momenta?

1. The duality between ordinary momentum space and area momentum space means that dual conformal transformations act on area momenta x_i as symmetries of the scattering amplitudes. At the level of ordinary momenta this symmetry extends conformal symmetry algebra to a Yangian algebra.

2. Is this possible in the case of M^8-H duality? Does SSA realized at CD boundaries have a counterpart at the $M^4 \subset H$ mass shells? The counterparts of SSA transformations in M^8 must map the mass shells to itself and leave the points of the cognitive representation invariant. In the interior of $X^4 \subset M^8$ they would induce a deformation of X^4 consistent with the assumption that X^4 is obtained as a local element of $CP_2 = SU(3)/U(2)$, i.e. the deformation is induced by $SU(3)$ element $g(x)$ acting as octonionic automorphism such that $U(2) \subset SU(3)$ leaves the image point invariant. This would guarantee M^8-H duality.

   This deformation at the mass shell would induce in $X^4 \subset H$ an action having interpretation in terms of a local $SU(3)$ (CP_2) transformation, or possibly an symplectic transformation of CP_2 local with respect to light-cone. At the level of H one has group symplectic transformations of $S^2 \times CP_2$ expressible in terms of Hamiltonian in irreps of $SU(3)$.

3. Could the local $SU(3)/U(2) = CP_2$ transformations be representable as symplectic transformations as the duality would suggest? Does this somehow relate to the facts that both CP_2 and its twistor space $SU(3)/U(1) \times U(1)$ have Kähler structure [A4] and therefore also symplectic structure: this in fact makes CP_2 and M^4 completely unique.

4. What about the M^8 counterparts S^2 Hamiltonians. Could they somehow correspond to quaternionic automorphism group $SO(3)$. Could $SO(3)$ correspond to the allowed symplectic (contact) transformations for the mass shell itself whereas $SU(3)$ would act in the interior of $X^4 \subset M^8$?

The dual conformal transformations induce bilocal transformations in the ordinary Minkowski space and this leads to the notion of Yangian, which also implies higher multi-local actions. Why would be the physical origin of this multilocality?

1. Quantum group structure is involved and bi-local elements should correspond to tensor products $f_{abc}T^b \otimes T_c$ of Lie-algebra generators. This generalizes to higher multilocal states. Galois confinement is a multilocal phenomenon in M^8. M^8-H duality maps this multilocality to H. The simplest bi-local state is the quark-antiquark pair with total momentum which is an ordinary integer (necessarily non-tachyonic even if the roots r_n had negative real parts). Leptons would be tri-local states of quarks in CP_2 scale.

   The multilocality of the Galois confined many quark states in M^8 strongly suggests that the total charges include, besides the 1-local contributions, there are also multilocal contributions to Noether charges.

2. Galois confinement should force the multilocality of the symmetry generators. In particular, since the total momenta of quarks sum up to an ordinary integer, one cannot perform Lorentz transformations for them independently but one must transform several momenta simultaneously in order to guarantee that the total momentum changes in such a manner that Galois confinement condition is satisfied.

   The Galois group acts also on spinors which can have number theoretic analogs of spinor space assignable to algebraic extensions as linear spaces and providing a finite-D number theoretic counterpart for WCW spinors. Therefore the generators of Lorentz transformations must contain bi-local and also n-local terms. Same applies to scalings and conformal transformations and in fact to all other symmetries.

3. In the case of energy, these multilocal contributions could have an interpretation as binding energy or potential energy depending on the distance between the image points of different momenta at the boundary of CD. The question is how these multilocal contributions would emerge in H for the super-symplectic algebra having a representation as classical Noether charges and fermionic Noether charges.

4. The notion of gravitational coupling constant suggests strongly that conserved quantities have besides the local contribution also bilocal contribution for which gravitional Planck constant defines unit of quantization. A possible identification is as a bilocal Yangian contribution.

In $\mathcal{N} = 4$ SUSY, scattering amplitudes are invariants of the Yangian defined by conformal transformations of M^4 and its dual acting in the space of area momenta. Since SSA is proposed to act as isometries of the "world of classical worlds" (WCW), also zero energy states having interpretation as scattering amplitudes should be Yangian invariants.

2.6 $M^8 - H$ duality and twistorialization of scattering amplitudes

The precise formulation of twistor amplitudes has remained a challenge although I have considered several proposals in this direction. The progress made in the understanding of the details of M^8-H duality [L35] motivate the attempts to find more explicit formulation for the scattering amplitudes. The following tries to give a brief overall vision.

1. In its recent form $M^8 - H$ duality predicts the twistor spaces of M^4 and CP_2 and their map to each other having interpretation in terms of 6-D twistor spaces of space-time surfaces as 6-surfaces in the product of the twistor spaces of M^4 and CP_2 replacing space-time surfaces with their twistor spaces in the twistor lift of TGD [L35].

2. Momentum twistors and space-time twistors are related by M^8-duality. M^8 momenta are identified as area momenta different from M^4-momenta in H. The notion of area momentum makes sense only for planar diagrams (it is not clear to me whether the imbedding of diagrams genus g topology could allow a definition of area momentum).

3. In the usual twistor Grassmann approach to massless QFTs, the momenta of internal lines are massless and thus on-mass-shell but complex. The simplest option conforming is that both area momenta x_i and H-momenta p_i are on-mass-shell. Area momenta are indeed in general complex as algebraic integers. For a given polynomial P area mass squared spectrum of quarks is fixed as - in general complex - roots of polynomial P.

4. What looks first like a problem is that H momenta have naturally integer valued components (periodic boundary conditions) and mass squared is integer using a suitable unit determined by the p-adic length L_p for the CD. However, at the M^8 side the momenta have components which are algebraic integers in the extension determined by the polynomial P.

   A natural solution of the problem is provided by Galois confinement requiring that momentum components of confined states, which are Galois singlets, are integer valued rather than algebraic integers. This provides a universal mechanism for the formation of bound states. This allows also to have identical spectra for area momenta and ordinary momenta.

   In this picture, the particle would be a Galois singlet formed as a composite of quarks. This notion of a particle is extremely general as compared to the QFT view about elementary particles. The external lines of twistor diagrams carrying H quantum numbers would correspond to states in the representations of super-symplectic algebra (SSA) with Yangian structure.

5. The second quantization for quark fields of H means an enormous simplification. One avoids all problems related to quantization in a curved background. Here an essential role is played by the Kähler structure of M^4 forced by the twistor lift. The generators of supersymplectic algebra and generalized Kac-Moody algebras can be expressed in terms of quark oscillator operators.

6. For given H momenta, the momentum transfers are fixed by $p_i = x_{i-1} - x_i$. The twistor sphere S^2 characterizes the momentum directions. Momentum plus S^2 point s characterized by helicity spinor, defines a point in the twistor space and the geometric interpretation for s is that it characterizes the direction of spin quantization axis.

   The direction of quantization axies is defined only apart from a sign and for spin 1/2 particles the interpretation is as the sign of the spin projection. For massless states the spin axis is parallel to momentum.

7. Galois confinement is crucial. The conditions allow integer valued H momenta only if the area momenta correspond to Galois bound states of quarks. Entire composite of quarks at

the same mass shell propagates as particle with total momentum which has integer components. By duality one can assign to the momentum p_i quantum numbers in supersymplectic representation.

Clearly the notion of a particle as a Galois singlet is very general and corresponds to a multilocal state in both M^8 and H leading also to the notion of Yangian. In H, a particle is a state of a super-symplectic representation. At the level of M^8 it is a Galois confined state. These states correspond to each other.

The basic ideas related to the construction of scattering amplitudes are as follows.

1. M^8-H duality remains as such. M^8-H duality maps. Total area momenta X_i of Galois confined states to points at the boundary of corresponding CD with size determined by the total area momentum by M^8-H duality.

2. Basic vertices for Galois confined states involve many-quark Galois singlet in H with total momentum P_i and 2 many-quark Galois singlets in M^8 involving area momenta X_i and X_{i+1} satisfying $P_i = X_{i+1} - X_i$. The scattering amplitude reduces to quark level and one can say that quark lines connect different mass shells of $X^4 \subset M^8$.

3. 3-vertices are between two M^8 Galois singlets and super-symplectic Galois singlet in H at different M^8 mass shells and lines connecting them carrying momenta calculated at the level of H. Quarks in Galois singlets have collinear rational parts which are analogous to SUSY where monomials of theta parameters assignable to higher spin states are analogous to collinear many-fermion states.

3 Are holomorphic twistor amplitudes for massive particles possible in TGD?

Massive particles are believed to make twistorialization impossible. For instance, for a scalar field theory with Yukawa coupling to fermions, the part of scattering amplitude involving vertex with Yukawa coupling plus scalar propagator gives $g < 12 > \times 1/(p_1-p_2)^2$. For massless particles, one has $(p_1-p_2)^2 =< pq > [pq]$ and the expression reduces to $g/ < pq >$. This is essential for the holomorphy in twistor components in turn reflecting conformal invariance.

In MHV construction the MHV amplitudes with 2 negative helicities are used as building bricks of twistorial representations of more complex planar tree amplitudes and loop amplitudes connecting them with off-mass-shell lines involving propagators. The obvious question is whether this construction could be generalized.

The simplest MHV diagrams would be replaced with diagrams assignable to single CD and involving only on-mass-shell area momenta in M^8 and on-mass-shell area momenta in H as external particles. One would take several diagrams of this kind and connect them by a line carrying off-mass-shell M^8 momentum and quantum numbers of a state in SSA representation. In a given vertex involving this kind of virtual H-line, the on-mass-shell fermion momenta would be replaced by two 2 on-mass-shell area momenta and off-mass-shell momentum of the scalar particle would correspond to M^8 momentum.

The intuitive idea is that somehow 8-D massless at the level of H solves the problem but it is not at all clear whether it is possible to obtain twistor holomorphy somehow. One hint comes from the fact that twistors associated with massive particles involve two independent helicity spinors μ and λ? Could one have holomorphy with respect to both? A further hint comes from the observation that at the level of H tachyonic right-handed neutrino makes possible the construction of massless states. A further hint comes from Galois confinement: could the external particles be Galois confined states and could the propagating particles be quarks in M^8 having complex masses coming as roots of the polynomial P?

3.1 Is it possible to have twistor holomorphy for massive scalar and fermions?

Consider first the simple example of massive fermions and a massive scalar field. Assume that fermions are on-mass-shell with masses m_1 and m_2 and scalar off-mass-shell with mass m.

1. Assume Dirac spinors expressible in terms of left and right handed components. For massive scalar particle, the propagator factor reads as $(p_1 - p_2)^2 - m^2 = m_1^2 + m_2^2 - m^2 - 2(p_1 \cdot p_2)$.

2. The completeness relation for spinor modes reads in massive case as $p^k\gamma_k + m = O(p)$, $O(p) = |p\rangle\, [p| + |p[\, \langle p|$

   One can express $O(p)$ as $p^k\gamma_k = O(p) - m$. One obtains for Dirac spinor with left and right handed parts

   $$2p_1 \cdot p_2 = \frac{1}{4}Tr[(O(p_1) - m)(O(p_2) - m)] = -m^2 - \frac{1}{4}Tr[O(p_1)O(p_2)] \ .$$

   For

   $$m_1^2 + m_2^2 = 2m^2 \ ,$$

   the propagator factor reduces to $1/(Tr(O(p)O(q)) =< pq > [pq]$ as if the particles were massless. The part of the amplitude considered would reduce to $g < pq >$.

3. Could the masses for the generalized twistor diagram satisfy a generalization of the condition $m_1^2 + m_2^2 = 2m^2$ guaranteeing the holomorphy with respect to $< .. >$ or $[..]$? The prediction for spinors would be an effective prediction of massless QFT. Note that this result is also true when the masses are identical. This in turn might relate to SUSY. The additivity of mass squared values might in turn relate to 2-D conformal invariance in which mass squared operator is scaling generator and mass squared values are conformal weights. 2-D conformal invariance would generalize to its 4-D counterpart.

   Could this picture generalize to TGD in such a manner that external on mass states correspond to states constructed in H area momenta are off-mass-shell? It is easy to see that this generalization does not work as such.

3.2 Scattering amplitudes in a picture based on $M^8 - H$ duality

The basic assumptions are inspired by $M^8 - H$ duality, ZEO, and geometric view about helicity spinors.

The first guess is that area momenta x_i are assignable to M^8 quarks and are at complex mass shells $m^2 = r_n$. x_i algebraic integers in the extension determined by a polynomial P. Galois confinement implies that the quark momenta associated with mass shells belong to quark composites forming Galois singlets and have a total momentum, which is integer valued with respect to the p-adic mass scale assignable to the mass shell. Also mass squared values would be integers. For general Galois singlets the momenta are assignable to several mass shells $m^2 = r_n$ and thus multi-local entities in M^8, which suggests possible origin of the Yangian symmetry. The mass shells are mapped to the boundaries of corresponding CD in H by $M^8 - H$ duality mapping p-adic mass scale m to its inverse defining p-adic length scale $L = \hbar_{eff}/m$ implying multi-locality in H. CDs form a Russian doll-like structure.Assume that the incoming momenta p_i are H assignable to supersymplectic representations constructed from spinor harmonics in H for a second quantized quark field. $M^8 - H$ duality suggests that the momentum and mass squared spectra are identical at M^8 and H sides. This conforms with Galois confinement at M^8 side. Particles would be Galois confined multi-quark states. Assume that twistors and momentum twistors have a geometric interpretation so that helicity spinors do not represent fermions but points in the CP_1 fiber of CP_3 as a bundle and the states with given spin correspond to wave functions in CP_2 having also half-integer spins. Twistor amplitudes would be constructed as contractions of these wave functions with the scattering amplitudes that the basic scattering amplitude would be independent of spin. In this framework, the many-quark states constructed by elements of Clifford algebra would be analogous to components of a super-field. By Galois confinement, the rational parts of quark momenta would be collinear, which conforms with the basic idea of SUSY that n-monomials of theta parameters are analogous to states of p collinear fermions. The spin of a given state would correspond to a product of spin 1/2 sherical harmonics in the space defined by the helicity spinor. A huge generalization of the notion of particle would be in question. Particle would correspond to an arbitrary Galois singlet assignable to single CD. This would conform with the WCW picture in which physical states of the Universe correspond to WCW spinor fields identified as zero energy states. Vertices would correspond to the states of Yangian supersymplectic representation identifiable as mode of WCW spinor field and representing general fermionic state analogous to a component of super field but without Majorana condition. In the standard model, all couplings except the coupling of Higgs to itself and to fermions respect helicity conservation. Assume that this is true also in TGD so that one can decompose quark spinors to left and right handed parts and that they can be described by spin wave functions in the fiber of twistor space corresponding to the momentum of the quark. Note however that the helicity twistors would be purely geometric quantities rather than representing spinor basis of a fermion. At the level of the twistor space of H, spin states would be described by partial waves at the twistor sphere. At the level of M^8 twistor space, a completely geometric description as a point of twistor space characterizing momentum and spin quantization axis and the sign of the spin 1/2 projection is possible. Helicity spinors μ and $\tilde{\lambda}$ would characterize the direction of the spin quantization axis as a point twistor sphere S^2. This conforms with the fact that for massive particles the direction of helicity spinor is not unique since the spin μ is determined only apart from a spinor proportional to λ. For massless particles the direction of the quantization axis is unique. Since only quarks with spin 1/2 are fundamental fermions, the twistor sphere with a fixed radius is enough. This interpretation is similar to the interpretation of the twistor sphere of $SU(3)/U(1) \times U(1)$ as a characterizer of the color quantization axes. For many-quark states a common quantization axis would force the spins to be parallel or antiparallel. The sum of spins associated with different momenta as different points of twistor space would be the sum of these spins.

The special twistorial role of quarks as spin 1/2 particles supports the idea that the construction of scattering amplitudes should be reduced to quark level although the physical states are Galois singlets. The situation would be very similar to that in QCD, where the challenge is to understand how the scattering amplitudes between hadrons are constructible in terms of scattering amplitudes for quarks and gluons. The basic problem in QCD is that a mechanism for the formation of bound states is missing: in TGD it is provided by Galois confinement.

The basic assumption is therefore that the quarks in M^8 are on-mass-shell states with $m^2 = r_n$. If Galois singlets were regarded as fundamental objects, one would encounter problems with the description of spin degrees of freedom. Situation is essentially the same as in hadron physics.

One can speak about Galois singlet states as a generalization of super-field but without Majorana conditions with oscillator operator monomials replacing the components of superfield: Galois singlets having quark momenta with parallel rational components would in this sense propagate linearly. Each quark Dirac operator $p^k\gamma_k$ is added to the vertex and is expressible in terms of a pair of holomorphic quantities $< .. >$ and [..] which are independent for massive quarks.

3.3 Twistor amplitudes using only mass shell M^8 momenta as internal lines

The simplest proposal for the twistor amplitudes assignable to single 4-surface assumes that the physical particles correspond to Galois singlets with integer valued momentum components p_i and integer valued mass squared spectrum. The components of quark momenta in M^8 would be algebraic integers.

$M^8 - H$ duality requires that physical states in M^8 and H correspond to each other and have the same mass and momentum spectrum. A stronger form of $M^8 - H$ duality would force the identification of the quark momenta in M^8 and H. Quark momenta would be virtual momenta. If the coupling to M^4 Kähler potential is not present, the twistor holomorphy is achieved if spinor modes satisfy $D(M^4)\Psi = 0$.

3.3.1 What could be the basic assumptions?

The following summarize the assumptions, which look plausible.

1. All quark states in both H and M^8 are on-mass-shell states with momenta which are algebraic integers in the extensions determined by polynomial P determining the quark mass shells $m^2 = r_n$ as its roots. Momenta for Galois singlets could also be rationals but periodic boundary conditions allow only integers.

   The physical states are Galois singlets with integer valued momenta in a given p-adic length scale. Mass squared values are integers and one obtains a stringy mass squared spectrum. By $M^8 - H$ duality the spectra at M^8 and H sides are identical.

2. The analog of the idea that the scattering amplitudes are poles of residue integral in momentum space is adopted. This means that in M^8 the purely algebraic 4-D quark Dirac operators $D(M^4)$, rather than propagators as in Feynman diagrams, act on the vertex defined by the trilinear of 3 Galois singlets (particles do not propagate in momentum space as they do in x-space!). The Galois singlets have an interpretation as representations of super-symplectic algebra.

   The Galois singlet with total momentum $P_i = \sum p_{i,k}$ corresponds to H-state and the two other Galois singlets corresponds to states with area momenta X_i, X_{i+1} having similar decompositions $X_k =\sim x_{i,r}$ in terms of in general complex algebraic integer valued area momenta x_i. The complex on-mass-shell area momenta are analogous to the complex on-mass- shell light-like virtual momenta in the twistor Grassmann approach.

3. The total momentum of the vertex is conserved and gives a constraint on the quark momenta associated with the 3 states. In each vertex one has sum over all possible quark momenta consistent with the Galois singlet property and the structure of the state. Momentum conservation at vertex does not make sense at quark level since fermion number conservation would fail unless one introduces fundamental bosons.

   Momentum conservation constraints $P_i = X_{i+1} - X_i$, which completely fixes the momentum exchanges as $2X_i \cdot X_j = P_i^2 - X_{i+1}^2 - X_i^2 - 2(X_i - X_j)^2$. Momentum conservation implies in ZEO that one can see scattering diagrams as polygons having momenta at mass shells at the half-light-cones of M^8.

4. An essential constraint is that the rational parts of the area momenta x_i are parallel to each other. This gives rise to an analogy with supersymmetry in which one could regard the higher components of the super field as parallelly propagating Majorana fermions.

5. The propagator lines correspond in M^8 to vertex factors with the analog of $D = x_i^k \gamma_k$ acting on Galois singlet i. This would mean that one has a residue of the Feynman propagator. By adding a multiplicative factor m^2, one could equally well use Feynman propagator $1/D = D/m^2$, where $m^2 = r_n$ is quark mass squared. The number of diagrams is limited by the number of roots and only the number of Galois singlets poses a limit to the summation if one considers only amplitudes for a single surface X^4.

   In principle all pairs of Galois singlets in M^8 with a non-vanishing trilinear overlap with a given Galois singlet in H are allowed in the vertex. Note that same Galois singlets can contain quarks assignable to different quark mass shells $m^2 = r_n$.

6. The details of the algebraic extension are not visible in the properties of Galois singlets as analogs of hadrons. The details of algebraic extension are however visible in the details of quark propagators and give rise to a number theoretic coupling constant evolution as will be found. Also the increase of the dimension of extension with the degree of P implies that the number of contributing diagrams increases.

   In principle, also roots r_n with negative rational parts are possible and one cannot exclude tachyonic states. From tachyonic states one can form non-tachyonic ones by requiring that the 3-momenta sum up to zero.

7. The big difference with respect to standard massive QFTs is that although the states are massive, they propagate with well-defined helicities. There is therefore a doubling of helicity spinors appearing as L-R degeneration. The division to positive and negative helicities corresponds to the presence of quarks and antiquarks.

8. It seems that quarks and antiquarks can correspond to the same CD and to the same diagram of the proposed kind. For a single space-time surface BCFW construction does not make sense since it would require an off-mass-shell H particle. One must however notice that the quark propagators bring in mind the $1/P^2$ lines connecting BCFW sub-diagrams and Galois singlets bring in mind the MHV diagrams.

   Can one construct Galois singlets from both quarks and antiquarks? It would seem that in this case the scattering amplitudes involve products of holomorphic and antiholomorphic monomials of the twistor variables. This option looks intuitively more plausible.

3.3.2 A possible solution of the mass problem

The basic problem of the twistor approach is that physical particles are not massless. The intuitive TGD based proposal has been that since quark spinors are massless in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes.

1. The first key observation stimulated by the recent findings about right-handed neutrino candidate [L33] was that although neutrinos are massive, their right-handed component has not been observed. This leads to a proposal that in H quarks should propagate with well-defined chiralities so that only the square of Dirac equation $D^2(H)\Psi = 0$ is satisfied.

2. At the level of M^8 the octonionic M^4 quark spinor reducing to a quaternionic spinor corresponds to H spinors. A spinor with a given chirality can be identified as a helicity spinor λ_{dota} and is annihilated by the operator $p^{a\dot{b}} = \mu^a \lambda^{\dot{a}}$. This makes sense by the fact that in the TGD Universe quarks are the only fundamental particles implying that all other particles, including elementary particles, emerge as their many particle states as Galois singlets.

   The M^8 counterpart of the 8-D massless condition in H is the restriction of the quark momenta to mass shells $m^2 = r_n$ determined as roots of P. The M^8 counterpart of Dirac equation in H is octonionic Dirac equation, which is algebraic. The solution is a helicity spinor $\tilde{\lambda}$ associated with the massive momentum p.

3.3.3 What about tachyons?

Polynomials P allow also roots r_n, which are negative and correspond to tachyonic mass shells. Should one restrict the roots inside the future light-cone? Should one require that the mass squared values of the masses of Galois singlets are non-negative integers? In principle, one can have integer valued momenta with tachyonic mass squared. The sum of this kind of momenta however gives always a non-tachyonic state if the energies are of the same sign as they are for a given half-light-cone.

1. M^4 Kähler structure implies that covariantly constant right- handed neutrino in CP_2 is a tachyon [L33]. This gives rise to the highly desired tachyon required by p-adic mass calculations [K5, K2]: with it the scale of mass spectrum would be huge and given by CP_2 mass. Tachyonic property is not consistent with the unitarity and ν_R cannot appear as a free particle.

2. Situation remains the same if the right-handed neutrino spinor mode is a good approximation for a Galois and color singlet of 3 quarks assignable to the same wormhole throat in H. ν_R as Galois singlet with tachyonic mass can be understood if tachyonic mass squared values are allowed for quarks.

   Could all quark masses could be tachyonic? Could this explain quark confinement? By generalizing slightly, also complex mass squared values for quarks could be seen as tachyonic so that Galois confinement would be essentially quark confinement.

3. A long-standing question has been whether ν_R could generate $N = 2$ SUSY. It seems that the tachyon property does not allow the analog of ordinary SUSY. States without ν_R would have huge masses of order CP_2 mass. One can also say that $calN = 2$ SUSY is broken in CP_2 scale.

3.3.4 Is the proposed picture consistent with coupling constant evolution?

Can one understand the discrete number theoretic coupling constant evolution in the proposed framework? As the number of roots of P increases, the number of scattering diagrams with N external particles with fixed momenta p_i increases since the number of Galois confined states characterized by mass shells $m_i^2 = n_i$ increases.

The number of diagrams contributing to the scattering increases and it becomes possible to speak about number theoretical coupling constant evolution. Otherwise the dependence on polynomials P is rather weak and brings in mind logarithmic coupling constant evolution replaced in TGD by discrete p-adic length scale evolution.

How does this relate to the p-adic coupling constant evolution and p-adic length scale hypothesis $p \simeq 2^k$, k some selected integer? For instance, could the p-adic primes preferred by a given extension correspond to the ramified primes of the extension dividing the product $\prod_i (r_i - r_j)$?

1. The dimensionless roots of $P(x)$ are of the form $r_n = R_n/M_p$, where R_n is the dimensional root of $P(M_p x)$. M_p would define the p-adic mass scale and the p-adic length scale of the corresponding CD. This would suggest that p-adic coupling constant evolution is not related to number theoretic coupling constant evolution.

2. On the other hand, the scattering amplitudes depend on the p-adic scale of the momenta. The reduction of scattering amplitudes to homogeneous functions of the factors $p_i \cdot p_j$ appearing in propagator denominators implies very simple dependence on momenta and the characteristic logarithmic dependence is absent. Does this mean that there should be a correlation between the p-adic length scale and algebraic extension? Why should a given extension prefer some p-adic primes, say ramified primes?

3. What about the vertices between Galois singlets, which involve a trilinear of an on-mass-shell state in H and two M^8 off-mass-shell states? How does the p-adic mass scale manifest itself in the properties of these Galois singlets? The conditions for Galois singlet property are scale invariant and the scale invariance is only broken by the condition that mass squared values are roots of polynomial P.

   $M^8 - H$ duality suggests the identification of the discriminant D of the polynomial as an exponent $exp(-K)$ of Kähler function defining vacuum functional and the identification of p-adic prime as a ramified prime dividing D. The real mass squared value would be determined by the canonical identification $\sum x_n p^n \to \sum x_n p^{-n}$ for ramified prime and depend on P.

4. p-Adic physics depends on the value of p-adic prime p. Could this bring in the p-adic coupling constant evolution and preferred p-adic primes number theoretically? The dimension of extensions of p-adics induced by a given extension of rationals depends on p since some roots exist as ordinary p-adic numbers. If p-adic physics as physics of cognition is essential also for real physics as p-adic mass calculations [K5, K2] suggest, it could force the natural selection of preferred p-adic primes and p-adic length scale evolution.

5. Only the identification of the preferred p-adic primes as ramified primes of extension comes into mind. What could make them so special? The p-adic variant of the polynomial P has a double root in order $O(p) = 0$ for a ramified prime. Double root is the mathematical counterpart of criticality and quantum criticality indeed is the basic dynamical principle of TGD. Could something which is of order $O(p^0)$ become order $O(p)$ for a ramified prime? The roots of P correspond to mass squared values: one would have $m_1^2 - m_2^2 = r_1 - r_2 = O(p)$ p-adically.

   For instance, could it be a generic mass squared scale defined by the difference $m_1^2 - m_2^2$ reduces from $M^2(CP_2)$ to $M(CP_2)^2/p$ for ramified primes or p-adic mass scale $M_p = M^2(CP)/p$ reduces to secondary $\mathrm{p}_a dic mass scale \mathrm{M}_{p,2} = M^2(CP)/p^2$. Could the interpretation be in terms of emergence of a massless excitation as counterpart of quantum criticality. Kind of number theoretic analog of Goldstone boson.

   There is some support for this idea. In the living matter, the 10 Hz biorhythm is fundamental. It corresponds to the secondary p-adic length scale of the electron characterized by Mersenne prime $M_{127} = 2^{127} - 1$ [K5]. 10 Hz biorhythm could correspond to a kind of Goldstone boson.

This argument still leaves open the question why ramified primes near powers of 2 (or of a small integer such as 3 [I1, I2]) should be so special?

6. One can even speculate with the possibility that a kind of natural selection takes place already at this level. A high number of zero energy states could be possible for Galois singlet states associated with very special polynomials. In the functional composition $P_1 \circ P_2$ of polynomials conservation of roots takes place if the condition $P_i(0) = 0$ is satisfied. This could make possible evolutionary hierarchies in which conserved roots would be analogous to conserved genes.

An open challenge is to formulate a precise criterion fixing what diagrams are allowed. The intuitive picture is that the lines of the diagrams connecting mass shells $m_i^2 = n_i$ diagrams define convex polygons.

3.4 How can one include the WCW degrees of freedom?

The above consideration has been restricted to a single cognitive representation defined by a polynomial P. Already the inclusion of color degrees of freedom requires color partial waves in H and the superposition over space-time surfaces related by color rotation and therefore WCW spinor fields.

3.4.1 "Objective" and "subjective" representations of physics

The usual understanding of Uncertainty Principle (UP) requires that one has a WCW spinor field providing for instance the analogs of the plane waves in the center of mass degrees of freedom for 3-surface. This representation at the level of WCW might be called "objective" representation since one looks at the system from the H or WCW perspective. The localization of particles to the space-time surface violates UP in this "objective" sense.

Discrete cognitive representations define in ZEO what might be called a "subjective" representation of the Poincare and color group since one looks at the system from the perspective of a single space-time surface.

1. The "subjective" representations of isometries would be realized as flows inside X^4 rather than in H. The flows would be defined by the projections of Killing vectors on the space-time surface [L35].

2. The "subjective" representation is actually highly analogous to quantum group representation. For instance, for many-sheeted space-time surface, rotation by 2π would not bring the particle to a different space-time sheet and one would obtain charge fractionalization closely related to the hierarchy of many-sheeted structure corresponding to $h_{eff}/h_0 = n$ hierarchy were n is the dimension of the extension of rationals determined by the polynomial P. This representation could be restricted to Cartan algebra and does not require a 2-D system since the Cartan algebra effectively replaces the 2-D system.

3. The notion of "subjective" representation allows to generalize the gravitational and inertial mass to all conserved charges. Inertial charges would relate to the action in H and gravitational charges to the quantum group charges for flows restricted to $X^4 \subset H$. $M^8 - H$ duality indeed maps the momenta at mass shells associated with $X^4 \subset M^8$ to positions at the boundaries of CD and the action of Lorentz symmetries keeps the image points at the boundaries of CD.

3.4.2 Is WCW needed at the level of M^8?

The inclusion of WCW degrees of freedom is necessary for several reasons. WCW provides the "objective" perspective extending the "subjective" perspective provided by scattering amplitudes at a single space-time surface. Also the understanding of classical physics as an exact correlate of quantum physics requires WCW.

WCW has been introduced at the level of H and the question whether the notion of WCW makes sense also at the level of M^8, has remained open for a long time.

It is now clear that the polynomials P alone determine only the mass shells as their roots [L35]. Could the adelization and p-adization alone serve as the counterpart of WCW for M^8?

On the other hand, the interiors of 4-surfaces in M^8 involve the local CP_2 element and at the mass shells one has a local $S^2 = SO(3)/SO(2)$ element. Hence WCW might be realized at both sides as $M^8 - H$ duality suggests. An interesting conjecture is that by $M^8 - H$ duality, the two WCWs are one and the same thing. Therefore it would seem that adelization does not provide the counterpart of WCW in M^8.

3.4.3 Summation over polynomials as M^8 analog for the WCW integration

What could be the "cognitive" M^8 analog of WCW and integration over WCW?

1. The preferred extremal property of space-time surface $X^4 \subset H$ means that it is defined by its intersections with the boundary of CD. $M^8 - H$ duality requires that this is the case also in M^8. This would mean that the polynomial P determines, not only the 3-D mass shells of selected M^4 as its roots contained in $X^4 \subset M^8_c$, but also the 4-surface as an $SU(3)/U(2)$ local deformation of M^4 containing them and mapped to H by $M^8 - H$ duality.

2. In the full theory, one has integration over WCW spinor fields. Number theoretical approach means number theoretically unique discretization using cognitive representation rather than its "active" points (containing quark) defining a representation of the Galois group.

   The natural proposal is that WCW integration reduces to a summation over some subset of polynomials and amplitudes associated with the corresponding cognitive representations for which the area momenta for quarks are algebraic integers. External momenta would be ordinary integers for a given p-adic prime p. Therefore the summation over polynomials of varying degree makes sense for amplitudes with fixed external momenta if one uses extension of rationals containing all extensions defined by the polynomials.

3. The rational coefficients of polynomials would serve as WCW coordinates for the polynomials. The assumption that they are rational, however, creates a problem since the summation over rationals defining the coefficients understood as real numbers does not define an analog of integration measure.

One can imagine two number theoretical solutions of the problem: both are inspired by p-adic thermodynamics [K9, K8].

1. One manner to overcome the problem would be a restriction of the coefficients of P to integers. This is natural if the polynomials are monic polynomials of the form $x^n + an - 1x^{n-1} + ...$ This would mean a loss of scaling invariance since $P(kx)$ is not a monic polynomial. The good news is that this might select preferred p-adic primes and explain even the p-adic length scale hypothesis.

2. For a monic polynomial of degree n, the summation would reduce to a summation over $n-1$ integers. The roots would be powers of a single generating root r_0 giving rise to a basis for algebraic integers, and one would have fractility since the quark mass shells correspond to the powers for the modulus of the generating root. The moduli for the differences of roots would be proportional to the power of the modulus of the root and it would be natural to assign p-adic prime to the root with the smallest modulus. This option is highly attractive both physically and mathematically.

3. One expects a rapid p-adic convergence in the sense that polynomials with coefficients, which differ by a large power of p give to scattering amplitudes p-adically very similar contributions. The sum over these contributions should converge rapidly.

   It would seem that the exponent of Kähler function must enter into the picture and give rise to something resembling p-adic thermodynamics with the Boltzmann weight $exp(-E/T)$ being replaced with p-adic number p^{S/T_p}, where the p-adic temperature T_p is inverse integer and S is integer valued. p-Adic number p^{S/T_p} would correspond to the exponent $exp(-K)$ of Kähler function for the H imasage of the surface associated with P. Canonical identification would map p^{S/T_p} to its p-adic norm p^{-S/T_p} identified with $exp(-K)$.

4. The values of S/T_p correspond to the maxima of the Kähler function K for preferred extremals. These exponents exist p-adically only if the value of Kähler coupling strength α_K as an analog of inverse of a critical temperature satisfies strong number theoretic conditions reducing the exponent to an integer power of p (unless one assumes that also the roots of p can appear in the extension considered). These conditions would give rise to a p-adic coupling constant evolution for α_K and also to a coupling constant evolution as a function of algebraic extension.

5. One expects that these conditions can be satisfied only in a very restricted subset of preferred extremals so that one should assume a localization of WCW spinor field to a subset of maxima of the Kähler function. TGD is analogous to a complex square root of thermodynamics and this kind of localization takes place quite generally (spontaneous magnetization) in thermodynamics and also in quantum field theories (Higgs mechanism).

   For spin glass discussed from the TGD point of view in [L34], this kind of localization occurs also and in the ultrametric topology of the spin glass energy landscape emerges naturally. p-Adic topologies represent basic examples about ultrametric topologies. The TGD inspired proposal indeed is that p-adic thermodynamics [K9, K5] allows the formulation of spin glass thermodynamics free of ad hoc assumptions.

   TGD is Universe is indeed highly analogous to a spin glass in long scales, where the action approaches Kähler action having a huge vacuum degeneracy involving classical non-determinism as the length scale dependent cosmological constant Λ predicted by the twistor lift [L7, L9] approaches zero. An attractive proposal is that this kind of localization has a purely number theoretic origin making p-adic thermodynamics for a suitably chosen value of α_K possible [L34].

6. Also the summation over amplitudes associated with different polynomials of various degrees is in principle possible and could correspond to the summation appearing in perturbation theory and to the summation appearing in p-adic thermodynamics.

One cannot exclude a more general option in which there is a summation over all polynomials with rational coefficients analogous to the summation over the valleys of the energy landscape for spin glass phase.

1. For general rational polynomials, one would have a scaling invariance $P(x) \to P(kx)$. There would be a summation over scaled roots of P and rationally scaled mass shells. For monic polynomials the scaling invariance is lost and this seems the only realistic possibility.

2. One might hope that the summations over rationals assigned to the coefficients of P with fixed degree reduce to a p-adic integration and that a p-adic integration measure for this integral exists and reduces essentially to summation over p-adic integers with a given norm p^k plus to a summation over the norms p^k at the limit when the norm approaches infinity (`https://cutt.ly/UUbit6f`). Here the problem is that there is no natural lower bound on the p-adic norm of the coefficients as for monic polynomials and the integral need not converge.

The restriction to monic polynomials looks highly attractive. Another possible restriction is that polynomials are proportional to x so that the roots of P are also the roots of the functional composite $P \circ Q$. This restriction might be also an outcome of a number theoretical evolution.

3.4.4 M^8 analog of vacuum functional

The vacuum functional as an exponent of the Kähler function determines the physics at WCW level. $M^8 - H$ duality suggests that it should have a counterpart at the level of M^8 and appear as a weight function in the summation. Adelic physics requires that weight function is a power of p-adic prime and ramified primes of the extension are the natural candidates in this respect.

1. The discriminant D of the algebraic extension defined by a polynomial P with rational coefficients (`https://en.wikipedia.org/wiki/Discriminant`) is expressible as a square for

the product of the non-vanishing differences $r_i - r_j$ of the roots of P. For a polynomial P with rational coefficients, D is a rational number as one can see for polynomial $P = ax^2 + bx + c$ from its expression $D = b^2 - 4ac$. For monic polynomials of form $x^n + a_{n-1}x^{m-1} + ...$ with integer coefficients, D is an integer. In both cases, one can talk about ramified primes as prime divisors of D.

If the p-adic prime $p$is identified as a ramified prime, D is a good candidate for the weight function since it would be indeed proportional to a power of p and have p-adic norm proportional to negative power of p. Hence the p-adic interpretation of the sum over scattering amplitudes for polynomials P is possible if p corresponds to a ramified prime for the polynomials allowed in the amplitude.

p-Adic thermodynamics [K5] suggest that p-adic valued scattering amplitudes are mapped to real numbers by applying to the Lorentz invariants appearing in the amplitude the canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adics to reals in a continuous manner

2. For monic polynomials, the roots are powers of a generating root, which means that D is proportional to a power of the generating root, which should give rise to some power of p. When the degree of the monic polynomial increases, the overall power of p increases so that the contributions of higher polynomials approach zero very rapidly in the p-adic topology. For the p-adic prime $p = M_{127} = 2^{127} - 1 \sim 10^{38}$ characterizing electrons, the convergence is extremely rapid.

   Polynomials of lowest degree should give the dominating contribution and the scattering amplitudes should be characterized by the degree of the lowest order polynomial appearing in it. For polynomials with a low degree n the number of particles in the scattering amplitude could be very small since the number n of roots is small. The sum $x_i + p_i$ cannot belong to the same mass shell for timelike p_i so that the minimal number of roots r_n increases with the number of external particles.

3. $M^8 - H$ duality requires that the sum over polynomials corresponds to a WCW integration at H-side. Therefore the exponent of Kähler function at its maximum associated to a given polynomial should be apart from a constant numerical factor equal to the discriminant D in canonical identification.

   The condition that the exponent of Kähler function as a sum of the Kähler action and the volume term for the preferred extremal $X^4 \subset H$ equals to power of D apart from a proportionality factor, should fix the discrete number theoretical and p-adic coupling constant evolutions of Kähler coupling strength and length scale dependent cosmological constant proportional to inverse of a p-adic length scale squared. For Kähler action alone, the evolution is logarithmic in prime p since the function reduces to the logarithm of D.

$M^8 - H$ duality suggests that the exponent $exp(-K)$ of Kähler function has an M^8 counterpart with a purely number theoretic interpretation. The discriminant D of the polynomial P is the natural guess. For monic polynomials D is integer having ramified primes as factors.

There are two options for the correspondence between $exp(-K)$ at its maximum and D assuming that P is monic polynomial.

1. In the real topology, one would naturally have $exp(-K) = 1/D$. For monic polynomials with high degree, D becomes large so that $exp(-K)$ is large.

2. In a p-adic topology defined by p-adic prime p identified as a ramified prime of D, one would have naturally $exp(-K) = I(D)$, where one has $I(x) = \sum x_n p^n = \sum x_n p^{-n}$.

   If p is the largest ramified prime associated with D, this option gives the same result as the real option, which suggests a unique identification of the p-adic prime p for a given polynomial P. P would correspond to a unique p-adic length scale L_p and a given L_p would correspond to all polynomials P for which the largest ramified prime is p.

   This might provide some understanding concerning the p-adic length scale hypothesis stating that p-adic primes tend to be near powers of integer. In particular, understanding about why Mersenne primes are favored might emerge. For instance, Mersennes could correspond to primes for which the number of polynomials having them as the largest ramified prime is especially large. The quantization condition $exp(-K) = D(p)$ could define which p-adic primes are the fittest ones.

The condition that $exp(-K)$ at its maximum equals to D via canonical identification gives a powerful number theoretic quantization condition. Is this condition realized for preferred extremals as extremals of both Kähler action and volume term, or should one regard these conditions as additional conditions?

1. P fixes only the mass-shells as its roots r_n. The real parts of these roots belong to the same M^4. $M^8 - H$ duality is realized by assuming that the mass shells are connected by a 4-surface X^4, which is a deformation of M^4 by a local $SU(3)$ element $g(x)$ such that the subgroup $U(2)$ leaves the points of deformation invariant: this condition gives rise to an explicit form of $M^8 - H$ duality.

   P itself poses no conditions on the local CP_2 element. Could the condition $exp(-K) = I(D)$ for the image of $X^4 \subset M^8$ in H fix the $g(x)$ and thus $X^4 \subset H$?

2. The twistor lift should determine the surface $X^4 \subset H$. The counterpart of twistor lift is defined also at the level of M^8. It maps 6-D surface connecting 5-D mass shells of M^8 as roots of P identified as a local $SU(3)$ deformation of M^6 remaining invariant under $U(1) \times U(1)$ at each point. Hence a point of CP_2 twistor space is assigned to M^6 identified locally as a point of M^4 twistor space.

   One can assign to the twistor space of X^4 as 6-surface $X^6 \subset T(M^4) \times T(CP_2)$ 6-D Kähler action reducing to 4-D Kähler action plus volume term by a dimensional reduction required by the bundle property. One can define the twistorial variant of WCW with the Kähler function K_6 defined by the 6-D Kähler action for X^6. The vacuum functional $exp(-K_6)$ would be the same as for WCW.

   Since S^2 degrees are non-dynamical, the two WCWs are more or less one and the same thing apart from delicacies of non-trivial windings numbers for the maps from the fiber S^2 of $T(X^4)$ to the fibers of $T(M^4)$ and $T(CP_2)$.

3. The $U(2)$ *resp.* $U(1) \times U(1)$ invariant points of the deformation of M^4 *resp.* M^6 would define X^4 *resp.* its twistor space $T(X^4)$. The condition that the image of the deformed M^6 is a preferred extremal of 6-D Kähler action, should determine $g(x)$. $I(D) = exp(-K)$ fixes the 6-D Kähler action action.

4. The formulation of the variational problem in H as a variational problem in $M^4 \subset M^8$ might provide some insight. The 6-D Kähler action for $X^6 \subset H$ naturally assigns an action to the deformed $M^6 \subset M^8$. At the level of M^8, the quantization condition $exp(-K) = I(D)$ plus the boundary conditions defined by the roots of P would select $X^6 \subset M^8$ as a preferred extremal of 6-D Kähler action. This condition could also induce a natural selection of p-adic primes explaining p-adic length scale hypothesis.

3.4.5 The evolution of α_K and of cosmological constant from number theory?

I have considered earlier the evolution of cosmological constant [L2, L7, L9] but it is interesting to look at it in a more detail from the number theoretic perspective.

1. There are three parameters involved: Kähler coupling strength α_K and the winding numbers n_1 and n_2 for the maps of the twistor sphere $T(X^4)$ of $X^4 \subset H$ to the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistor spaces $T(M^4)$ and $T(CP_2)$: these maps essentially identify the latter twistor spheres.

2. The 6-D Kähler action for $X^6 = T(X^4) \subset T(M^4) \times T(CP_2)$ is proportional to Kähler coupling strength and the scale factor $1/R^2$, which is equal to CP_2 radius squared. The recent interpretation is that CP_2 radius corresponds to the Planck length L_{Pl} scaled up by h_{eff}/h_0. So that for $h_{eff} = h_0$, the CP_2 radius would reduce to Planck length apart from a numerical constant.

3. Dimensional reduction is necessary in order that X^6 has the structure of the induced twistor bundle with $X^4 \subset H$ as a base-space. This requires maps of the twistor sphere S^2 of the twistor space $T(X^4)$ of $X^4 \subset H$ to the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$: this map identifies these twistor spheres locally.

4. Dimensional reduction gives rise to the usual 4-D Kähler action and a volume term with a cosmological constant Λ determined by the Kähler action for the S^2 part of 6-D Kähler action. The induced Kähler form in S^2 is the sum of the contributions from $S^2(M^4)$ and $S^2(CP_2)$.

   Unless the winding numbers of the maps differ from unit, the induced Kähler form is zero or twice the Kähler form of $S^2(CP_2)$ depending on the relative sign of the Kähler forms, whose normalization is fixed by the condition that the magnetic flux is quantized to unity. The form of the maps in spherical coordinates (θ, ϕ) for $S^2(X^4)$ is given by $\theta(M^4) = \theta(CP_2) = \theta$ and $\phi(M^4) = n_1\phi$ and $\phi(CP_2) = n_2\phi$.

5. If the winding numbers n_i are different and of opposite sign (assuming the same sign for Kähler forms), the induced Kähler form is given by $J = (n_2 - n_1)J(S^2(CP_2))$, where n_i are positive.

   The induced line element is $ds^2 = d\theta^2 + sin^2(\theta)(n_1^2 + n_2^2)\phi^2$. The determinant $\sqrt{g}$ of the induced metric of S^2 is $\sqrt{g} = \sqrt{n_1^2 + n_2^2}\sqrt{g(CP_2)}$. The contravariant induced Kähler form is given by

   $$J^{\theta\phi} = \frac{g^{\theta\theta}g^{\phi\phi}}{J}_{\theta\phi} = (n_1 - n_2)/n_1^2 + n_2^2 J^{\theta\phi}(CP_2) \ . \tag{3.1}$$

   The Kähler action for S^2 is given by

   $$J^{\theta\phi}J_{\theta\phi}\sqrt{g} = \frac{n_1 - n_2}{\sqrt{n_1^2 + n_2^2}} J^{\theta\phi}(CP_2)J_{\theta\phi}(J(CP_2))\sqrt{g(CP_2)} \ .$$

   For small values of $n_1 - n_2$ and large values of $n_1 \sim n_2$ the contribution to action behaves like $\Delta n/n_1$ and can become arbitrarily small. This would predict that cosmological constant approaches to zero in long p-adic length scales.

   This poses a condition on the integers n_i depending on the p-adic prime p identified as a ramified prime: $\Delta n/n_1$ should behave like the inverse of the p-adic length scale scale. The p-adic length scale evolution of both α_K and integers n_i should follow from the condition that the total action equals to the discriminant D (also a polynomial of discriminant can in principle be considered but this seems artificial). The best one can hope is that $M^8 - H$ duality completely fixes both coupling constant evolutions.

6. For the cosmological constant Λ in cosmological scales, the dark energy density is parameterized as $\rho_{vac} = 1/L^4$, $L \sim L_{neuron}$, where $L_{neuron} \simeq 10^{-4}$ m corresponds to the size scale of neuron.

   This rough estimate follows from the identification $\Lambda/8\pi G = 1/L^4$ giving $L(8\pi G/\Lambda)^{1/4}$. Λ itself would correspond to an inverse of p-adic length square, which is of order of the horizon size (naturally the size of cosmological CD).

3.4.6 Do Grassmannians emerge at the QFT limit of TGD?

There is no obvious use for Grassmannians and related concepts in the construction of twistor amplitudes for a space-time surface associated with a given polynomial P.

However, a given scattering amplitude is a sum of contributions associated with monic polynomials P with an increasing number of roots such that a given p-adic prime p appears as their ramified prime. The discriminant D is assumed to play the role of the vacuum functional $exp(-K)$. This picture is highly analogous to a perturbation theory in a given p-adic length scale.

This suggests that QFT with massless particles is a reasonable approximation of TGD at the QFT limit and that the basic twistorial structures could appear at this limit.

Apart from masses given by p-adic thermodynamics [K5, K2], elementary particles, to be distinguished from fundamental quarks, correspond to massless states so that massless QFT is a good guess for the QFT limit.

The emergence of the massless states requires M^4 Kähler structure forced by the twistor lift [L33]. This breaks the Lorentz symmetry to that of $M^2 \times E2$ and the transversal degrees of freedom correspond to harmonic oscillator type degrees of freedom just as in string model and are characterized by two conformal weights. This spontaneous breaking of Lorentz symmetry characterizes massless particles and hadronic quarks. It makes possible the required tachyonic ν_R making it possible to construct massless ground states in p-adic mass calculations.

1. In M^8, the mass shells in general correspond to complex roots. It is possible to have tachyonic Galois confined states. Covariantly constant right-handed neutrino ν_R would be such a state and needed to construct massless Galois confined physical states in H.

2. In H, only the ν_R constructed from quarks is tachyonic in the approximation that H-spinor mode with Kähler charge $Q_K = 3$ describes leptons as 3-quark Galois singlets. $M^8 - H$ duality suggests that there are no other tachyonic quark states and that all Galois confined states are non-tachyonic so that the momenta belong to the interior of the light-cone in M^8.

3. If the amplitudes in the massless sector are indeed Yangian invariants, Grassmannians would emerge naturally at the QFT limit.

The following series of questions is an attempt to crystallize my ignorance.

1. Could a QFT based on twistorial amplitudes for massless Galois confined external particles in H provide a QFT limit of TGD?

2. Could the sum over amplitudes for different polynomials having a given p-adic prime p as a ramified prime correspond to structure resembling that produced in BCFW recursion?

3. Or could MHV structure emerge at the level of a single polynomial P: this is the case if the quark propagators connecting Galois singlets in the amplitudes be regarded as analogs of the propagators $1/P^2$ connecting parts of MHV amplitudes?

4. How the coupling constant evolution emerges at the QFT limit. Number theoretic approach does not allow logarithmic contributions coming from loops but it would not be surprising if the discrete p-adic coupling constant evolution would allow a logarithmic coupling evolution as a reasonable approximation.

   This is also suggested by the fact that the expression of α_K in terms of discriminant D involves logarithm of the p-adic length scale (, that is p). If $exp(-K)$ equals to the image $I(D)$ under canonical identification, one has $\alpha_K = S/log(I(D))$, where $S = K\alpha_K$ is the total action without the proportionality factor $1/\alpha_K$. For ramified primes α_K is proportional to $1/log(p)$.

3.5 What about the twistorialization in CP_2 degrees of freedom?

The proposed picture does not use CP_2 twistor space at all. One should understand why this is the case.

The treatment of color degrees of freedom involves several new aspects. First of all, color is not a spin-like quantum number in the TGD framework.

1. One can identify colored states as color partial waves in WCW degrees of freedom associated with the center of mass degrees of freedom of 3-surface. H spinor modes can be indeed regarded as color partial waves in H.

   It would seem that one cannot speak of color for a single space-time surface. This is indeed true for an "objective" view about the isometries of H. One can however define the "subjective" representations of the isometries by replacing them with flows defined by the projections of Killing vectors to the space-time surfaces [L35].

For cognitive representations the "subjective" representations could in some situations be reduced to those for the discrete Galois group. One can wonder whether color confinement could reduce to Galois confinement.

2. "Subjective" representations are analogous to quantum group representations [L35]. Objective-subjective dichotomy coud also generalize the inertial-gravitational dichotomy. Note that one can also assign Noether charges to the projected flows. This applies also to supersymplectic symmetries.

The treatment of CP_2 degrees of freedom for the twistor amplitudes remains a challenge and in the following I can only try to clarify my thoughts.

1. Twistor lift strongly suggests that M^8-H duality defines a map of the twistor spaces of H and M^8 to each other. The M^8 counterparts of 6-D twistor space as a surface $X^6 \subset T(M^4) \times T(CP_2)$ would be 6-D surface with a commutative normal space defined by a deformation of complexified Minkowski space M^6 by a local $SU(3)$ element, which is left-invariant under $U(1) \subset U(1)$. This would give a 6-surface Y^6 as a counterpart of the 6-surface. It would seem that M^6 should correspond to the twistor $T(M^4)$, perhaps via the identification with a projective space of M^8 by 2-D projective scalings (perhaps by hypercomplex numbers).

2. This map would preserve S^2 bundle structure so that the twistor spheres of $T(M^4)$ and $T(CP_2)$ would be mapped to each other. This looks strange at first but conforms with the general picture.

   At the level of $T(H)$ twistor wave functions at the twistor spheres S^2 of $T(M^4)$ and $T(CP_2)$, which have been identified, describe spin and color or electroweak quantum numbers (the holonomy group of the spinor connection of CP_2 defining weak gauge group can be identified as $U(2) \subset SU(3)$). This implies a correlation for spin and electroweak spin doublets defined quarks apart from the sign factors.

   In the algebraic picture a single point of M^8 does not define only the momentum of quark momentum: rather quark momentum and spin corresponds to a single point of $X^6 \subset M^8$. Fermi statistics boil down to the condition that each point of X^6 can contain only a single quark. Also now directions of the quantization axis characterize the sign of spin and electroweak spin.

3. Spin-isospin correspondence makes sense only because quarks are both spin and weak isospin doublets. The fact that spin value $\pm 1/2$ corresponds to the two directions of the quantization axis allows all possible pairings of spin and electroweak (or color) isospin.

   This map between $T(M^4)$ and $T(CP_2)$ can be understood at M^8 level and generalizes the mapping of M^4 to CP_2 for a space-time surface with 4-D M^4 projection. There are 4-suraces X^4 for which the dimensions of the projections M^4 or CP_2 projection are not maximal. These 4-surfaces correspond to singularities in which normal space at the points of the singularity is not unique [L38].

   It is enough that the twistor spheres of $T(M^4)$ and $T(CP_2)$ are mapped to each other by locally 1-to-1 projection to the twistor sphere of $T(X^4)$: the base space of the twistor space X^6 need not have 4-D projection to M^4 or CP_2.

4. CP_2 twistors can be regarded as functions of M^4 twistors for a given space-time surface with 4-D M^4 projection. The implications for the construction of scattering amplitudes remain to be understood.

How color degrees of freedom are described at M^8 level? There are two equivalent manners to understand the emergence of CP_2 in M^8-H duality.

1. The normal spaces of $X^4 \subset M^8$ define an integrable distribution. Normal space of X^4 is regarded as a CP_2 point characterizing the deformation of fixed M^4 [L35, L20, L21] so that one obtains M^8-H duality.

This distribution contains an integrable distribution of commutative 2-surfaces in turn defining a 6-D surface X^6, which is a good candidate for the counterpart of twistor space. The assignment of the normal space defines a point of the twistor space $SU(3)/U(1) \times U(1)$.

2. Second view [L35, L20, L21], which emerged only quite recently from the detailed study of the surfaces determined by polynomials P, is that the element of local $SU(3)$ naturally defines a deformation of X^4, which is invariant under left or right action by $U(2) \subset SU(3)$ so that local element of $CP_(2)$ is in question. This means that color $SU(3)$ corresponds to a subgroup of the automorphism group G_2 of octonions. P as such does not etermines the local CP_2 element. What determines P, will be discussed later.

   The counterpart for the distribution of commutative normal spaces of X^6 is a deformation of M^6, or its variant with some signature of metric, defined by a local element of $SU(3)$ such that the imagine point remains invariant by $U(1) \times U(1) \subset SU(3)$ so that it assigns a point of the twistor space $SU(3)/U(1)U(1)$ to each point of X^6.

3. The equivalence of these views is not rigorously proven. Note that the polynomial P itself defines only 3-D complex mass shells as its roots and the 4-surface connecting them is determined from the condition that $M^8 - H$ duality makes sense.

There is an objection against CP_2 type extremal as a blow-up of 1-D singularity of $X^4 \subset M^8$. Is it really possible to describe CP_2 type extremal as 1-D singularity of $X^{\subset} M^8$ using the $U(2)$ invariant map $M^4 \rightarrow CP_2$?

1. The line singularity can be identified as an 1-D intersection of 2 Minkowskian space-time sheets as roots of P. At H level, this leads to a generation of wormhole contact with an Euclidean signature of metric, CP_2 type extremal, connecting the space-time sheets. The Minkowskian space-time becomes Euclidean at the wormhole throats.

2. At each point of 1-D curve L the singularity should be 3-D surface in CP_2. This requires that the normal space is non-unique and the normal spaces at a point x of L form a 3-D surface in CP_2. If one however thinks about how this could be achieved, one ends up with a problem. One can think that the images of an arbitrarily small sphere S^2 around the point of L is a sphere of CP_2. At the limit one would obtain 2-D rather than 3-D surface of CP_2.

3. The $U(2)$ invariant local $SU(3)$ transformation as a deformation of M^4 defining a local CP_2 transformation is not quite enough to describe the situation. The solution is to consider its inverse as a map from CP_2 to M^4 having a singularity at which a 4-D region of CP_2 is mapped to a line of M^4.

4 What about unitarity?

Unitarity is a poorly understood problem of the twistor approach and also of TGD.

4.1 What do we mean with time evolution?

The first questions relate to the identification of the TGD counterpart of S-matrix.

1. Zero energy states correspond to superpositions of pairs of ordinary 3-D states assignable to the opposite boundaries of CD. The simplest assumption corresponds to the idea about state preparation is that the states are unentangled. Unitarity would mean that the 3-D zero energy states at the active boundary of CD are orthogonal if the 3-D states at the passive boundary of CD are orthogonal. The scattering amplitudes considered in this article would naturally correspond to zero energy states. Is there any reason for zero energy states to satisfy this kind of orthogonality?

2. The time evolutions between "small" state function reductions (SSFRs) are assumed to increase the size of CD in a statistical sense at least and affect the states at the active boundary of CD but leave the "visible" part of the state at the passive boundary unaffected. These time

evolutions are proposed to correspond to the scalings of CD rather than time translations. In this case unitarity would look a reasonable property.

The sequence of (ordinary) "big" SFRs (BSFRs) could allow approximate description as being associated with unitary time evolutions with time translations rather than scalings and followed by BSFR changing the arrow of time. The characteristic features of these time evolutions would be polynomial and exponential decay and the relaxation of spin glass would be a key example about time evolution by SSFRs [L34].

4.2 What really occurs in BSFR?

It has been assumed hitherto that a time reversal occurs in BSFR. The assumption that SSFRs correspond to a sequence of time evolutions identified as scalings, forces to challenge this assumption. Could BSFR involve a time reflection T natural for time translations or inversion $I : T \rightarrow 1/T$ natural for the scalings or their combination TI?

I would change the scalings increasing the size of CD to scalings reducing it. Could any of these options: time reversal T, inversion I, or their combination TI take place in BSFRs whereas arrow would remain as such in SSFRs? T (TI) would mean that the active boundary of CD is frozen and CD starts to increase/decrease in size.

There is considerable evidence for T in BSFRs identified as counterparts of ordinary SFRs but could it be accompanied by I?

1. Mere I in BSFR would mean that CD starts to decrease but the arrow of time is not changed and passive boundary remains passive boundary. What comes to mind is blackhole collapse.

   I have asked whether the decrease in size could take place in BSFR and make it possible for the self to get rid of negative subjective memories from the last moments of life, start from scratch and live a "childhood". Could this somewhat ad hoc looking reduction of size actually take place by a sequence of SSFRs? This brings into mind the big bang and big crunch. Could this period be followed by a BSFR involving inversion giving rise to increase of the size of CD as in the picture considered hitherto?

2. If BSFR involves TI, the CD would shift towards a fixed time direction like a worm, and one would have a fixed arrow of time from the point of view of the outsider although the arrow of time would change for sub-CD. This modified option might be consistent with the recent picture, in particular with the findings made in the experiments of Minev *et al* [L12] [L12].

   This kind of shifting must be assumed in the TGD inspired theory of consciousness. For instance, after images as a sequence of time reversed lives of sub-self, do not remain in the geometric past but follow the self in travel through time and appear periodically (when their arrow of time is the same as of self). The same applies to sleep: it could be a period with a reversed arrow of time but the self would shift towards the geometric future during this period: this could be interpreted as a shift of attention towards the geometric future. Also this option makes it possible for the self to have a "childhood"".

3. However, the idea about a single arrow of time does not look attractive. Perhaps the following observation is of relevance. If the arrow of time for sub-CD correlates with that of sub-CD, the change of the arrow of time for CD, would induce its change for sub-CDs and now the sub-CDs would increase in the opposite direction of time rather than decrease.

4.3 Should unitarity be replaced with the Kähler-like geometry of the fermionic state space?

After these preliminaries we can state the key question. Is unitarity possible at all and should it be replaced with some deeper principle? I have considered these questions several times and in [L27] a rather radical solution was proposed.

Assigning an S-matrix to a unitary time evolution works in non-relativistic theory but fails already in the generic QFT and correlation functions replace S-matrix.

1. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space could replace the unitary S-matrix.

2. An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of the Teichmüller matrix.

   Teichmüller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong form of holography (SH), the most natural candidate would be Cartesian product of Teichmüller spaces of partonic 2 surfaces with punctures and string world sheets.

3. Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

4. In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmüller condition guaranteeing non-negative probabilities.

5. Equivalence Principle generalizes to the level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

6. There is also an objection. The transition probabilities would be given by $P(A,B) = g^{A,\overline{B}} g_{\overline{B},A}$ and the analogs for unitarity conditions would be satisfied by $g^{A,\overline{B}} g_{\overline{B},C} = \delta^A_C$. The problem is that $P(A,B)$ is not real without further conditions. Can one imagine any physical interpretation for the imaginary part of $Im(P(A,B))$?

In this framework, the twistorial scattering amplitudes as zero energy states define the covariant Kähler metric $g_{A\overline{B}}$, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric. The problem with the unitarity would disappear.

4.3.1 Explicit expressions for scattering probabilities

The proposed identification of scattering probabilities as $P(A \to B) = g^{A\overline{B}} g_{\overline{B}A}$ in terms of components of the Kähler metric of the fermionic state space.

Contravariant component $g^{A\overline{B}}$ of the metric is obtained as a geometric series $\sum_{n\geq 0} T^n$ from from the deviation $T_{A\overline{B}} = g_{A\overline{B}} - \delta_{A\overline{B}}$ of the covariant metric $g_{A\overline{B}}$ from $\delta A\overline{B}$.

g this is not a diagonal matrix. It is convenient to introduce the notation Z^A, $A \in \{1, ..., n\}$ $Z^{\overline{A}} = Z^{n+k}$, $k = n+1, ..., 2n$. So that the $g_{\overline{B}C}$ corresponds to $g_{k+n,l} = \delta_{k,l} + T_{k,l}$. and one has $g^{A\overline{B}}$ to $g^{k,l+n} = \delta_{k,l} + T^1_{k,l}$.

The condition $g^{A\overline{B}} g_{\overline{B}C} = \delta^A_C$ gives

$$g^{k,l+n}g_{l+n,m} = \delta^k_m \quad . \tag{4.1}$$

giving

$$\sum_l (\delta_{k,l} + T^1_{k,l})(\delta_{l,m} + T_{l,m}) = \delta_{k,m} + (T^1 + T + T^1T)_{km} = \delta_{k,m} \quad . \quad . \tag{4.2}$$

which resembles the corresponding condition guaranteeing unitarity. The condition gives

$$T_1 = -\frac{T}{1+T} = -\sum_{n>1}((-1)^n T^n. \quad . \tag{4.3}$$

The expression for $P(A \to B)$ reads as

$$\begin{array}{l} P(A \to B) = g^{A\overline{B}}g_{\overline{B}A} \\ = [1 - \frac{T}{1+T} + T^\dagger - (\frac{T}{1+T})_{AB}T^\dagger]_{AB} \quad . \end{array} \tag{4.4}$$

It is instructive to compare the situation with unitary S-matrix $S = 1+T$. Unitarity condition $SS^\dagger = 1$ gives

$$T^\dagger = -\frac{T}{1+T} \quad ,$$

and

$$P(A \to B) = \delta_{AB} + T_{AB} + T^\dagger_{AB} + T^\dagger_{AB}T_{AB} = [\delta_{AB} - (\frac{T}{1+T})_{AB} + T_{AB} - (\frac{T}{1+T})_{AB}T_{AB} \quad .$$

The formula is the same as in the case of Kähler metric.

4.4 Critical questions

One can pose several critical questions helping to further develop the proposed number theoretic picture.

4.4.1 Is mere recombinatorics enough as fundamental dynamics?

Fundamental dynamics as mere re-combination of free quarks to Galois singlets is attractive in its simplicity but might be an over-simplification. Can quarks really continue with the same momenta in each SSFR and even BSFR?

1. For a given polynomial P, there are several Galois singlets with the same incoming integer-valued total momentum p_i. Also quantum superpositions of different Galois singlets with the same incoming momenta p_i but fixed quark and antiquark numbers are in principle possible. One must also remember Galois singlet property in spin degrees of freedom.

2. WCW integration corresponds to a summation over polynomials P with a common ramified prime (RP) defining the p-adic prime. For each P of the Galois singlets have different decomposition to quark momenta.

   One can even consider the possibility that only the total quark number as the difference of quark and antiquark numbers is fixed so that polynomials P in the superposition could correspond to different numbers of quark-antiquark pairs.

3. One can also consider a generalization of Galois confinement by replacing classical Galois singlet property with a Galois-singlet wave function in the product of quark momentum spaces allowing classical Galois non-singlets in the superposition.

   Hydrogen atom serves as an illustration: electron at origin would correspond to classical ground state and s-wave correspond to a state invariant under rotations such that the position of electron is not anymore invariant under rotations. The proposal for transition amplitudes remains as such otherwise.

Note however that the basic dynamics at the level of a single polynomial would be recombinatorics for all these options.

4.4.2 General number theoretic picture of scattering

Only the interaction region has been considered hitherto. One must however understand how the interaction region is determined by the 4-surfaces and polynomials associated with incoming Galois singlets. Also the details of the map of p-adic scatting amplitude to a real one must be understood.

The intuitive picture about scattering is as follows.

1. The incoming particle "i" is characterized by p-adic prime p_i, which is RP for the corresponding 4-surface in M^8. Also the "adelic" option that all RPs characterize the particle, is considered below.

2. The interaction region corresponds to a polynomial P. The integration over WCW corresponds to a sum over several P:s with at least one common RP allowing to map the superposition of amplitudes to real amplitude by canonical identification I: $\sum x_n p^n \to \sum x_n p^{-n}$.

   If one gives up the assumption about a shared RP, the real amplitude is obtained by applying I to the amplitudes in the superposition such that RP varies. Mathematically, this is an ugly option.

3. If there are several shared RPs, in the superposition over polynomials P, one can consider an adelic picture. The amplitude would be mapped by I to a product of the real amplitudes associated with the shared RP:s. This brings in mind the adelic theorem stating that rational number is expressible as a product of the inverses of its p-adic norms. The map I indeed generalizes the p-adic norm as a map of p-adics to reals. Could one say that the real scattering amplitude is a product of canonical images of the p-adic amplitudes for the shared RP:s? Witten has proposed this kind of adelic representation of real string vacuum amplitude.

   Whether the adelization of the scattering amplitudes in this manner makes sense physically is far from clear. In p-adic thermodynamics one must choose a single p-adic prime p as RP. Sum over ramified primes for mass squared values would give CP_2 mass scale if there are small p-adic primes present.

The incoming polynomials P_i should determine a unique polynomial P assignable to the interaction regions as CD to which particles arrive. How?

1. The natural requirement would be that P possess the RPs associated with P_i:s. This can be realized if the condition $P_i = 0$ is satisfied and P is a functional composite of polynomials P_i. All permutations π of $1, ..., n$ are allowed: $P = P_{i_1} \circ P_{i_2} \circ ....P_{i_n}$ with $(i_1, ...i_n) = (\pi(1), ..., \pi(n))$. P possesses the roots of P_i.

   Different permutations π could correspond to different permutations of the incoming particles in the proposal for scattering amplitudes so that the formation of area momenta $x_{i+1} = \sum_{k=1}^{i} p_k$ in various orders would corresponds to different orders of functional compositions.

2. Number theoretically, interaction would mean composition of polynomials. I have proposed that so-called cognitive measurements as a model for analysis could be assigned with this kind of interaction [L26, L28]. The preferred extremal property realized as a simultaneous extremal property for both Kähler action and volume action suggests that the classical non-determinism due to singularities as analogs of frames for soap films serves as a classical correlate for quantum non-determinism [L38].

3. If each incoming state "i" corresponds to a superposition of P_i:s with some common RPs, only the RP:s shared by all compositions P from these would appear in the adelic image. If all polynomials P_i are unique (no integration over WCW for incoming particles), the canonical image of the amplitude could be the product over images associated with common RPs.

   The simplest option is that a complete localization in WCW occurs for each external state, perhaps as a result of cognitive state preparation and reduction, so that P has the RP:s of P_is as RP:s and adelization could be maximal.

4.4.3 Do the notions of virtual state, singularity and resonance have counterparts?

Is the proposal physically acceptable? Does the approach allow to formulate the notions of virtual state, singularity and resonance, which are central for the standard approach?

1. The notion of virtual state plays a key role in the standard approach. On-mass-shell internal lines correspond to singularities of S-matrix and in a twistor approach for $\mathcal{N} = 4$ SUSY, they seem to be enough to generate the full scattering amplitudes.

   If only off-mass-shell scattering amplitudes between on- mass-shell states are allowed, one can argue that only the singularities are allowed, which is not enough.

2. Resonance should correspond to the factorization of S-matrix at resonance, when the intermediate virtual state reduces to an on-mass-shell state. Can the approach based on Kähler metric allow this kind of factorization if the building brick of the scattering amplitudes as the deviation of the covariant Kähler metric from the unit matrix $\delta_{A\overline{B}}$ is the basic building bricks and defined between on mass shell states?

   Note that in the dual resonance model, the scattering amplitude is some over contribution of resonances and I have proposed that a proper generalization of this picture could make sense in the TGD framework.

The basic question concerns the number theoretical identification of on-mass-shell and off-mass-shell states.

1. Galois singlets with integer valued momentum components are the natural identification for on-mass-shell states. Galois non-singlet would be off-mass-shell state naturally having complex quark masses and momentum components as algebraic integers.

   Virtual states could be arbitrary states without any restriction to the components of quark momentum except that they are in the extension of rationals and the condition coming from momentum conservation, which forces intermediate states to be Galois singlets or products of them.

   Therefore momentum conservation allows virtual states as on mass shell states, that is intermediate states, which are Galois singlets but consist of Galois non-singlets identified as off-mass-shell lines. The construction of bound states formed from Galois non-singlets would indeed take place in this way.

2. The expansion of the contravariant part of the scattering matrix $T_1 = T/(1+T)$ appearing in the probability

$$\begin{array}{l} P(A \to B) = g^{A\overline{B}} g_{\overline{B}A} \\ = [1 - \frac{T}{1+T} + T^\dagger - (\frac{T}{1+T})_{AB} T^\dagger]_{AB} \quad . \end{array}$$

   would give a series of analogs of diagrams in which Galois singlets of intermediate states are deformed to non-singlets states.

3. Singularities and resonances would correspond to the reduction of an intermediate state to a product of Galois singlets.

4.4.4 What about the planarity condition in TGD?

The simplest proposal inspired by the experience with the twistor amplitudes is that only planar polygon diagrams are possible since otherwise the area momenta are not well-defined. In the TGD framework, there is no obvious reason for not allowing diagrams involving permutations of external momenta with positive energies *resp.* negative energies since the area momenta $x_{i+1} = \sum_{k=1}^{i} p_k$ are well-defined irrespective of the order. The only manner to uniquely order the Galois singlets as incoming states is with respect to their mass squared values given by integers.

4.4.5 Generalized OZI rule

In TGD, only quarks are fundamental particles and all elementary particles and actually all physical states in the fermionic sector are composites of them. This implies that quark and antiquark numbers are separately conserved in the scattering diagrams and the particle reaction only means the-arrangement of the quarks to a new set of Galois singlets.

At the level of quarks, the scattering would be completely trivial, which looks strange. One would obtain a product of quark propagators connecting the points at mass shells with opposite energies plus entanglement coefficients arranging them at positive and negative energy light-cones to groups which are Galois singlets.

This is completely analogous to the OZI role. In QCD it is of course violated by generation of gluons decaying to quark pairs. In TGD, gauge bosons are also quark pairs so that there is no problem of principle.

There is an objection against this picture.

1. If particle reactions are mere recombinations of Galois singlets with Galois singlets, the quark and antiquark numbers N_q and $N_{\overline{q}}$ of quark and antiquark numbers are separately conserved (as also their difference $N_q - N_{\overline{q}}$). This forbids many reactions, for instance those in which a gauge boson is emitted unless one assumes that many quark states are superpositions of states with a varying total quark number N. This would mean that the extremely simple re-combinatorics picture is lost.

2. Crossing symmetry, which is a symmetry of standard QFTs, suggests a solution to the problem. Crossing symmetry would mean that one can transfer quarks between initial and final states by changing the sign of the quark four-momentum so that momentum conservation is not violated. Crossing means analytic continuation of the scattering amplitude by replacing incoming (outgoing) momentum p with outgoing (incoming) momentum $-p$. The scattering amplitudes for reactions for which the quark number is conserved can be constructed using mere recombinatorics, and the remaining amplitudes would be obtained by crossing.

3. Crossing must respect the Galois singlet property. For instance, the crossing of a single quark destroys Galois singlet. Unless one allows destruction and recombination of Galois singlets, the crossing can apply only to Galois singlets. These rules bring to mind the vanishing of twistor amplitudes when one gluon has negative helicity and the remaining gluons have positive helicity.

4.5 Western and Eastern ontologies of physics

This picture forces us to ask whether something deeper might lurk behind the usual ideas about particle physics in which scattering rates encode the information. Could the imaginary part of $P(A, B)$ have a well-defined physical meaning in some more general framework?

1. In ZEO, single classical time evolution and zero energy state as a pair of initial and final states becomes the basic entity. One can even ask whether it might make sense to speak about probability density for different zero energy states as time evolutions, events.

   Could the "western" view about existing reality evolving in time be replaced with an ontology in which events in both classical sense (zero energy states) and quantum transitions would be what really exists.

   In the "eastern" view, the relevant probabilities would not be for transitions $A \to B$ for a given state A but for the occurrence of these transitions $A \to B$ in given state, whatever its definition might be, and one would measure the relative rates for occurrence for the various transitions $A \to B$.

The ensemble would not consist of entities A but transitions $A \to B$. In biology and neuroscience, the states are indeed replaced with behaviors. IKn computer science the program, rather than the state of the computer, is the basic notion.

2. In order to develop this picture at the level of scattering amplitudes, one could start from the QFT description for the n-point correlation functions used to construct S-matrix. One adds to the exponent of action a term, which is a combination of small current terms assignable to external particles and calculates functional Taylor series with respect to the small parameters. The Taylor coefficients are identified as n-point functions.

   In QFTs this is regarded as a mere calculational trick and the "state" defined by the exponential as an analog of that in statistical physics is defined by the exponential of action when the values of the parameters vanish.

   One can of course ask what it would mean if these parameters do not vanish. In perturbation theory one actually has this situation. These deformed states look formally like coherent states. Could the physical states at a deeper level correspond to these analogs of coherent states as analogs of thermo-dynamical states?

3. TGD can be formally regarded as a complex square root of thermodynamics, which suggests a generalization of the formulation of quantum theory as algebraic QFT promoted for instance by Connes [A3], and this is what this new interpretation would mean also physically.

4. In the TGD framework, one would add to the exponent of $exp(-K)$ a superposition of oscillator operator monomials of quark oscillator operators creating positive and negative energy parts of the zero energy states with complex coefficients Z_i as parameters and essentially defining coordinates for the Hilbert space. Z_i would be analogous to the complex numbers defining coherent states.

   The exponential can be expanded and fermionic vacuum expectation forces conservation of quark number and the combination of the positive and negative energy parts to give a non-vanishing result. At the limit of infinitely large CD conservation of 4-momentum is obtained.

5. The ordinary transition amplitudes are obtained by performing the limit $Z_i \to 0$, and calculating Taylor coefficients as transition amplitudes. The analog of $G_{A,\overline{B}}$ would be obtained for the analogs 2-point functions having as arguments the parts of zero energy states and $P(A,B) = Re(G_{A,\overline{B}}G_{\overline{B},A})$ would give transition probabilities. For Kähler geometry the analog of probability conservation and unitarity would hold true.

6. That these amplitudes are obtained as second derivatives with respect to the fermionic Hilbert space complex coordinates Z_i and $\overline{Z}_j$ conforms with the interpretation of the exponential containing the additional terms as a generalization of an exponential of Kähler function associated with the fermionic degrees of freedom. Kähler metric indeed corresponds to $\partial_{Z_I}\partial_{\overline{Z}_J}K$, where K is the Kähler function.

7. Could the expressions of higher n-point functions in fermionic degrees of freedom boil down to the curvature tensor and its covariant derivatives so that quantum theory would be geometrized? If one has a constant curvature space, as strongly suggested by the mere existence of infinite-D Kähler metric, then only $G_{A,\overline{B}}$ would be needed so that it is enough to measure only the scattering probabilities (rates at infinite-volume limit for CD).

   Could the parameters Z_i be non-vanishing and define a square root of a thermodynamic state as an analog of a coherent state? If a constant curvature metric is in question, the scattering rates for non-vanishing Z_i could be expressed in terms of those for $Z_i = 0$. Could different phases of quantum theory correlate with the value ranges of the parameters Z_i?

4.6 Connection with the notion of Fisher information

The notion of Fisher information (`https://cutt.ly/GUPvF37`) relates in an interesting manner to the proposed Kähler geometrization of quantum theory.

1. Fisher information matrix F is associated with a probability density function $f(X,Z)$ for random variables X_i depending on the parameters Z_i (Z_i are denoted by θ_i in the Wikipedia article at `https://cutt.ly/GUPvF37`). Matrix F gives information about the $f(X,\theta)$, which must be deduced from the measurements of X. The matrix element F_{ij} is essentially integral over X for the the quantity $\langle \partial_{\theta_i}\partial_{\theta_j} log(f)\rangle$, where $\langle ..\rangle$ denotes the expectation obtained by integrating over X. F_{ij} determines a statistical metric and for complex parameters Z_i one obtains a Kähler metric.

2. In TGD, X would correspond to WCW coordinates and f would be analogous to the vacuum functional $exp(-K)$ but containing also a parameter dependent part defined by the combination of positive and negative energy parts of the fermionic zero energy states. The complex coefficients Z_i *resp.* $\overline{Z}_i$ of monomials of creation *resp.* annihilation operators would define the parameters. Fermionic Kähler metric would have an interpretation as Fisher information, which can be also complex valued.

3. Also the higher derivatives with respect to coefficients of zero energy states would provide information about the vacuum functional. One would have n-point functions for zero energy states possibly reducing to covariant derivatives of the analog curvature tensor. If the space of fermionic zero energy states is analog of a constant curvature space, the scattering amplitudes at the limit $Z_i = 0$ would give all the needed information needed to calculate the scattering amplitudes for $Z_i \neq 0$. $P(A,B)$ would be complex as components of the Fisher information matrix.

4. Basically, the information provided by the scattering amplitudes would be about the generalization of the vacuum functional of WCW including also the fermionic part. Scattering amplitudes would give information Kähler function of the WCW metric and about parameters Z_i.

   The scattering amplitudes indeed correlate strongly with the properties of space-time surfaces determined by polynomials. The p-adic prime p, crucial for the real scattering amplitudes as canonical images of p-adic amplitudes, corresponds to a ramified prime for P and this means localization of the vacuum functional to polynomials having a ramified prime equal to p. The number of Galois singlets in the scattering amplitude means lower bound for the degree of P.

4.7 About the relationship of Kähler approach to the standard picture

The replacement of the notion of unitary S-matrix with Kähler metric of fermionic state space generalizes the notion of unitarity. The rows of the matrix defined by the contravariant metric are orthogonal to the columns of the covariant metric in the inner product $(T\circ U)_{A\overline{B}} = T_{A\overline{C}}\eta^{\overline{C}D}U_{D\overline{B}}$, where $\eta^{\overline{C}D}$ is flat contravariant Kähler metric of state space. Although the probabilities are complex, their real parts sum up to 1 and the sum of the imaginary parts vanishes.

4.7.1 The counterpart of the optical theorem in TGD framework

The Optical Theorem generalizes. In the standard form of the optical theorem $i(T-T^\dagger)_{mm} = 2Im(T) = TT^\dagger_{m,m}$ states that the imaginary part of the forward scattering amplitude is proportional to the total scattering rate. Both quantities are physical observables.

In the TGD framework the corresponding statement

$$T^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}T_{\overline{B}C} + T^{A\overline{B}}T_{\overline{B}C} = 0 \ . \tag{4.5}$$

Note that here one has $G = \eta + T$, where G and T are hermitian matrices. The correspondence with the standard situation would require the definition $G = \eta + iU$. The replacement $T \to T = iU$, where U is antihermitian matrix, gives

One has

$$i[U^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}I_{\overline{B}C}] = U^{A\overline{B}}U_{\overline{B}C} \ . \tag{4.6}$$

This statement does not reduce to single condition but gives two separate conditions.

1. The first condition is analogous to Optical Theorem:

$$Im[\eta^{A\overline{B}}U_{C\overline{B}}+U^{A\overline{B}}\eta_{\overline{B}C}]=-Re[U^{A\overline{B}}U_{\overline{B}C}]=Re[U^{A\overline{B}}U_{C\overline{B}}] \ . \tag{4.7}$$

2. Second condition is new and reflects the fact that the probabilities are complex. It is necessary to guarantee that the sum of the probabilities reduces to the sum of their real parts.

$$Re[\eta^{A\overline{B}}U_{C\overline{B}}+U^{A\overline{B}}\eta_{\overline{B}C}]=-Im[U^{A\overline{B}}U_{C\overline{B}}] \ . \tag{4.8}$$

   The challenge would be to find a physical meaning for the imaginary parts of scattering probabilities. This is discussed in [L27]. The idea is that the imaginary parts could make themselves visible in a Markov process involving a power of the complex probability matrix.

In the applications of the optical theorem, the analytic properties of the scattering matrix T make it possible to construct the amplitude as a function of mass shell momenta using its discontinuity at the real axis. Indeed, $2Im(T)$ for the forward scattering amplitude can be identified as the discontinuity $Disc(T)$. In the recent case, this identification would suggests the generalization

$$Disc[T^{A\overline{B}}\eta_{\overline{B}C}]=T^{A\overline{B}}\eta_{\overline{B}C}+\eta^{A\overline{B}}T_{C\overline{B}} \ . \tag{4.9}$$

Therefore covariant and contravariant Kähler metric could be limits of the same analytic function from different sides of the real axis. One assigns the hermitian conjugate of S-matrix to the time reflection. Are covariant and contravariant forms of Kähler metric related by time reversal? Does this mean that T symmetry is violated for a non-flat Kähler metric.

4.7.2 The emergence of QFT type scattering amplitudes at long length scale limit

The basic objection against the proposal for the scattering amplitudes is that they are non-vanishing only at mass shells with $m^2=n$. A detailed analysis of this objection improves the understanding about how the QFT limit of TGD emerges.

1. The restriction to the mass shells replaces cuts of QFT approach with a discrete set of masses. The TGD counterpart of unitarity and optical theorem holds true at the discrete mass shells.

2. The p-adic mass scale for the reaction region is determined by the largest ramified prime RP for the functional composite of polynomials characterizing the Galois singlets participating in the reaction. For large values of ramified prime RP for the reaction region, the p-adic mass scale increases and therefore the momentum resolution improves.

3. For large enough RP below measurement resolution, one cannot distinguish the discrete sequence of poles from a continuum, and it is a good approximation to replace the discrete set of mass shells with a cut. The physical analogy for the discrete set of masses along the real axis is as a set of discrete charges forming a linear structure. When their density becomes high enough, the description as a line charge is appropriate and in 2-D electrostatistics this replaces the discrete set of poles with a cut.

This picture suggests that the QFT type description emerges at the limit when RP becomes very large. This kind of limit is discussed in the article considering the question whether a notion of a polynomial of infinite degree as an iterate of a polynomial makes sense [L30]. It was found that the number of the roots is expected to become dense in some region of the real line so that effectively the QFT limit is approached.

1. If the polynomial characterizing the scattering region corresponds to a composite of polynomials participating in the reaction, its degree increases with the number of external particles. At the limit of an infinite number of incoming particles, the polynomial approaches a polynomial of infinite degree. This limit also means an approach to a chaos as a functional iteration of the polynomial defining the space-time surface [L22]. In the recent picture, the iteration would correspond to an addition of particles of a given type characterized by a fixed polynomial. Could the characteristic features for the approach of chaos by iteration, say period doubling, be visible in scattering in some situations. Could p-adic length scale hypothesis stating that p-adic primes near powers of two are favored, relate to this.

2. For a large number of identical external particles, the functional composite defining RG involves iteration of polynomials associated with particles of a particular kind, if their number is very large. For instance, the radiation of IR photons and IR gravitons in the reaction increases the degree of RP by adding to P very high iterates of a photonic or gravitonic polynomial.

   Gravitons could have a large value of ramified prime as the approximately infinite range of gravitational interaction and the notion of gravitational Planck constant [L8, L37] originally proposed by Nottale [E1] suggest. If this is the case, graviton corresponds to a polynomial of very high degree, which increases the p-adic length scale of the reaction region and improves the momentum resolution. If the number of gravitons is large, this large RP appears at very many steps of the SFR cascade.

4.7.3 A connection with dual resonance models

There is an intriguing connection with the dual resonances models discussed already in [L9].

1. The basic idea behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. The 4-particle amplitude of Veneziano was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged.

2. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have a representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

3. The resonances have zero width and the imaginary part of the amplitude has a discontinuity only at the resonance poles, which is not consistent with unitarity so that one must force unitarity by hand by an iterative procedure. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of the twistor Grassmann approach.

It is interesting to compare this picture with the twistor Grassman approach and TGD picture.

1. In the TGD framework, one just picks up the residue of what would be analogous to stringy scattering amplitude at mass shells. In the dual resonance models, one keeps the entire amplitude and encounters problems with the unitarity outside the poles. In the twistor Grassmann approach, one assumes that the amplitudes are completely determined by the singularities whereas in TGD they *are* the residues at singularities. At the limit of an infinite-fold iterate the amplitudes approach analogs of QFT amplitudes.

2. In the dual resonance model, the sums over s- and t-channel resonances are the same. This guarantees crossing symmetry. An open question is whether this can be the case also in the TGD framework. If this is the case, the continuum limit of the scattering amplitudes should have a close resemblance with string model scattering amplitudes as the $M^4 \times CP_2$ picture having magnetic flux tubes in a crucial role indeed suggests.

3. In dual resonance models, only the cyclic permutations of the external particles are allowed. As found, the same applies in TGD if the scattering event is a cognitive measurement [L26], only the cyclic permutations of the factors of a fixed functional composite are allowed. Non-cyclic permutations would produce the counterparts of non-planar diagrams and the cascade of cognitive state function reductions could not make sense for all polynomials in the superposition simultaneously. Remarkably, in the twistor Grassmann approach just the non-planar diagrams are the problem.

5 Some useful objections

The details of the proposed construction of the scattering amplitudes starting from twistors are still unclear and the best way to proceed is to invent objections and critical questions.

5.1 How the quark momenta in M^8 and H relate to each other?

The relationship between quark momenta in M^8 and H is not clear. There are four options to consider corresponding to the Dirac propagators in H and M^4 with or without coupling to $A(M^4)$. I assign to these options attributea $D(H,A)$, $D(H)$, $D(M^4,A)$ and $D(M^4)$. For all options something seems to go wrong.

Consider fits the list of criteria that the correct option should satisfy.

1. M^8-H duality suggests the same momentum and mass spectrum for quarks in M^8 and H.

    (a) However, the mass spectrum of color partial waves for quark spinors for $D(H)$ and $D(H,A)$ is very simple and characterized by 2 integers labeling triality $t=1$ representations of $SU(3)$ [L1]. Neither $D(H)$ or $D(H,A)$ allows a mass spectrum as algebraic roots of polynomials and seems to be excluded.

    (b) If M^8-H duality holds true in a strong sense so that these spectra are identical, the only possible conclusion seems to be that the propagator in both M^8 and H is just the M^4 Dirac propagator $D(M^4)$ and that the roots of the polynomial P give the spectrum of off-mass-shell masses. Also tachyonic mass squared values are allowed as roots of P. The real on-shell masses would be associated with Galois singlets.

2. Twistor holomorphy and associativity leave only the $D(M^4)$ option. The couplings to $A(M^4)$ and presence of $D(CP_2)$ spoil these properties. $D(M^4)$ option has very nice features. The integration over the momentum space reduces to a finite summation over virtual mass shells defined by the roots of P and one avoids divergences. This tightens the connection with QFTs. For $D(M^4()$ this nice property is lost. Massless quarks are also consistent with the QCD picture about quarks.

3. The predictions of p-adic mass calculations [K5, K2] were sensitive to the negative ground state conformal weight h_{vac} depending on the electroweak isospin and gave rise to electroweak symmetry breaking. h_{vac} could be generated by conformal generators with weights h coming as algebraic integers determined by P. This favors $D(H)$ and $D(H,A)$. $D(H,A)$ predicts tachyonic ν_R, which was necessary for the calculation. Only $D(H,A)$ survives.

4. For some years ago, I found that the space-time propagators for points of H connected by a light-like geodesic behave like massless propagators irrespective of mass. CP_2 type extermals have a light-like geodesic as an M^4 projection. This would suggest that quarks associated with CP_2 type extremals effectively propagate as massless particles even if one assumes that they correspond to modes of the full H Dirac operator. This allows us to consider $D(H)$ as an alternative. For this option most quarks in the interior of the space-time surface would be extremely massive and practically absent.

5. Suppose that one takes seriously the idea that the situation can be described also by using massless M^8 momenta. This implies that for some choices of $M^4 \subset M^8$ the momentum is parallel to M^4 and therefore massless in 4-D sense. Only the quarks associated with the same

M^4 can interact. Hence M^4 can be always chosen so that the on mass-shell 4-momenta are light-like. $D(H,A)$ option could be correct but $D(M^4)$ option would appear as an effective option obtained by a suitable choice of $M^4 \subset M^8$.

6. The consideration of problems related to right-handed neutrino [L33] led to the question whether the quark spinor modes in H are annihilated only by the H d'Alembertian $D^2(H,A(M^4))$ but not by the H Dirac operator [L33]. The assumption that on mass shell H-spinors are annihilated by $D(M^4,A)$ leads to the same outcome.

   D^2 options allow different M^4 chiralities to propagate separately and solves problems related to the notion of right-handed neutrino ν_R (assumed to be 3-antiquark state and modellable using leptonic spinors in H. This also conforms with the right and left-handed character of the standard model couplings. However, the mixing of M^4 chiralities serves as a signature for the massivation and is lost.

   If leptons are allowed as fundamental fermions, $D(H)$ allows ν_R as a spinor mode, which is covariantly constant in CP_2. If leptons are not allowed, one can argue that ν_R as a 3-quark state can be modeled as a mode of H spinor with Kähler coupling yielding correct leptonic charges.

   The M^4 Kähler structure favored by the twistor lift of TGD [L9] implies that ν_R with negative mass squared appears as a mode of $D(H)$. This mode allows the construction of tachyonic ground states. This is lost for $D(M^4)$ with coupling to $A(M^4)$.

   For $D(M^4,A)$, one obtains for all spinor modes states with both positive and negative mass squared from the $J_{kl}\Sigma^{kl}$ term. Physical on-mass- shell states with negative mass squared cannot be allowed. These would however allow to construct tachyonic ground states needed in the p-adic mass calculations. Now the problem is that $D(M^4,A)$ as propagator spoils twistor holomorphy.

7. Since the color group acts as symmetries, one can assume that spinor modes correspond to color partial waves as eigen states of CP_2 spinor d'Alembertian $D^2(CP_2)$. This predicts that different M^4 chiralities propagate independently. $D(M^4)$ and $D(M^4,A)$ options make the same prediction. For the $D(H)$ and $D(H,A)$ option one obtains a mixing of M^4 chiralities having interpretation in terms of massivation.

   For all options the correlation between color and electroweak quantum numbers is "wrong". This is however not a problem for off-mass-shell fundamental quarks since the physical states are obtained as SSA representations.

To sum up, $D(H,A)$ is strongly favored by the p-adic thermodynamics, by the possibility to build the physical quarks using SSA, by the fact that propagators over-light-like distances do not depend on mass, and also by the freedom to choose $M^4 \subset M^8$ in such a manner that on mass shell spinor mode is massless. $D(M^4)$ is strongly favoured by M^8-H duality (associativity) and by twistor analyticity. Both options seem to be both right and wrong. This suggests that something is wrong with the interpretation of the notion of the Dirac propagator.

1. From the view point of H, M^8 quarks are off-mass-shell whereas from the M^8 point of view they are on-mass-shell. Suppose that off-mass shell quarks in the sense of $D(H,A)$ differ from on-mass-shell quarks only in that they have M^4 momentum $p_{off} = p_{on} + \Delta p$ differing by Δp from the on-mass shell momentum p_{on} with integer components and satisfying mass shell condition for $D(H)$. In CP_2 these states are on-mass-shell. Suppose that p_{off} is on M^8 mass shell determined as a root of P.

   With these assumptions, one can write Dirac operator as $D(H,A,off) = D(H,A,on) + \Delta p^k gamma_k$, whose action to incoming Galois singlets reduces to $D(H,A,off) = \Delta p^k gamma_k = D(M^4)$. This is just the free massless propagator.

2. The propagating entities would be basically solutions of $D(H,A)$ with an off-mass-shell M^4-momentum with Δp having mass. In particular, they are superpositions of components with left- and right-handed M^4 chiralities having opposite CP_2 chiralities and the mixing of M^4 chiralities can be seen as a signature of massivation. On the other hand, $D(M^4)$ does not depend on M^4 chirality. Maybe this option could avoid all objections!

5.2 Can one allow "wrong" correlation between color and electroweak quantum numbers for fundamental quarks?

For CP_2 harmonics, the correlation between color and electroweak quantum numbers is wrong [K5]. Therefore the physical quarks cannot correspond to the solutions of $D^2(H)\Psi = 0$. The same applies also to the solutions of $D(M^4)\Psi = 0$ if one assumes that they belong to irreducible representations of the color group as eigenstates of $D(CP_2)$.

How to construct quark states, which are physical in the sense that they are massless and color-electroweak correlation is correct?

1. The reduction of quark masses to zero requires a tachyonic ground state in p-adic mass calculations [K5]. The assumption that physical states are constructed using quarks, which are on-mass-shell in the M^8 sense but off-mass-shell in the H sense.

   Colored operators with non-vanishing conformal weight are required to make all quark states massless color triplets. This is possible only if the ground state is tachyonic, which gives strong support for M^4 Kähler structure.

2. This is achieved by the identification of physical quarks as states of super-symplectic representations. Also the generalized Kac-Moody algebra assignable to the light-like partonic orbits or both of these representations can be considered. These representations could correspond to inertial and gravitational representations realized at "objective" embedding space level and "subjective" space-time level.

   Supersymplectic generators are characterized by a conformal weight h completely analogous to mass squared. The conformal weights naturally correspond to algebraic integers associated with P. The mass squared values for the Galois singlets are ordinary integers.

3. It is plausible that also massless color triplet states of quarks can be constructed as color singlets. From these one can construct hadrons and leptons as color singlets for a larger extension of rationals. This conforms with the earlier picture about conformal confinement. These physical quarks constructed as states of super-symplectic representation, as opposed to modes of the H spinor field, would correspond to the quarks of QCD.

   One can argue that Galois confinement allows to construct physical quarks as color triplets for some polynomial Q and also color singlets bound states of these with extended Galois group for a higher polynomial $P \circ Q$ and with larger Galois group as representation of group $Gal(P)/Gal(Q)$ allowing representations of a discrete subgroup of color group.

5.3 Can one allow complex quark masses?

One objection relates to unitarity. Complex energies and mass squared values are not allowed in the standard picture based on unitary time evolution.

1. Here several new concepts lend a hand. Galois confinement could solve the problems if one considers only Galois singlets as physical particles. ZEO replaces quantum states with entangled pairs of positive and negative energy states at the boundaries of CD and entanglement coefficients define transition amplitudes.

   The notion of the unitary time evolution is replaced with the Kähler metric in quark degrees of freedom and its components correspond to transition amplitudes. The analog of the time evolution operator assignable to SSFRs corresponds naturally to a scaling rather than time translation and mass squared operator corresponds to an infinitesimal scaling.

2. The complex eigenvalues of mass squared as roots of P be allowed when unitarity at quark level is not required to achieve probability conservation. For complex mass squared values, the entanglement coefficients for quarks would be proportional to mass squared exponents $exp(im^2\lambda)$, λ the scaling parameter analogous to the duration of time evolution. For Galois singlets these exponentials would sum up to imaginary ones so that probability conservation would hold true.

5.4 Are M^8 spinors as octonionic spinors equivalent with H-spinors?

At the level of M^8 octonionic spinors are natural. $M^8 - H$ duality requires that they are equivalent with H-spinors. The most natural identification of octionic spinors is as bi-spinors, which have octonionic components. Associativity is satisfied if the components are complexified quaternionic so that they have the same number of components as quark spinors in H. The H spinors can be induced to $X^4 \subset M^8$ by using $M^8 - H$ duality. Therefore the M^8 and H pictures fuse together.

The quaternionicity condition for the octonionic spinors is essential. Octonionic spinor can be expressed as a complexified octonion, which can be identified as momentum p. It is not an on-mass shell spinor. The momenta allowed in scattering amplitudes belong to mass shells defined by the polynomial P. That octonionic spinor has only quaternionic components conforms with the quaternionicity of $X^4 \subset M^8$ eliminating the remaining momentum components and also with the use of $D(M^4)$.

5.5 Two objections against p-adic thermodynamics and their resolution

Unlike the Higgs mechanism, p-adic thermodynamics provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry, which guarantees the existence of p-adic Boltzmann weights.

There are two basic objections against p-adic thermodynamics. The mass calculations require the presence of states with negative conformal weights giving rise to tachyons. Furthermore, by conformal invariance L_0 should annihilate physical states so that all states should have vanishing mass squared! In this article a resolution of these objections, based on the very definition of thermodynamics and on number theoretic vision predicting quark states with discretized tachyonic mass, which are counterparts for virtual states in QFTs, is discussed.

Physical states for the entire Universe would be indeed massless but for subsystems such as elementary particles the thermal expectation of the mass squared is non-vanishing. This conforms with the formula of blackhole entropy stating that it is proportional to the mass square of the blackhole and vanishes for vanishing mass: this would indeed correspond to a pure state.

5.5.1 p-Adic thermodynamics

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L5, L6], and classical number fields [K18]. p-Adic mass calculations is a rather successful application of p-adic thermodynamics for the mass squared operator identified as conformal scaling generator L_0. p-Adic thermodynamics can be also understood as a constraint on a real thermodynamics for the mass squared from the condition that it can be also regarded as a p-adic thermodynamics.

The motivation for p-adicization came from p-adic mass calculations [K5, K2].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Boltzmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1 + i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [K9] is interpreted as a correlate for cognition. Motivation comes from the observation that piecewise constant functions depending on a finite number of pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic

differential equations. This could provide a classical correlate for the non-determinism of imagination.

5.5.2 Objections and their resolution

The number theoretic picture leads to a deeper understanding of a long standing objection against p-adic thermodynamics [K5] as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra.

If one requires super-Virasoro symmetry and identifies mass squared with a scaling generator L_0, one can argue that only massless states are possible since L_0 must annihilate these states! All states of the theory would be massless, not only those of fundamental particles as in conformally invariant theories to which twistor approach applies! This looks extremely beautiful mathematically but seems to be in conflict with reality already at single particle level!

The resolution of the objection is that *thermodynamics* is indeed in question.

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and describes approximately the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors. The excited states of the entire system indeed are massless [L42].

2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment and the entire mass squared would remain vanishing. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

3. In the p-adic mass calculations [K5, K2], the effective environment for quarks and leptons would in a good approximation consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons). The many-quark state many-quark state associated with the wormhole throat (single quark state for quarks and 3-quark-state for leptons [L29].

4. In M^8 picture [L20, L21], tachyonicity is unavoidable since the real part of the mass squared as a root of a polynomial P can be negative. Also tachyonic real but algebraic mass squared values are possible. At the H level, tachyonicity corresponds to the Euclidean signature of the induced metric for a wormhole contact.

   Tachyonicity is also necessary: otherwise one does not obtain massless states. The super-symplectic states of quarks would entangle with the tachyonic states of the wormhole contacts by Galois confinement.

5. The massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-symplectic representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Galois confinement is satisfied. The tachyonic mass squared is assigned with wormhole contacts with the Euclidean signature of the induced metric, whose throats in turn carry the fermions so that the wormhole contact would form the nearby environment.

   The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state remains massless and massive single particle states receive a compensating negative mass squared from the wormhole contact. Thermal mass squared corresponds to a single particle mass squared and does not take into account the contribution of wormhole contacts except for the ground state.

6. There is a further delicate number theoretic element involved [L33, L38]. The choice of $M^4 \subset M^8$ for the system is not unique. Since M^4 momentum is an M^4 projection of a massless M^8 momentum, it is massless by a suitable choice of $M^4 \subset M^8$. This choice must be made for the environment so that both the state of the environment and the single particle ground state are massless. For the excited states, the choice of M^4 must remain the same, which forces the massivation of the single particle excitations and p-adic massivation.

5.5.3 All physical states are massless!

These arguments strongly suggest that pure states, in particular the state of the entire Universe, are massless. Mass would reflect the statistical description of entanglement using a density matrix. The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural.

This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K16] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and $M^8 - H$ duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L39, L40].

5.5.4 Relationship with Higgs mechanism

Polynomials P have two kinds of solutions depending on whether their roots determine either mass or energy shells. For the energy option a space-time region corresponds by $M^8 - H$ duality to a solution spectrum in which the roots correspond to energies rather than mass squared values and light-cone proper time is replaced with linear Minkoski time [L20, L21]. The physical interpretation of the energy shell option has remained unclear.

The energy shell option gives rise to a p-adic variant of the ordinary thermodynamics and requires integer quantization of energy. This option is natural for massless states since scalings leave the mass shell invariant in this case. Scaling invariance and conformal invariance are not violated.

One can wonder what the role of these massless virtual quark states in TQC could be. A good guess is that the two options correspond to phases with broken *resp.* unbroken conformal symmetry. In gauge theories they correspond to phases with broken and unbroken gauge symmetries. The breaking of gauge symmetry indeed induces breaking of conformal symmetry and this breaking is more fundamental.

1. Particle massivation corresponds in gauge theories to symmetry breaking caused by the generation of the Higgs vacuum expectation value. Gauge symmetry breaking induces a breaking of conformal symmetry and particle massivation. In the TGD framework, the generation of entanglement between members of state pairs such that members having opposite values of mass squared determined as roots of polynomial P in the most general case, leads to a breaking of conformal symmetry for each tensor factor and the description in terms of p-adic thermodynamics gives thermal mass squared.

2. What about the situation when energy, instead of mass squared, comes as a root of P. Also now one can construct physical states from massless virtual quarks with energies coming as algebraic integers. Total energies would be ordinary integers. This gives massless entangled states, if the rational integer parts of 4-momenta are parallel. This brings in mind a standard twistor approach with parallel light-like momenta for on-mass shell states. Now however the virtual states can have transversal momentum components which are algebraic numbers (possibly complex) but sum up to zero.

   Quantum entangled states would be superpositions over state pairs with parallel massless momenta. Massless extremals (topological light rays) are natural classical space-time correlates for them. This phase would correspond to the phase with unbroken conformal symmetry.

3. One can also assign a symmetry breaking to the thermodynamic massivation. For the energy option, the entire Galois group appears as symmetry of the mass shell whereas for the mass squared option only the isotropy group does so. Therefore there is a symmetry breaking of

the full Galois symmetry to the symmetry defined by the isotropy group. In a loose sense, the real valued argument of P serves as a counterpart of the Higgs field.

If the symmetry breaking in the model of electroweak interaction corresponds to this kind of symmetry breaking, the isotropy group, which presumably involves also a discrete subgroup of quaternionic automorphisms as an analog of the Galois group. Quaternionic group could act as a discrete subgroup of $SU(2) \subset SU(2)_L \times U(1)$. The hierarchy of discrete subgroups associated with the hierarchy of Jones inclusions assigned with measurement resolution suggests itself. It has the isometry groups of Platonic solids as the groups with genuinely 3-D action. $U(1)$ factor could correspond to Z_n as the isotropy group of the Galois group. In the QCD picture about strong interactions there is no gauge symmetry breaking so that a description based on the energy option is natural. Hadronic picture would correspond to mass squared option and symmetry breaking to the isotropy group of the root.

To sum up, in the maximally symmetric scenario, conformal symmetry breaking would be only apparent, and due to the necessity to restrict to non-tachyonic subsystems using p-adic thermodynamics. Gauge symmetry breaking would be replaced with the replacement of the Galois group with the isotropy group of the root representing mass squared value. The argument of the polynomial defining space-time region would be the analog of the Higgs field.

5.6 Some further comments about the notion of mass

In the sequel some further comments related to the notion of mass are represented.

5.6.1 $M^8 - H$ duality and tachyonic momenta

Tachyonic momenta are mapped to space-like geodesics in H or possibly to the geodesics of X^4 [L20, L21, L35]. This description could allow to describe pair creation as turning of fermion backwards in time [L40]. Tachyonic momenta correspond to points of de Sitter space and are therefore outside CD and would go outside the space-time surface, which is inside CD. Could one avoid this?

1. Since the points of the twistor spaces $T(M^4)$ and $T(CP_2)$ are in 1-1 correspondence, one can use either $T(M^4)$ or $T(CP_2)$ so that the projection to M^4 or CP_2 would serve as the base space of $T(X^4)$. One could use CP_2 coordinates or M^4 coordinates as space-time coordinates if the dimension of the projection is 4 to either of these spaces. In the generic case, both dimensions are 4 but one must be very cautious with genericity arguments which fail at the level of M^8.

2. There are exceptional situations in which genericity fails at the level of H. String-like objects of the form $X^2 \times Y^2 \subset M^4 \subset CP_2$ is one example of this. In this case, X^6 would not define 1-1 correspondence between $T(M^4)$ or $T(CP_2)$.

   Could one use partial projections to M^2 and S^2 in this case? Could $T(X^4)$ be divided locally into a Cartesian product of 3-D M^4 part projecting to $M^2 \subset M^4$ and of 3-D CP_2 part projected to $Y^2 \subset CP_2$.

3. One can also consider the possibility of defining the twistor space $T(M^2 \times S^2)$. Its fiber at a given point would consist of light-like geodesics of $M^2 \times S^2$. The fiber consists of direction vectors of light-like geodesics. S^2 projection would correspond to a geodesic circle $S^1 \subset S^2$ going through a given point of S^2 and its points are parametrized by azimuthal angle Φ. Hyperbolic tangent $tanh(\eta)$ with range $[-1, 1]$ would characterize the direction of a time like geodesic in M^2. At the limit of $\eta \rightarrow \pm\infty$ the S^2 contribution to the S^2 tangent vector to length squared of the tangent vector vanishes so that all angles in the range $(0, 2\pi)$ correspond to the same point. Therefore the fiber space has a topology of S^2.

   There are also other special situations such as $M^1 \times S^3$, $M^3 \times S^1$ for which one must introduce specific twistor space and which can be treated in the same way.

During the writing of this article I realized that the twistor space of H defined geometrically as a bundle, which has as H as base space and fiber as the space of light-like geodesic starting from a given point of H need not be equal to $T(M^4) \times T(CP_2)$, where $T(CP_2)$ is identified as $SU(3)/U(1) \times U(1)$ characterizing the choices of color quantization axes.

1. The definition of $T(CP_2)$ as the space of light-like geodesics from a given point of CP_2 is not possible. One could also define the fiber space of $T(CP_2)$ geometrically as the space of geodesics emating from origin at $r = 0$ in the Eguchi-Hanson coordinates [K1] and connecting it to the homologically non-trivial geodesic sphere S^2_G $r = \infty$. This relation is symmetric.

   In fact, all geodesics from $r = 0$ end up to S^2. This is due to the compactness and symmetries of CP_2. In the same way, the geodesics from the North Pole of S^2 end up to the South Pole. If only the endpoint of the geodesic of CP_2 matters, one can always regard it as a point S^2_G.

   The two homologically non-trivial geodesic spheres associated with distinct points of CP_2 always intersect at a single point, which means that their twistor fibers contain a common geodesic line of this kind. Also the twistor spheres of $T(M^4)$ associated with distinct points of M^4 with a light-like distance intersect at a common point identifiable as a light-like geodesic connecting them.

2. Geometrically, a light-like geodesic of H is defined by a 3-D momentum vector in M^4 and 3-D color momentum along CP_2 geodesic. The scale of the 8-D tangent vector does not matter and the 8-D light-likeness condition holds true. This leaves 4 parameters so that $T(H)$ identified in this way is 12-dimensional.

   The M^4 momenta correspond to a mass shell H^3. Only the momentum direction matters so that also in the M^4 sector the fiber reduces to S^2 . If this argument is correct, the space of light-like geodesics at point of H has the topology of $S^2 \times S^2$ and $T(H)$ would reduce to $T(M^4) \times T(CP_2)$ as indeed looks natural.

5.6.2 Conformal confinement at the level of H

The proposal of [L45], inspired by p-adic thermodynamics, is that all states are massless in the sense that the sum of mass squared values vanishes. Conformal weight, as essentially mass squared value, is naturally additive and conformal confinement as a realization of conformal invariance would mean that the sum of mass squared values vanishes. Since complex mass squared values with a negative real part are allowed as roots of polynomials, the condition is highly non-trivial.

$M^8 - H$ duality [L20, L21] would make it natural to assign tachyonic masses with CP_2 type extremals and with the Euclidean regions of the space-time surface. Time-like masses would be assigned with time-like space-time regions. In [L43] it was found that, contrary to the beliefs held hitherto, it is possible to satisfy boundary conditions for the action action consisting of the Kähler action, volume term and Chern-Simons term, at boundaries (genuine or between Minkowskian and Euclidean space-time regions) if they are light-like surfaces satisfying also $detg_4 = 0$. Masslessness, at least in the classical sense, would be naturally associated with light-like boundaries (genuine or between Minkowskian and Euclidean regions).

5.6.3 About the analogs of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

1. Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L44] define a unique discretization of 4-surface in M^4 and, by M^8-H duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L44, L41] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L32]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3)\setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

   The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

   Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and M^8-H duality in terms of associativity [L20, L21]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

2 .De Sitter manifolds as tachyonic analogs of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ. (`https://cutt.ly/XVsdLwY`).

3.Do complexified hyperbolic manifolds as analogs of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

   Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T = 1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H^3_c with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogs of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogs of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H^3_c would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H^3_c is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$ contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H^3_c could be such in a complexified sense.

5.7 Is pair creation really understood in the twistorial picture?

Twistorialization leads to a beautiful picture about scattering amplitudes at the level of M^8 [L39, L40]. In the simplest picture, scattering would be just a re-organization of Galois singlets to new Galois singlets. Fundamental fermions would move as free particles.

The components of the 4-momentum of virtual fundamental fermion with mass m would be algebraic integers and therefore complex. The real projection of 4-momentum would be mapped by $M^8 - H$ duality to a geodesic of M^4 starting from either vertex of the causal diamond (CD) . Uncertainty Principle at classical level requires inversion so that one has $a = \hbar_{eff}/m$, where ab denotes light-cone proper time assignable to either half-cone of CD and m is the mass assignable to the point of the mass shell $H^3 \subset M^4 \subset M^8$.

The geodesic would intersect the $a = \hbar_{eff}/m$ 3-surface and also other mass shells and the opposite light-cone boundaries of CDs involved. The mass shells and CDs containing them would have a common center but Uncertainty Principle at quantum level requires that for each CD and its contents there is an analog of plane wave in CD cm degrees of freedom.

One can however criticize this framework. Does it really allow us to understand pair creation at the level of the space-time surfaces $X^4 \subset H$?

1. All elementary particles consist of fundamental fermions in the proposed picture. Conservation of fermion number allows pair creation occurring for instance in the emission of a boson as fermion-antifermion pair in $f \rightarrow f + b$ vertex.

2. The problem is that if only non-space-like geodesics of H are allowed, both fermion and antifermion numbers are conserved separately so that pair creation does not look possible. Pair creation is both the central idea and source of divergence problems in QFTs.

3. One can identify also a second problem: what are the anticommutation relations for the fermionic oscillator operators labelled by tachyonic and complex valued momenta? Is it possible to analytically continue the anticommutators to complexified $M^4 \subset H$ and $M^4 \subset M^8$? Only the first problem will be considered in the following.

Is it possible to understand pair creation in the proposed picture based on twistor scattering amplitudes or should one somehow bring the bff 3-vertex or actually $ff\overline{ff}$ vertex to the theory at the level of quark lines? This vertex leads to a non-renormalizable theory and is out of consideration.

One can first try to identify the key ingredients of the possible solution of the problem.

1. Crossing symmetry is fundamental in QFTs and also in TGD. For non-trivial scattering amplitudes, crossing moves particles between initial and final states. How should one define the crossing at the space-time level in the TGD framework? What could the transfer of the end of a geodesic line at the boundary of CDs to the opposite boundary mean geometrically?

2. At the level of H, particles have CP_2 type extremals -wormhole contacts - as building bricks. They have an Euclidean signature (of the induced metric) and connect two space-time sheets with a Minkowskian signature.

   The opposite throats of the wormhole contacts correspond to the boundaries between Euclidean and Minkowskian regions and their orbits are light-like. Their light-like boundaries, orbits of partonic 2-surfaces, are assumed to carry fundamental fermions. Partonic orbits allow light-like geodesics as possible representation of massless fundamental fermions.

   Elementary particles consist of at least two wormhole contacts. This is necessary because the wormhole contacts behave like Kähler magnetic charges and one must have closed magnetic field lines. At both space-time sheets, the particle could look like a monopole pair.

3. The generalization of the particle concept allows a geometric realization of vertices. At a given space-time sheet a diagram involving a topological 3-vertex would correspond to 3 light-like partonic orbits meeting at the partonic 2-surface located in the interior of X^4. Could the topological 3-vertex be enough to avoid the introduction of the 4-fermion vertex?

Could one modify the definition of the particle line as a geodesic of H to a geodesic of the space-time surface X^4 so that the classical interactions at the space-time surface would make it possible to describe pair creation without introducing a 4-fermion vertex? Could the creation of a fermion pair mean that a virtual fundamental fermion moving along a space-like geodesics of a wormhole throat turns backwards in time at the partonic 3-vertex. If this is the case, it would correspond to a tachyon. Indeed, in M^8 picture tachyons are building bricks of physical particles identified as Galois singlets.

1. If fundamental fermion lines are geodesics at the light-like partonic orbits, they can be light-like but are space-like if there is motion in transversal degrees of freedom.

2. Consider a geodesic carrying a fundamental fermion, starting from a partonic 2-surface at either light-like boundary of CD. As a free fermion, it would propagate to the opposite boundary of CD along the wormhole throat.

   What happens if the fermion goes through a topological 3-vertex? Could it turn backwards in time at the vertex by transforming first to a space-like geodesic inside the wormhole contact leading to the opposite throat and return back to the original boundary of CD? It could return along the opposite throat or the throat of a second wormhole contact emerging from the 3-vertex. Could this kind of process be regarded as a bifurcation so that it would correspond to a classical non-determinism as a correlate of quantum non-determinism?

3. It is not clear whether one can assign a conserved space-like M^4 momentum to the geodesics at the partonic orbits. It is not possible to assign to the partonic 2-orbit a 3-momentum, which would be well-defined in the Noether sense but the component of momentum in the light-like direction would be well-defined and non-vanishing.

   By Lorentz invariance, the definition of conserved mass squared as an eigenvalue of d'Alembertian could be possible. For light-like 3-surfaces the d'Alembertian reduces to the d'Alembertian for the Euclidean partonic 2-surface having only non-positive eigenvalues. If this process is possible and conserves M^4 mass squared, the geodesic must be space-like and therefore tachyonic.

   The non-conservation of M^4 momentum at single particle level (but not classically at n-particle level) would be due to the interaction with the classical fields.

4. In the M^8 picture, tachyons are unavoidable since there is no reason why the roots of the polynomials with integer coefficients could not correspond to negative and even complex mass squared values. Could the tachyonic real parts of mass squared values at M^8 level, correspond to tachyonic geodesics along wormhole throats possibly returning backwards along the another wormhole throat?

How does this picture relate to p-adic thermodynamics [L45] as a description of particle massivations?

1. The description of massivation in terms of p-adic thermodynamics [L45] suggests that at the fundamental level massive particles involve non-observable tachyonic contribution to the mass squared assignable to the wormhole contact, which cancels the non-tachyonic contribution.

   All articles, and for the most general option all quantum states could be massless in this sense, and the massivation would be due the restriction of the consideration to the non-tachyonic part of the mass squared assignable to the Minkowskian regions of X^4.

2. p-Adic thermodynamics would describe the tachyonic part of the state as "environment" in terms of the density matrix dictated to a high degree by conformal invariance, which this description would break. A generalization of the blackhole entropy applying to any system emerges and the interpretation for the fact that blackhole entropy is proportional to mass squared. Also gauge bosons and Higgs as fermion-antifermion pairs would involve the tachyonic contribution and would be massless in the fundamental description.

3. This could solve a possible and old problem related to massless spin 1 bosons. If they consist of a collinear fermion and antifermion, which are massless, they have a vanishing helicity and would be scalars, because the fermion and antifermion with parallel momenta have opposite helicities. If the fermion and antifermion are antiparallel, the boson has correct helicity but is massive.

   Massivation could solve the problem and p-adic thermodynamics indeed predicts that even photons have a very small thermal mass. Massless gauge bosons (and particles in general) would be possible in the sense that the positive mass squared is compensated by equally small tachyonic contribution.

4. It should be noted however that the roots of the polynomials in M^8 can also correspond to energies of massless states. This phase would be analogous to the Higgs=0 phase. In this phase, Galois symmetries would not be broken: for the massive phase Galois group permutes different mass shells (and different $a = constant$ hyperboloids) and one must restrict Galois symmetries to the isotropy group of a given root. In the massless phase ,Galois symmetries permute different massless momenta and no symmetry breaking takes place.

6 Antipodal duality and TGD

I learned of a new particle physics duality from the popular article "Particle Physicists Puzzle Over a New Duality" published in Quanta Magazine (`https://cutt.ly/jZOaDhd`). The article describes the findings of Dixon et al reported in the article "Folding Amplitudes into Form Factors:

An Antipodal Duality" [B19] (`https://cutt.ly/EZOsfGl`) This work relies on the calculations of Goncharov et al published in the article "Classical Polylogarithms for Amplitudes and Wilson Loops" [B24] (`https://cutt.ly/sZOsuu6`).

The calculations of Goncharov et al lead to an explicit formula for the loop contributions to the 6-gluon scattering amplitude in $\mathcal{N} = 4$ SUSY. The new duality is called antipodal duality and relates 6-gluon amplitude for the forward scattering to a 3-gluon form factor of stress tensor analogous to a quantum field describing a scalar particle. This amplitude can be identified as a contribution to the scattering amplitude $h + g \rightarrow g + g$. The result is somewhat mysterious since in the standard model strong and electroweak interactions are completely separate.

6.1 Findings of Dixon et al

Consider first the findings of Dixon et al [B19].

1. One considers [B24] twistor amplitudes in $\mathcal{N} = \triangle$ SUSY. Only the maximally helicity violating amplitudes (MHV) are considered and one restricts the consideration to planar diagrams (to my best understanding, non-planar diagrams are still poorly understood). The contribution of the loop corrections is studied and the number of loops is rather high in the computations checking the claimed result.

   6-gluon forward scattering amplitude and 3-gluon form factor of stress energy tensor regarded as a quantum field are discussed. Conformal invariance fixes the Lorentz invariants appearing in the 6-gluon forward amplitude and in the 3-gluon form factor of stress tensor to be 3 conformally invariant cross ratios formed from the 3 gluon momenta.

   The claimed antipodal duality is found to hold true for each number of loops separately at the limit when one of conformal invariants approaches zero: the interpretation is that momentum exchange between 2 gluons vanishes at this limit. For 6-gluon forward amplitudes, this limit corresponds to in the 3-D space of conformal invariants to the edges of a tetrahedron.

2. $3g \rightarrow 3g$ scattering amplitude is studied at the limit when the scattering is in forward direction. One has effectively 3 gluons but not 3-gluon scattering since there is no momentum conservation constraining the total momentum of 3 gluons except effectively for the forward scattering of the stress tensor.

   As far as total quantum numbers are considered, the stress tensor can give rise to a quantum field behaving like Higgs as far as QCD is considered. The surprising finding is that the so-called antipodal duality applied to the 6-gluon amplitude gives a 3-gluon form factor of the stress tensor, which is scalar having no spin and vanishing color quantum numbers.

3. The antipodal transformation is carried for the 6-gluon amplitude in forward direction so that only 3 gluon momenta are involved. One starts from the 6-gluon amplitude constructed using the standard rules, which require that the amplitude involves only cyclic permutations of the gluons (elements of S_6 of the gluons.

   One considers permutation group $S_3 \subset S_6$ acting in the same way on the first 3 first and 3 remaining gluons, and constructs an S_3 singlet as a sum of the amplitudes obtained by applying S_3 transformations. S_3 operations are not allowed in the twistor diagrammatics since only planar amplitudes are considered usually (the construction of twistor counterparts of non-planar amplitudes is not well-understood).

4. One also constructs the 3-gluon form factor of stress energy tensor by using the twistor rules and considers the so-called soft limit at which the sum of the 3 gluon momenta vanishes so that the effectite particle assignable to the stress tensor scatters in the forward direction. It comes as a surprise that this amplitude is related to the amplitude obtained from the forward 6-gluon amplitude by the antipodal transformation.

5. The duality also involves a simple transformation of the 3 conformal invariants formed from the gluon momenta involved to the 3-gluon form factor of the energy momentum tensor. The antipodal duality holds true at the edges of the 2-D tetrahedron surface defined by the image of the 3-gluon form factor in the space of 3 conformal invariants characterizing the 6-gluon forward amplitude.

The term antipodal derives from the fact that the 6-gluon amplitude can be expressed as a "word" formed from 6 "letters" and the above described transformation reverses the order of the letters.

6. It is conjectured that this result generalizes to large values of n so that antipodal images of $2n$-gluon scattering amplitude in forward direction could correspond to $n-$-gluon form factor for stress tensor energy and this in turn would be associated with scattering of Higgs and n gluons.

6.2 Questions

Since the stress tensor is a scalar, it is not totally surprising that a term proportional to this amplitude contributes to the scattering amplitude $h + g \to g + g$, where h denotes Higgs particle. What looks somewhat mysterious is that Higgs is an electro-weakly interacting particle and has no direct color interactions. The description of the scattering in the standard model involves electroweak interactions and involves at least one decay of a gluon to a quark pair in turn interacting with the Higgs.

This inspires several questions.

1. Can one consider more general subgroups $S_m \subset S_{2n}$ and by forming S_m singlets construct amplitudes with a physical interpretation?

2. Can one imagine a deep duality between color and electroweak interactions such that $\mathcal{N} = 4$ SUSY would reflect this duality? Could one even think that the strong and electroweak interactions are in some sense dual?

In TGD color interactions and electroweak interactions are related to the isometries and holonomies of $CP2$ and there indeed exists quite a number of pieces of evidence for this kind of duality. However, the possibility that electroweak or color interactions alone could provide a full description of scattering amplitudes looks unrealistic: both electroweak and color quantum numbers are needed. The number-theoretical view of TGD [L35, L6, L39, L40] could however come into rescue.

6.3 In what sense could electroweak and color interactions be dual?

Some kind of duality of electroweak and color interactions is suggested by the antipode duality having an interpretation in terms of Hopf algebras (`https://en.wikipedia.org/wiki/Hopf_algebra`): antipode generalizes the notion of inverse for an element of algebra.

TGD contains several mysterious looking and not-well understood features suggesting some kind of duality between electroweak and color interactions. What could make this duality possible in the TGD framework, would be the presence of Galois symmetry, which would allow us to describe electroweak or color particle multiplets number-theoretically using representations of the Galois group.

1. The electric-magnetic duality or Montonen-Olive duality (`https://en.wikipedia.org/wiki/Montonen\OT1\textendashOlive_duality`) is inspired by the homology of CP_2 in TGD [?]. The generalization of this duality in gauge theories relates the perturbative description of gauge interactions for gauge group G to a non-perturbative description in terms of magnetic monopoles associated with the dual gauge group G_L. Langlands duality [?, ?] discussed from the TGD perspective in [?, ?] relates the representations of Galois groups and those of Lie groups, and involves Lie group and its Langlands dual. Therefore gauge groups, magnetic monopoles and the corresponding dual gauge group, and number theory seem to be mathematically related, and TGD suggests a physical realization of this view.

2. The dual groups G and G_L should be very similar but electroweak gauge group $U(2)$ and color group $SU(3)$, albeit naturally related as holonomy and isometry groups of CP_2, do not satisfy this condition. Here the Galois group could come into rescue and provide the missing quantum numbers.

3. Depending on the situation, Galois confinement could relate to color confinement or electroweak confinement. In the context of electric-magnetic duality [K4, K10, K6], I have discussed electroweak confinement and as a possible dual description for the electroweak massivation, involving summation of electroweak $SU(2)$ quantum numbers to zero in the scale of monopole flux tubes assignable to elementary particles. The screening of weak isospin would take place by a pair of neutrino and right-handed neutrino in the Compton scale of weak boson or fermion: $h_{eff} > h$ allows longer scales.

4. Also magnetic charge or flux assignable to the flux tubes could make possible a topological description of color hypercharge topologically whereas color isospin could might have description in terms of weak iosospin. I considered this idea already in my thesis. As a matter of fact, already before the discovery of CP_2 around 1980, I proposed that magnetic (homology-) charges 2,-1,-1 for cP_2 could correspond to em charges 2/3,-1/3,-1/3 of quarks and that quark confinement could be a topological phenomenon. Maybe these almost forgotten ideas might find a place in TGD after all.

Consider now the possible duality between electroweak and color interactions.

6.3.1 H level

At the level of H spinors do not couple classically to gluons and color is not spin-like quantum number.

1. The proposal is that the zero energy states are singlets either with respect to the Galois group or the isotropy group of a given root. Z_3 as a subgroup or possibly normal subgroup of the Galois group would act on the space of fermion momenta for which components are algebraic integers belonging to the extension of rationals defined by P.

2. Color confinement could correspond to Galois confinement. Alternatively, the confinement of color isospin could correspond to Galois confinement whereas the confinement of color hypercharge would have a description in terms of the already mentioned monopole confinement. Both number theoretic and topological color would be invisible.

Could antipodal duality be understood number-theoretically?

1. The antipodal duality produces an S_3 singlet from a twistor amplitude. Could color singlets correspond to Z_3 Galois-singlets and electroweak singlets above Compton scale to Z_2 singlets.

2. Could Z_2 be realized as an exchange of two gluons ordered cyclically in the amplitude? Could one think that S^6 acts as a Galois group or its isotropy group?

   The stress tensor as a Higgs like state is not a doublet. Could one obtain Higgs as a Z_2 doublet by allowing the entire orbit of S_3 but requiring only that Z_3 singlet property holds true?

3. Could all isotropy groups or even all subgroups of S^3 be allowed. Could S_n quite generally have a representation as a Galois group? This picture applies also to $2n$-gluon amplitudes but also more general conditions for Galois singlet property can be imagined.

6.3.2 M^8 level

The roles of color and electroweak quantum numbers are changed in $M^8 - H$ duality [L20, L21].

1. At the level of M^8, complexified octonionic 2-spinors [L16, L20, L21] decompose to the representations of the subgroup $SU(3) \subset G_2$ of octonionic automorphisms as $1 + \overline{1} + 3 + \overline{3}$. One obtains leptons and quarks with spin but electroweak quantum numbers do not appear as spin-like quantum numbers. This would suggests that one should assume both lepton and quark spinors at the level of H although the idea about leptons as 3-quark composites in CP_2 scale is attractive [L29].

   One can however construct octonionic spinor fields $M^4 \times E^4$ with the spinor partial waves belonging to the representations of $SO(4) = SU(2) \times SU(2)$ decomposing to representation of $U(2)$ with quantum numbers having interpretation as orbital angular momentum like electroweak quantum numbers.

2. At the level of 4-surfaces of M^8, weak isospin doublet could correspond to Galois doublet associated with a Z_2 factor of the Galois group.

6.3.3 Twistor space level

Also at the level of twistor spaces, the roles of electroweak and color numbers are changed in $M^8 - H$ duality.

1. At the level of H, $M^4 \times CP_2$ is replaced by the product of the twistor spaces $T(M^4)$ and $T(CP_2) = SU(3)/U(1)\times U(1)$. Since spinors are not involved anymore, electroweak quantum numbers disappear. Number theoretic description should apply. Here Galois subgroup Z_2 could help.

   This suggests that $U(2) \subset SU(3)$ must be interpreted in terms of electroweak quantum numbers. There indeed exists a natural embedding of the holonomy group of CP_2 to its isometry group. At the level of space-time, surface color hyper-charge and isopin could correspond to electroweak hyper-charge and isospin. This works if, for given electroweak quantum numbers, the choice of the quantization axes of color quantum numbers depends on the state so that the regions of space-time surface assignable to a fermion depends on its color quantum numbers in H. This would give a correlation between space-time geometry and quantum numbers.

2. At the level of M^8 the twistor space $T(E^4)$ contains information about weak quantum numbers but no information of color quantum numbers since octonionic spinors are given up. Z_6 as a subgroup of the Galois group could help now.

Also the induced twistor structure at the level of space-time surface in H and at the level of 4-surface in M^8 gives strong consistency conditions.

1. The induced twistor structure for the surface $T(X^4) \subset T(H)$ has S^2 bundle structure characterizing twistor space. This structure is obtained by dimensional reduction to $X^6 = X^4 \times S^2$ locally such that S^2 corresponds to the twistor sphere of both $T(M^4)$ and $T(CP_2)$.

2. For cognitive representations as unique number theoretic discretizations of the space-time surface, the twistor spheres S^2 of $T(M^4)$ *resp.* $T(CP_2)$ must correspond to each other. The point of S^2 represents the direction of the quantization axis and the value $\pm 1/2$ of spin *resp.* color isospin or appropriately normalized color hypercharge respectively.

   For quark triplets this kind of correlation can make sense between spin and color hypercharge only and only at the level of the space-time surface. Since the quantization directions of color isospin are not fixed, only the correlation between representations, rather states, is required and makes sense for quarks. This suggests that color isospin at the space-time level must correspond to Galois quantum number.

3. What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color [K15].

The induced twistor structure for 4-surfaces in M^8 does not correspond to dimensional reduction but one expects an analogous correlation between spin and electroweak quantum numbers induced by the mapping of the twistor spheres S^2 to each other.

1. This correlation spin H-spinors correspond to tensor products of spin and electroweak doublets and all elementary particles are constructed from these.

2. Something seems to be however missing: also M^4 spinors should have a $U(1)$ charge to make the picture completely symmetric. The spinor lift strongly suggests that also M^4 has the analog of Kähler structure [L33] and this would give rise to $U(1)$ charge for M^4 spinors [L7] [K10]. This coupling could give rise to small CP breaking effects at the level of fundamental spinors [L33].

The experimental picture about strong and electroweak interactions suggests that the description of standard model interactions as either color interactions or electroweak interactions combined with a number theoretic/topological description of the missing quantum numbers is enough.

1. In hadron physics, only electroweak quantum numbers are visible. Color could be described using number-theory and topology and also these descriptions might be dual. In the QCD picture at high energies only color quantum numbers are visible and electroweak quantum numbers could be described number-theoretically. For a given particle, electroweak confinement would work above its Compton scale of weak scale.

2. In the old fashioned hadron physics conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) relate hadron physics and electroweak physics in a manner which is not fully understood since also quark confinement is still poorly understood. PCAC reflects the massivation of hadrons and can be also seen as caused by the massivation of quarks and leptons and makes successful predictions. In the TGD framework PCAC is applied to the model of so-called lepto-hadrons [K20].

   One can say that hadronic description uses $SO(4) = SU(2)_L \times U(2)_R$ or rather, $U_{ew}(2)$ as a symmetry group whereas QCD uses $SU(3)$ in accordance with the duality between color and electroweak interactions. This conforms with the $M^8 - H$ duality.

3. What about CP_2 type extremals (wormhole contacts), which have Euclidean metric. Could electroweak spin be described as the spin of an octo-spinor and could M^4 spin be described number-theoretically.

What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color.

7 How could Julia sets and zeta functions relate to Galois confinement?

In this section the limit of large particle number of identical particles for the scattering is considered. It is found that the mass spectrum belongs to the Julia set of an infinitely iterated polynomial defining the many-particle state. Also a generalization replacing polynomials with real analytic functions is discussed and it is found that zeta functions and elliptic functions are especially interesting concerning conformal confinement as analog of Galois confinement.

7.1 The mass spectrum for an iterate of polynomial and chaos theory

Suppose that the number theoretic interaction in the scattering corresponds to a functional composition of the polynomials characterizing the external particles. If the number of the external particles is large, the composite can involve a rather high iterate of a single polynomial. This motivates the study of the scattering of identical particles described by the same polynomial P at the limit of a large particle number. These particles could correspond to elementary particles, in particular IR photons and gravitons. This situation leads to an iteration of a complex polynomial.

If the polynomials satisfy $P(0) = 0$ requiring $P(x) = xP_1(x)$, the roots of P are inherited. In this case fixed points correspond to the points with $P(x) = 1$. Assume also that the coefficients are rational. Monic polynomials are an especially interesting option.

For a k:th iterate of P, the mass squared spectrum is obtained as a union of spectra obtained as images of the spectrum of P under iterates P^{-r}, $r \leq k$, for the inverse of P, which is an n-valued algebraic function if P has degree n. This set is a subset of Fatou set (`https://cutt.ly/hOgq6Yy`)and for polynomials a subset of filled Julia set.

At the limit of large k, the limiting contributions to the spectrum approach a subset of Julia set defined as a P-invariant set which for polynomials is the boundary of the set for which the iteration divergences. The iteration of all roots except $x = 0$ (massless particles) leads to the Julia set asymptotically.

All inverse iterates of the roots of P are algebraic numbers. The Julia set itself is expected to contain transcendental complex numbers. It is not clear whether the inverse iterates at the limit are algebraic numbers or transcendentals. For instance, one can ask whether they could consist of n-cycles for various values of n consisting of algebraic points and forming a dense subset of the Julia set. The fact that the number of roots is infinite at this limit, suggests that a dense subset is in question.

The basic properties of Julia set deserve to be listed.

1. At the real axis , the fixed points satisfying $P(x) = x$ with $|dP/dx| > 1$ are repellers and belong to the Julia set. In the complex plane, the definition of points of the Julia set is $|P(w) - P(z)| \geq |w - z|$ for point w near to z.

2. Julia set is the complement of the Fatou set consisting of domains. Each Fatou domain contains at least one critical point with $dP/dz = 0$. At the real axis, this means that P has maximum or minimum. The iteration of P inside Fatou domain leads to a fixed point inside the Fatou set and inverse iteration to its boundary. The boundaries of Fatou domains combine to form the Julia set. In the case of polynomials, Fatou domains are labeled by the $n - 1$ solutions of $dP/dz = P_1 + zdP_1/dz = 0$.

3. Julia set is a closure of infinitely many periodic repelling orbits. The limit of inverse iteration leads towards these orbits. These points are fixed points for powers P^n of P.

4. For rational functions Julia set is the boundary of a set consisting of points whose iteration diverges to infinity. For polynomials Julia set is the boundary of the so-called filled Julia set consisting of points for which the iterate remains finite.

Chaos theory also studies the dependence of Julia set on the parameters of the polynomials. Mandelbrot fractal is associated to the polynomial $Q(z) = a + z^2$ for which origin is an stable critical point and corresponds to the boundary of the region in a-plane containing origin such that outside the boundary the iteration leads to infinity and in the interior to origin.

The critical points of P with $dP/dz = 0$ for $z = z_{cr}$ located inside Fatou domains are analogous to point $z = 0$ for $Q(z)$ associated with Fatou domains and quadratic polynomial $a + b(z - z_{cr})^2$, $b > 0$, would serve as an approximation. The variation of a is determined by the variation of the coefficients of P required to leave z_{cr} invariant.

Feigenbaum studied iteration of a polynomial $a - x^2$ for which origin is unstable critical point and found that the variation of a leads to a period doubling sequence in which a sequence of 2^n-cycles is generated (`https://cutt.ly/pOgwuqj`). Origin would correspond to an unstable critical point $dP(z)/dz = 0$ belonging to a Julia set.

The physical implications of this picture are highly interesting.

1. For a large number of interacting quarks, the mass squared spectrum of quarks as roots of the iterate of P in the interaction region would approach the Julia set as infinite inverse iterates of the roots of P. This conforms with the idea that the complexity increases with the particle number.

   Galois confinement forces the mass squared spectrum to be integer valued when one uses as a unit the p-adic mass scale defined by the larger ramified prime for the iterate. The complexity manifests itself only as the increase of the microscopic states in interaction regions.

2. Julia set contains a dense set consisting of repulsive n-cycles, which are fixed points of P and the natural expectation is that the mass spectrum decomposes into n-multiplets. Whether all values of n are allowed, is not clear to me. The limit of a large quark number would also mean an approach to (quantum) criticality.

To sum up, it would seem that chaos (or rather complexity-) theory could be an essential part of the fundamental physics of many-quark systems rather than a mere source of pleasures of mathematical aesthetics.

7.2 A possible generalization of number theoretic approach to analytic functions

M^8-H duality also allows the possibility that space-time surfaces in M^8 are defined as roots of real analytic functions. This option will be considered in this subsection.

7.2.1 Are polynomials 4-surfaces only an approximation

One of the open problems of the number-theoretic vision is whether the space-time surfaces associated with rational or even monic polynomials are an approximation or not.

1. One could argue that the cognitive representations are only a universal discretization obtained by approximating the 4-surface in M^8 by a polynomial. This discretization relies on an extension of rationals and more general than rational discretizations, which however appear via Galois confinement for the momenta of Galois singlets.

   One objection against space-time surfaces as being determined by polynomials in M^8 was that the resulting 4-surfaces in M^8 would bre algebraic surfaces. There seems to be no hope about Fourier analysis. The problem disappeared with the realization that polynomials determine only the 3-surfaces as mass-shells of M^4 and that M^8-H duality is realized by an explicit formula subject to $I(D)=exp{-K}$ condition.

2. Galois confinement provides a universal mechanism for the formation of bound states. Could evolution be a development of real states for which cognitive representations in terms of quarks become increasingly precise.

   That the quarks defining the active points of the representation are at 3-D mass shells would correspond to holography at the level of M^8. At the level of H they would be at the boundaries of CD. This would explain why we experience the world as 3-dimensional.

Also the 4-surfaces containing quark mass shells defined in terms of roots of arbitrary real analytic functions are possible.

1. Analytic functions could be defined in terms of Taylor or Laurent series. In fact, any representation can be considered. Also now one can consider representation involving only integers, rationals, algebraic numbers, and even reals as parameters playing a role of Taylor coefficients.

2. Does the notion of algebraic integers generalize? The roots of the holomorphic functions defining the meromorphic functions as their ratios define an extension of rationals, which is in the general transcendental. It is plausible that the notion of algebraic integers generalizes and one can assume that quarks have momentum components, which are transcendental integers. One can also define the transcendental analog of Galois confinement.

3. One can form functional composites to construct scattering amplitudes and this would make possible particle reactions between particles characterized by analytic functions. Iteration of analytic functions and approach to chaos would emerge as the functions involved appear very many times as one expects in case of IR photons and gravitons.

What about p-adicization requiring the definition discriminant D and identification of the ramified primes and maximal ramified prime? Under what conditions do these notions generalize?

1. One can start from rational functions. In the case of rational functions R, one can generalize the notion of discriminant and define it as a ratio $D=D_1/D_2$ of discriminants D_1 and D_2 for the polynomials appearing as a numerator and denominator of R. The value of D is finite irrespective of the values of the degrees of polynomials.

2. Analytic functions define function fields. Could a generalization of discriminant exist. If the analytic function is holomorphic, it has no poles so that D could be defined as the product of squares of root differences.

   If the roots appear as complex conjugate pairs, D is real. This is guaranteed if one has $f(\overline{z}) = \overline{f(z)}$. The real analyticity of f guarantees this and is necessary in the case of polynomials. A stronger condition would be that the parameters such as Taylor coefficients are rational.

   If the roots are rationals, the discriminant is a rational number and one can identify ramified primes and p-adic prime if the number of zeros is finite.

3. Meromorphic functions are ratios of two holomorphic functions. If the numbers of zeros are finite, the ratio of the discriminants associated with the numerator and denominator is finite and rational under the same assumptions as for holomorphic functions.

4. $M^8 - H$ duality leads to the proposal that the discriminant interpreted as a p-adic number for p-adic prime defined by the largest ramified prime, is equal to the exponent of $exp(-K)$ of Kähler function for the space-time surface in H.

   If one can assign ramified primes to D, it is possible to interpret D as a p-adic number having a finite real counterpart in canonical identification. For instance, if the roots of zeta are rationals, this could make sense.

7.2.2 Questions related to the emergence of mathematical consciousness

These considerations inspire further questions about the emergence of mathematical consciousness.

1. Could some mathematical entities such as analytic functions have a direct realization in terms of space-time surfaces? Could cognitive processes be identified as a formation of functional composites of analytic functions? They would be analogs of particle reactions in which the incoming particles consist of quarks, which are associated with mass-shells defined by the roots of analytic function.

   These composites would decay to products of polynomials in cognitive measurements involving a cascade of SSFRs reducing the entanglement between a relative Galois group and corresponding normal group acting as Galois group of rationals [L26].

2. Could the basic restriction to cognition come from the Galois confinement: momenta of composite states must be integers using p-adic mass scale as a unit.

   Or could one think that the normal sub-group hierarchies formed by Galois groups actually give rise to hierarchies of states, which are Galois confined for an extension of the Galois group.

   Could these higher levels relate to the emergence of consciousness about algebraic numbers. Could one extend computationalism allow also extensions of rationals and algebraic integers as discussed in [L25].

   Galois confinement for an extension of rationals would be analogous to the replacement of a description in terms of hadrons with that in terms of quarks and mean increase of cognitive resolution. Also Galois confinement could be generalized to its quantum version. One could have many quark states for which wave function in the space of total momenta is Galois singlet whereas total momenta are algebraic integers. S-wave states of a hydrogen atom define an obvious analog.

3. During the last centuries the evolution of mathematical consciousness has made huge steps due to the discovery of various mathematical concepts. Essentially a transformation of rational arithmetics with real analysis and calculus has taken place since the times of Newton. Could these evolutionary explosions correspond to the emergence of space-time surfaces defined by analytic functions or is it that only a conscious awareness about their existence has emerged?

7.2.3 Space-time surfaces defined by zeta functions and elliptic functions

Several physical interpretations of Riemann zeta have been proposed. Zeta has been associated with chaotic systems, and the interpretation of the imaginary parts of the roots of zeta as energies has been considered. Also an interpretation as a formal analog of a partition function has been considered. The interpretation as a scattering amplitude was considered by Grant Remmen [B23] (`https://cutt.ly/TID1kjU`).

1.Conformal confinement as Galois confinement for polynomials?

TGD suggests a totally different kind of approach in the attempts to understand Riemann Zeta. The basic notion is conformal confinement [K7].

1. The proposal is that the zeros of zeta correspond to complex conformal weights $s_n = 1/2+iy_n$. Physical states should be conformally confined meaning that the total conformal weight as the sum of conformal weights for a composite particle is real so that the state would have integer value conformal weight n, which is indeed natural. Also the trivial roots of zeta with $s = -2n$, $n > 0$, could be considered.

2. In $M^8 - H$ duality, the 4-surfaces $X^4 \subset M^8$ correspond to roots of polynomials P. M^8 has an interpretation as an analog of momentum space. The 4-surface involves mass shells $m^2 = r_n$, where r_n is the root of the polynomial P, algebraic complex number in general.

   The 4-surface goes through these 3-D mass-shells having $M^4 \subset M^8$ as a common real projection. The 4-surface is fixed from the condition that it defines $M^8 - H$ duality mapping it to $M^4 \times CP_2$. One can think X^4 as a deformation of M^4 by a local $SU(3)$ element such that the image points are $U(2)$ invariant and therefore define a point of CP_2. $SU(3)$ has an interpretation as octonionic automorphism.

3. Galois confinement states that physical states as many-quark states with quark momenta as algebraic integers in the extension defined by the polynomial have integer valued momentum components in the scale defined by the causal diamond also fixed by the p-adic prime identified as the largest ramified prime associated with the discriminant D of P.

   Mass squared in the stringy picture corresponds to conformal weight so that the mass squared values for quarks are analogous to conformal weights and the total conformal weight is integer by Galois confinement.

2.Conformal confinement for zeta functions

At least formally, TGD also allows a generalization of real polynomials to analytic functions. For a generic analytic function it is not possible to find superpositions of roots that would be integers and this could select Riemann Zeta and possible other analytic functions are those with infinite number of roots since they might allow a large number of bound states and be therefore winners in the number theoretic selection.

Riemann zeta is a highly interesting analytic function in this respect.

1. Actually an infinite hierarchy of zeta functions, one for any extension of rationals and conjectured to have zeros at the critical line, can be considered. Could one regard these zetas as analogous to polynomials with an infinite degree so that the allowed mass squared values for quarks would correspond to the roots of zeta?

2. Conformal confinement [K7] requires integer valued momentum components and total conformal weights as mass squared values. The fact that the roots of zetas appear as complex conjugates allows for a very large number of states with real conformal weights. This is however not enough. The fact that the roots are of the form $z_n = 1/2 + iy_n$ or $z = -2n$ implies that the conformal weights of Galois/conformal singlets are integer-valued and the spectrum is the same as in conformal field theories.

3. Riemann zeta has only a single pole at $s = 1$. Discriminant would be the product $\prod_{m\neq n}(y_m - y_n^2)\prod_{m\neq n} 4(m-n)^2 \prod_{m,n}(4m^2 + y_n^2)$ since the pole gives $D = 1$. D would be infinite.

4. Fermionic zeta $\zeta_F(s) = \zeta(s)/\zeta(2s)$ is analogous to the partition function for fermionic statistics and looks more appropriate in the case of quarks. In this case, the zeros are z_n *resp.* $z_n/2$ and the ratio of determinants would reduce to an infinite power of 2. The ramified prime would be the smallest possible: $p = 2$!

   $D = D_1/D_2$ would be infinite power of 2 and 2-adically zero so that $exp(-K)$ should vanish and Kähler function would diverge. 3-adically it would be infinite power of -1. If one can say that the number of roots is even, one has $D = 1$ 3-adically. Kähler function would be equal to zero, which is in principle possible.

   For Mersenne primes $M_n = 2^n - 1$, 2^n would be equal to $1 + M_n = 1 \ modM_n$ and one would obtain an infinite power $1 + M_n$, which is equal to 1 mod M_n. Could this relate to the special role of Mersenne primes?

5. What about Riemann Hypothesis? By $\zeta(\overline{s}) = \overline{zeta}(s)$, the zeros of zeta appear in complex conjugate pairs. By functional equation, also s and $1 - s$ are zeros. Suppose that there is a zero $s_+ = s_0 + iy_n$ with $s_0 \neq 1/2$ in the interval $(0, 1)$. This is accompanied by zeros $\overline{s}_+$, $1 - s_+$, $s_- = 1 - \overline{s}_+$. The sum of these four zeros is equal to $s = 2$. Therefore Galois singlet property does not allow us to say anything about the Riemann hypothesis.

3.Conformal confinement for elliptic functions

Elliptic functions (`https://cutt.ly/dINxAeQ`) provide examples of analytic functions with infinite number of roots forming a doubly periodic lattice and are therefore candidates for analogs of polynomials with infinite degree.

1. Weierstrass $\mathcal{P}(z)$-function $\mathcal{P}(z) = \sum_\lambda 1/(z - \lambda)^2$, where the summation is over the lattice defined by a complex modular parameter τ, is the fundamental elliptic function. The basic objection is that $\mathcal{P}(z)$ is not real analytic. Despite this it is interesting to look at its properties so that conformal weights do not appear in complex conjugate pairs. Therefore it is not clear whether conformal confinement is possible. One can also ask whether the notion of integer could be replaced with that of "modular" integers $m + n\tau$.

2. Elliptic functions are doubly periodic and characterized by the ratio τ of complex periods ω_1 and ω_2. One can assume the convention $\omega_1 = 1$ giving $\omega_2 = \tau$. The roots of the elliptic function for an infinite lattice and complex rational roots are of obvious interest concerning the generalization of Galois/conformal confinement.

3. The fundamental set of zeros is associated with a cell of this lattice. The finite number of zeros (with zero with multiplicity m counted as m zeros) in the cell is the same as the number poles and characterizes partially the elliptic function besides τ.

4. Weierstrass $\mathcal{P}$-function and its derivative $d\mathcal{P}/\lceil\ddagger$ are the building blocks of elliptic functions. A general elliptic function is a rational function of $\mathcal{P}$ and $d\mathcal{P}/\lceil\ddagger$. In even elliptic functions only the even funktion $\mathcal{P}$ appears.

5. The roots of Weierstrass $\mathcal{P}$-function $\mathcal{P}(z) = \sum_\lambda 1/(z - \lambda)^2$ appear in pairs $\pm z$ whereas the double poles at at the points of the modular lattice: see the article "The zeros of the Weierstrass $\mathcal{P}$-function and hypergeometric series" of Duke and Imamoglu [A6] (`https://cutt.ly/uIZSK4T`).

   The roots are given by Eichler-Zagier formula $z_\pm(m, n) = 1/2 + m + n\tau \pm z_1$, where z_1 contains an imaginary transcendental part $log(5 + 2\sqrt{6})/2\pi)$ plus second part, which depends on τ (see formula 6) of `https://cutt.ly/uIZSK4T`).

6. Conformally confined states with conformal weights $h = 1 + (m_1 + m_2) + (n_1 + n_2)\tau$ can be realized as pairs with conformal weights $(z_+(m_1, n_1), z_-(m_2, n_2))$. The condition $n_1 = -n_2$ guarantees integer-valued conformal weights and conformal confinement for a general value of τ.

7. A possible problem is that the total conformal weights can be also negative, which means tachyonicity. This is not a problem also in the case of Riemann zeta if trivial zeros are included.

   As a matter of fact, already at the level of M^8, M^4 Kähler structure implies that right-handed neutrino ν_R is a tachyon [L33]. However, ν_R provides the tachyon needed to construct massless super-symplectic ground states and also allows us to understand why neutrinos can be massive although right-handed neutrinos are not detected. The point is that only the square of Dirac equation in H is satisfied so that different M^4 chiralities can propagate independently.

   In M^8-H duality, non-tachyonicity makes it possible to map the momenta at mass shell to the boundary of CD in H. Hence the natural condition would be that the total conformal weight of a physical state is non-negative.

What about the notion of discriminant and ramified prime? One can assign to the algebraic extensions primes as prime ideals for algebraic integers and this suggests that the generalization of p-adicity and p-adic prime is possible. If this is the case also for transcendental extensions, it would be possible to define transcendental p-adicity.

One can however ask whether the discriminant is rational under some conditions. D could also allow factorization to the primes of the transcendental extension.

1. Elliptic functions are meromorphic and have the same number of poles and zeros in the basic cell so that there are some hopes that the ratio of discriminants is finite and even rational or integer for a suitable choice of the modular parameter τ as the ratio of the periods and the other parameters. Discriminant D as the ratio D_1/D_2 of the discriminants defined by the products of differences of roots and poles could be finite although they diverge separately.

2. For the Weierstrass $\mathcal{P}$-function, the zeros appear as pairs $\pm z_0$ and also as complex conjugate pairs. Complex pairs are required by real analyticity essential for the number theoretical vision. It might be possible to define the notion of ramified prime under some assumptions.

   For $z_+(m,n)$ or $z_-(m,n)$, the defining D_1 in D_1/D_2 would reduce to a product $\prod_{m,n}\Delta_{m,n})^2(\Delta_{m,n}+2z_1)(\Delta_{m,n}-2z_1)$, $\Delta_{m,n}=\Delta m+\Delta n\tau$, which is a complex integer valued if τ has integer components. D_1 would be a product of Gaussian integers.

3. The number of poles and zeros for the basic cell is the same so that D_2 as a product of the pole differences would have an identical general form. For large values of m,n, the factors in the product approach $\Delta_{m,n}$ for both zeros and poles so that the corresponding factors combine to a factor approaching unity.

   The double poles of $\mathcal{P}(z)=\sum_\lambda 1/(z-\lambda)^2$ are at points of the lattice. One has $D_2=\prod_{m,n}\Delta_{m,n})^4$. This gives

$$D=\frac{D_1}{D_2}=\prod_{m,n}(1+\frac{2z_0}{\Delta_{m,n}})(1-\frac{2z_0}{\Delta_{m,n}})=\prod_{m,n}(1-4(\frac{2z_0}{\Delta_{m,n}})^2)\ .$$

   Therefore D is finite and in general complex and transcendental so that the notion of ramified prime does not make sense as an ordinary prime. z_0 contains a transcendental constant term plus a term depending on τ (`https://cutt.ly/uIZSK4T`). Whether values of τ for which D is rational, might exist, is not clear.

In the number theoretic vision, the construction of many-particle states corresponds to the formation of functional composites of polynomials P. If the condition $P(0)=0$ is satisfied, the $n-fold$ composite inherits the roots of $n-1$-fold composites and the roots are like conserved genes. If one multiplies zeta functions and elliptic functions by z, one obtains similar families and the formation of composites gives rise to iteration sequences and approach to chaos [L22].

7.2.4 Riemann zeta, quantum criticality, and conformal confinement

There are strong indications Riemann zeta (https://cutt.ly/iVTV1kqs) has a deep role in physics, in particular in the physics of critical systems. TGD Universe is quantum critical. What quantum criticality would mean at the space-time level is discussed in [L43]. This raises the question whether Riemann zeta could have a deep role in TGD.

First some background relating to the number theoretic view of TGD.

1. In TGD, space-time regions are characterized by polynomials P with rational coefficients [L20, L21]. Galois confinement defines a universal mechanism for the formation of bound states. Momenta for virtual fermions have components, which are algebraic integers in an extension of rationals defined by a polynomial P characterizing space-time region. For the physical many fermion states, the total momentum as the sum of fermion momenta has components, which are integers using the unit defined by the size of the causal diamond (CD).

   This defines a universal number theoretical mechanism for the formation of bound states. The condition is very strong but for rational coefficients it can be satisfied since the sum of all roots is always a rational number as the coefficient of the first order term.

2. Galois confinement implies that the sum of the mass squared values, which are in general complex algebraic numbers in E, is also an integer. Since the mass squared values correspond to conformal weights as also in string models, one has conformal confinement: states are conformal singlets. This condition replaces the masslessness condition of gauge theories [L45].

Riemann zeta is not a polynomial but has infinite number of root. How could one end up with Riemann zeta in TGD? One can also consider the replacement of the rational polynomials with analytic functions with rational coefficients or even more general functions [L40].

1. For real analytic functions roots come as pairs but building many-fermion states for which the sum of roots would be a real integer, is very difficult and in general impossible.

2. Riemann zeta and the hierarchy of its generalizations to extensions of rationals (Dedekind zeta functions) is however a complete exception! If the roots are at the critical line as the generalization of Riemann hypothesis assumes, the sum of the root and its conjugate is equal to 1 and it is easy to construct many fermion states as 2N fermion states, such that they have integer value conformal weight.

   One can wonder whether one could see Riemann zeta as an analog of a polynomial such that the roots as zeros are algebraic numbers. This is however not necessary. Could zeta and its analogies allow it to build a very large number of Galois singlets and they would form a hierarchy corresponding to extensions of rationals. Could they represent a kind of second abstraction level after rational polynomials?

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