What Scattering Amplitudes Should Look Like?

M. Pitkänen,

February 2, 2024
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland. ORCID: 0000-0002-8051-4364.

Contents

Abstract

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the ZEO applied already earlier in TGD inspired cosmology. ZEO motivated the replacement of the term "S-matrix" with "M-matrix".

The general mathematical concepts are not enough to get to the level of concrete scattering amplitudes. The notion of preferred extremal inspiring the notion of generalized Feynman diagram is central in bringing in this concretia. The very notion of preferred extremals means that ordinary Feynman diagrams providing a visualization of path integral are not in question. Generalized Feynman diagrams have 4-D Euclidian space-time regions (wormhole contacts) as lines, and light-like partonic orbits of 2-surfaces as 3-D lines. String world sheets carrying fermions are also present and have 1-D boundaries at the light-like orbits of partonic 2-surfaces carrying fermion number and light-like 8-momenta suggesting strongly 8-D generalization of twistor approach.

The resulting objects could be indeed seen as generalizations of twistor diagrams rather than Feynman diagrams. The preferred extremal property strongly encourages the old and forgotten TGD inspried idea as sequences of algebraic operations with product and co-product representing 3-vertices. The sequences connect given states at the opposite boundaries of CD and have minimal length. The algebraic structure in question would be the Yangian of the super-symplectic algebra with generators identified as super-symplectic charges assignable to strings connecting partonic 2-surfaces.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction requires the introduction of generalization of twistor approach to 8-D context.

1 Introduction

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant [K19, K9] a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the zero energy ontology applied already earlier in TGD inspired cosmology [K1]. ZEO motivated the replacement of the term "S-matrix" with "M-matrix". This led to the discovery that rather stringy formulas for M-matrix elements emerge in TGD framework.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix scattered in the chapters of books about TGD and often drowned into details and plagued by side tracks and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction is suggested in the chapters about twistors and TGD.

My hope is that this chapter might provide a kind of bird's eye of view and help the reader to realize how fascinating and profound and near to physics the mathematics of hyper-finite factors is.

The goal is to sketch an overall view about the ideas which have led to the recent view about the construction of M-matrix. First the basic philosophical ideas are discussed. These include the basic ideas behind TGD inspired theory of consciousness [K16]. the identification of p-adic physics as physics of cognition forcing the central idea of number theoretic universality, quantum classical correspondence, and the crucial notion of zero energy ontology.

The understanding of the fundamental variational principles of TGD is so detailed that one can sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to Euclidian regions of 4-D surfaces - preferred extremals - defined by orbits of wormhole contacts plus the string world sheets connecting them and carrying spinor modes. Fermioaction contains also a part associated with the boundaries of string world sheets at partonic orbits. As a consequence, fundamental fermions propagate as particles with momenta which are light-like in 8-D sense along the light-like geodesics defined by the boundaries of string world sheets at which spinor modes are localized. This strongly suggests 8-D generalization of twistor approach.

The topological identification of the basic interaction vertices is as partonic 2-surfaces at which the orbits of partonic 2-surfaces meet. Fermions behave like free massless (in 8-D sense) particles during propagation along boundaries of string world sheets but interact at partonic surfaces and associated wormhole contacts by classical induced gauge fields. The naive guess would be that the conformal scaling generator L_0 for super-symplectic algebra could serving as propagator mediating the interaction between fermions at opposite wormhole throats.

The notion of preferred extremal does not favor ordinary Feynman diagrammatics resulting from path integral approach. The picture suggested by twistorialization looks more natural. Scattering amplitudes would be analogous to a minimal sequences of calculations transforming a given initial state to a given final state located at boundaries of CD. I proposed this vision for many years ago in terms of bi-algebras and related structures but gave it up as too speculative, and the only remnant of the enthusiasism period is a little appendix [K2]. The basic operations would be product and co-product in the Yangian associated with the super-symplectic algebra. Interaction vertices would correspond product and co-product for the generators of the Yangian algebra. The generators of this algebra would be Noether super charges associated with strings connecting partonic two surfaces.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

2 General Vision Behind Matrices

In the following I summarize the basic notions and ideas discussed in previous chapters.

2.1 Basic Principles

My original intention was to summarize the basic principles of Quantum TGD first. The problem is however where to start from since everything is so tightly interwoven that linear representation proceeding from principles to consequences seems impossible. Therefore it might be a good idea to try to give a summary with emphasis on what has happened during the few months in turn of 2008 to 2009 assuming that the reader is familiar with the basic concepts discussed in previous chapters. This summary gives also a bird's eye of view about what I believe M-matrix to be. Later this picture is used to answer the questions raised in the earlier version of this chapter.

2.1.1 Zero energy ontology

One of the key notions underlying the recent developments is zero energy ontology.

1. Zero energy ontology leads naturally to the identification of light-like 3-surfaces interpreted as a generalization of Feynman diagrams as the most natural dynamical objects (equivalent with space-like 3-surface by holography).

2. The fractal hierarchy of causal diamonds (CD) with light like boundaries of CD interpreted as carriers of positive and negative energy parts of zero energy state emerges naturally. If the scales of CDs come as powers of 2, p-adic length scale hypothesis follows as a consequence.

3. The identification of M-matrix as time-like entanglement coefficients between zero energy states identified as the product of positive square root of the density matrix and unitary

S-matrix emerges naturally and leads to the unification of thermodynamics and quantum theory.

4. The identification of M-matrix in terms of Connes tensor product means that the included algebra $\mathcal{N} \subset \mathcal{M}$ acts effectively like complex numbers and does not affect the physical state. The interpretation is that $\mathcal{N}$ corresponds to zero energy states in size scales smaller than the measurement resolution and thus the insertion of this kind of zero energy state should not have any observable effects. The uniqueness of Connes tensor product gives excellent hopes that the M-matrix could be unique apart from the square root of of density matrix.

5. The unitary U-matrix between zero energy states assignable to quantum jump has nothing to do with S-matrix measured in particle physics experiments. A possible interpretation is in terms of consciousness theory. For instance, U-matrix could make sense even for p-adic-to-real transitions interpreted as transformations of intentions to actions making sense since zero energy state is generated ("Everything is creatable from vacuum" is the basic principle of zero energy ontology) [K12] . One can express U-matrix as a collection of M-matrices labeled by zero energy states and unitaritity conditions for U-matrix boil down to orthogonality conditions for the zero energy states defined by M-matrices.

2.1.2 The notion of finite measurement resolution

The notion of finite measurement resolution as a basic dynamical principle of quantum TGD might be seen by a philosophically minded reader as the epistemological counterpart of zero energy ontology.

1. As far as length scale resolution is considered, finite measurement resolution implies that only CDs above some size scale are allowed. This is not an approximation but a property of zero energy state so that zero energy states realize finite measurement resolution in their structure. One might perhaps say that quantum states represent only the information that we can becomes conscious of.

2. In the case of angle resolution the hierarchy of Planck constants accompanied by a hierarchy of algebraic extensions of rationals by roots of unity, and realized in terms of the book like structures assigned with CD and CP_2, is a natural outcome of this thinking.

3. Number theoretic braids implying discretization at parton level can be seen as a space-time correlate for the finite measurement resolution. Zero energy states should contain in their construction only information assignable to the points of the braids. Note however that there is also information about tangent space of space-time surface at these points so that the theory does not reduce to a genuinely discrete theory. Each choice of M^2 and geodesic spheres defines a selection of quantization axis and different choice of the number theoretic braid. Hence discreteness does not reduce to that resulting from the assumption that space-time as the arena of dynamics is discrete but reflects the limits to what we can measure, perceive, and cognize in continuous space-time. Zero energy state corresponds to wave-function in the space of these choices realized as the union of copies of the page $CD \times CP_2$. Quantum measurement must induce a localization to single point in this space unless one is ready to take seriously the notion of quantum multiverse.

4. Finite measurement resolution allows a realization in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 (HFFs) about which the WCW Clifford algebra provides standard example. Also the factor spaces $\mathcal{M}/\mathcal{N}$ are suggestive and should correspond to quantum variants of HFFs with a finite quantum dimension. p-Adic coupling constant evolution can be understood in this framework and corresponds to the inclusions of HFFs realized as inclusions of spaces of zero energy states with two different scale cutoffs.

2.1.3 Number theoretical compactification and $M^8 - H$ duality

The closely related notions of number theoretical compactification and $M^8 - H$ duality have had a decisive impact on the understanding of the mathematical structure of quantum TGD.

1. The hypothesis is that TGD allows two equivalent descriptions using either M^8- the space of hyper-octonions- or $H = M^4 \times CP_2$ as embedding space so that standard model symmetries have a number theoretic interpretation. The underlying philosophy is that the world of classical worlds and thus H is unique so that the symmetries of H should be something very special. Number theoretical symmetries indeed fulfil this criterion.

2. In M^8 description space-time surfaces decompose to hyper-quaternionic and co-hyperquaternionic regions. The map assigning to $X^4 \subset M^8$ the image in $X^4 \subset H$ must be a isometry and also preserve the induced Kähler form so that the Kähler action has same value in the two spaces. The isometry groups of E^4 and CP_2 are different, and the interpretation is that the low energy description of hadrons in terms of $SO(4)$ symmetry and high energy description in terms of $SU(3)$ gauge group reflect this duality.

3. Number theoretic compactification implies very detailed conjectures about the preferred extremals of Kähler action implying dual slicings of the M^4 projection of space-time surface to string world sheets Y^2 and partonic 2-surfaces X^2 for Minkowskian signature of induced metric. This occurs for the known extremals of Kähler action of this kind [K3, K15, K18]. These slicings allow to understand how Equivalence Principle emerges via its stringy variant in TGD framework through dimensional reduction. The tangent spaces of Y^2 and X^2 define local planes of physical and un-physical polarizations and M^2 defines also the plane for the four-momentum assignable to the braid strand so that gauge symmetries are purely number theoretical interpretation.

4. Also a slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 parallel to X_l^3 giving equivalent space-time representations of partonic dynamics is predicted. This implies holography meaning an effective reduction of space-like 3-surfaces to 2-D surfaces. Number theoretical compactification leads also to a dramatic progress in the construction of quantum TGD in terms of the second quantized induced spinor fields. The holography seems however to be not quite simple as one might think first. Kac-Moody symmetries respecting the light-likeness of X_l^3 and leaving X^2 fixed act as gauge transformations and all light-like 3-surfaces with fixed ends and related by Kac-Moody symmetries would be geometrically equivalent in the sense that WCW Kähler metric is identical for them. These transformations would also act as zero modes of Kähler action.

5. A physically attractive realization of the braids - and more generally- of slicings of space-time surface by 3-surfaces and string world sheets, is discussed in [K11] by starting from the observation that TGD defines an almost topological QFT of braids, braid cobordisms, and 2-knots. The boundaries of the string world sheets at the space-like 3-surfaces at boundaries of CDs and wormhole throats would define space-like and time-like braids uniquely.

   The idea relies on a rather direct translation of the notions of singular surfaces and surface operators used in gauge theory approach to knots [A3] to TGD framework. It leads to the identification of slicing by 3-surfaces as that induced by the inverse images of $r = constant$ surfaces of CP_2, where r is $U(2)$ invariant radial coordinate of CP_2 playing the role of Higgs field vacuum expectation value in gauge theories. $r = \infty$ surfaces correspond to geodesic spheres and define analogs of fractionally magnetically charged Dirac strings identifiable as preferred string world sheets. The union of these sheets labelled by subgroups $U(2) \subset SU(3)$ would define the slicing of space-time surface by string world sheets. The choice of $U(2)$ relates directly to the choice of quantization axes for color quantum numbers characterizing CD and would have the choice of braids and string world sheets as a space-time correlate.

2.1.4 WCW spinor structure

The construction of WCW ("world of classical worlds", configuration space) spinor structure in terms of second quantized induced spinor fields is certainly the most important step made hitherto towards explicit formulas for M-matrix elements.

1. Number theoretical compactification ($M^8 - H$ duality) states that space-time surfaces can be equivalently regarded as 4-dimensional surfaces of either $H = M^4 \times CP_2$ or of 8-D Minkowski

space M^8, and consisting of hyper-quaternionic and co-hyper-quaternionic regions identified as regions with Minkowskian and Euclidian signatures of induced metric. Duality preserves induced metric and Kähler form. This duality poses very strong constraints on the geometry of the preferred extremals of Kähler action implying dual slicings of the space-time surface by string worlds sheets and partonic 2-surfaces as also by light-like 1-surfaces and light-like 3-surfaces. These predictions are consistent what is known about the extremals of Kähler action. The predictions of number theoretical compactification lead to dramatic progress in the construction of configurations space spinor structure and geometry.

2. The construction of WCW geometry and spinor structure in terms of induced spinor fields leads to the conclusion that finite measurement resolution is an intrinsic property of quantum states basically due to the vacuum degeneracy of Kähler action. This gives a justification for the notion of number theoretic braid effectively replacing light-like 3-surfaces. Hence the infinite-dimensional WCW is replaced with a finite-dimensional space $(\delta M^4_\pm \times CP_2)^n/S_n$. A possible interpretation is that the finite fermionic oscillator algebra for given partonic 2-surface X^2 represents the factor space $\mathcal{M}/\mathcal{N}$ identifiable as quantum variant of Clifford algebra. $(\delta M^4_\pm \times CP_2)^n/S_n$ would represent its bosonic analog.

3. The isometries of the WCW corresponds to X^2 local symplectic transformations $\delta M^4_\pm \times CP_2$ depending only on the value of the invariant $\epsilon^{\mu\nu}J_{\mu\nu}$, where $J_{\mu\nu}$ can correspond to the Kähler form induced from $\delta M^4_\pm$ or CP_2. This group parameterizes quantum fluctuating degrees of freedom. Zero modes correspond to coordinates which cannot be made complex, in particular to the values of the induced symplectic form which thus behaves as a classical field so that WCW allows a slicing by the classical field patterns $J_{\mu\nu}(x)$ representing zero modes.

4. By the effective 2-dimensionality of light-like 3-surfaces X^3_l (holography) the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. Quantum classical correspondence suggests that gauge fixing in Kac-Moody degrees of freedom takes place and implies correlation between the quantum numbers of the physical state and X^3_l or equivalently any light-like 3-surface Y^3_l parallel to X^3_l. There would be no path integral over X^3_l and only functional integral defined by WCW geometry over partonic 2-surfaces.

5. The condition that the Noether currents assignable to the modified Dirac equation are conserved requires that space-time surfaces correspond to extremals for which second variation of Kähler action vanishes. A milder condition is that the rank of the matrix defined by the second variation of Kähler action is less than maximal. Preferred extremals of Kähler action can be identified as this kind of 4-surface and the interpretation is in terms of quantum criticality.

   For given preferred extremal one expects the existence of an infinite number of deformations with a vanishing second variation of Kähler action. These deformations act as conformal gauge symmetries realizing quantum criticality at space-time level. The natural assumption is that the number, call it n, of conformal gauge equivalence classes of space-time surfaces with fixed 3-surfaces at their ends at the boundaries of CD is finite. This integer would characterize the effective value of Planck constant $h_{eff} = n \times h$.

6. The physically most transparent formulation of criticality as a hierarchy of broken super-symplectic conformal symmetries emerged rather recently. Super-symplectic algebra has an infinite fractal hierarchy of isomorphic sub-algebras with conformal weights coming as multiple of integer n for a given sub-algebra. The natural hypothesis is that the sub-algebra labelled by n acts as a conformal gauge algebra. This gives rise to infinite number of hierarchies of super-symplectic breakings labelled by sequences of integers $n_{i+1} = \prod_{k<i+1} m_k$. In a given symmetry breaking criticality is reduced as gauge degrees of freedom transform to physical ones. At quantum level the gauge sub-algebra labelled by n annihilates the physical states. At space-time level the corresponding super-symplectic Noether charges vanish. This defines precisely what it means to be a preferred extremal in zero energy ontology (ZEO).

2.1.5 Hierarchy of Planck constants

The hierarchy of Planck constants realized as a replacement of CD and CP_2 of $CD \times CP_2$ with book like structures labeled by finite subgroups of $SU(2)$ assignable to Jones inclusions is now relatively well understood as also its connection to dark matter, charge fractionization, and anyons [K9, K13].

1. This notion leads also to a unique identification of number theoretical braids as intersections of CD (CP_2) projection of X_l^3 and the back M^2 (the backs S_I^2 and S_{II}^2) of M^4 (CP_2) book. The spheres S_I^2 and $S_{II}^2$are geodesic spheres of CP_2 orthogonal to each other).

2. The formulation of M-matrix should involve the local data from the points of number theoretic braids at partonic 2-surfaces. This data involves information about tangent space of $X^4(X^3)$ so that the theory does not reduce to 2-D theory. The hierarchy of CDs within CDs means that the improvement of measurement resolution brings in new CDs with smaller size.

3. The points of number theoretical braids are by definition quantum critical with respect to the phase transitions changing Planck constant and meaning leakage between different pages of the books in question. This quantum criticality need not be equivalent with the quantum criticality in the sense of the degeneracy of the matrix like entity defined by the second variation of Kähler action. Note that the entire partonic 2-surface at the boundary of CD cannot be quantum critical unless it corresponds to vacuum state with only topological degrees of freedom excited (that is have as its CD (CP_2) projection at the back of CD (CP_2) book or both) since Planck constant would be ill-defined in this kind of situation.

2.1.6 Super-conformal symmetries

The attempts to understand super-conformal symmetries has been unavoidably a guess work and produced several alternative scenarios. The consistency with p-adic mass calculations requiring five tensor factors to Super-Virasoro algebra has been the basic experimental constraint. The work with Kähler-Dirac equation has helped dramatically in the attempts to understand of super-conformal symmetries. Also the understanding of Super-Kac-Moody symmetries acting as gauge symmetries and made possible by the non-determinism of Kähler action has helped a lot.

There have been a considerable progress also in the understanding of super-conformal symmetries [K20, K6].

1. Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW .

2. One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

3. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

4. A further step of progress relates to the understanding of the fusion rules of symplectic field theory [K4]. These fusion rules makes sense only if one allows discretization that is number theoretic braids. An infinite hierarchy of symplectic fusion algebras can be identified with nice number theoretic properties (only roots of unity appear in structure constants). Hence

there are good hopes that symplecto-conformal N-point functions defining the vertices of generalized Feynman diagrams can be constructed exactly.

5. The possible reduction of the fermionic Clifford algebra to a finite-dimensional one means that super-conformal algebras must have a cutoff in conformal weights. These algebras must reduce to finite dimensional ones and the replacement of integers with finite field is what comes first in mind.

6. The conserved fermionic currents implied by vanishing second variations of Kähler action for preferred extremal define a hierarchy of super-conformal algebras assignable to zero modes. These currents are appear in the expression of measurement interactions added to the Kähler-Dirac action in order to obtain stringy propagators and the coding of super-conformal quantum numbers to space-time geometry.

2.2 Various Inputs To The Construction Of M-Matrix

It is perhaps wise to summarize briefly the vision about M-matrix.

2.2.1 Zero energy ontology and interpretation of light-like 3-surfaces as generalized Feynman diagrams

1. Zero energy ontology is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X^3_l connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Light-like 3-surfaces are the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

2.2.2 Identification of TGD counterpart of S-matrix as time-like entanglement coefficients

1. The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix-call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several good arguments suggesting that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

2.2.3 Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW .

2.2.4 Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix

in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

2.2.5 Conformal symmetries and stringy diagrammatics

The Kähler-Dirac equation has rich super-conformal symmetries helping to achieve concrete vision about the structure of M-matrix in terms of generalized Feynman diagrammatics.

Both super-conformal symmetries and the effective reduction of space-time sheet to string world sheets at Minkowskian regions as a consequence of finite measurement resolution suggest that the generalized Feynman diagrams have as vertices N-point functions of a conformal field theory assignable to the partonic 2-surfaces at which the lines of Feynman diagram meet. The vertices can be assigned with wormhole contacts with Euclidian signture of induced metric. In Minkowskian regions fundamental fermions propagate like massless particles along boundaries of string world sheets. One can say that a hybrid of Feynman and stringy diagrammatics results.

Finite measurement resolution means that this conformal theory is defined in the discrete set defined by the intersections of braids defined by boundaries of string worlds sheets with partonic two-surfaces. The presence of symplectic invariants in turn suggest a symplectic variant of conformal field theory leading to a concrete construction of symplectic fusion rules relying in crucial manner to discretization.

2.2.6 TGD as almost topological QFT

The idea that TGD could be regarded as almost topological QFT has been very fruitful although the hypothesis that Chern-Simons term for induced Kähler gauge potential assignable to light-like 3-surfaces identified as regions of space-time where the Euclidian signature of induced metric assignable to the interior or generalized Feynman diagram changes to Minkowskian one turned out to be too strong. The reduction of WCW and its Clifford algebra to finite dimensional structures due to finite measurement resolution however realizes this idea but in different manner.

1. There is functional integral over the small deformations of Feynman cobordisms corresponding to the maxima of Kähler function which is finite-dimensional if finite measurement resolution is taken into account. Almost topological QFT property of quantum suggests the identification of M-matrix as a functor from the category of generalized Feynman cobordisms (generalized Feynman diagrams) to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces.

2. The limit at which momenta vanish is well-defined for M-matrix since the Kähler-Dirac action contains measurement interaction term and at this limit one indeed obtains topological QFT.

3. Almost TQFT property suggests that braiding S-matrices should have important role in the construction. It is indeed possible to assign the with the lines of the generalized Feynman diagram. The reduction of quantum TGD to topological QFT should occur at quantum criticality with respect to the change of Planck constant since in this situation the M-matrix should not depend at all on Planck constant. Factoring QFTs in 1+1 dimensions give examples of this kind of theories.

2.2.7 Heuristic picture about generalized Feynman rules

Concerning the understanding of the relationship between HFFs and M-matrix the basic implications are following.

1. General visions do not allow to provide explicit expressions for M-matrix elements. Therefore one must be humble and try to feed in all understanding about quantum TGD and from the quantum field theoretic picture. In particular, the dependence of M-matrix on Planck constant should be such that the addition of loop corrections as sub- CDs corresponds to an expansion in powers of $1/\hbar$ as in quantum field theory whereas for tree diagrams there is no dependence on $\hbar$.

2. The vacuum degeneracy of Kähler action and the identification of Kähler function as Dirac determinant strongly suggest that fermionic oscillator operators define what could be interpreted as a finite quantum-dimensional Clifford algebra identifiable as a factor space $\mathcal{M}/\mathcal{N}$, $\mathcal{N} \subset \mathcal{M}$. One must be however very cautious since also an alternative option in which excitations of labeled by conformal weight are present cannot be excluded. Finite-dimensionality would mean an enormous simplification, and together with the unique identification of number theoretic braids as orbits of the end points of string world sheets this means that the dynamics is finite-quantum-dimensional conforming with the fact effective finite-dimensionality is the defining property of HFFs. Physical states would realize finite measurement resolution in their structure so that approximation would cease to be an approximation.

3. An interesting question is whether this means that M-matrix must be replaced with quantum M-matrix with operator valued matrix elements and whether the probabilities should be determined by taking traces of these operators having interpretation as averaging over $\mathcal{N}$ defining the degrees of freedom below measurement resolution. This kind of picture would conform with the basic properties of HFFs.

4. To the strands of number theoretic braids one would attach fermionic propagators. Since bosons correspond to fermion pairs at the throats of wormhole contact, all propagators reduce to fermionic ones. As found, the addition of measurement interaction term fixes fermionic propagator completely and gives it a stringy character.

5. Similar correlation function in WCW degrees of freedom would be given in lowest order - and perhaps exact - approximation in terms of the contravariant metric of the configuration space proportional to g_K^2. Besides this the exponent of Kähler action would be involved. For elementary particles it would be the exponent of Kähler action for CP_2 type vacuum extremal. In this manner something combinatorially very similar to standard perturbation theory would result and there are excellent hopes that p-adic coupling constant evolution in powers of 2 is consistent with the standard coupling constant evolution.

6. Vertices correspond to n-point functions. The contribution depending on fermionic fields defines the quantum number dependent part of the vertices and comes from the fermion field and their conjugats attached to the ends of propagator lines identified as braid strands. Besides this there is a symplecto-conformal contribution to the vertex.

7. The stringy variant of twistor Grassmannian approach is highly suggestive since the necessary conditions are satisfied. In particular, the fundamental fermions propagate in the internal lines effectively as massless on-mass shell states but with non-physical polarization. M^4 *resp.* CP_2 is the unique 4-D manifold *resp.* compact manifold with Minkowskian *resp.* Euclidian signature of metric allowing twistor space with Kähler structure [A4]. This suggests that a generalization of twistorialization to 8-D context makes sense. The twistor space for CP_2 is 6-dimensional flag manifold $SU(3)/U(1) \times U(1)$ parameterizing the choice of color quantization axes and has popped up earlier in TGD inspired theory of consciousness.

2.2.8 The expansion of M-matrix in powers of $\hbar$

One should understand how the proportionality of gauge couplings to g_K^2 emerges and how loops give rise to powers of α_K. In zero energy ontology one does not calculate M-matrix but tries to construct zero energy state in the hope that QFT wisdom yields cold help to construct Connes tensor product correctly.

1. The basic rule of quantum field theory is that each loop gives $\alpha = g^2/4\pi$ and thus $1/\hbar$ factor whereas in tree diagrams only g^2 appears so that they correspond to the semiclassical approximation.

2. This rule is obtained if one assumes loops correspond to a hierarchy of sub- CDs and that in loop one can distinguish one line as "base line" and other lines as radiative corrections. To each internal line one must one must assign the factor $r^{-1/2} = (\hbar_0/\hbar)^{1/2}$ and factor g_K^2 except to the portion of base line appearing in loop since otherwise double counting would

result. This dictates the expansion of M-matrix in powers of $r^{-1/2}$. It would not be too surprising to have this kind of expansion.

3. g_K^2 factor comes from the functional integral over the partonic 2-surface selected by stationary phase approximation using the exponent of Kähler action. The functional integral over the WCW degrees of freedom is carried out using contravariant Kähler metric as a propagator and this gives g_K^2 factor in the lowest non-trivial order since one must develop a perturbation theory with respect to the deformations at the partonic 2-surfaces at the ends of line.

   If the analogs of radiative corrections to this functional integral vanish - as suggested by quantum criticality and required by number theoretic universality - the resulting dependence on g_K^2 is exact and completely analogous to the free field theory propagator. The numerical factors give the appropriate gauge coupling squared.

4. Besides this one must assign to the ends of the propagator line positive and negative energy parts of quantum state representing the particle in question. These give a contribution which is zeroth order in $\hbar$. For instance, gauge bosons correspond to fermionic bilinears. Essentially fermion currents formed from spinor fields at the two light-like wormhole throats of the wormhole contact at which the signature of the induced metric changes are in question. Correct dimension requires the presence of $1/\hbar$ factor in boson state and $1/\sqrt{\hbar}$ factor in fermion state. The correlators between fermionic fields at the end points of the line are proportional to $\hbar$ so that normalization factors cancel the $\hbar$ dependence. Besides this one would expect N-points function of symplecto-conformal QFT with $N = N_{in} + N_{out}$ having no dependence on $\hbar$.

2.3 But What About The Concrete Feynman Rules?

The skeptic reader can say that all this is just an endless list of general principles. I dare however claim that the only manner to proceed is to try to identify the general principles first. At this moment the understanding of the fundamental variational principled of TGD understood at such level of detail that one can indeed sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to the 4-D surfaces defined by the Euclidian regions defined by wormhole contacts plus the string world sheets connecting them and carrying spinor modes. One might also talk about combination of Feynman diagrams and stringy diagrams or even about generalization of Wilson loops. The lines of these diagrams form also braids.

1. The boundaries of string world sheets at which the modes of induced spinor field are localized (by well-definedness of em charge) carry fermion number and are identifiable as braid strands within partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. 1-D Dirac action for induced metric and its bosonic counterpart - must be assigned with partonic orbits in order to obtain non-trivial fermionic propagator. Massles fermion propagator emerges if light-like portions of string world sheet boundary contain 1-D Dirac action in induced metric. The bosonic part of this action implied by supersymmetry implies that light-like geodesic of embedding space is in question and there is a conserved light-like four-momentum associated with the fermion line.

2. The fundamental interaction is the scattering of fermions at opposite wormhole throats of wormhole contact. With string model based intuition one can argue that this interaction must correspond essentially to the stringy propagator $1/L_0$ so that one would obtain a combination of Feynman rules and stringy rules. The vertices correspond topologically to a fusion of 4-D lines along the 3-surfaces at their ends and this means deviation from string model picture: stringy diagrams correspond at topological level to what happens when particle travels between A and B along two different routes and has nothing to do with particle decay.

   One can criticize this idea about ad hoc character. Furthermore, super-symmetry requires also the presence of super-generator G and its hermitian conjugate. In TGD however these operators carry baryon or lepton number and cannot appear as propagators unless they appear as pairs $GG^{dagger} \propto L_0$.

The vision about scattering amplitudes as sequences of algebraic operations with 3-vertices identified as product and co-products in super-sympelectic Yangian of super-symplectic algebra looks much more feasible option [K17].

3. Physical particles are bound states of massless fundamental fermions and correspond to pairs of wormhole contacts: a pair is required since wormhole throats behave effectively as magnetic monopoles and closed flux tube consisting of pieces at the two space-time sheets and wormhole contacts is required. This resolves the infrared difficulties of twistor approach. Twistor Grassmann approach strongly suggests that the residue integral over the virtual four-momenta reduces the propagators of fundamental fermions to their inverses at mass-shell so that only non-physical fermion helicities appear as virtual fermions.

The reader wishing for a brief summary of TGD might find the three articles about TGD, TGD inspired theory of consciousness, and TGD based view about quantum biology helpful [L3, L2, L1].

3 How To Define Generalized Feynman Diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in ZEO (ZEO) [K14] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K17] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. ZEO (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also embedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K17] . Kähler-Dirac gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D embedding space parameterized by quaternionic space-time surfaces.

5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K20]. It took long time to realize that in ZEO the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

   One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K20, K17] .

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B5] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of Kähler-Dirac operator.

2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K10] in infinite-dimensional context already in the case of much simpler loop spaces [A2] .

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II_1 defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

3.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to this goal is by making questions.

3.1.1 What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the embedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the Kähler-Dirac action [K20] suggests however a de-localization of braid points, that is wave function in space

of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

   In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. The light-like 8-momenta p^k have same M^4 and CP_2 mass squared and latter correspond to the the eigenvalues of the CP_2 spinor d'Alembertian by quantum-classical correspondence.

5. One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

6. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

3.1.2 How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case

would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

3.1.3 How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the Kähler-Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub-CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for ZEO.

4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

3.2 Generalized Feynman Diagrams At Fermionic And Momentum SpaceLevel

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. ZEO encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

3.2.1 Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_\pm$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

3.2.2 Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the Kähler-Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^\alpha p_\alpha + \hat{\Gamma}^\alpha D_\alpha \ , \\ p_\alpha &= p_k \partial_\alpha h^k \ . \end{aligned} \tag{3.1}$$

   The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is Kähler-Dirac gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D Kähler-Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

4. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [?] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

3.2.3 Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B1] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_\pm$ pairs ($X_\pm$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_\pm$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and Kähler-Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [?] .

3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \to F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).

4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K8] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

3.3 Harmonic Analysis In WCW As a way To Calculate WCWFunctional Integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

3.3.1 Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator

a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2-m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the Kähler-Dirac operator D at propagator lines [K20] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i=1,2 \ . \end{aligned} \tag{3.2}$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \tag{3.3}$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the Kähler-Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \tag{3.4}$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

3.3.2 Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K7, K20]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \tag{3.5}$$

   works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/ 4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

   The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of embedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

   Recall that also the construction of number theoretic braids and symplectic QFT [K5] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

   This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \ . \tag{3.6}$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J_{kl}^- \ , \\ J_\pm^{kl} &= (1+K_\pm)\partial_\alpha s^k \partial_\beta s^l J_\pm^{\alpha\beta} \ . \end{aligned} \tag{3.7}$$

The tensors are lifts of the induced Kähler form of $X_\pm^2$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the Kähler-Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1, s^2)/\partial(x_\pm^1, x_\pm^2)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

3.3.3 Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

3.3.4 Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

2. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

3. In ZEO this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

3.3.5 Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the Kähler-Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

4 A More Detailed View About The Construction Of Scattering Amplitudes

The following represents an update view about construction of scattering amplitudes at the level of "world of classical worlds" (WCW).

4.1 Basic Principles

In order to facilitate the challenge of the reader I summarize basic ideas behind the construction of scattering amplitudes.

4.1.1 Construction of scattering amplitudes as functional integrals in WCW

The decomposition of space-time surface to Minkowskian and Eucldian regions is the basic distinction from ordinary quantum field theories since it replaces path integral with mathematically well-defined functional integral over WCW .

1. Space-time surface decomposes to regions with Minkowskian or Euclidian signature of the induced metric. The regions with Euclidian metric are identified as lines of generalized Feynman diagrams. The boundaries between two kinds of regions - to be called parton orbits - can be regarded as carriers of elementary particle quantum numbers such as fermion number assignable to the boundaries of string world sheets at them. Induced spinor fields are localized at them from the well-definedness of electromagnetic charge requiring that induced W boson fields vanish. Hence strings emerge from TGD. Note that at boundary between Euclidian and Minkowskian regions the metric determinant vanishes. Unlike the name would suggest, generalized Feynman diagrams are analogous to twistor diagrams, and instead of infinite number of superposed diagrams there might just single diagram.

2. Weak form of electric magnetic duality together with the assumption that the term $j^\alpha A_\alpha$ in Kähler action vanishes imply that Kähler action reduces to 3-D Chern-Simons term. This hypothesis is inspired by TGD as almost topological quantum field theory conjecture. In Minkowskian regions this conjecture is very natural. In the Euclidian region the contribution to Kähler action need not reduce to a mere Chern-Simons term associated with its boundary. This would be due to the non-triviality of the U(1) bundle defined by Kähler form giving also Chern-Simons terms inside the CP_2 type vacuum extremal.

3. Scattering amplitude is a functional integral over space-time surfaces: the data about these space-time surfaces are coded by their ends about the opposite light-like boundaries of causal diamond (CD) of given scale. The weight function in the functional integral is exponential of Kähler function of "world of classical worlds" coming from Euclidian regions of the space-time surface representing lines of generalized Feynman diagram and being deformation of CP_2 type vacuum extremals representing wormhole contacts connecting two space-time sheets with Minkowskian signature of induced metric. Kähler function is the exponent of Kähler action from Euclidian regions. The real exponent takes care that the functional integral is obtained instead of path integral so that the outcome is mathematically well-defined.

4. Euclidian region would give only the analog of thermodynamics but there is also an imaginary exponential coming from the exponential of the imaginary Kähler action from Minkowskian regions. Space-time surfaces are extremals of Kähler action and for very general ansatz Minkowskian contribution to Kähler action reduces to imaginary Chern-Simons term at the light-like 3-D boundary between regions at which the 4-D metric is degenerate. This term makes possible interference of different contributions to the functional integral which is absolutely essential in quantum field theory.

5. The details of the theory in fermionic sector have turned out to be crucial. From the well-definedness of the electric charge for the modes of the induced spinor field - and also by number theoretic arguments - spinor modes are localized at 2-D string world sheets carrying vanishing W gauge fields. Preferred extremals can be constructed by fixing first partonic 2-surfaces, string world sheets, and possibly also the light-like orbits of partonic 2-surfaces and posing the condition that the canonical momentum densities have no components normal to string world sheets. Also the condition that a sub-algebra of super-symplectic algebra gives rise to vanishing Noether charges at the space-like ends of preferred extremal is natural.

   This construction would conform with the strong form of holography. The boundaries of string world sheets at the light-like orbits of partonic 2-surfaces carry 1-D Dirac action for induced gamma matrices. The bosonic counterpart of this action gives as solutions light-like geodesics of embedding space - light-likeness in 8-D sense. 1-D Dirac equation for induced gamma matrices is satisfied. A very twistorial picture emerges and suggests 8-D generalization of twistor approach. M^4 and CP_2 are indeed twistorially completely unique.

6. The generators of super-symplectic algebra can be represented as Noether charges for the fermionic strings and the supercharges identifiable as WCW gamma matrices are natural identification for fermionic oscillator operators. Since one expects that a given partonic 2-surface is connected to a large number of partonic 2-surfaces a generalization to Yangian [A1] [B4, B2, B3] of super-symplectic algebra seems necessary and is in spirit with twistorialization. It seems possible to identify the fundamental vertices assignable to partonic 2-surfaces at which three lines of diagram meet in terms of product and co-product for Yangian so that there are hopes about realizing the already forgotten TGD inspired dream about reduction of scattering amplitudes to sequences of algebraic operations of Yangian with minimal length and connecting chosen initial and final states at the boundaries of CD. Universe would be Yangian algebraist!

So what one expects vertices and propagators to be? Fermionic propagators would be massless in 8-D sense and they should be contracted with the legs of the vertices defined by product tor co-product involving three Yangian generators. Structure constants would define the coupling constants. Each Yangian generator would involve a collection of fermions fields associated with strings and with each fermion field propagator would contract. The only modification of the ordinary vertex is that partonic 2-surfaces carry many-fermion states and the vertices involve 3 multi- fermion states. Fermion lines can also turn backwards in time: this gives rise to virtual bosons.

4.1.2 Why it might work?

There are many reasons encouraging the hopes about calculable theory.

1. The theory has huge super-conformal symmetries dramatically reducing the dynamical degrees of freedom by the choice of conformal gauge. This implies that both the space-like 3-surfaces at the ends of space-time surface and partonic orbits satisfy classical Super conformal conditions for generalizations of ordinary super-conformal algebras perhaps extending to multilocal Yangian with loci identified as strings connecting partonic 2-surfaces at the light-like boundary of CD. This algebra extends also to include both boundaries of CD. Fermionic anticommutation relations which allow by 2-dimensionality of string world sheet also quantum group variant determine the anticommutations between all generators.

   Yangian symmetry in turn gives excellent hopes about twistorialization: in fact, $M^4 \times CP_2$ is completely unique choice for the embedding space by twistorial considerations and the product of the twistor spaces of M^4 and CP_2 allows to constructed the twistor spaces of space-time surfaces as liftings of the extremals of Kähler action to 6-D sphere bundles over space-time surface.

2. The integrand in the functional integral represents the analog of ordinary Feynman diagrams involving only fermions and 1-D lines. Indeed, by bosonic emergence all bosons (in fact all elementary particles) can be regarded as composites of fundamental fermions. The only purely fermionic vertices are 2-fermion vertices. 3-vertices correspond to space-time surfaces

meeting along common 3-surface and are thus purely topological, and as already mentioned could correspond to product and co-product for Yangian. This is of course excellent news from the point of view of finiteness. The fermionic vertices are represented by the discontinuity of the Kähler-Dirac operator associated with the string boundary line at partonic 2-surface so that there are no coupling constants involved. The only fundamental coupling parameter is Kähler strength whose value is dictated by quantum criticality as the analog of critical temperature.

One must have a view about what elementary particles - as opposed to fundamental fermions - are, how the ordinary view about scattering based on exchanges of elementary particles emerges from this picture and how say BFF vertex reduces to a diagram at for fundamental fermions involving only 2-fermion vertices.

4.2 Elementary Particles In TGD Framework

The notion of elementary particles involves two aspects: elementary particles as space-time surfaces and elementary particles as many-fermion states with fundamental fermions localized at the wormhole throats and defining elementary particles as their bound states (including physical fermions).

Let us first summarize what kind of picture ZEO suggests about elementary particles.

1. Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [**?**].

2. The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see **Fig.** `http://tgdtheory.fi/appfigures/wormholecontact.jpg` or **Fig. ??** in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.

3. A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

   The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the Kähler-Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is sparticle of left-handed neutrino.

   It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

4.3 Scattering Amplitudes

The basic challenge is to introduce vertices and fermionic propagators. The recent based on stringy realization of Yangian algebra allows to do this.

4.3.1 Fermionic propagators

How fermionic propagators emerge? The first explanation coming in mind is based on the discontinuity associated with the Dirac operator at the partonic 2-surfaces defining vertices.

Discontinuities can be of two different types. Fermionic lines has discontinuous tangent at the partonic 2-surfaces meaning local non-conservation of light-like 8-momentum. Also second kind of discontinuity in which two lines belonging to orbits of distinct partonic 2-surfaces emerge at single point. Their 8-momenta need not be opposite if one requires only global momentum conservation. If it is assumed one can say that fermionic line turns backwards in time. These kind of pairs of lines forming closed curves with peaks at ends are associated with bosonic propagators- say those describing boson exchange between two fermions.

The discontinuities of the induced spinor along the fermionic line making a turn at the partonic 2-surface give rise to delta function singularities under the action of 1-D Dirac operator. This would give Dirac equation with a source term and its solution would be given by Dirac propagator convoluted with the discontinuity.

4.3.2 Vertices

Vertices can be considered at both space-time level and fermionic level.

1. At space-time level vertices correspond to the fusion of space-surfaces representing particles along common 3-surface defining the vertex. At the parton level 3-light-like parton orbits fuse together along partonic 2-surface. In these vertices particle number changes this change correspond the change of particle number for elementary particles.

2. At fermion level vertices are localized at the partonic 2-surfaces. The above argument would suggest that vertices corresponds to the discontinuity of the Kähler Dirac operator at the corner of the line representing the boundary of string world sheet. The creation of fermion pair from vacuum corresponds to an corner of string boundary at which the boundaries of string world sheets associated with two outgoing or incoming sheets meet. The creation/annihilation of a fermion pair is essential for the realization of say tree diagrams describing fermion scattering by virtual boson exchange.

The identification of vertex as a product or co-product in Yangian looks the most promising approach. The charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator at each leg plus vertex factor. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge - essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormfhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place. One obtains integration of the light-like 8-momenta of fermions in natural manner and something resembling very strongly standard QFT. The integration interpreted as residue integral should give only inverse of the propagator actin on on mass shell states with wrong helicity. Virtual fermions would have wrong helicity unlikes incoming ones.

4.4 What One Should Obtain At QFT Limit?

After functional integration over WCW of one should obtain a scattering amplitude in which the fermionic 2- vertices defined as discontinuities of the Kähler-Dirac operator at partonic 2-surfaces should boil down to a contraction of an M^8 vector with gamma matrices of M^8. This vector has dimension of mass. This basic parameter should characterize many different physical situations. Consider only the description of massivation of elementary particles regarded as bound states of fundamental massless fermions and the mixing of left and right-handed fermions. Also CKM mixing should involve this parameter. These vectors should also appear in Higgs couplings, which in QFT description contain Higgs vacuum expectation as a factor.

In twistor approach virtual particles have complex light-like 8-momenta. Fundamental fermions have most naturally real and light-like momenta. $\mathcal{N} = 4$ SUSY describes gauge bosons which correspond to bound states of fundamental fermions in TGD. This suggests that the four-momenta of bound states of massless fermions - be they hadrons, leptons, or gauge bosons - can be taken to be complex.

There is an intriguing connection with TGD based notion of space-time. In TGD one obtains at space-time level complexified light-like 8-momenta since the 8-momentum from Minkowskian/Euclidian region is real/imaginary. In the case of physical particle necessary involving two wormhole contacts and two flux tubes connecting them the total complexifies four momentum would be sum of two real and two imaginary contributions. Every elementary particle should have also imaginary part in its 8-momentum and would be massless in complexified sense allowing mass in real sense given by the length of the imaginary four-momentum. In twistor approach complex light-like momenta indeed appear in BCFW bridge.

TGD predicts Higgs boson although Higgs expectation does not have any role in quantum TGD proper. Higgs vacuum expectation is however a necessary part of QFT limit (Higgs decays to WW pairs require that vacuum expectation is non-vanishing). Higgs vacuum expectation must correspond in TGD framework to a quantity with dimensions of mass. In TGD Higgs cannot be scalar but a vector in CP_2 degrees of freedom. The problem is that CP_2 does not allow covariantly constant vectors. The imaginary part of classical four-momentum gives a parameter which has interpretation as a vector in the tangent space of which is same as that of $M^4 \times CP_2$. Could $M^8 - H$ duality be realized at the level of tangent space and for relate four-momentum and color quantum numbers to the E^4 part of 8-momentum?

Elementary particles of course need not be eigenstates of the CP_2 part of 8-momentum. For a fixed mass one can have wave functions in the space of CP_3 part of 8-momentum analogous to S^3 spherical harmonics at the sphere of E^4 with radius defined by the length of imaginary four-momentum (mass). These harmonics are characterized by $SO(4)$ quantum numbers. Could one interpret this complexification in terms of M^8-duality and say that SO(4) defines the symmetries for the low energy dual of WCW defining high energy description of QCD based on SU(3) symmetry. SO(4) would corresponds to the symmetry group assigned to hadrons in the approach based on conserved vector currents and partially conserved axial currents. SO(4) would be much more general and associated also with leptons.

The anomalous color hyper-charge of leptonic spinors would imply that one can have also in the case of leptons a wave function in S^3. Higher harmonics would correspond to color excitations of leptons and quarks. If one considers gamma matrices, complexification of M^4 means introduction of gamma matrix algebra of complexified M^4 requiring 8 gamma matrices. This suggests a connection with $M^8 - H$ duality. All elementary particles have also imaginary part of four-momentum and the 8-momentum can be interpreted as M^8-momentum combining the four-momentum and color quantum numbers together.

REFERENCES

Mathematics

[A1] Yangian symmetry. Available at: `https://en.wikipedia.org/wiki/Yangian`.

[A2] Freed DS. The Geometry of Loop Groups, 1985.

[A3] Witten E. Fivebranes and Knots, 2010. Available at: https://arxiv.org/pdf/1101.3216v1.

[A4] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc*, 8(43):133–151, 1981.. Available at: https://tinyurl.com/pb8zpqo.

Theoretical Physics

[B1] Montonen Olive Duality. Available at: https://en.wikipedia.org/wiki/Montonen-Olive_duality.

[B2] Witten E Dolan L, Nappi CR. Yangian Symmetry in $D = 4$ superconformal Yang-Mills theory, 2004. Available at: https://arxiv.org/abs/hep-th/0401243.

[B3] Plefka J Drummond J, Henn J. Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory, 2009. Available at: https://cdsweb.cern.ch/record/1162372/files/jhep052009046.pdf.

[B4] Arkani-Hamed N et al. The All-Loop Integrand For Scattering Amplitudes in Planar N=4 SYM, 2010. Available at: https://arxiv.org/abs/1008.2958.

[B5] Cutkosky RE. *J Math Phys*, (1):429–433, 1960.

Particle and Nuclear Physics

Neuroscience and Consciousness

Books related to TGD

[K1] Pitkänen M. Cosmology and Astrophysics in Many-Sheeted Space-Time. In *Topological Geometrodynamics: Overview: Part II. https://tgdtheory.fi/tgdhtml/Btgdview2.html*.

[K2] Pitkänen M. Appendix A: Quantum Groups and Related Structures. In *Hyper-finite Factors and Dark Matter Hierarchy: Part I.* Available at: https://tgdtheory.fi/pdfpool/bialgebra.pdf, 2023.

[K3] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/class.pdf, 2023.

[K4] Pitkänen M. Category Theory and Quantum TGD. In *TGD and Hyper-finite Factors. https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/categorynew.pdf, 2023.

[K5] Pitkänen M. Construction of Quantum Theory: M-matrix. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/towards.pdf, 2023.

[K6] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/quthe.pdf, 2023.

[K7] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/compl1.pdf, 2023.

[K8] Pitkänen M. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG: Part I. https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/eegdark.pdf, 2023.

[K9] Pitkänen M. Does TGD Predict a Spectrum of Planck Constants? In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/Planck, 2023.

[K10] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/kahler.pdf., 2023.

[K11] Pitkänen M. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/knotstgd.pdf, 2023.

[K12] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness: Part I. https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/nmpc.pdf, 2023.

[K13] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *TGD and Condensed Matter. https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/anyontgd.pdf, 2023.

[K14] Pitkänen M. *Quantum TGD.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/tgdquantum.html., 2023.

[K15] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionb.pdf, 2023.

[K16] Pitkänen M. *TGD Inspired Theory of Consciousness.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/tgdconsc.html., 2023.

[K17] Pitkänen M. The classical part of the twistor story. In *Quantum TGD: Part III. https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/twistorstory.pdf, 2023.

[K18] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/numbervision.pdf, 2023.

[K19] Pitkänen M. Was von Neumann Right After All? In *TGD and Hyper-finite Factors. https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/vNeumann.pdf, 2023.

[K20] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/cspin.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. TGD Inspired Quantum Model of Living Matter. Available at: https://tgdtheory.fi/quantumbio.pdf., 2008.

[L2] Pitkänen M. TGD Inspired Theory of Consciousness. Available at: https://tgdtheory.fi/tgdconsc.pdf., 2008.

[L3] Pitkänen M. Topological Geometrodynamics: What Might Be the Basic Principles. Available at: https://tgdtheory.fi/public_html/articles/tgd2008.pdf., 2008.

[L4] Pitkänen M. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. Available at: https://www.tgdtheory.fi/tgdglossary.pdf., 2014.