Various General Ideas Related to Quantum TGD

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Contents

Abstract

I have gathered to this chapter those ideas related to quantum TGD which are not absolutely central and whose status is not clear in the long run. I have represented earlier these ideas in chapters and the outcome was a total chaos and reader did not have a slightest idea what is they real message. I hope that this organization of material makes it easier for the reader to grasp the topology of TGD correctly.

1 Introduction

I have gathered to this chapter those ideas related to quantum TGD which are not absolutely central and whose status is not clear in the long run. I have represented earlier these ideas in chapters and the outcome was a total chaos and reader did not have a slightest idea what is they real message. I hope that this organization of material makes it easier for the reader to grasp the topology of TGD correctly. The representation includes various ideas and notions such as weak form of electric magnetic duality, $M^8 - H$ duality, hierarchy of Planck constants, and the notion of number theoretic braid. Sections related to WCW integration and about possible topological invariances defined by geometric invariants for preferred extremals of Kähler action are included.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L2]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L3]. The topics relevant to this chapter are given by the following list.

- Quantum theory [L8]
- Emergent ideas and notions [L4]
- Weak form of electric-magnetic duality [L11]
- $M^8 - H$ duality [L7]
- Hierarchy of Planck constants [L5]
- Hyperfinite factors and TGD [L6]
- The unique role of twistors in TGD [L9]
- Twistors and TGD [L10]

2 $M^8 - H$ Duality, Preferred Extremals, Criticality, And Mandelbrot Fractals

$M^8 - H$ duality [K10] represents an intriguing connection between number theory and TGD but the mathematics involved is extremely abstract and difficult so that I can only represent conjectures. In the following the basic duality is used to formulate a general conjecture for the construction of preferred extremals by iterative procedure. What is remarkable and extremely surprising is that the iteration gives rise to the analogs of Mandelbrot fractals and space-time surfaces can be seen as fractals defined as fixed sets of iteration. The analogy with Mandelbrot set can be also seen as a geometric correlate for quantum criticality.

2.1 $M^8 - H$ Duality Briefly

$M^8 - M^4 \times CP_2$ duality [**?**]tates that certain 4-surfaces of M^8 regarded as a sub-space of complexified octonions can be mapped in a natural manner to 4-surfaces in $M^4 \times CP_2$: this would mean that $M^4 \times CP_2$ and therefore also the symmetries of standard model would have purely number theoretical meaning.

Consider a distribution of two planes $M^2(x)$ integrating to a 2-surface $\tilde{M}^2$ with the property that a fixed 1-plane M^1 defining time axis globally is contained in each $M^2(x)$ and therefore in $\tilde{M}^2$. M^1 defines real axis of octonionic plane M^8 and M^2(x) a local hyper-complex plane. Quaternionic subspaces with this property can be parameterized by points of CP_2: this leads to $M^8 - H$ duality as can be shown by a simple argument.

1. Hyper-octonionic subspace of complexified octonions is obtained by multiplying octonionic imaginary units by commuting imaginary unit. This does not bring anything new as far as automorphisms are considered so that it is enough to consider octonions (so that M^2 is replaced with C). Octonionic frame consists of orthogonal octonionic units. The space of octonionic frames containing sub-frame spanning fixed C is parameterized by $SU(3)$. The reason is that complexified octonionic units can be decomposed to the representations of $SU(3) \subset G_2$ as $1 + 1 + 3 + \overline{3}$ and the sub-frame 1+1 spans the preferred C.

2. The quaternionic planes H are represented by frames defined by four unit octonions spanning a quaternionic plane. Fixing $C \subset H$ means fixing the 1+1 part in the above decomposition. The sub-group of $SU(3)$ leaving the plane H invariant can perform only a rotation in the plane defined by two quaternionic units in 3. This sub-group is $U(2)$ so that the space of quaternionic planes $H \supset C$ is parameterized by $SU(3)/U(2) = CP_2$.

3. Therefore quaternionic tangent plane $H \supset C$ can be mapped to a point of CP_2. In particular, any quaternionic surface in E^8, whose tangent plane at each point is quaternionic and contains C, can be mapped to $E^4 \times CP_2$ by mapping the point $(e_1, e_2) \in E^4 \times E^4$ to $(e_1, s) \in e^4 \times CP_2$. The generalization from E^8 to M^8 is trivial. This is essentially what $M^8 - H$ duality says.

This can be made more explicit. Define quaternionic surfaces in M^8 as 4-surfaces, whose tangent plane is quaternionic at each point x and contains the local hyper-complex plane $M^2(x)$ and is therefore labelled by a point $s(x) \in CP_2$. One can write these surfaces as union over 2-D surfaces associated with points of $\tilde{M}^2$:

$$X^4 = \cup_{x \in \tilde{M}^2} X^2(x) \subset E^6 \quad .$$

These surfaces can be mapped to surfaces of $M^4 \times CP_2$ via the correspondence $(m(x), e(x)) \to (m, s(T(X^4(x)))$. Also the image surface contains at given point x the preferred plane $M^2(x) \supset M^1$. One can also write these surfaces as union over 2-D surfaces associated with points of $\tilde{M}^2$:

$$X^4 = \cup_{x \in \tilde{M}^2} X^2(x) \subset E^2 \times CP_2 \quad .$$

One can also ask what are the conditions under which one can map surfaces $X^4 = \cup_{x \in \tilde{M}^2} X^2 \subset E^2 \times CP_2$ to 4-surfaces in M^8. The map would be given by $(m, s) \to (m, T^4(s)$ and the surface would be of the form as already described. The surface X^4 must be such that the distribution of 4-D tangent planes defined in M^8 is integrable and this gives complicated integrability conditions. One might hope that the conditions might hold true for preferred extremals satisfying some additional conditions.

One must make clear that the conditions discussed above do not allow most general possible surface.

1. The point is that for preferred extremals with Euclidian signature of metric the M^4 projection is 3-dimensional and involves light like projection. Here the fact that light-like line $L \subset M^2$ spans M^2 in the sense that the complement of its orthogonal complement in M^8 is M^2. Therefore one could consider also more general solution ansatz for which one has

$$X^4 = \cup_{x \in L(x) \subset \tilde{M}^2} X^3(x) \subset E^2 \times CP_2 \quad .$$

2. One can also consider co-quaternionic surfaces as surfaces for which tangent space is in the dual of a quaternionic subspace. This says that the normal bundle rather than tangent bundle is quaternionic. The space-time regions with Euclidian signature of induced metric correspond naturally to co-quaternionic surfaces. Quaternionic surfaces are maximal associative sub-manifolds of octonionic space and one of the key ideas of the number theoretic vision about TGD is that associativity (co-associativity) defines the dynamics iof space-time surfaces. That this dynamics gives preferred extremals of Kähler action remains to be proven.

2.2 The Integrability Conditions

The integrability conditions are associated with the expression of tangent vectors of $T^(X^4)$ as a linear combination of coordinate gradients ∇m^k, where m^k denote the coordinates of M^8. Consider the 4 tangent vectors $e_{i)}$ for the quaternionic tangent plane (containing $M^2(x)$) regarded as vectors of M^8. $e_{i)}$ have components $e^k_{i)}$, $i = 1,..,4$, $k = 1,...,8$. One must be able to express $e_{i)}$ as linear combinations of coordinate gradients ∇m^k:

$$e^k_{i)} = e^\alpha_{i)}\partial_\alpha m^k \quad .$$

Here x^α and e^k denote coordinates for X^4 and M^8. By forming inner products of of $e_{i)}$ one finds that matrix $e^\alpha_{i)}$ represents the components of vierbein at X^4. One can invert this matrix to get $e^{i)}_\alpha$ satisfying $e^{i)}_\alpha e^\beta_{i)} = \delta^\beta_\alpha$ and $e^{i)}_\alpha e^\alpha_{j)} = \delta^i_j$. One can solve the coordinate gradients ∇m^k from above equation to get

$$\partial_\alpha m^k = e^{i)}_\alpha e^k_{i)} \equiv E^k_\alpha \quad .$$

The integrability conditions follow from the gradient property and state

$$D_\alpha E^k_\beta = D_\beta E^k_\alpha \quad .$$

One obtains $8 \times 6 = 48$ conditions in the general case. The slicing to a union of two-surfaces labeled by $M^2(x)$ reduces the number of conditions since the number of coordinates m^k reduces from 8 to 6 and one has 36 integrability conditions but still them is much larger than the number of free variables- essentially the six transversal coordinates m^k.

For co-quaternionic surfaces one can formulate integrability conditions now as conditions for the existence of integrable distribution of orthogonal complements for tangent planes and it seems that the conditions are formally similar.

2.3 How To Solve The Integrability Conditions And Field Equations For Preferred Extremals?

The basic idea has been that the integrability condition characterize preferred extremals so that they can be said to be quaternionic in a well-defined sense. Could one imagine solving the integrability conditions by some simple ansatz utilizing the core idea of $M^8 - H$ duality? What comes in mind is that M^8 represents tangent space of $M^4 \times CP_2$ so that one can assign to any point (m, s) of 4-surface $X^4 \subset M^4 \times CP_2$ a tangent plane $T^4(x)$ in its tangent space M^8 identifiable as subspace of complexified octonions in the proposed manner. Assume that $s \in CP_2$ corresponds to a fixed tangent plane containing $M^2(x)$, and that all planes $M^2(x)$ are mapped to the same standard fixed hyper-octonionic plane $M^2 \subset M^8$, which does not depend on x. This guarantees that s corresponds to a unique quaternionic tangent plane for given $M^2(x)$.

Consider the map $T \circ s$. The map takes the tangent plane T^4 at point $(m, e) \in M^4 \times E^4$ and maps it to $(m, s_1 = s(T^4)) \in M^4 \times CP_2$. The obvious identification of quaternionic tangent plane at (m, s_1) would be as T^4. One would have $T \circ s = Id$. One could do this for all points of the quaternion surface $X^4 \subset E^4$ and hope of getting smooth 4-surface $X^4 \subset H$ as a result. This is the case if the integrability conditions at various points $(m, s(T^4)(x)) \in H$ are satisfied. One could equally well start from a quaternionic surface of H and end up with integrability conditions in M^8 discussed above. The geometric meaning would be that the quaternionic surface in H is image of quaternionic surface in M^8 under this map.

Could one somehow generalize this construction so that one could iterate the map $T \circ s$ to get $T \circ s = Id$ at the limit? If so, quaternionic space-time surfaces would be obtained as limits of iteration for rather arbitrary space-time surface in either M^8 or H. One can also consider limit cycles, even limiting manifolds with finite-dimension which would give quaternionic surfaces. This would give a connection with chaos theory.

1. One could try to proceed by discretizing the situation in M^8 and H. One does not fix quaternionic surface at either side but just considers for a fixed $m_2 \in M^2(x)$ a discrete collection X $\{(T^4_i\} \supset M^2(x)$ of quaternionic planes in M^8. The points $e_{2,i} \subset E^2 \subset M2 \times$

$E^2 = M^4$ are not fixed. One can also assume that the points $s_i = s(T_i^4)$ of CP_2 defined by the collection of planes form in a good approximation a cubic lattice in CP_2 but this is not absolutely essential. Complex Eguchi-Hanson coordinates ξ^i are natural choice for the coordinates of CP_2. Assume also that the distances between the nearest CP_2 points are below some upper limit.

2. Consider now the iteration. One can map the collection X to H by mapping it to the set $s(X)$ of pairs $((m_2, s_i)$. Next one must select some candidates for the points $e_{2,i} \in E^2 \subset M^4$ somehow. One can define a piece-wise linear surface in $M^4 \times CP_2$ consisting of 4-planes defined by the nearest neighbors of given point $(m_2, e_{2,i}, s_i)$. The coordinates $e_{2,i}$ for $E^2 \subset M^4$ can be chosen rather freely. The collection $(e_{2,i,i})$ defines a piece-wise linear surface in H consisting of four-cubes in the simplest case. One can hope that for certain choices of $e_{2,i}$ the four-cubes are quaternionic and that there is some further criterion allowing to choose the points $e_{2,i}$ uniquely. The tangent planes contain by construction $M^2(x)$ so that the product of remaining two spanning tangent space vectors (e_3, e_4) must give an element of M^2 in order to achieve quaternionicity. Another natural condition would be that the resulting tangent planes are not only quaternionic but also as near as possible to the planes T_i^4. These conditions allow to find $e_{2,i}$ giving rise to geometrically determined quaternionic tangent planes as near as possible to those determined by s^i.

3. What to do next? Should one replace the quaternionic planes T_i^4 with geometrically determined quaternionic planes as near as possible to them and map them to points s_i slightly different from the original one and repeat the procedure? This would not add new points to the approximation, and this is an unsatisfactory feature.

4. Second possibility is based on the addition of the quaternionic tangent planes obtained in this manner to the original collection of quaternionic planes. Therefore the number of points in discretization increases and the added points of CP_2 are as near as possible to existing ones. One can again determine the points $e_{2,i}$ in such a manner that the resulting geometrically determined quaternionic tangent planes are as near as possible to the original ones. This guarantees that the algorithm converges.

5. The iteration can be stopped when desired accuracy is achieved: in other words the geometrically determined quaternionic tangent planes are near enough to those determined by the points s_i. Also limit cycles are possible and would be assignable to the transversal coordinates e_{2i} varying periodically during iteration. One can quite well allow this kind of cycles, and they would mean that e_2 coordinate as a function of CP_2 coordinates characterizing the tangent plane is many-valued. This is certainly very probable for solutions representable locally as graphs $M^4 \to CP_2$. In this case the tangent planes associated with distant points in E^2 would be strongly correlated which must have non-trivial physical implications. The iteration makes sense also p-adically and it might be that in some cases only p-adic iteration converges for some value of p.

It is not obvious whether the proposed procedure gives rise to a smooth or even continuous 4-surface. The conditions for this are geometric analogs of the above described algebraic integrability conditions for the map assigning to the surface in $M^4 \times CP_2$ a surface in M^8. Therefore $M^8 - H$ duality could express the integrability conditions and preferred extremals would be 4-surfaces having counterparts also in the tangent space M^8 of H.

One might hope that the self-referentiality condition $s \circ T = Id$ for the CP_2 projection of (m, s) or its fractal generalization could solve the complicated integrability conditions for the map T. The image of the space-time surface in tangent space M^8 in turn could be interpreted as a description of space-time surface using coordinates defined by the local tangent space M^8. Also the analogy for the duality between position and momentum suggests itself.

Is there any hope that this kind of construction could make sense? Or could one demonstrate that it fails? If s would fix completely the tangent plane it would be probably easy to kill the conjecture but this is not the case. Same s corresponds for different planes $M^2(x)$ to different point tangent plane. Presumably they are related by a local G_2 or $SO(7)$ rotation. Note that the construction can be formulated without any reference to the representation of the imbedding space gamma matrices in terms of octonions. Complexified octonions are enough in the tangent space of M^8.

2.4 Connection With Mandelbrot Fractal And Fractals As Fixed Sets For Iteration

The occurrence of iteration in the construction of preferred extremals suggests a deep connection with the standard construction of 2-D fractals by iteration - about which Mandelbrot fractal [A2, A1] is the canonical example. $X^2(x)$ (or $X^3(x)$ in the case of light-like $L(x) \subset M^2(x)$) could be identified as a union of orbits for the iteration of $s \circ T$. The appearance of the iteration map in the construction of solutions of field equation would answer positively to a long standing question whether the extremely beautiful mathematics of 2-D fractals could have some application at the level of fundamental physics according to TGD.

X^2 (or X^3) would be completely analogous to Mandelbrot set in the sense that it would be boundary separating points in two different basis of attraction. In the case of Mandelbrot set iteration would take points at the other side of boundary to origin on the other side and to infinity. The points of Mandelbrot set are permuted by the iteration. In the recent case $s \circ T$ maps X^2 (or X^3) to itself. This map need not be diffeomorphism or even continuous map. The criticality of X^2 (or X^3) could be seen as a geometric correlate for quantum criticality.

In fact, iteration plays a very general role in the construction of fractals. Very general fractals can be defined as fixed sets of iteration and simple rules for iteration produce impressive representations for fractals appearing in Nature. The book of Michael Barnsley [A4] gives fascinating pictures about fractals appearing in Nature using this method. Therefore it would be highly satisfactory if space-time surfaces would be in a well-defined sense fixed sets of iteration. This would be also numerically beautiful aspect since fixed sets of iteration can be obtained as infinite limit of iteration for almost arbitrary initial set. This construction recipe would also give a concrete content for the notion measurement resolution at the level of construction of preferred extremals.

What is intriguing is that there are several very attractive approaches to the construction of preferred extremals. The challenge of unifying them still remains to be met.

3 Hierarchy Of Planck Constants And The Generalization Of The Notion Of Imbedding Space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

3.1 The Evolution Of Physical Ideas About Hierarchy Of Planck Constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E2] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K8, K5] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K9]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K12].

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E2] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr} G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L1, K12], [L1].

3.2 The Most General Option For The Generalized Imbedding Space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is

like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the Kähler-Dirac gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP_2}\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP_2}/G_b$, $\hat{CD}/G_a \times (\hat{CP_2}\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP_2}/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

3.3 About The Phase Transitions Changing Planck Constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

3.4 How One Could Fix The Spectrum Of Planck Constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to n_an_b sheets and multiplies YM action action by n_an_b which is equivalent with the $\hbar = n_an_b\hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_an_b\hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	n_an_b	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_an_b}$

3.5 Preferred Values Of Planck Constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [K1].

3.6 How Planck Constants Are Visible In Kähler Action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various super-conformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

3.7 Could The Dynamics Of Kähler Action Predict The Hierarchy Of Planck Constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

3.7.1 1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

1. In canonical quantization canonical momentum densities $\pi_k^0 \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

2. Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multiplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

3. One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the space-time surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \rightarrow CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multi-valued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknowns. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

4. One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

3.7.2 Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multi-valuedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

1. The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   (a) One should fix also the values of $\pi_k^n = \partial L_K / \partial h_n^k$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi_k^n = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π_k^0 and π_k^n. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

(b) This picture is not quite correct yet. The fixing of π_k^0 and π_k^n should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π_k^0 and π_k^n must be fixed at X^3 and X_l^3 in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X_l^3 as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π_0^k would give n_b branches in CP_2 degrees of freedom and the conditions for π_k^n would split each of these branches to n_a branches.

(c) The existence of these two kinds of conserved charges (possibly vanishing for π_k^n) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

2. Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

3. How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = n g_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

4. The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

5. The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For space-like 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

3.7.3 Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

3.8 Updated View About The Hierarchy Of Planck Constants

The original hypothesis was that the hierarchy of Planck constants is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. In this formulation the singular covering of the imbedding space became only a convenient auxiliary tool. It is no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

The formulation based on multi-furcations of space-time surfaces to N branches. For some reason I assumed that they are simultaneously present. This is too restrictive an assumption. The N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural. As a matter fact, this interpretation was the original one, and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

3.8.1 Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard

value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K13].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K6] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E2] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

   It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the Kähler-Dirac equation. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

3.8.2 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the Kähler-Dirac gamma matrices [K16] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

3.8.3 Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elemenetary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless.

Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \to k\hbar$ in the formula $E_n = (n+1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.

2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

3.8.4 Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N + 1$: th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N - 1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N - 1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

   The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

3.8.5 What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the Kähler-Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{3.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \quad .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar / K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0} \frac{1}{L} \quad . \tag{3.2}$$

   For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta} L_\alpha L_\beta = g_{eff}^{kl} L_k L_l = l(l+1)\hbar^2 \quad . \tag{3.3}$$

   This would give under the same simplifying assumptions

$$L^2 = l(l+1) \frac{\hbar^2}{K^2} \quad . \tag{3.4}$$

   This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = K m^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

3.8.6 Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E1, E3] discussed in [K19] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

1. This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

2. That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

3. The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K18, K17] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   (a) If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K1] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   (b) For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

(c) Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (`http://en.wikipedia.org/wiki/Ionization_energy`). The energies of molecular bonds are in the range 2-10 eV (`http://en.wikipedia.org/wiki/Bond-dissociation_energy`). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to.62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies.

Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.

It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

(d) The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is.2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

4. An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

5. The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38, .99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

6. There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times

stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

3.8.7 Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K16]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level, to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K11] so that conformal dynamics represents conformal evolution) [K10].

4 Number Theoretic Braids And Global View About Anti-Commutations Of Induced Spinor Fields

The anti-commutations of the induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-D partonic surface and the effective 2-dimensionality means that partonic 2-surfaces plus there 4-D tangent space take the role of fundamental dynamical objects. This is expressed concretely by the condition that the ends of the space-time surface and wormhole throats are extremals of Chern-Simons action. Conformal invariance would in turn make the 2-D partons 1-D objects (analogous to Euclidian strings) and braids, which can be regarded as the ends of string world sheets with Minkowskian signature, in turn would discretize these Euclidian strings. It must be however noticed that the status of Euclidian strings is uncertain.

Somehow these views should be unifiable into a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales.

1. The notions of measurement resolution and braid concept indeed provides the needed physical insights in this respect. The precise definition of the notion of braid and its number theoretic counterpart has however remained open and I have considered several alternatives. The topological character of braid indeed allows flexibility in its definition but it would be nice to have some canonical definition with a clear physical meaning.

2. It turned out that the braid concept emerges automatically from the localization of the modes of Kähler-Dirac action to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - with vanishing induced W fields and above weak scale also induced Z^0 fields. The boundaries of string world sheets can be identified as braids and string world sheets as 2-braids. Hence the identification of braids is unique although their topological character does not necessitate this. The attribute "number theoretic" would mean that the intersections

of braids with partonic 2-surfaces corresponds to points with preferred imbedding space coordinates having values which are algebraic numbers in some extension of rational numbers. This selects preferred extremals among all extremals and they could perhaps be said to belong to the intersection of real and p-adic space-time sheets.

4.1 Quantization Of The Kähler-Dirac Action And Configuration Space Geometry

The quantization of the Kähler-Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

One can imagine two alternative forms of the anti-commutation relations.

1. The standard canonical anti-commutation relations for the induced the spinor fields would be given by

$$\{\overline{\Psi}\hat{\Gamma}^0(x),\Psi(y)\} = \delta^2_{x,y} \ . \tag{4.1}$$

The factor that $\hat{\Gamma}^0(x)$ corresponds to the canonical momentum density associated with Kähler action. The discrete variant of the anti-commutation relations applying in the case of non-stringy space-time sheets is

$$\{\overline{\Psi}\hat{\Gamma}^0(x_i),\Psi(x_j)\} = \delta_{i,j} \ . \tag{4.2}$$

where x_i and x_j label the points of the number theoretic braid. These anti-commutations are are inconsistent at the limit of vacuum extremal and also extremely non-linear in the imbedding space coordinates.

2. The construction of WCW gamma matrices leads to a nonsingular form of anti-commutation relations given by

$$\{\overline{\Psi}(x)\gamma^0,\Psi(x)\} = (1+K)J\delta_{x,y} \ . \tag{4.3}$$

Here J denotes the Kähler magnetic flux J_m and Kähler electric flux relates to via the formula $J_e = KJ_m$, where K is symplectic invariant. What is nice that at the limit of vacuum extremals the right hand side vanishes so that spinor fields become non-dynamical. Therefore this option- actually the original one - seems to be the only reasonable choice.

For the latter option the super counterparts of local flux Hamiltonians can be written in the form

$$\begin{aligned}
H_{A,+,n} &= H_{A,+,q,n}+H_{A,+,L,n} \ , \ H_{A,-,n}=H_{A,-,q,n}+H_{A,-,L,n} \ , \\
H_{A,+,q,n} &= \oint \overline{\Psi}J^A_+ q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J^A_- \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi}J^A_+ L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J^A_- \Psi d^2x \ , \\
J^A_+ &= j^{Ak}\Gamma_k \ , \ J^A_- = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \tag{4.4}$$

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m, i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$\begin{aligned} q_n = q_{m,i} &= \Phi_m q_i \ , \\ L_n = L_{m,i} &= \Phi_m L_i \ , \\ \overline{q}_i \gamma^0 q_j &= \overline{L}_i \gamma^0 L_j = g_{ij} \ . \end{aligned} \tag{4.5}$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$\{H_{A,+,n}, H_{A,-,n}\} = g_{ij} \oint (1+K) \overline{\Phi}_m \Phi_n H_A J d^2x \ . \tag{4.6}$$

The product of scalar functions can be expressed as

$$\overline{\Phi}_m \Phi_n = c_{mn}^{\ \ k} \Phi_k \ . \tag{4.7}$$

Note that the notion of symplectic QFT led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to WCW metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras $S^2 \times S$ associated with the regions in question. As far as coordinatization of the reduced configuration space is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of WCW .

4.2 Expressions For WCW Super-Symplectic Generators In Finite Measurement Resolution

The expressions of WCW Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

1. In p-adic context integrals do not makes sense so that this representation fails in p-adic context. Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

2. The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

3. Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_\pm$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_\pm$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_\pm \times CP_2$ contribute to WCW Hamiltonians and super Hamiltonians and therefore to the WCW metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of WCW as one can expect on basis of the fact that WCW Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that the infinite-dimensional WCW can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and WCW spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} . \quad (4.8)$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should ge general coordinate invariant. As already described, the localization of the modes of the Kähler-Dirac action to 2-D surfaces resolves this problem: the points x_m correspond to points of imbedding space which in preferred imbedding space coordinates have values in some algebraic extension of rationals.

4.3 QFT Description Of Particle Reactions At The Level Of Braids

The overall view conforms with zero energy ontology in which hierarchy of causal diamonds (CDs) within CDs gives rise to a hierarchy of generalized Feynman diagrams and geometric description of the radiative corrections. Each sub-CD gives also rise to to zero energy states and thus particle reactions in its own time scale so that improvement of the time resolution brings in also new physics as it does also in reality.

The natural question is what happens to the braids at vertices.

1. The vision based on infinite primes led to the conclusion that the selection rules of arithmetic quantum field theory based on the conservation of the total number theoretic momentum $P = \sum n_i log(p_i)$ dictate the selection rules at the vertices. For given p_i the momentum $n_i log(p_i)$ can be shared between the outgoing lines and this allows several combinations of infinite primes in outgoing lines having interpretations in terms of singular coverings of CD and CP_2.

2. What happens then to the braid strands? If the bosons and fermions with given p_i are shared between several outgoing particles, does this require that the braid strands replicate? Or is their number preserved if one regards each braid strand as having n_a *resp.* n_b copies at the sheets of the corresponding coverings? This is required by the conservation of number theoretic momentum if one accepts the connection between the hierarchy of Planck constants and infinite primes.

3. The question raised already earlier is whether DNA replication could have a counterpart at the level of fundamental physics. The interpretation of the incoming lines of generalized Feynman diagram as representations of topological quantum computations and the virtual particle lines as representations of quantum communications would support this picture. The no-cloning theorem [B1] would hold true since exact copies of quantum states would not be possible by the conservation of the number theoretical momentum. One could however say that the bosonic occupation number n_i means the presence of n_i-fold copy of same piece of information so that the sharing of information by sharing the pages of the singular covering associated with n_i would be possible in the limits posed by the values of n_i. Note again that the identification $n_i = n_a$ or $n_i = n_b$ (two infinite primes characterize the quantum state) makes sense only if only one of the p-adic primes associated with the 3-surface is realized as a physical state since the identification forces the selection of the covering. The quantum model for DNA based on hierarchy of Planck constants [K14] inspires the question whether DNA replication could be actually accompanied by its proposed counterpart at the fundamental level defining the fundamental information transfer process.

4. The localization of the quantum numbers to braid strands suggests that braid ends of a given braid continue to one particular line or more generally, are shared between several lines. This condition is quite strong since without additional quantization conditions the ends of the braids of outgoing particles do not co-incide with the ends of the incoming braid. These kind of quantization conditions would conform with the generalized Bohr orbit property of light-like 3-surfaces.

5. Without these quantization conditions one meets the challenge of calculating the anti-commutators of fermionic oscillator operators associated with non-co-inciding points of the incoming and outgoing braids. This raises the question whether one should regard the quantizations of induced spinor fields based on the L_{min} as one possible gauge only and allow the variation of L_{min} in some limits. If these quantizations are equivalent, the fermionic oscillator operators would be unitarily related. How to deduce this unitary transformation would be the non-trivial problem and it seems that the simpler picture is much more attractive.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

4.4 How Do Generalized Braid Diagrams Relate To The Perturbation Theory?

The association of generalized braid diagrams characterized by infinite primes to the incoming and outgoing partonic legs and internal lines of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The basic question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams.

1. If one believes in perturbation theoretic approach, the basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancelation could occur only for critical values of Kähler coupling strength α_K: for general values of α_K the cancellation would require separate vanishing of each term in the sum and does not occur.

   In perturbative approach the expression of Kähler function as Chern-Simons action could be used and propagator would correspond to the inverse of the 1-1 part of the second variation of the Chern-Simons action with respect to complex WCW coordinates evaluated allowing only the extrema of Chern-Simons action for the ends of space-time surface and for wormhole throats. One would have perturbation theory for a sum over maxima of Kähler function. From the expression of the Kähler function as Dirac determinant the maxima would correspond to the local minima of $L_p = \sqrt{p}L_{min}$ for a given infinite prime. The connection between Chern-Simons representation and Dirac determinant representation of Kähler function would be obviously highly desirable.

2. The possibility to define WCW functional integral in terms of harmonic analysis for infinite-dimensional spaces leads to a non-perturbative approach to functional integration allowing also a generalization the p-adic context [K10]. In this approach there is no need to make additional assumptions.

For both cases the assignment of the collection of braids characterized by pairs of infinite primes allows to organize the generalized Feynman diagrams into a sum of generalized Feynman diagrams and for each diagram type the exponent of Kähler function - if given by the Dirac determinant- would be simply the product $\prod_i L_{p_i}^{-1}$, $L_p = \sqrt{p}L_{min}$. One should perform a sum over different infinite primes in the internal lines subject to the conservation of the total number theoretic momenta. The conservation of the incoming number theoretic momentum would allow only a finite number of configurations for the intermediate lines. For the approach based on harmonic analysis the expression of the Kähler function in terms of the Dirac determinant would be optimal since it is manifestly algebraic function.

Both approaches involve a perturbative summation in the sense of introducing sub-CDs with time scales coming as 2^{-n} powers of the time scale of CD defining the infrared cutoff.

1. The addition of zero energy insertions corresponding to sub-CDs as radiative corrections allows to improve measurement resolution. Hence a connection with QFT type Feyman diagram expansion would be obtained and Connes tensor product would have a practical computational realization.

2. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

4.5 How Could P-Adic Coupling Constant Evolution And P-Adic Length Scale Hypothesis Emerge?

The condition $T_n = 2^n T_0$ would assign to the hierarchy of CDs as hierarchy of time scales coming as octaves. A weaker condition would be $T_n = nT_0$, n integer or $T_p = pT_0$, p prime, and would assign all secondary p-adic time scales to the size scale hierarchy of CDs.

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution. Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks like an attractive idea but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^kT_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around.1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW .

5 Scattering Amplitudes At The Level Of WCW

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in zero energy ontology (ZEO) [K7]. This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K15] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the “world of classical worlds” (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. Zero energy ontology (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K15]. Kähler-Dirac gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces.

5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K16]. It took long time to realize that in zero energy ontology the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck

constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of super-symplectic algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K2, K15].

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B2] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of Kähler-Dirac operator.

2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K3] in infinite-dimensional context already in the case of much simpler loop spaces [A3].

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿ defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW . Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

5.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

5.1.1 What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the Kähler-Dirac action [K16] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

   A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world

sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. The light-like 8-momenta p^k have same M^4 and CP_2 mass squared and latter correspond to the the eigenvalues of the CP_2 spinor d'Alembertian by quantum-classical correspondence.

5. One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

6. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

5.1.2 How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

5.2 Harmonic Analysis In WCW As A Manner To Calculate WCW Functional Integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW .

5.2.1 Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW .

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the Kähler-Dirac operator D at propagator lines [K16]. G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \tag{5.1}$$

   Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

   Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \tag{5.2}$$

   such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the Kähler-Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \quad (5.3)$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

5.2.2 Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

5.2.3 Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the Kähler-Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

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