The Recent View about SUSY in TGD Universe

M. Pitkänen,

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Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

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Abstract

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge would appear as space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with the precise understanding of quantum criticality and understand the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials. The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is indeed bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \to (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

Super-Dirac action vanishes on-mass-shell. The proposed construction relying on ZEO allows however to get scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation is however needed and makes possible to express the derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution conforms with the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

What SUSY is in TGD framework is a longstanding question, which found a rather convincing answer rather recently. In twistor Grassmannian approach to $\mathcal{N} = 4$ SYM [B5, B2, B3, B4, B7, B6, B1] twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions suggests that super-twistors are realized both at the level of M^8 geometry and momentum space.

In TGD framework M^8-H duality allows to geometrize the notion of super-twistor in the sense that at the level of M^8 different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

0.1 New view about SUSY

The progress in understanding of $M^8 - H$ duality [L9] throws also light to the problem whether SUSY is realized in TGD [L11] and what SUSY breaking cold mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them. Super-octonion components of polynomials have different orders so that also the extension of rational assignable to them is different and therefore also the ramified primes so that p-adic prime as one them can be different for the members of SUSY multiplet and mass splitting is obtained.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and perhaps also hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge could appear as a space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with an improved understanding of quantum criticality and the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials: the criticality is criticality for the polynomials interpreted as p-adic polynomials in $O(p) = 0$ approximation meaning the presence of multiple roots in this approximation.

0.2 Connection of SUSY and second quantization

The linear combinations monomials of theta parameters appearing in super-fields are replaced in case of hermitian H super coordinates consisting of combinations of monomials with vanishing quark number. For super-spinors of H the monomials carry odd quark number with quark number 1. Monomials of theta parameters are replaced by local monomials of quark oscillator operators labelled besides spin and weak isospin also by points of cognitive representation with imbedding space coordinates in an extension of rationals defining the adele. Discretization allows anti-commutators which are Kronecker deltas rather than delta functions. If continuum limit makes sense, normal ordering must be assumed to avoid delta functions at zero coming from the contractions. The monomials (not only the coefficients appearing in them) are solved from generalized classical field equations and are linearly related to the monomials at boundary of CD playing the role of quantum fields and classical field equations determine the analogs of propagators.

The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals. The polynomial composition hierarchies correspond to inclusion hierarchies for isomorphic sub-algebras of super-symplectic algebra having interpretation in terms of inclusions of hyper-finite factors of type II_1.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

0.3 Proposal for S-matrix

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces.

1. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

2. Super-Dirac equation implies that super-Dirac action vanishes on-mass-shell. The proposed construction however allows to get also scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation however makes possible to express derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution follows from the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

3. S-matrix is trivial unless CD contains the images of 6-D analogs of branes as universal special solutions of the algebraic equations determining space-time surfaces at the level of M^8. 4-D space-time surfaces representing particle orbits meet at the partonic 2-surfaces associated with the 3-D surfaces at $t = r_n$ hyper-surfaces of M^4. The values of $t = r_n$ correspond to the roots of the real polynomial with rational coefficients determining the space-time surface. These transitions are analogs of weak measurements, and in TGD theory of consciousness they give rise to the experience flow of time and can be said to represent "very special moments" in the life of self [L8].

4. The creation and annihilation operators at vertices associated with the monomials would be connected to the points assignable to cognitive representations at opposite boundaries of CD and also to partonic 2-surfaces in the interior of CD possibly accompanied by sub-CDs. This would give analogs of twistor Grassmannian diagrams containing finite number of partonic 2-surfaces as topological vertices containing in turn finite number ordinary vertices defined by the monomials. The diagrams would be completely classical objects in accordance with the fact that quantum TGD is completely classical theory apart from state function reduction.

5. This view allows also a formulation of continuum theory since the monomials appearing in the action density in the interior of CD are linear superposition of the monomials at the points of boundary of CD involving 3-D integral so that contractions of oscillator operators only reduce one integration without introducing divergence. One can also normal order the

monomials at boundary of CD serving as initial values. If preferred extremals are analogs of Bohr orbits, one can express extremals using either boundary as the seat of initial data.

1 How to formulate SUSY at the level of $H = M^4 \times CP_2$?

In the following I will represent the recent trial for constructing SUSY at the level of $H = M^4 \times CP_2$. The first trial replaced theta parameters of SUSY with quark oscillator operators labelled by spin and isospin and had rather obvious shortcomings: in particular, one did not obtain many-quark states with large quark numbers. The second trial allows quark oscillator operators to have as labels also the points of space-time surface in cognitive representation and thus having coordinates of H belonging to an extension of rationals defining the adele [?]

1.1 First trial

If SUSY is realized at the level of M^8, it should have a formulation also at the level of H. The basic elements of the first trial form part of also second trial. The basic modification made in the second trial is that finite number of theta parameters replaced with the fermionic oscillator operators labelled by the points of cognitive representations so that they are analogous to fermion fields in lattice, and only local composites of the oscillator operators appear in the super coordinates and super-spinors. This means that SUSY is essential element of the second quantization of fermions in TGD.

1. $M^8 - H$ duality is non-local and means that the dynamics at the level of H is not strictly local but dictated by partial differential equations for super-fields having interpretation as describing purely local many-fermion states made of fundamental fermions with quantum numbers of leptons and quarks (quarks do not possess color as spin like quantum number) ad their antiparticles.

2. Classical field equations and modified Dirac equation must result from this picture. Induction procedure for the spinors of H must generalize so that spinors are replaced by super-spinors Ψ_s having multi-spinors as components multiplying monomials of theta parameters θ. The determinant of metric and modified gamma matrices depend on imbedding space coordinates h replaced with super coordinates h_s so that monomials of θ appear in two different manners. Hermiticity requires that sums of monomial and its hermitian conjugate appear in h_s. Monomials must also have vanishing fermion numbers. Otherwise one can obtain fermionic states propagating like bosons. For Dirac action one must assume that Ψ_s involves only odd monomials of θ with quark number 1 involving monomials appearing in h_s to get only states with quark number 1 and correct kind of propagators.

3. One Taylor expands both bosonic action density (6-D Kähler action dimensionally reducing to 4-D Kähler action plus volume term) and Super-Dirac action with respect to the super-coordinates h_s. In Super-Dirac action one has also the expansion of super-spinor in odd monomials with total quark number 1. The coefficients of the monomials of θ:s are obtained are partial derivatives of the action. Since the number of θ parameters is finite and corresponds to the number of spin-weak-isopin states of quarks and leptons, the number of terms is finite if the θ parameters anti-commute to zero. If not, one can get an infinite number of terms from the Taylor series for the action to the coefficient given monomial. Number theoretical considerations do not favor this and there should exist a cancellation mechanism for the radiative corrections coming from fermionic Wick contractions if thetas correspond to fermionic oscillator operators as it seems to be.

4. One can interpret the superspace as the exterior algebra of the spinors of H. This reminds of the result that the sections of the exterior algebra of Riemann manifold codes for the Riemann geometry (see `http://tinyurl.com/yxrcr8xv`). This generalizes the observation that one can hear the shape of a drum since the sound spectrum is determined by its frequency spectrum defined by Laplacian.

   Super-fields define a Clifford algebra generated by θ parameters as a kind of square root of exterior algebra which corresponds to the Clifford algebra of gamma matrices. Maybe this algebra could code also for the spinor structure of imbedding space or even that of space-time surface so that the super-fields could be seen as carriers of geometric information about space-time surface as a preferred extremal. In 8-D case there is also $SO(1,8)$ triality suggesting that corresponding three Clifford algebras correspond to exterior algebra fermionic and anti-fermionic algebras.

What about the situation at the level of M^8?

1. At M^8 level the components of super-octonion correspond to various derivatives of the basic polynomial $P(t)$ so that space-time geometry correlates with the quantum numbers assignable to super-octonion components - this is in accordance with QCC (quantum-classical correspondence). This is highly desirable at the level of H too.

   Could the space-time surface in M^8 be same for super-field components with degree $d < d_{max}$ in some special cases? The polynomial associated with super octonion components are determined by the derivatives of the basic polynomial $P(t)$ with order determined by the degree of the super-monomial. If they have decomposition $P(t) = P_1^k(t)$, the monomials with degree $d < k$ the roots corresponding to the roots $P_1(t)$ co-incide. Besides this there are additional roots of d^rP_1/dt^r for super-octonion component with r θ parameters.

   A possible interpretation could be as quantum criticality in which there is no SUSY breaking for components having $d < k$ (masses in p-adic thermodynamics could be the same since the extension defined by P_1 and corresponding ramified primes would be same). This would conform with the general vision about quantum criticality.

2. Usual super-field formalism involves Grassmann integration over θ parameters to give the action. M^8 formalism does not involve the θ integral at all. Should this be the case also at the level of H? This would guarantee that different components of H- coordinates as super-field would give rise to different space-time surface and QCC would be realized. θ integration produces SUSY invariants naturally involved with the definition of vertices involving components of super-fields. Also vertices involving fermionic and bosonic states emerge since bosonic super-field components appear in super-coordinates in super-Dirac action.

This approach does not say anything about second quantization. There is a strong temptation to replace the theta parameters with fermionic oscillator operators. One cannot however obtain second quantization of fermions in this manner since the maximal quark number (and lepton number if leptons are present as fundamental fermions) of the states is 4. To achieve second quantization, one must replace the theta parameters with fermionic oscillator operators labelled besides spin and weak isospin by the coordinates of points of 3-surface, most naturally the points belonging to a cognitive representation characterizing space-time surface for given extension of rationals.

1.2 Second trial

I have already earlier considered a proposal for how SUSY could be realized in TGD framework. As it often happens, the original proposal was not quite correct. The following discussion gives a formulation solving the problems of the first proposal and suggests a concrete formulas for the scattering amplitudes in ZEO based on super-counterparts of preferred extremals. In the sequel I will talk about super Kähler function as functional of 3-surfaces and - super Kähler function action. By holography allowing to identify 3-surfaces with corresponding space-time surfaces as analogs of Bohr orbits, these notions have the same meaning.

1.2.1 Could the exponent of super-Kähler function as vacuum functional define S-matrix as its matrix elements

Consider first the key ideas - some of them new - formulated as questions.

1. Could one see SUSY in TGD sense as a counterpart for the quantization in the sense of QFT so that oscillator operators replace theta parameters and would become fermionic oscillator operators labelled by spin and electroweak spin - as proposed originally - and by selected

points of 3-surface of light-cone boundary with imbedding space coordinates in extension of rationals? One would have analog of fermion field in lattice identified as a number theoretic cognitive representation for given extension of rationals. The new thing would be allowance of local composites of oscillator operators having interpretation in terms of analogs for the components of super-field.

SUSY in TGD sense would be realized by allowing local composites of oscillator operators containing 4+4 quark oscillator operators at most. At continuum limit normal ordering would produce delta functions at origin unless one assumes normal ordering from beginning. For cognitive representations one would have only Kronecker deltas and one can consider the possibility that normal ordering is not present. The vanishing of normal ordering terms above some number of them suggested to be the dimension for the extension of rationals would give rise to a discrete coupling constant evolution due to the contractions of fermionic oscillator operators.

2. What is dynamical in the superpositions of oscillator operator monomials? Are the coefficients dynamical? Or are the oscillator operators themselves dynamical - this would mean a QFT type reduction to single particle level? The latter option seems to be correct. Oscillator operators are labelled by points of cognitive representation and in continuum case define an analog of quantum spinor field, call it q. This suggests that this field satisfies the super counter part of modified Dirac equation and must involve also super part formed from the monomials of q and $\overline{q}$. This however requires the replacement of q with q_s in super-Dirac operator and super-coordinates h_s and one ends up with an iteration $q \rightarrow q_s \rightarrow ...$

   The only solution to the paradoxical situation is that one has self-referential equation $q = q_s$ having interpretation in terms of quantum criticality fixing the coefficients of terms in $q = q_s$. Analogous condition $h_s = (h_s)_s$ must be satisfied by h_s under substitution $h_s \rightarrow (h_s)_s$. These conditions fix coefficients of terms in H super-coordinate h_s and q_s interpreted as coupling constants so that quantum criticality implying a discrete coupling constant evolution as function of extension of rationals follows. Also super-Dirac equation $D_s q_s = 0$ and field equations $D_{s,\alpha}\Gamma^{\alpha,s} = 0$ for Kähler action guaranteeing hermiticity are satisfied.

3. Could one interpret the time reversal operation taking creation- and annihilation operators to each other as time reflection permuting the points at the opposite boundaries of CD? The positive *resp.* negative energy parts of zero energy states would be created by creation *resp.* annihilation operators from respective vacuums assigned to the opposite boundaries of CD.

4. Could one regard preferred extremal regarded as 4-surface in super imbedding space parameterized by the hermitian imbedding coordinates plus the coefficients of the monomials of quarks and antiquarks with vanishing quark number, whose time evolution follows from dimensionally reduced 6-D super-Kähler action? Could one assume similar interpretation for super spinors consisting of monomials with total quark number equal to 1 and appearing in super-Dirac action?

5. In WKB approximation the exponent of action defines wave function. In QFTs path integral is defined by an exponent of action and scattering operator can be formally defined as action exponential. Could the matrix elements for the exponent of the super counterpart of Kähler function plus super Dirac action between states at opposite boundaries of CD between positive and negative energy parts of zero energy states define S-matrix? Could the positive and negative energy parts of zero energy states be identified as many particles states formed from the monomials associated with imbedding space super-coordinates and super-spinors?

6. Could the construction of S-matrix elements as matrix elements of super-action exponential reduce to classical theory? Super-field monomials in the interior of CD would be linear superpositions of super-field monomials at boundary of CD. Note that oscillator operator monomials rather than their coefficients would be the basic entities and the dynamics would reduce to that for oscillator operators as in QFTs. The analogs of propagators would relate the monomials to those at boundary ly to the monomials at the boundary of CD, and would be determined by classical field equations so that in this sense everything would be classical.

Note however that the fixed point condition $q = q_s$ and super counterpart of modified Dirac equation are non-linear.

Vertices would be defined by monomials appearing in super-coordinate and super-spinor field appearing in terms of those at boundary of CD. If two vertices at interior points x and y of CD are connected there is line leading from x to a point z at boundary of CD and back to y and one would have sum over points z in cognitive representation. This applies also to self energy corrections with $x = y$. At the possibly existing continuum limit integral would smoothen the delta function singularities and in presence of normal ordering at continuum would eliminate them.

In the expressions for the elements of S-matrix annihilation operators appearing in the monomials would be connected to the passive boundary P of CD and creation operators to the active boundary. If no partonic 2-surfaces appear as topological vertices in the interior of CD, this would give trivial S-matrix!

$M^8 - H$ duality however predicts the existence of brane like entities as universal 6-D surfaces as solutions of equations determining space-time surfaces. Their M^4 projection is $t = r_n$ hyperplane, where r_n corresponds to a root of a real polynomial with algebraic coefficients giving rise to octonion polynomial, and is mapped to similar surface in H. 4-D space-time surfaces representing incoming and outgoing lines would meet along their ends at these partonic 2-surfaces.

Partonic 2-surfaces at these hyper-surfaces would contain ordinary vertices as points in cognitive representation. Given vertex would have at most 4+4 incoming and outgoing lines assignable to the monomial defining the vertex. This picture resembles strongly the picture suggested by twistor Grassmannian approach. In particular the number of vertices is finite and their seems to be no superposition over different diagrams. In this proposal, the lines connecting vertices would correspond to 1-D singularities of the space-time surfaces as minimal surfaces in H. Also stringy singularities can be considered but also these should be discretized.

By fixing the set of monomials possibly defining orthonormal state basis at both boundaries one would obtain given S-matrix element. S-matrix elements would be matrix elements of the super-action exponential between states formed by monomials of quark oscillator operators. Also entanglement between the monomials defining initial and finals states can be allowed. Note that this in principle allows also initial and final states not expressible using monomials but that monomials are natural building bricks as analogs of field operators in QFTs.

7. The monomials associated with imbedding space coordinates are imbedding space vectors constructible from Dirac currents (left- or right-handed) with oscillator operators replacing the induced spinor field and its conjugate. The proposed rules for constructing S-matrix would give also scattering amplitudes with odd quark number at boundaries of CD. Could the super counterpart of the bosonic action (super Kähler function) be all that is needed to construct the S-matrix?

   In fact, classically Dirac action vanishes on mass shell: if this is true also for super-Dirac action then the addition of Dirac action would not be needed. The super-Taylor expansion of super- Kähler action gives rise to the analogs of perturbation theoretic interaction terms so that one has perturbation theory without perturbation theory as Wheeler might state it. The detailed study of the structure of the monomials appearing in the super-Kähler action shows that they have interpretation as currents assignable to gauge bosons and scalar and pseudo-scalar Higgs.

   Super Dirac action is however needed. Super-Dirac equation for q and $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ allow to reduce ordinary divergences $\partial_\alpha j^\alpha$ of fermionic currents appearing in super-Kähler action to commutators $[A_{\alpha,s} j^\alpha]$. Therefore no information about q at nearby points is needed and one avoids lattice discretization: cognitive representation is enough.

8. Topological vertices represent discontinuities of the space-time surface bringing strongly in mind the non-determinism of quantum measurement, and one can ask whether the 3-branes and associated partonic 2-surfaces. Could the state function reductions analogous to weak

measurements correspond to these discontinuities? Ordinary state function reductions would change the arrow of time and the roles of active and passive boundaries of CD [L6]. In TGD inspired theory of consciousness these time values would correspond to "very special moments" in the life of self [L8].

9. The unitarity of S-matrix can be understood from the structure of the exponent of Kähler action. The exponent decomposes to a sum of real and purely imaginary parts. The exponent of the hermitian imaginary part is a unitary operator for a given space-time surface. Real exponent containing also radiative corrections from the normal ordering gives exponent of Kähler function as vacuum functional in WCW (sum in the case of cognitive representations) and by choosing the normalization factor of the state appropriately one obtains unitary S-matrix.

1.3 More explicit picture

The following sketch tries to make the picture of the second trial more explicit.

1. The construction of S-matrix should reduce to super-geometry coded by super Kähler function determined by the 6-D Kähler action for twistor lift by dimensional reduction. This might be possible since zero energy states have vanishing total conserved charges and exponent of super-Kähler function has matrix elements only between states at opposite boundaries of CD having same total charges.

2. Construction should reduce to preferred extremals and their super-deformations determined by variational principle with boundary conditions. The boundary values of super-deformations at either boundary could be also interpreted as initial values for preferred extremals analogous to Bohr orbits. The expectations for the super action with fixed initial values between positive and negative energy parts would give the scattering amplitudes assignable to a given space-time surface. There would be functional integral over space-time surfaces using exponent of Kähler function as weight. In number theoretic vision this would reduce to sum over preferred extremals labelled by cognitive representations serving as WCW coordinates.

3. Number theoretic vision suggests a discretization in terms of cognitive representation consisting of points with coordinates in extension of rationals defining the adele. This representation could be associated with the boundaries of CD and possibly with M^4 time=constant hyper-planes assignable with the universal special solutions in M^8. At the partonic 2-surfaces associated with these hyper-planes 4-D extremals would meet along their ends: topological particle vertices would be in question. Is string world sheets and partonic 2-surfaces correspond to singularities, the boundaries of strings world sheets as intersections of the string world sheets and orbits of partonic 2-surfaces should represent fermion lines.

4. Creation operators would be assigned with the passive boundary of CD - call it P - and annihilation operators as their conjugates would act as creation operators at the opposite boundary, active boundary - call it A. Time reversal symmetry of CD suggests that annihilation operator as conjugate of creation operator labelled by the a point of boundary of CD corresponds to the same point in common coordinates for light-cone boundary. This would conform also with the basic character of the half-algebras associated with super-symplectic symmetries.

   The original proposal was that oscillator operators have only spin and electroweak spin as indices but the standard view about spin and statistics requires that also the points of the 3-surface must label them. Also the fact that the total quark number can be larger than 4 of course requires this too. Algebraically the only difference with respect to this proposal is that one allows also the points of 3-surface at the boundary of CD as labels.

5. Number theoretical vision requires that only points of 3-surface having imbedding space coordinates in the extension of rationals defining the adelic physics are allowed. In the generic case the number of points in the cognitive representation would be finite and would increase with the dimension of extension so that at the limit of algebraic numbers they form a dense set of 3-surface.

Since action has infinite expansion in powers of super coordinates the contractions of oscillator operators would give rise to a renormalization of the coefficients of the monomials and of classical action. For cognitive representations one would avoid normal ordering problems sine the number of contractions is limited by the number of points in cognitive representation. This would give rise to discrete coupling constant evolution as function of the extension of rationals.

6. In continuum theory all points of 3-D boundary would label quark oscillator operators and one must normal order the oscillator operators in given local monomial. Also now the idea about connecting creation and annihilation operators to opposite boundaries of CD would allow to get rid of infinities due to contractions.

The action exponential would lead to a rather concrete proposal for the coefficients of the monomials appearing in super-fields.

1. The deformations of imbedding space coordinates would be expressible as WCW-local superpositions of isometry generators or as WCW-global superpositions of Hamiltonian currents contracted with the coordinate deformations. The latter would conform with super-symplectic symmetries of WCW. CP_2 Hamiltonian currents would give color quantum numbers. S^2 Hamiltonian currents would be also present. One could see space-time local Kac-Moody symmetries assignable to light-like partonic orbits and string world sheets as a dual representations at space-time level of symplectic symmetries at imbedding space level.

2. Spinor modes would be expressible as superpositions of imbedding space spinor modes having expansion as super-Taylor series at the boundaries of CD. This would give spin and electroweak quantum numbers.

Does one really obtain description of gauge bosons and gravitons by using the exponent?

1. Could the coefficients of super-monomials at boundary of CD allow interpretation in terms of gauge bosons? These entities could have well-defined quantum numbers so that this might be possible. Quark spin and isospin would represent additional spin degrees of freedom. The Hamiltonians of H of CP_2 expressible for given 3-surface as local superpositions of SU(3) Killing vector fields would represent color degrees of freedom.

   For string world sheets one would naturally have transversal M^4 super-coordinates and CP_2 super-coordinates as analogs of fields. Could this allow to get gauge bosons as excitations of strings as in string theories.

2. Gauge bosons could be also bi-local composites of fermion and anti-fermion at opposite boundaries of wormhole contact or at opposite wormhole contacts of wormhole flux tube. Gravitons could be 4-local composites. Baryons and mesons could be this kind of non-local composites. One can consider also the analog of monopole phase of QFTs in which particles would be multilocal composites.

3. The bosonic action is for induced metric and induced Kähler form. QFT wisdom would suggest that their super-analogs could correspond to external particles. One could indeed take the induced gauge potentials or -fields at boundary and form their contractions with Killing vectors of isometries to obtain general coordinate invariant quantities and form their super-analogs as normal ordered local composites. One can consider the same idea for induced gravitational field or its deviation from Minkowski metric.

   Formally this would correspond to an addition to the action exponential of perturbative terms of type jA appearing in QFTs representing coupling to external currents and take the limit $j \to 0$. In QFT picture this works since various gauge fields are functionally independent but in TGD framework this is not the case. Second problem is to to construct a complete orthonormalized set of states in this manner. Therefore it seems this description can make sense only at QFT limit of TGD.

1.3.1 Dimensionally reduced 6-D Kähler action as an analog of SYM action

The 6-D dimensionally reduced Kähler action reduces to a sum of 4-D Kähler action and volume term and will be simply referred to as Kähler action. The super variant of this action is obtained by replacing imbedding space coordinates with their super counterparts. Super-Kähler action is analogous to pure SYM action.

1. Space-time would be super-surface in super counterpart of $H = M^4 \times CP_2$ with coordinates h^k having super components proportional to multi-spinors multiplying the monomials of oscillator operators. The ocillator operator monomials rather than only the multi-spinor coefficients of the oscillator monomials transforming like vectors of H are regarded as analogs of quantum fields expressible by classical field equations as linear superpositions of their values at the boundary of CD for preferred extremals. The dynamics of monomials would reduce to that for oscillator operators labelled by points of cognitive representation and having interpretation as restriction of quantized quark field satisfying super-Dirac equation and the quantum criticality condition $q = q_s$.

2. Fermionic creation operators and annihilation operators labelled not only by spin and weak isospin as in the original proposal but also by the finite number of points of the cognitive representation. Therefore oscillator operators are analogous to the values of fermion field in discretization obeying super variant of modified Dirac equation. Both leptonic and quark like oscillator operators corresponding to two different H-chiralities and having different couplings to Kähler gauge potential could be present but octonionic triality allows only quarks. The vacuum expectation value of the action action exponentials contains only monomials with vanishing B (and L if leptons are present as fundamental fields). The matrix elements between positive and negative energy parts of zero energy states gives S-matrix.

   Real super-coordinates can be assumed to be hermitian and thus contain only sums of monomials and their conjugates having vanishing fermion numbers. This guarantees super-symmetrization respecting bosonic statistics at the level of propagators since all kinetic terms involve two covariant derivatives - one can indeed transform ordinary derivatives of monomials coming from the Taylor expansion to covariant derivatives involving also the coupling to Kähler form since the total Kähler charge of terms vanishes.

The lack of anti-commutativity of fermionic oscillator operators implies the presence of terms resulting in contractions.

1. The super-Taylors series would involve a finite number of partial derivatives of action. Wick contractions of oscillator operators would give rise to an infinite number of terms in continuum case. The appearance of infinite Taylor series defining the coefficients of super-polynomial is however troublesome from the point of view of number theoretic vision since there is no guarantee that the coefficients are rational functions. The finite number of points in the cognitive representation implying finite number of oscillator operators however allows only finite number of terms in the super-Taylor expansion.

   The monomials appearing in action in the interior of CD can be expressed as linear superpositions of those at boundary also in continuum case. Therefore each monomial is 3-D integral over the monomials at the boundary of CD. As a consequence, the contractions giving delta functions only eliminate one integration but do not give rise to infinities. A general solution to the divergence problems emerges.

   This is actually nothing new: one of the key ideas behind the notion of WCW is that path integral over space-time surfaces is replaced by a functional integral over 3-surfaces in WCW holographically equivalent with preferred extremals as analogs of Bohr orbits. The non-locality of the theory due to the replacement of point-like particles with 3-surfaces would solve the divergence problems.

An interesting possibility in line with the speculations of Nima-Arkani Hamed and others is that the action defining space-time as a 4-surface of imbedding space could emerge from the anti-commutators of the oscillator operator monomials as radiative corrections so that the bosonic action would vanish when the super-part of h_s vanishes.

1.3.2 Super-Dirac action

Before doing anything one can recall what happens in the case of modified Dirac action.

1. One has separate modified Dirac actions $\overline{\Psi}D\Psi$, $D = \Gamma^\alpha D_\alpha$ for quarks and leptons (later it will be found that modified Dirac action for quarks might be enough) and the covariant derivatives differ since there is a coupling to n-ple of included Kähler potential. For leptons one has $n = -3$ and for quarks $n = 1$. This guarantees that em charges come out correctly. This coupling appears in the covariant derivative D_α of fermionic super field.

2. One obtains modified Dirac equations for quarks and leptons by variation with respect to spinors. The variation with respect to the imbedding space coordinates gives quantized versions of classical conservation laws with respect to isometries. One also obtains and infinite number of super-currents as contractions of modes of the modified Dirac operator with Ψ.

3. Classical field equations for the space-time surface emerge as a consistency condition guaranteeing that the modified Dirac operator is hermitian: canonical momentum currents of classical action must be conserved and define conserved quantum when contracted with Killing vectors of isometries. Quantum-classical correspondence (QQC) requires than for Cartan algebra of symmetry algebra the classical Noether charges are same as the fermionic Noether charges.

   It turns out that the super-symmetrization of modified Dirac equation gives only fermions and they fermionic superpartners in this manner if one requires that propagators are consistent with statistics.

Consider first the situation without the quantum criticality condition $q = q_s = \Psi_s$. H coordinates are super-symmetrized and induced spinor field becomes a super-spinor $\Psi_s = \Psi^N O_N(q, \overline{q})$ with Ψ_N depending on h_s (summation over N is understood).

1. As in the case of bosonic action the vacuum expectation value gives modified Dirac action conserving fermion numbers but one could assume that the monomials in the leptonic (quark) modified Dirac action have either non-vanishing L (B) and vanishing B (L). It seems that the lepton (baryon -) number of monomials can vary from 1 to maximum value. A more restrictive condition would be that the value is 1 for all terms.

2. Super-Dirac spinor is expanded in monomials $O_N(q, \overline{q})$ of q and its conjugate $\overline{q}$, whose anti-commutator is non-trivial. One can equally well talk about quark like oscillator operators. The sum $\Psi = \Psi^N O_N$ defining super-spinor field. The multi-spinors Ψ_N are functions of space-time coordinates, which are ordinary numbers. Quark oscillator operators are same as appearing in the imbedding space super-coordinates. Only monomials O_N having total quark number equal to 1 are allowed. Super-spinor field however contains terms involving quark pairs giving rise to spartners of multiquark states with fixed quark number. The conjugate of super-spinor is defined in an obvious manner.

3. The metric determinant and modified gamma matrices appearing in the Dirac action are expanded as Taylor series in hermitian super-coordinate $h_s + \overline{h}_s$ with $h = h^N O_N$. This as as in the case of bosonic action.

   There are also couplings to gauge potentials defined by the spinor connection of CP_2 and the expansion of them with respect to the imbedding space coordinates gives at the first step rise covariant derivatives of gauge potentials giving spinor curvature. At next steps one obtains covariant derivatives of spinor curvature, which however vanish so that the number of terms coming from the dependence of spinor connection on CP_2 coordinates is expected to be finite. Constant curvature property of CP_2 is therefore be essential (not that also M^4 would have covariantly constant spinor curvature in twistor lift and give rise to CP breaking).

   The super-coordinate expansion of the metric determinant $\sqrt{g}$ and modified gamma matrices Γ^α and covariant derivatives D_α involving dependence on H coordinates give additional monomials of q parameters appear as hermitian monomials. Classical field equations correspond to $D_\alpha \Gamma^\alpha = 0$ guaranteeing the hermiticity of $D = \Gamma^\alpha D_\alpha$.

4. When super-coordinates of H are replaced with ordinary imbedding space coordinates the only Wick contractions are between O^N and $\overline{O}^N$ in the vacuum expectation of Dirac action, and the action reduces to super-Dirac action with components satisfying modified Dirac equation. Propagator is Dirac propagator for all terms and the presence of only odd components in Ψ with quark number 1 and even components in h^s guarantees that Fermi statistics is not violated at the level of propagators. The dependence on h_s induces coupling between different components of the super-spinor. The components of super-spinor are interpreted as second quantized objects.

5. The terms in the action would typically involve n-tuples of partial derivatives $L_{k_1\alpha_1 \ldots k_n 1\alpha_n}$ defined earlier for $L = \sqrt{g}$ coming from super-Taylor expansions. Similar derivatives come from the modified gamma matrices Γ^α.

   Also now one obtains loops from the self contractions in the terms coming from the expression of action and gamma matrices. These terms should vanish and as already found this would requires vanishing of currents perhaps identifiable as Noether currents of symmetries. This guarantees that the Taylor expansion contains only finite number of terms as required by number theoretic vision.

The multi-fermion vertices defined by the action would be non-trivial but involve always contraction of all fermion indices between monomials formed from oscillator operators in Ψ and their conjugates in $\overline{\Psi}$ if the loop contractions sum up to zero. One could interpret these supersymmetric vertices as a redistribution of fermions of a local many-fermion state between external local many-fermion states particles represented by the monomials appearing in the vertices. The fermions making the initial state would be same as in final state and all distributions of fermion number between sfermion lines would be allowed. The action obtained by contraction would has SUSY as symmetry but the propagation of different sfermions is fermionic and does not look like that for ordinary spartners.

The quantum criticality condition $q = q_s$ makes the situation non-linear and should fix the coefficients of various terms in super-Taylor expansions as fixed point values of coupling constants.

1.3.3 Could super-Kähler action alone give fermionic scattering amplitudes?

The concrete study of the super-counterpart of Kähler action led to a realization of an astonishing possibility: super-Kähler action alone could give also fermionic scattering amplitudes.

1. In principle this is possible if in S-marix one has contractions of quark creation operator and annihilation operator appearing in quark-antiquark bilinear with different partonic 2-surfaces. This would give fermionic line connecting the points of the cognitive representation at the boundary of CD with points at partonic 2-surfaces in $t = r_n$ hyper-planes in the interior of CD or at the opposite boundary of CD.

   As a matter of fact, this must be the case if the exponent for the sum of super-Kähler and super-Dirac action gives the scattering amplitudes as its matrix elements! The reason is that super-Dirac action vanishes or its solutions.

   The super-Dirac equation must be however present and corresponding variational principle must be satisfied. The hermiticity of the modified Dirac operator requires the vanishing of the covariant derivatives of the modified gamma matrices meaning that bosonic field equations are satisfied. This must be true also for the super variants of the modified gamma matrices.

   If super-Dirac equation is satisfied, the action of modified Dirac operator without connection (ordinary rather than covariant derivative) terms on the discretized quark fields can be expressed in terms of spinor connection as $\Gamma^\alpha - s\partial_\alpha \Psi = \Gamma^\alpha_s A_{\alpha,s} \Psi$ and there is no need for explicit information about the behavior of quark field in the nearby points so that cognitive representation is enough. Otherwise one must have the usual lattice type discretization.

2. The super expansion of super-Kähler action contains only ordinary derivatives of 4-currents defined by quark bi-linears. If the quark field operators with continuous arguments are behind those with discretized arguments and satisfy modified Dirac equation, one can transform the action on quark and antiquark fields to a multiplication with induced gauge potential. This

gives nothing but the coupling terms to the gauge potentials in the standard perturbation theory, where one assumes free solutions of Dirac action as approximate solutions. One therefore obtains on mass shell variant of the perturbation theory! Perturbation theory without perturbation theory, might Wheeler say. Or more concretely: the fact that one can treat super-coordinates only perturbatively.

3. The natural guess is that all terms in the expansion of super-Kähler can be transformed to interaction terms and super-Kähler action gives the analog of perturbation theory as a discretized version. The leptonic terms associated with $(3,3)$ term in super-Kähler action should transfrom to the analog of interaction terms for leptonic Dirac action. Whether Kähler gauge potential and spinor connection are developed in super-Taylor series in ordinary manner or remains an open questions.

1.4 What super-Dirac equation could mean and does one need super-Dirac action at all?

What does super-Dirac equation actually mean? Super Dirac action vanishes on mass shell and super-Kähler action would give all scattering amplitudes. Are super-Dirac action and super-spinor field needed at all? Should one interpret the oscillator operators defining analog of quark field q as the super-Dirac field Ψ_s as conceptual economy suggests. But doesn't this imply $q = q_s$?

One can consider 3 options as an attempt to answer these questions. Options I and II are not promising. Option III leads to very nice concrete realization of quantum criticality.

1.4.1 Option I: No super-Dirac action and constant oscillator operators

1. If oscillator operators can be regarded as constant, the super Taylor expansion for super Kähler action would give ordinary divergences of the fermionic currents and the action of derivative would be on modified gamma matrices and charge matrix A commutator of $[A_\alpha, \Gamma^\alpha Q]$ and the outcome would be non-vanishing so that one would obtain the coupling terms also now. Could the commutator $[A_\alpha, \Gamma^\alpha]$ be interpreted in terms of gravitational interaction and the commutator $[A_\alpha, Q]$ as electro-weak interaction? In any case, there would be no need for super-Dirac action!

2. There is however an objection. Quark oscillator operators are labelled by the points of cognitive representation and in continuum case they are analogous to the values of quantized spinor field. Should one identify this spinor field with super-spinor field and solve it using a generalization of modified Dirac equation to super-Dirac equation? Can one argue that oscillator operators labelled by points represent superpositions of constant oscillator operators involving integration over 3-D surface at light-cone boundary and are indeed constant?

This option does not look promising.

1.4.2 Option II: q satisfies ordinary Dirac equation

1. Could one assume that the solution q_0 of ordinary Dirac equation defines the solution to be used as q in the super-Kähler action. The coupling terms of super-Kähler action obtained using $D_0 q_0 = 0$ would be proportional to the classical spinor connection. Classical Kähler action does not involve gauge potentials so that internal consistency would not be lost at this level. The super-variant of Kähler action however involves derivatives of the analogs of fermion currents and there transformation to purely local objects requires the introduction of electroweak gauge potentials so that the symmetry between super-Kähler and super-Dirac would be lost.

2. This would save from developing gauge potentials A_k to super Taylor series - as found this would give only 2 terms by the covariant constancy of spinor curvature. The divergence would reduce to a term involving only a commutator $[A_{alpha}, Q]$, where A_α is purely classical. If Q is Kähler charge, this commutator would vanish, which looks strange since electroweak hypercharge is proportional to Q_K. This could be seen as a failure. If Kähler gauge potential is replaced with its super-variant $A_\alpha + J_{\alpha l} \delta h^l_s$ the commutator is non-vanishing as it should be.

3. Leptons would not appear in $q = q_0$ but since the exponent of super-Kähler action would define the scattering amplitudes by the vanishing of (super-)Dirac action, one could say that leptons emerge as 3-quark composites. SUSY would be dynamical after all!

Mathematically this option looks awkward and must be dropped from consideration.

1.4.3 Option III: q is a solution of super-Dirac equation

It is best to start from an objection.

1. Assume that q is given Super-Dirac equation

$$D_s(q)q = 0 \ .$$

This non-linear equation involves powers of q and its conjugate. The problem is that super-Dirac equation is non-linear in q and there are actually 7 separate equations for the part of q with quark number one. 7 equations is too much. The only manner to solve the problem is to replace q with q_s to get $D_s q_s = 0$. But this would require replacing q with q_s in $D_s(q)$ and it would seem that one has an infinite recursion.

2. Could q be self-referential in the sense that one has

$$q_s = q \ . \tag{1.1}$$

q would be invariant under iteration $q \to q_s$. This would give excellent hopes of fixing q uniquely. This allows also physical interpretation. The fixed points of iteration give typically fractals and quantum criticality means indeed fractality. This condition could therefore realize quantum criticality, and would give hopes about unique solution for $q = q_s$ for given extension of rationals.

Also h_s should satisfy similar self-referentiality condition expressing quantum criticality:

$$h_s = (h_s)_s \ . \tag{1.2}$$

The general ansatz for h_s involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

The general ansatz for h_s^k involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

3. Many consciousness theorists love the idea of self-referentiality described by Douglas Hofstadter in fascinating manner in his book "Gödel, Escher, Bach". They might get enthusiastic about the naive identification of q_s and h_s with field of consciousness. In TGD inspired theory of consciousness the self-referentiality of consciousness is understood in different manner but $q = q_s$ and $h_s = (h_s)_s$ as quantum correlated for the self-referentiality is certainly a fascinating possibility.

Consider now a more detailed picture.

1. What does one really mean with q_s? q_s could contain parts with quark number 1 and 3 but a very natural requirement is that it has well-defined fermion number and thus has only a part with quark number 1. The part with quark number 3 is not needed since super-Kähler action would contain it: leptons would emerge as local 3-quark composites from super-Kähler action.

2. Super-Dirac equation would be given by

$$\begin{aligned} D_s(q)q &= 0 \ , \\ D_s(q) &= \Gamma^{\alpha,s}(q)D_{\alpha,s}(q) \ . \end{aligned} \quad (1.3)$$

   $D_s(q)$ is super-Dirac operator and

$$\Gamma^{\alpha}_s = T^{\alpha k}_s \gamma_k \quad (1.4)$$

   are super counterparts of the modified gamma matrices $\Gamma^{\alpha} = T^{\alpha k}\gamma_k$ defined by the contractions of canonical momentum currents of Kähler action with the gamma matrices γ_k of H:

$$T^{\alpha}_k = \frac{\partial L_K}{\partial(\partial_{\alpha}h^k)} \ . \quad (1.5)$$

   One would have $\gamma_{k,s} = \gamma_k$ by covariant constancy. L_K denotes Kähler action density, which is sum of 4-D Kähler action and volume term. The field equations of super Kähler action give

$$D_{\alpha,s}\Gamma^{\alpha}_s = 0 \quad (1.6)$$

   guaranteeing the hermiticity of the super Dirac operator.

3. The basic equations would thus reduce to

$$\begin{aligned} q &= q_s \ , \\ D_{\alpha,s}\Gamma^{\alpha}_s &= 0 \ , \\ D_s(q)q &= 0 \ . \end{aligned} \quad (1.7)$$

In the continuum case one could think of solving the field equations iteratively.

1. One would first by solve $q = q_0$ for classical modified Dirac operator $D(h_0)$ defined by the ordinary coordinates h_0 of H. Next one would solve $q_1 = q_0 + \Delta q_1$ for the super version $D_1 = D(q_0)$. This would allow to solve next iterate $h_1 = h_0 + \Delta h_1$ using $D(q_1)$. One could continue this process in the hope that the iteration converges. At each step one have group of equations $D_n q_n = 0$ for q_n and for h_{n+1}.

2. An objection is that the iteration could lead outside the extension of rationals if it involves infinite number of iterates. This could occur for space-time surface itself if the normal ordering terms affect the classical action and force to modify the preferred extremal and also cognitive representation at each step. Remaining inside the extension of rationals could also mean that the coefficients of the monomials at points of cognitive representation belong to the extension.

   It is not of course completely clear whether these equations make sense in the interior of CD or can be solved unlike the lowest equation. It however seems that for each independent monomial m_n the equation would be of form $D_0 m_n = ...$ so that other terms would define kind of sources term and the equation super-Dirac equation could be written as non-linear equation $D_0 q = -\Delta D(q)q$.

3. Each order of bosonic monomials would give its own group of equations making sense also for the cognitive representations and the same would be true for quark monomials and monomials of different orders would be coupled but different quark numbers in q (quarks and leptons) would decouple. These equations are analogous to those appearing in QFT in a gauge theory involving gauge fields and fermion fields.

For cognitive representations the situation is much simpler.

1. All that is needed is the transformation of the ordinary divergences of fermionic currents to a form in which derivative ∂_α is replaced with the linear action of super-gauge potential $A_{\alpha,s}$. Therefore there is no need to solve the non-linear modified Dirac equation in this case and it would become necessary only at the continuum limit. The full solution of non-linear super-Dirac equation would be necessary only in the continuum theory.
2. Could one think that q has vanishing derivatives at the points of cognitive representation: $\partial_\alpha q = 0$ implying $\Gamma^\alpha A_\alpha q = 0$ If the condition holds true then q would be effectively constant for cognitive representations and the situation would effectively reduce to that for option I. This condition is is diffeo-invariant but not gauge invariant. If the points of cognitive representation correspond to singularities of the space-time surface at which several roots of the octonionic polynomial co-incide, the tangent space at the level of M^8 parameterized by a point of CP_2 is not unique and the singular point is mapped to several points in H, and the conditions $\partial_\alpha q = 0$ would make sense at the level of M^8 at least.
3. If one assumes that the quarks correspond to singular points defined by intersections of roots also in the continuum case, one obtains discretization also in this case irrespective of whether one assumes $\partial_\alpha q = 0$ at singularities. Allowing analytic functions with rational Taylor coefficients one obtains also now roots which can be however transcendental and one can identify intersections of roots in the similar manner.

To sum up, there are many uncertainties involved but to my opinion the most satisfactory option is Option III. If one assumes that condition at continuum case, one would obtain also now the discretization.

1.4.4 What information is needed to solve the scattering amplitudes?

One can look the situation also from a more practical point of view. Are there any hopes of actually calculating something? Is it possible to have the information needed?

1. The condition that super-Dirac equation is satisfied would remove the need to have a lattice and cognitive representation would be enough. If the condition $\partial_\alpha q = 0$ holds true, the situation simplifies even more but this condition is not essential. The condition that the points of the cognitive representation assignable to quark oscillator operators correspond to singularities of space-time surface at which several space-time sheets intersect, would make the identification of these points of cognitive representation easier. Note that the notion of singular point makes sense also at the continuum limit giving cognitive representation even in this case in terms of possibly transcendental roots of octonion analytic functions.

   If the singular points correspond to solution to 4 polynomial conditions on octonionic polynomials besides the 4 conditions giving rise to the space-time surfaces. The intersections for two branches representing two roots of polynomial equation for space-time surface indeed involve 4 additional polynomial conditions so that the points would have coordinates in an extension of rationals, which is however larger than for the roots $t = r_n$. One could of course consider an additional condition requiring that the points belong to the extension defined by r_n but this seems un-necessary.

   The octonionic coordinates used at M^8-side are unique apart from a translation of real coordinate and value of the radial light-like coordinate $t = r_n$ corresponds to a root of the polynomial defining the octonionic polynomial as its algebraic continuation. At this plane the space-time surfaces corresponding to polynomials defining external particles as space-time surfaces would intersect at partonic 2-surfaces containing the shared singular points defined as intersections.

2. The identification of cognitive representations goes beyond the recent knowhow in algebraic geometry. I have considered this problem in [L10] in light of some recent number theoretic ideas. If the preferred extremals are images of octonionic polynomial surfaces and $M^8 - H$ duality the situation improves, and one might hope of having explicit representation of the images surfaces in H-side as minimal surfaces defined by polynomials.

1.5 About super-Taylor expansion of super-Kähler and super-Dirac actions

The study of the details of of the general vision reveals several new rather elegant features and clarifies the connections with QFT picture.

1.5.1 About the structure of bosonic and fermionic monomials

The super part of the imbedding space coordinates is H-vector and this allows to pose strong conditions on the form of the monomials.

1. One can construct the simplest monomials as bilinears of quarks and anti-quarks. Since oscillator operators are analogs of quark fields, one can construct analogs of left- and right-handed electroweak currents $\overline{q}(1 \pm \gamma_5)\gamma^k Q q$ involving charge matrix Q naturally assignable to electroweak interactions. The charge matrices Q should reflect the structure of CP_2 spinor connection so that analogs of electroweak currents would be in question. One can multiply the objects Hamiltonians HA_A of the isometries and even symplectic transformations at the boundary of CD.

2. One can obtain higher monomials of q and $\overline{q}$ by multiplying these vectorial currents by bilinears, which are scalars and pseudo-scalars obtained by contracting some symmetry related vector field j^k_A of H with gamma matrices of H to give $\overline{q}(1 \pm \gamma_5) j^k_A Q \gamma_k q$ giving rise to analogs of scalar and pseudoscalar Higgs. The Killing vector fields of isometries of H and symplectic vector fields assignable to the Hamiltonians of $\delta CD \times CP_2$ are a natural choice for j^k_A.

   One can construct also scalar currents for which gamma matrices contract with gradient of Hamiltonian to give $\overline{q}(1 \pm \gamma_5)\gamma^k \partial_k H_A Q \gamma_k q$ as kind of duals of symplectic currents. These do not define symplectic transformations.

   These vector fields make sense at the boundaries of CD and this is enough (they could make sense also at shifted boundaries) since the field equations would allow to express monomials as linear superpositions of the monomials at boundary of CD. Oscillator would always be assigned with the boundaries of CD.

3. If the spin of graviton is assigned with spinor indices, the vector nature of the monomials excludes the analog of graviton. One can however consider also the possibility that the second spin index of graviton like state corresponds to the Hamilton of a symplectic isometry of S^2: for small enough size scales of CD this angular momentum would look like spin. In CP_2 degrees this would give rise to an analog of gluon. Also gluon with spin zero would be obtained.

   An alternative option is to assume that graviton corresponds to a non-local state with vectorial excitations at opposite throats of wormhole contact or at different wormhole contacts of closed flux tube. All these states are in principle possible and the question is which of them correspond to ordinary gravitons.

The super counterpart of Dirac spinor consists of odd monomials of quark spinor. Well-defined fermion number allows only monomials with quark number 1 and with definite H-chirality. Quark spinors allow leptons like stats as local 3-quark composites appearing in the super-Kähler action determining the scattering amplitudes since super-Dirac action vanishes at mass shell.

1. In the bosonic case one has vectorial entities and now it is natural to require that one has an object transforming like spinor of H. This poses strong conditions on the monomials since one should have spin 1/2-isospin 1/2 representation.

2. The lowest monomial correponds to quark-antiquark current. What about leptonic analog. The number of oscillator operators at given point is 4+4=8. Leptonic part of super-Kähler action must have 3+3 indices. Therefore also leptonic bilinear seems to be possible and pairs of quarks and lepton like states are possible.

   Intuitively it is clear that leptonic term exists and corresponds to an entity completely anti-symmetric in spin-isospin index pairs (s_3, i_3) of quark spinors. The construction of baryons without color symmetry indeed gives proton and neutron. In order to obtain Δ resonance from u and d quarks, one must have color degrees of freedom and perform anti-symmetrization in these.

   The general condition is that the tensor product of 3 8-D spin representation of $SO(1,7)$ contains 8-D representation in its decomposition. The existence of lepton representation is clear from the fact that the completely antisymmetric representation formed from 4 quarks is $SO(1,7)$ singlet and is product of lepton representation with 3 fold tensor product which must therefore contain spin-isospin 4-plet . The coupling to Kähler gauge potential would correspond to leptonic coupling, which is 3 times the quark coupling.

3. Quarks and lepton monomials have also satellites obtained by adding scalars and pseudo-scalars constructible as quark-anti-quark bi-linears in the manner already discussed. The interpretation as analogs of Higgs fields might make sense.

1.5.2 Normal ordering terms from contractions of oscillator operators

Normal ordering terms from contractions of oscillator operators is a potential problem. In the discretization based on cognitive representations this problem disappears.

1. Contraction terms could induce discrete coupling constant evolution by renormalizing the local monomials. Infinite number these terms would spoil number theoretical vision since a sum over infinite number of terms in general leads outside the extension of rationals involved. If the number of contractions is finite, there are no problems. This is the case in the number theoretical vision since contraction involves always a pair of points. If the rule for construction of S-matrix holds true these points are at opposite boundaries of CD. In the general case they can be at the same boundary. The number of contracted points cannot be larger than the number of points in cognitive representation, which is finite in the generic situation.

   This would give discrete coupling constant evolution as function of extension of rationals since the contractions renormalize the coefficients of the 4+4 terms in the local composites of oscillator operators. The original proposal that additional symmetries are needed to obtain discrete coupling constant evolution is not needed.

2. One could argue that algebraic numbers as a limit for extension is enough to get the continuum limit since the points of cognitive representation would be dense subset of 3-surface. For continuum theory 3-D delta functions would replace Kronecker deltas in anti-commutators implying in ordinary QFT divergences coming as powers of 3-D delta function at zero.

   In the proposed vision one can allow contractions even in the continuum case. The monomials in the interior are linear multilocal composites of those at either boundary of CD involving 3-D integration over boundary points. Contractions associated with two monomials in the interior means an appearance of delta function cancelling the second integration so that there is no divergence.

1.5.3 About the super-Taylor expansions of spinor connection and -curvature

There are also questions related to the details of the expansion of of spinor connection and -curvature in powers of monomials of quark oscillator operators.

1. The rule is that one develops Kähler function as Taylor series with argument shifted by super-part of the super-coordinate. This involves expansion in powers of coordinate gradients and also the expansion of Kähler gauge potential. In the case of modified Dirac action one must expand also the spinor connection of CP_2.

A potential problem is that the Taylor expansions of Kähler gauge potential and spinor connection have infinite number of terms. Since the monomials in the interior can be expressed linearly in terms of those at boundary of CD by classical field equations, number theoretic discretization based on cognitive representation implies that only a finite number of terms are obtained by using normal ordering and the fact that the number of oscillator operators at same point is 4+4=8. Normal ordering terms would represent radiative corrections giving rise to renormalization depending on the extension of rationals.

2. Is this enough or should one modify the Taylor expansion of Kähler gauge potential A? The idea that $A_k dh^k$ is the basic entity suggests that one must form super Taylor series for both A_k and dh^k. This would give $A_k dh^k \rightarrow A_k \partial_k \delta h^k + A_l \partial_(\delta h^l))dh^k$. By performing an infinitesimal super gauge transformation $A_l \rightarrow A_l + \partial_l (A_l \delta h^k)$ one obtains $A_k \rightarrow A_k + J_{kl} \Delta h^k_s$, where Δh^k_s denotes super part of super-coordinate. The next term would vanish by covariant constancy of J_{kl}.

   The same trick could be applied to spinor connection and since also spinor curvature is covariantly constant, one would obtain only 2 terms in the expansion also in the continuum case. This provides an additional reason for why S $(= CP_2)$ must be constant curvature space.

   This applies also to M^4: in fact, twistor approach strongly suggests that also M^4 has the analog of covariantly constant Kähler form. This conforms with the breakdown of Poincare symmetry at M^8 level forced by the selection of the octonion structure. Poincare invariance is gained by integrating over the moduli space of octonion structures in the construction of scattering amplitudes. What is remarkable that one could use the irreps of Lorentz group at boundaries of CD, which for obvious reasons are much more natural than than those of Poincare group.

3. In the case of imbedding metric the same trick would give only the c-number term and only the gradients of imbedding space coordinates would contribute to the super counterpart of the induced metric. In this case general gauge super-coordinate transformation would allow to treat the components of metric as constants.

1.5.4 What is the role of super-symplectic algebra?

This picture is not the whole story yet. Super-symplectic approach predicts that the super-symplectic algebra (SSA) generated essentially by the Hamiltonians of $S^2 \times CP_2$ assignable to the representations of $SO(3) \times SU(3)$ localized with the respect to the light-like radial coordinate of light-cone boundary characterize the states besides electro-weak quantum numbers. Color quantum numbers would correspond to Hamiltonians in octet representation. This would predict huge number of additional states.

There are however gauge conditions stating that sub-algebra of SSA having radial conformal weights coming as n-ples of SSA and isomorphic to SSA and its commutator with SSA annihilate physical states. This reduces the degrees of freedom considerably but the number of symplectic Hamiltonians is still infinite: measurement resolution very probably makes this number to finite.

2 Other aspects of SUSY according to TGD

In this section other aspects of SUSY according to the present proposal are discussed.

2.1 $M^8 - H$ duality and SUSY

$M^8 - H$ duality and $h_{eff}/h_0 = n$ hypothesis pose strong constraints on SUSY in TGD sense.

1. $h_{eff}/h_0 = n$ interpreted as dimension of extension of rationals gives constraints. Galois extensions are defined by irreducible monic polynomials $P(t)$ extended to octonionic polynomials, whose roots correspond to 4-D space-surfaces and in special case 6-spheres at 7-D light-cones of M^8 taking the role of branes.

The condition that the roots of extension defined by Q are preserved for larger extension $P \circ Q$ is satisfied if P has zero as root:

$$P(0) = 0 \ .$$

This simple observation is of crucial importance, and suggests an evolutionary hierarchy $P \circ Q$ with simplest possible polynomials Q at the bottom of the hierarchy are very naturally assignable to elementary particles. These polynomials have degree two and are of form $Q = x^2 \pm n$. Discriminant equals to $D = 2n$ and has the prime factors of n as divisors defining ramified primes identified as p-adic primes assignable to particles.

Remark: Also polynomials $P(t) = t - c$ are in principle possible. The corresponding space-time surfaces at the level of H would be M^4 and CP_2 and they are extremals of Kähler action but do not have particle interpretation.

It turns out the normal ordering of oscillator operators renormalizes the coefficients of P. In particular P can be shifted by a constant term and this deforms the roots of the real polynomial. Also the action principle to be discussed allows $RE(P) = c$ and $IM(P) = c$ surfaces as solutions.

2. The key idea is that the powers o^n of octonion are associative. If the coefficients of $P(o)$ are real or possibly even complex rationals $m + in$ commuting with octonions, associativity is not lost. Octonion o would be replaced by super-octions o_s with (possibly complex-) rational coefficients. o_s is octonion shifted by oscillator operator polynomial analogous to a real number. The conjugate octonion $\overline{o}$ would be treated analogously. Associativity would be preserved.

3. One could assign oscillator operators to both leptons and quarks but the option identifying leptons as local 3-quark local composites and in this sense spartners of quarks allows only baryon number zero composites of quarks and anti-quarks to appear in the octonionic polynomial, which is also hermitian. This would conform with $SO(1,7)$ triality.

   Remark: Anti-leptons are spartners of quarks in the sense of being their local composites but not in the sense that they would appear as local composites in q_s. Leptonic currents can appear in super-Kähler action so that anti-leptons are spartners of quarks in this sense.

   Oscillator operators would transform like components of 8-D spinor *resp.* its conjugate and have interpretation as quark *resp.* anti-quark like spinors. $SO(1,7)$ triality allows only leptonic or quark-like spinors and quark-like spinors are the only physical choice. Also the super-quark q_s which must satisfy self-referential condition $q_s = q$ must have components behaving like $8-D$ spinors with quark number 1. o_s should satisfy analogous condition $o_s = (o_s)_s$.

4. Super-polynomial $P_s(o)$ would be defined by super-analytic continuation as $P(o_s)$ by Taylor expanding it with respect to the super-part of o_s. The outcome is super-polynomial with coefficients of oscillator operator monomials containing k quark-antiquark pairs given by ordinary octonionic polynomials $P_{n-k}(o)$. Each $P_{n-k}(o)$ obtained by algebraically continuing the k:th derivative of the real polynomial $P(t)$ would define 4-surface by requiring that the imaginary or real part of $P_{n-k}(o)$ (in quaternionic sense) vanishes or is constant. Normal ordering of oscillator operators renormalizes the coefficients of P_{n-k}. The interpretation would be as radiative corrections.

   Octonionic super-polynomials obtained from octonionic polynomials of degree n as super-Taylor series decompose to a sum of products of octonionic polynomials $P_k(o)$ with degree $k = n - d$ with oscillator operator monomials consisting of d quark-antiquark pairs. If the degree n of the octonionic polynomial is smaller than the maximal number $N = 4$ of oscillator operator pairs in super-polynomial, only a fraction of spartners are possible. SUSY is realized only partially and one can say that part of spartners are absent at the lowest levels of evolutionary hierarchy. At the lowest level of hierarchy corresponding to $n = 2$ only fermions (quarks) would be present as local states and would form non-local states such as baryons and mesons. Gauge bosons and Higgs like state would be bi-local states and graviton 4-local state.

Remark: Gauge bosons and Higgs like states as local fermion-anti-fermion composites at level $n = 2 \times 2$. For the option involving only quarks (color is not spin like quantum number). Note that the value of $n_0 = 3 \times 2 = 6$ in $h = n_0 \times h_0$ suggested by the findings of Randel Mills [L1, L4] would allow the known elementary particles.

5. The geometric description of SUSY would be in terms of super-octonions and polynomials and the components of SUSY multiplet would correspond to components of a real polynomial continued to that of super-octonion and would in general give rise to minimal space-time surfaces as their roots: one space-time sheet for each component of the super-polynomial.

   The components would have different degrees so that the minimal extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles could be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected. The components of super-polynomial would have different degrees so that the extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles would be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected.

2.2 Can one construct S-matrix at the level of M^8 using exponent of super-action?

The construction of S-matrix in H picture in terms of exponential of action defining Kähler function of WCW forces to ask whether M^8 really is an alternative picture as the term "duality" would suggest or is it only part of a description necessitating both M^8 and H. If the duality holds true in strict sense the proposed construction of S-matrix at the level of H should make sense also at the level of M^8. Is this possible at all or could it be that S-matrix emerges the level of H and that M^8 level provides only a tool to describe preferred extremals in H by using what I have called M^8 duality? In the sequel I will look what one obtains if the duality holds true in strict sense.

1. The original idea was to identify space-time-surfaces in M^8 as roots of polynomial equations generalizing ordinary polynomial conditions. Could this makes sense also when octonions are replaced by super-octonions and what super-octonions and quark oscillator operators could mean?

2. The oscillator operators are interpreted as a discretized version of second quantized quark field q allowing local composites of q defining analogs of SUSY multiplets. One can indeed define second quantization for cognitive representations also now. Quark oscillator operators would be analogs of complex coefficients commuting with octonionic units ($i = \sqrt{-1}$ commute with them). The gamma matrices appearing in the quark-antiquark bi-linears would be ordinary gamma matrices of M^8.

   Remark: I have also considered the possibility that M^8 spinors correspond to octonionic spinors with octonionic units defining sigma matrices.

3. One could define simplest contribution the octonionic super-coordinate o_s as sum of M^8 octonion and super-part defined as contraction of 8-component quark current $\overline{q}\gamma^k q$ with contracted with octonionic units e_k to give $\Delta o_s = \overline{q}\gamma^k Q q e_k$. Charge matrices Q are linear combinations of sigma matrices of M^8 in the currents. Gamma matrices should be ordinary gamma matrices and q would transform like ordinary M^8 spinor. The entity $o_s = o + \Delta o_s$ would replace octonionic coordinate o in polynomial equations expressing the vanishing of the real or imaginary part (in quaternionic sense) for $P(0_s)$.

The contractions of Killing vector fields of translations with gamma matrices would give scalars $j^k\gamma_k$ giving in turn scalars $S = \overline{q}j^k\gamma_k Qq$ and these could be used to build higher monomials. Octonion analyticity in the proposed sense does not allow to use Killing vector fields of rotations and symplectic currents. On the other hand, for cognitive representations these vector fields are restricted to single point of cognitive representation: could this mean that one can allow also the more general scalars.

Leptons should emerge from o_s. This is the case if one allows also higher monomials in o_s. Also leptonic tri-linears and their conjugate could be built and these would give leptonic bi-linears $\overline{L}\gamma^k QL$. Therefore all (covariantly) constant contributions to super-octonion are possible. The coefficients of various monomials in o_s would be derivatives of polynomial P since they are obtained as super-Taylor series and the coefficients of these polynomials would have interpretation as coupling constants.

4. At the level of H one can construct much larger number of monomials of quark oscillator operators transforming like vector in H. The scalars and pseudo-scalars constructed from the Killing vector fields and symplectic currents can be used to build higher monomials. At the level of H the super-symplectic Hamiltonian currents except those associated with isometries could however annihilate physical states.

   The quark currents defined by symplectic isometries are however not constant so that there seems to be a slight inconsistency. Could one assume that also color isometries at the level of H annihilate states quite generally as also S^2 isometries associated with the "heavenly" sphere S^2 in the decomposition $\delta M^4_+ = S^2 \times R_+$? Or can one argue that the restriction to translations is enough because one considers only points of cognitive representation?

5. What about quantum super-spinors q_s (analog of quantized quark field). q would be ordinary rather than octonionic spinor. q_s would be constructed using q and the scalars already discussed. These monomials would carry information about couplings constants. If they are identifiable as the spinors appearing in o_s, one must have $q = q_s$ realizing quantum criticality in quark sector. This would pose strong conditions on the coefficients of the monomials appearing in q interpreted as coupling constants. The conditions would depend on the extension of rationals defined by the polynomia $P(o)$.

   The discretization by cognitive representations at the level of H is made possible by super-Dirac equation. At M^8 level there is no need to get rid of partial derivatives acting on currents and super-Dirac equation is not needed.

6. The polynomial equations are purely local algebraic equations and the notions of propagation and boundary value problem do not make sense at the level of M^8. $M^8 - H$ correspondence should lead to the emergence of these notions by mapping surfaces to minimal surfaces natural by quantum criticality. Octonion analyticity and associativity of tangent or normal space inducing dynamics should induce M^8 analog of propagation.

Could one imagine a counterpart for the action exponential and a construction of S-matrix similar to that in the case of H?

1. The action principle should be purely local involving no derivatives of the super-octonionic polynomial $P(o_s)$. It should produce $RE(P) = 0$ and $IM(P) = 0$ as solutions. One might allow also solution $RE(P) = c$, where c is rational number. This would shift of the real polynomial continued algebraically to octonionic polynomial modifying the roots. One should obtain also 6-spheres as universal solutions and identifiable as subsets of 7-D light cones. Now one would have $IM(P) = 0$, $RE(P) = c$ modifying the roots $t = r_n$ defining hyper-surfaces in M^4.

2. Action should be sum over contributions over the points of cognitive representation, perhaps identifiable as the set of singular points at which two roots co-incide.

   (a) Could one minimize the action with respect to the components of $RE(P)$ or $IM(P)$? If this were the case one obtains one would have either $RE(P) = 0$ or $IM(P) = 0$. Surfaces with associative tangent and normal space should have different action and this does not look nice.

(b) Could one require stationarity of the action with respect to the small deformations of the points of cognitive representation so that they would represent local extrema of action density? These points indeed change, when the polynomial is modified. Since only the deformations of these points are the visible trace of variation for cognitive representations, one could require that the value of action is stationary against these variations rather than variations of the values of $RE(P)$ and or $IM(P)$. This would give rise a condition involving derivatives of $RE(P)$ and $IM(P)$ at singular points with respect to space-time components of octonion. This option will be considered in the sequel.

3. The action density should be finite, and allow both solution types. One can imagine two options.

   Option I: If one requires that the action density is dimensionless, the simplest guess for the "action density" L is

$$L = \frac{(RE, IM)}{[(RE, RE) + (IM, IM)]} \ ,$$

   where one has $RE \equiv RE(P(o))$ and $IM \equiv IM(P(o))$ and the inner product is quaternionic inner product. The problem is that denominator gives infinite series giving rise to infinite number of normal ordering terms which may lead out of extension. For exceptional solutions $RE = 0, IM = 0$ the denominator also diverges.

   Option II: The alternative avoiding these problems is analogous to the action density of completely local free field theory given by

$$L = K(RE, IM) \ . \tag{2.1}$$

   K is constant with dimensions of inverse length squared and should relate to the CP_2 length squared. This is not dimensionless but can remain bounded if the quantity (RE, IM) remains bounded for large values of $(RE, RE) + (IM, IM)$.

4. For **Option I** L is a generalization of conformally invariant action from 2-D complex case, in which L reduces to $L = w_1w_2/(w_1^2 + w_2^2) = sin(\phi)cos(\phi)$, $w_1 = Re(w(z))$,$w_2 = Im(w(z))$. (ϕ) is the conformally invariant direction angle associated with w.

   The variation of 2-D action with respect to position of the point of cognitive representation gives

$$\frac{[(\partial_u w_1 w_2 + w_1 \partial_u w_2)(w_1^2 + w_2^2) + w_1 w_2 (w_1 \partial_u w_1 + w_2 \partial_u w_2)]}{(w_1^2 + w_2^2)^2} \ , \ u \in \{x, y\} \ .$$

   The general solutions are $w_i = c_i \neq 0$, where c_i are constant rational numbers.

   The criticality of the action density (maybe it could be seen as a manifestation of quantum criticality) is essential and means that the graph of L as function of w_1 and w_2 is analogous to saddle $w_1w_2/((w_1^2 + w_2^2)$. The condition that L is well-defined requires $c_1 \neq 0$. c_1 could in principle depend on point of cognitive representation. **Option II** gives the same equations in complex case.

5. For **Option II** one obtains 8 equations in the octonionic case and the outcome is that the derivatives of RE or IM or both with respect to components of o vanish. One can have $RE(P(o)) = c_1 \neq 0$ or $IM(P(o)) = c_2 \neq 0$, where c_i is rational. Both conditions are true for the special 6-D solution at 7-D light-cone boundary. Also now both options give the same equations.

What about the super variant of the variational principle?

1. Super-Taylor expansion must be carried out and normal ordering reduces the action to 5 independent terms according to the number $k \in \{0, ..., 4\}$ of quark pairs involved. It seems that only **Option II** is free of number theoretical problems due to normal ordering. Also in this case one has renormalization corrections to various terms in RE and IM. Inner product does not however give rise to additional terms. The degree of the polynomial $P_{n-k}(o_s)$ is equal to $n-k$ and decreases as the degree h of the monomial increases and normal ordering terms are present.

2. One can decompose action action density as $L = \sum L_k$ corresponding to different numbers k of quark pairs. The stationarity conditions hold true for the polynomial coefficient $P_{n-k}(o)$ of each oscillator operator monomial appearing in RE and IM. One has both $RE(P_{n-k}) = c_k \neq$ and $IM(P_{n-k}) = c_k \neq 0$ options. Both conditions are true for the special solutions. Without further conditions the option can depend on k and on the point of cognitive representation. $c_k \neq 0$ for some values of k guarantees that L to be non-vanishing so that the exponential of S can define a non-trivial S-matrix.

   Since an approximation of continuous case should be in question, the options should be same all points of the cognitive representation. In the lowest order approximation one obtains $k = 0$ solution obtained without super-symmetry. Normal ordering terms however modify the coefficients of $P(o)$ so that this solution is not exact.

3. Each monomial $P_{n-k}(o)$ defines its own space-time surface and conditions should hold true independently for each super-component L_k. Second option would be to consider vacuum expectation value of the action in which case one would have only single surface.

4. One would have purely local free field theory and the construction of S-matrix would be extremely simple. One could introduce CDs and the identification of hermitian conjugates of fermionic oscillator operators labelled by points at given boundary of CD as creation operators at time reflected points at opposite boundary. If one can talk about sub-CDs assignable to partonic 2-surfaces in M^8 picture one obtains similar identification for them. Also leptons would emerge from S-matrix.

To sum up, the second trial has a generalization although octonionic picture allows only the Killing vectors of translations of E^8 in the construction of o_s and q_s. The action principle replaces the earlier ansatz with solution in which one has roots of polynomials of $RE(P)$ and $IM(P)$ shifted by rational number. Also a renormalization of P takes place.

2.3 How the earlier vision about coupling constant evolution would be modified?

In [L7, L5] I have considered a vision about coupling constant evolution assuming twistor space $T(M^4) = M^4 \times S^2$. In this model the interference of the Kähler form made possible by the same signature of $S^2(M^4)$ and $S^2(CP_2)$ gives rise to a length scale dependent cosmological constant appearing defining the running mass squared scale of coupling constant evolution.

For $T(M^4)$ identified as $CP_3(3,h)$ the signatures of twistor spheres are opposite and Kähler forms differ by factor i (imaginary unit commuting with octonion units) so that the induced Kähler forms do not interfere anymore. The evolution of cosmological constant must come from the evolution of the ratio of the radii of twistor spaces (twistor spheres). This forces to modify the earlier picture.

1. $M^8 - H$ duality has two alternative forms with $H = CP_{2,h} \times CP_2$ or $H = M^4 \times CP_2$ depending on whether one projects the twistor spheres of $CP_{3,h}$ to $CP_{2,h}$ or M^4. Let us denote the twistor space $SU(3)/U(1) \times U(1)$ of CP_2 by F.

2. The key idea is that the p-adic length scale hierarchy for the size of 8-D CDs and their 4-D counterparts is mapped to a corresponding hierarchy for the sizes of twistor spaces $CP_{3,h}$ assignable to M^4 by M^8-H-duality. By scaling invariance broken only by discrete size scales of CDs one can take the size scale of CP_2 as a unit so that $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ becomes an evolution parameter.

Coupling constant evolution must correspond to a variation for the ratio of $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ and a reduction to p-adic length scale evolution is expected. A simple argument shows that Λ is inversely proportional to constant magnetic energy assignable to $S^2(X^4)$ divided by $1/\sqrt{g_2(S^2)}$ in dimensional reduction needed to induce twistor structure. Thus one has $\Lambda \propto 1/r^2 \propto 1/L_p^2$. Preferred p-adic primes would be identified as ramified primes of extension of rationals defining the adele so that coupling constant evolution would reduce to number theory.

3. The induced metric would vanish for $R(S^2(CP_{3,h}) = R(S^2(F))$. Λ would be infinite at this limit so that one must have $R(S^2(CP_{3,h}) \neq R(S^2(F))$. The most natural assumption is that one $R(S^2(CP_{3,h}) > R(S^2(F))$ but one cannot exclude the alternative option. Λ behaves like $1/L_p^2$. Inversions of CDs with respect to the values of the cosmological time parameter $a = L_p$ would produce hierarchies of length scales, in particular p-adic length scales coming as powers of $\sqrt{p}$. CP_2 scale and the scale assignable to cosmological constant could be seen as inversions of each other with respect to a scale which is of order 10^{-4} meters defined by the density of dark energy in the recent Universe and thus biological length scale.

4. The original model for the length scale evolution of coupling parameters [L7] would reduce to that along paths at $S^2(CP_2)$ and would depend on the ends points of the path only. This picture survives as such. Also in the modified picture the zeros of Riemann zeta could naturally correspond to the quantum critical points as fixed points of evolution defining the coupling constants for a given extension of rationals.

   Space-time surfaces the level of M^8 would be determined by octonionic polynomials determined by real polynomials with rational coefficients. The non-critical values of couplings might correspond to the values of the couplings for space-time surfaces associated with octonion analytic functions determined by real analytic functions with rational Taylor coefficients.

2.4 How is the p-adic mass scale determined?

p-Adic prime identified as a ramified prime of extension of rationals is assumed to determine the p-adic mass scale. There are however several ramified primes and somehow the quantum numbers of particle should dictate with ramified prime is chosen. There are two options to consider depending on whether both the extension and ramified prime are same for all spartners Option 1) or whether spartners can have different ramified primes (Option 2)). There also options depending on whether both leptons and quarks appear in their own super-Dirac actions (Option a) or whether only quarks appear in super-Dirac action (Option b implied by quark number conservation) . Call the 4 composite options Option 1a), 2a), 1b), 2b) respectively.

1. Consider first Options 1a) and 1b). The ramified prime is same for all states corresponding to the same degree of θ monomial and thus same value of $F + \overline{F}$. At the lowest $k = 2$ level containing only fermions as local states the p-adic thermal masses of quarks and leptons are same for Option 1a) at least for single generation and for all generations if Q_2 does not depend on the genus g of the partonic 2-surface. For Option 1b) the masses would *not* be same for leptons and quarks since they would correspond to different degrees of super-octonionic polymials. For both options would have $n = n(g)$.

2. For Option 2 ramified prime depends on the state of the SUSY multiplet. This would require that for fermions with $k = 2$ the integer n in $Q_2(x) = x^2 \pm n$ has the p-adic primes assignable to leptons and quarks as factors.

   There are 6 different quarks and 6 different leptons with different p-adic mass scales. For Option 2a) n should have 12 prime factors which are near to power of 2. For leptons the factors correspond to Mersenne primes M_k, $k \in \{107, 127\}$ and Gaussian Mersenne $k = 113$. Gaussian Mersenne is complex integer. TGD requires complexification of octonions with imaginary unit i commuting with octonionic units so that also Gaussian primes are possible. This would resolve the question whether $P(t)$ can have complex coefficients $m + in$.

   For option 2b) quarks and leptons as local proton and neutron would have different extensions since the polynomials would be different. The p-adic primes for 6 quark states quarks would depend on genus. The value of n need not depend on genus g since the ramified primes p depends on g: $p = p(g)$.

Since the polynomials describing higher levels of the dark hierarchy would be composites $P \circ Q_2$ with $P(0) = 0$, Q_2 would be a really fundamental polynomial in TGD Universe. For Option 2b) it would be associated with quarks and would code for the elementary particles physics. The higher levels such as leptons would represent dark matter levels.

3. The crucial test is whether the mass scales of gauge bosons can be understood. If one assumes additivity of p-adic mass squares so that the masses for 2-local bosons would be p-adically sums of mass squared at the "ends" of the flux tube. If the discriminant $D = 2n$ of Q_2 contains high enough number of factors this is possible. The value of the factor p for photon would be rather larger from the limits on photon mass. For graviton the value p would be even larger.

To sum up, the vision about dark phases suggests that the monopole phase is possible already for the minimal value $n = 2$ involving only fundamental quarks for Option 2b), which is the simplest one and could solve the probelm of matter antimatter asymmetry. Bosons and leptons as purely local composites of quarks are possible for $n = 6$. Rather remarkably, also empirical constraints [L1, L4] led to the conclusion $h = 6h_0$. The condition is actually weaker: $h/h_0 \ mod \ 6 = 0$.

2.5 Super counterpart for the twistor lift of TGD

Twistor lift of TGD is now relatively well understood. I have made somewhat adhoc attempts to construct TGD analog of the Grassmannian approach so super-twistors. The proposed formalism for constructing scattering amplitudes seems to generalize as such to the twistor lift of TGD.

2.5.1 Could twistor Grassmannian approach make sense in TGD?

By $M^8 - H$ duality [L3] there are two levels involved: M^8 and H. These levels are encountered both at the space-time level and momentum space level. Do super-octonions and super-twistors make sense at M^8 level?

1. At the level of M^8 the high uniqueness and linearity of octonion coordinates makes the notion of super-octonion natural. By $SO(8)$ triality octonionic coordinates (bosonic octet 8_0), octonionic spinors (fermionic octet 8_1), and their conjugates (anti-fermionic octet 8_{-1}) would for triplet related by triality. A possible problem is caused by the presence of separately conserved B and L. Together with fermion number conservation this would require $\mathcal{N} = 4$ or even $\mathcal{N} = 4$ SUSY, which is indeed the simplest and most beautiful SUSY.

2. At the level of the 8-D momentum space octonionic twistors would be pairs of two quaternionic spinors as a generalization of ordinary twistors. Super octo-twistors would be obtained as generalization of these.

Also Grassmannian is replaced with super-Grassmannian and super-coordinates as matrix elements of super matrices are introduced.

1. The integrand of the Grassmannian integral defining the amplitude can be expanded in Taylor series with respect to θ parameters associated with the super coordinates C as rows of super $G(k, n)$ matrix.

2. The delta function $\delta(C, Z)$ factorizing into a product of delta functions is also expanded in Taylor series to get derivatives of delta function in which only coordinates appear. By partial integration the derivatives acting on delta function are transformed to derivatives acting on integrand already expanded in Taylor series in θ parameters. The integration over the θ parameters using the standard rules gives the amplitudes associated with different powers of θ parameters associated with Z and from this expression one can pick up the scattering amplitudes for various helicities of external particles.

The super-Grassmannian formalism is extremely beautiful but one must remember that one is dealing with quantum field theory. It is not at all clear whether this kind of formalism generalizes to TGD framework, where particle are 3-surfaces [L3]. The notion of cognitive representation effectively reducing 3-surfaces to a set of point-like particles strongly suggests that the generalization exists.

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It seems now clear that sparticles are predicted and SUSY remains in the simplest scenario exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The increased undestanding of what twistorialization leads to an improved understanding of what twistor space in TGD could be. It turns out that the hyperbolic variant $CP_{3,h}$ of the standard twistor space CP_3 is a more natural identification than the earlier $M^4 \times S^2$ also in TGD framework but with a scale corresponding to the scale of CD at the level of M^8 so that one obtains a scale hierarchy of twistor spaces [L12]. Twistor space has besides the projection to M^4 also a bundle projection to the hyperbolic variant $CP_{2,h}$ of CP_2 so that a remarkable analogy between M^4 and CP_2 emerges. One can formulate super-twistor approach to TGD using the same formalism as will be discussed in this article for the formulation at the level of H. This requires introducing besides 6-D Kähler action and its super-variant also spinors and their super-variants in super-twistor space. The two formulations are equivalent apart from the hierarchy of scales for the twistor space. Also M^8 allows analog of twistor space as quaternionic Grassmannian HP_3 with signature (6,6). What about super- variant of twistor lift of TGD? consider first the situation before the twistorialization.

1. The parallel progress in the understanding SUSY in TGD framework [L11] leads to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.

2. In fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of imbedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.

3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

2.5.2 Super-counterpart of twistor lift using the proposed formalism

The construction of super-coordinates and super-spinors suggests a straightforward twistorialization. One would only replace the super-imbedding space and super-spinors with super-twistor space and corresponding super-spinors. Dimensional reduction should give essentially the 4-D theory apart from the variation of the radius of the twistor space predicting variation of cosmological constant. The size scale of CD would correspond to the size scale of the twistor space for M^4 and for CP_2 the size scale would serve as unit and would not vary.

1. Replace the coordinates of twistor space with superspinors expressed in terms of quark and anti-quark spinors lifted to the corresponding spinors of twistor space. Express 6-D Kähler action in terms of super-coordinates.

2. Replace H-spinors with the spinors of 12-D twistor space and assume only quark chirality. By the bundle property of the twistor space one can express the spinors as tensor products of spinors of the twistor spaces $T(M^4)$ and $T(CP_2)$. One can express the spinors of $T(M^4)$ tensor products of spinors of M^4 - and S^2 spinors locally and spinors of $T(CP_2)$ as tensor products of CP_2 - and S^2 spinors locally. Chirality conditions should reduce the number of 2 spin components for both $T(M^4)$ and $T(CP_2)$ to one so that there are no additional spin degrees of freedom.

   The dimensional reduction can be generalized by identifying the two S^2 fibers for the preferred extremals so that one obtains induced twistor structure. In spinorial sector the dimensional reduction must identify spinorial degrees of freedom of the two S^2s by the proposed chirality conditions also make them non-dynamical. The S^2 spinors covariantly constant in S^2 degrees of freedom.

   Define the twistor counterpart of the analog of modified Dirac action using same general formulas as in case of H.

3. Identify super spinors as sum of odd monomials of theta parameters with quark number 1 identified as oscillator operators. Identify super-Dirac action for twistor space by replacing $T(H)$ coordinates with their super variants and Dirac spinors with their super variants.

3 Are quarks enough to explain elementary particle spectrum?

TGD based SUSY involves super-spinors and super-coordinates. Suppose that one has a cognitive representation defined by the points of space-time surface with coordinates in an extension of rationals defining adele and belonging to the partonic 2-surfaces defined by the intersections of 6-D roots of octonionic polynomials with 4-D roots. This representation has H counterpart.

Cognitive representation gives rise to a tensor product of these algebras and the oscillator operators define a discretized version of fermionic oscillator operator algebra of quantum field theories. One would have interpretation as many-fermion states but the local many-fermion states would have particle interpretation. This would replace fermions of the earlier identification of elementary particles with SUSY multiplets in the proposed sense. This brings in large number of new particles. One can however ask whether the return to the original picture in which single partonic 2-surface corresponds to elementary particle could be possible. Certainly it would simplify the picture dramatically.

Could this picture explain elementary particle spectrum and how it would modify the recent picture?: these are the questions.

3.1 Attempt to gain bird's eye of view

Rather general arguments suggest that SYM action plus Super-Dirac action could explain elementary particle spectrum. Some general observations help to get a bird's eye of view about the situation.

1. The antisymmetric tensor products for fermions and anti-fermions produce states with same spectrum of electro-weak quantum numbers irrespectively of whether the fermion and anti-fermion are at same point or at different points. Which option is correct or are these options correspond analogous to two different phases of lattice gauge theory in which nodes *resp.* links determine the states? Only multi-local states containing fermions with identical spin and weak isospin at different points are not possible as local states.

   There is no point in denying the existence of either kind of states. What suggests itself is the generalization of electric-magnetic duality relating perturbative Coulomb phase in which ordinary particles dominate and the non-perturbative phase in which magnetic monopoles dominate. I have considered what I have called weak form of electic-magnetic duality already earlier [K6] but as a kind of self-duality stating that for homologically charged partonic 2-surfaces electric and magnetic fluxes are identical. The new picture would conform with the view of ordinary QFT about this duality.

2. The basic distinction between TGD and standard model is that color is not spin-like quantum number but represented as color partial waves basically reducing to the spinor harmonics plus super-symplectic generators carrying color quantum numbers. Spinor harmonics as such have non-physical correlation between color and electro-weak quantum numbers [K2] although quarks and leptons correspond to triality $t = 1$ and triality $t = 0$ states.

3. It turns out that one could understand quarks, leptons, and electro-weak gauge bosons and their spartners as states involving only single partonic 2-surface [K1]: this would give essentially the original topological model for family replication in which partonic 2-surfaces were identified as boundary components of 3-surface. In principle one can allow also quarks and gluons with unit charge matrix with color partial waves defining Lie-algebra generator as bosonic states. Could these states correspond to free partons for which perturbative QCD applies at high energies?

   Also color octet partial waves of electro-weak bosons and Higgs and the predicted additional pseudo-scalar - something totally new - are possible as both local and bi-local states. There would be no mixing of $U(1)_Y$ state and neutral $SU(2)_w$ states for color octet gluon. In this sense electro-weak symmetry breaking would be absent.

4. Electro-weak group as holonomy group of CP_2 can be mapped to the Cartan group of color group, and electro-weak and color quantum numbers would relate like spin and angular momentum to each other. This encourages to think that there are deep connections between electro-weak physics and color physics, which have remained hidden in standard model.

   The conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) of hadron physics suggests a strong connection between color physics and electro-weak physics. There is also evidence for so called X bosons with mass 16.7 MeV [C4] [L2] suggesting in TGD framework that weak physics could have fractally scaled down copy in hadronic and even nuclear scales.

   Could ordinary gluons be responsible for CVC whereas colored variants of weak bosons and Higgs/pseudo-scalar Higgs would be responsible for PCAC? Usually strong force in hadronic sense is assigned with pion exchange. This approach does not work perturbatively. Could one assign strong force with the exchange of pseudo-scalar, and colored variants of gluons, pseudo-scalar, and Higgs?

5. Hitherto it has been assumed that homology charges (Kähler magnetic charges) characterize flux tubes connecting the two wormhole throats associated with the monopole flux of elementary particle. Could one understand the bi-local or multi-local objects of this kind as exotic phase analogous to magnetic monopole dominated phase of gauge theories as dual of Coulomb phase?

   Hadrons would certainly be excellent candidates for monopole dominated phase. Gluons would be pairs of quarks associated with homologically charged partonic 2-surfaces with opposite homology charges. Gluons would literally serve as "glue" in the spirit of lattice QCD. Gluons and hadrons would be multi-local states made from quarks and gluons as homologically trivial configurations with vanishing total homology charge.

6. Is there a correlation between color hyper-charge and homology charge forcing quarks and gluons to be always in this phase and forcing leptons to be homologically neutral? This could provide topological realization of color confinement. The simplest option is that valence quarks have homology charges $2, -1, -1$ summing up to zero. This was one of the first ideas in TGD about 38 years ago.

   One can also imagine that the homological quark charges $(3, -2, -1)$ summing up to zero define a classical correlate for the color triplet of quarks, a realization of Fermi statistics, and allow to understand color confinement topologically. The color partial waves in H would emerge at the imbedding space level and characterize the ground states of super-symplectic representations. Color triplets of quarks and antiquarks could thus correspond to homology charges $(3, -2, -1)$ and $(-3, 2, 1)$ and neutral gluons could be superpositions of pairs of form $(q, -q)$, $q = 3, -1, -1$. Charged gluons as flux tubes would not be possible in the confined phase.

7. Is monopole phase possible also for leptons as general QFT wisdom suggests? For instance, could Cooper pairs could be flux tubes having members of Cooper pair - say electrons - at its ends and photons in this phase be superposition of fermion and anti-fermion at the ends of the flux tube and monopole confinement would make the length of flux tube short and photon massive in superconducting phase.

3.2 Comparing the new and older picture about elementary particles

The speculative view held hitherto about elementary particles in TGD Universe correspond to the TGD analog of the magnetic monopole dominated phase of QFTs. This view is considerably more complicated than the new view and involves unproven assumptions.

1. Identification of elementary particles

   Old picture: Ordinary bosons (and also fermions) are identified as multilocal many-fermion states. The fermions and anti-fermions would reside at different throats of the 2 wormhole contacts associated with a closed monopole flux tube associated with the elementary particle and going through wormhole contact to second space-time sheet. All elementary particles are analogous to hadron-like entities involving closed monopole flux tubes.

   One can raise objections against this idea. Leptons are known to be very point-like. One must also assume that the topologies of monopole throats are same for given genus in order that p-adic mass calculations make sense. The assumption that quarks correspond to monopole pairs makes things unnecessarily complex: it would would be enough to assume that they correspond to partonic 2-surfaces with monopole charge at the "ends" of flux tubes at given space-time sheet.

   One must assume that the genus of the 4 throats is same for known elementary particles: this assumption looks rather natural but can be criticized. The correlations forced by preferred extremal property should of course force the genera of wormhole throats to be identical.

   New picture: Elementary fermions would be partonic 2-surfaces. Leptons would have vanishing homology charge. Elementary bosons could be simply pairs of fermion anti-fermion located at the opposite ends of flux tubes. This would dramatically simplify the topological description of particle reactions. In the case of quarks however the homological space-time correlate of color confinement is attractive and would force monopole flux tubes. It turns out that this picture corresponds to the simplest level in the $h_{eff} = nh_0$ hierarchy. One could also see leptons and quarks as analogs of perturbative and non-perturbative monopole dominated phases of gauge theories.

   Flux tubes could allow to understand phases like super-conductivity involving massivation of photons (Meissner effect). For instance, Cooper pairs could correspond closed flux tubes involving charged fermions at their "ends". In high Tc super-conductivity Cooper pairs in this sense would be formed at higher critical temperature and at lower critical temperature they would form quantum coherent phase [K4, K5]. Flux tube picture could also allow to understand strongly interacting phases of electrons.

2. Electroweak massivation

   Old picture: Electro-weak massivation has been assumed to involve screening of electro-weak isospin by a neutrino pair at the second wormhole contact. The screening is not actually necessary in p-adic thermodynamics in its recent form since the thermal massivation is due to the mixing of different mass eigenstates.

   New picture: There is no need to add pairs of right- and left-handed neutrino to screen the weak charges in the scale of flux tube.

3. Identification of vertices

   Old picture: In old picture one could do almost without vertices: in the simplest proposal particle reactions would correspond to re-arrangements of fermions and antifermions so that fermion and antifermion number would be conserved separately. Therefore one needs an analog of vertex in which partonic 2-surface turns back in time in order to describe creation of particle pairs and emission of bosons identified as fermion-antifermion pairs.

New picture: In vertices fermions and antifermions assignable to super spinor component would be redistributed between different orbits of partonic 2-surfaces meeting along their ends at the 6-D braney object in M^8 picture or turn backwards in time - the interpretation for this might be in terms of interaction with classical induce gauge field. What is new are the new vertices corresponding to the monomials of oscillator operators in the super-spinor. The original identification of particles (given up later) as single partonic 2-surface predicts genus-generation correspondence without additional assumptions. Both old and new picture predict also higher gauge boson genera for which some evidence exists: TGD predictions for the masses are correct [K3].

3.3 Are quarks enough as fundamental fermions?

For the first option - call it Option a) - quarks and leptons would define their own super-spinors. Whether only quark or lepton-like spinors are enough remains still an open question.

1. I have also considered the possibility that quarks are actually anti-leptons carrying homology charge and have anomalous em charge equal to $-1/3$ units. One might perhaps say that quarks are kind of anyonic states [K7]. It is however difficult to understand how the coupling to Kähler form could be dynamical and have values $n = -3$ and $n = 1$ for homologically neutral and charged states respectively. This would mean that only lepton like θ parameters appear in super-coordinates and only leptonic Dirac action is needed.

2. For this option proton would be bound state of homologically charged leptons. This in principle allows decays of type $p \rightarrow e^+ ...$ and $p \rightarrow e^+ + e^+ + \overline{\nu}$ requiring that the 3 partonic 2-surfaces fused with non-trivial homology charges fuse to single homologically trivial 2-surface. This form of proton instability would be different from that of GUTs. The topology changing process is expected to be slow. Is the introduction of two super-octonionic θ parameters natural assignable to B and L or is single parameter enough?

3. The coupling to Kähler form is not explicitly visible on the bosonic action but is visible in modified Dirac action. Could leptonic modified Dirac action transform to quark type modified Dirac action? This does not seem plausible.

The super-Dirac action for quarks however suggests another option, call it Option b). Leptons could be local 3-quark states.

1. Could one identify leptons as local 3 quark composites - essentially anti-baryons as far as quantum numbers are considered - but with different p-adic scale and emerging from the super-Dirac action for quarks as purely local states with super-degree $d = 3$? Could one imagine totally new approach to the matter antimatter asymmetry?

   Leptons would be purely local 3-quark composites and baryons non-local 3-quark composites so that charge neutrality alone would would guarantee matter-antimatter symmetry at fundamental level. Anti-quark matter would slightly prefer to be purely local and quark matter 3-local. The small CP violation due to the M^4 part of Kähler action forced by twistor lift should explain this asymmetry.

   Leptons and anti-leptons would drop from thermal equilibrium with quarks at some stage in very early cosmology. The reason would be the slowness of the reactions producing local 3-quark composites from quarks. This slowness is required also by the stability of proton. Opposite matter anti-matter asymmetries at the level of both leptons and quarks would have been generated at this stage by CP violation and would have become visible after annihilation.

2. The local baryons would have much simpler spectrum and would correspond for given genus g (lepton generation) to the baryons formed from u and d quarks having however no color. There would be no counterparts for higher quarks. This would suggests that (L, ν_L) could be local analog of (p, n).

   For ordinary baryons statistics is a problem and this led to the introduction of quark color absent for local states. The isospin structure of the local analogs of p and n is not a problem. In uud (udd) type states allowed by statistics the spins of the u (d) quarks must have opposite spin. The analogs of Δ resonances are not possible so that one would obtain only the analogs of p and n!

3. The widely different mass scales for leptons and quarks would be due to locality making possible different ramified primes for the extension of rationals. The widely differing p-adic length scales of leptons and neutrinos could be undersood if the ramified prime for given extension can be different for the particles super-multiplets with same degree of octonionic polynomial. This could be caused by electroweak symmetry breaking. The vanishing electroweak quantum numbers of right-handed neutrino implies a dynamics in sharp contrast with that of neutron, whose dynamics would be dictated by non-locality.

   Also local pions are possible. The lepto-pions of lepto-hadron hypothesis [K8] could correspond to either local pions or to pion-like bound states of lepton and anti-leptons. There is evidence also for the muon- and tau-pions.

4. This idea might provide a mathematically extremely attractive solution to the matter antimatter asymmetry: matter and antimatter would be staring us directly into eyes. The alternative TGD inspired solution would be that small CP breaking would induce opposite matter-antimatter asymmetries inside long cosmic strings and in their exteriors so that annihilation period would lead to the observed asymmetry.

The decay $p \to e^+ + X$ could in principle take place and also the reverse decay $e^+ \to p + X$ can be considered in higher energy collisions of electron. The life-time for the decay modes predicted by GUTs is extremely long - longer than 1.67×10^{34} years (see `http://tinyurl.com/nqco2j7`). This fact provides a killer test for the proposal.

One should estimate the life-time of proton in number theoretic approach. The corresponding SUSY vertex corresponds to a Wick contraction involving 4 terms in super-Dirac action: the trilinear term for quarks and 3 linear terms.

1. The vertex would associated with a partonic 2-surface at which 3 incoming quark space-time sheets and outgoing electron space-time sheet meet. At quark level the vertex means an emanation of 3 quark lines from single 3-quark line at a point of partonic 2-surface in the intersection of the ends of 4 space-time surfaces with 6-sphere $t = r_n$ defining a universal root of octononic polynomial $P(o)$. t is M^4 time coordinate [L9]. The vertex itself does not seem to be small.

2. A fusion of 3 homologically non-trivial partonic 2-surfaces to single partonic 2-surface with trivial homology charge cannot occur since partonic 2-surfaces with different homology charge cannot co-incide.

   The reaction $p \to e^+ + ..$ can occur only if the quark-like partonic 2-surface fuse first to single homologically trivial partonic 2-surface: this would correspond to de-confinement phase transition for quarks. After that the 3 quark lines would fuse to single e^+ line.

   (a) To gain some intuition consider two oppositely oriented circles around a puncture of a plane with opposite homology charges. The circles can reconnect to homologically trivial circle. Instead of circles one would now have 3 homologically trivial quark-like 2-surfaces at three light-like boundaries between Minkowskian and Euclidian regions of the space-time surface representing proton. First 2 quark-like 2-surfaces would touch and develop a wormhole contact connecting them. After that the resulting di-quark 2-surface and third quark 2-surface would fuse. The 3 quarks would be now analogous to de-confined quarks.

   (b) At the next step the 3 separate quark lines would fuse to single one. This process must occur in single step since di-quark cannot correspond to single point because the Dirac super-polynomial is odd in oscillator operators and has quark number 1. The fusion point would correspond to 3 degenerate roots of the octonionic polynomial associated with the partonic 2-surface. This partonic 2-surface would be associated with $t = r_n$ hyperplane of M^4 and it would become leptonic 3-surface.

(c) 3 4-D sheets defined by the roots of the octonionic polynomial should meet at the vertex assignable to $t = r_n$ hyper-plane. This gives 2 additional conditions besides the conditions defining space-time sheets. This for both the protonic and positronic space-time sheets. One would have double quantum criticality. The tip of a cusp catastrophe serves as an analog. Since the coefficients of the octonionic polynomial are rational numbers, it might be possible to estimate the probability for this to occur: the probability could be proportional to the ratio N_2/N_0 of the number N_2 of doubly critical points to the number N_0 of all points with coordinates in the extension. This could make the process very rare.

It must be however emphasized that also the option in which also leptons are fundamental fermions cannot be excluded.

3.4 What bosons the super counterpart of bosonic action predicts?

It has been already noticed that the spectra of fermion-antifermion states are identical for local and bi-local states if one assumes that the wave function in the relative coordinate of fermion and anti-fermion is symmetric. This does not yet imply that the particle spectrum is realistic in the case of the bosonic action.

The situation is simplified considerably by the facts that color is not spin-like quantum number but analogous to momentum and can therefore be forgotten, family replication can be explained topologically, and depending B and L are separately conserved for Option a) but for Option b) L reduces to B since leptons would be local 3-quark composites. Let us restrict first the considered to Option b).

1. What kind of spectrum would be predicted? Consider first quark Clifford algebra formed by the oscillator operators defining the spartners of quark without any conditions on total quark number of the monomial Forgetting color, one has 8 states coming from left and right handed weak doublet and their anti-doublets. The numbers of elements $N(k)$ in Clifford algebra with given quark number $B = k = N(q) - N(\overline{q})$ is given by $N(k) = \sum 0 \leq q \leq 4 - kB(4, q + k) \times B(4, q)$ in terms of binomial coefficients.

   For $B = 0$ one obtains $N(0) = \sum 0 \leq q \leq 4B(4, q)^2 = 70$ states. The states corresponding to the same degree of oscillator operator polynomial and therefore having fixed $q + \overline{q} = B + \overline{B}$ have same masses. For $q - \overline{q} = 0$ bosonic state having $q = \overline{q} = 0$ with fixed k one has $q + \overline{q} = 4 + k$ so that one has $N(k) = B(4, k)^2$ ($N(k)$ states with same mass even after p-adic massivation). The numbers $N(k)$ are $(1, 4^2 = 16, 6^2 = 36, 4^2 = 16, 1)$.

2. The number of $q\overline{q}$ type states in super-Kähler action is 16. If one considers super-symmetrization of the bosonic action, these states would correspond to bosons. Could these states allow an interpretation in terms of the known gauge bosons and Higgs? Weak bosons correspond to 4 helicity doublets giving 8 states. Higgs doublet corresponds to doublet and its conjugate. There is also a pseudo-scalar doublet and its conjugate.

   Gluon cannot belong to this set of states, which actually conforms with the fact that gluon corresponds to CP_2 isometries rather than holonomies and gluon corresponds to CP_2 partial wave since color is not spin-like quantum number. Known particle would give 8+2+2=12 states and pseudo-scalar doublets the remaining 4. This kind of pseudo-scalar states are predicted both as local and the bi-local states. As already explained, one can however also understand gluons in this picture as octet color partial waves. Also color octet variants of $SU(2)_w$ weak bosons are predicted.

3. There are actually some indications for a Higgs like state with mass 96 GeV (see `http://tinyurl.com/yxnmy8c7`) . Could this be the pseudo-scalar state. Higgs mass 125 GeV is very nearly the minimal mass for $k = 89$. The minimal mass for $k = 90$ would be 88 GeV so that the interpretation as pseudo-scalar with $k = 90$ might make sense. The proposal that gluons could have also weak counterparts suggests that also the pseudo-scalar could have this kind of counterpart. The scaling of the mass of the Higgs like state with $k = 90$ to $k = 112$ ($k = 113$ corresponds to nuclear p-adic scale) would give mass $m(107) = 37.5$ MeV. Kh.U.

Abraamyan et al have found evidence for pion like boson with mass 38 MeV [C1, C2, C3] (see `http://tinyurl.com/y7zer8dw`).

4. For Option b) only monomials with $N(q)-N(\overline{q}) = k = 1$ are allowed in q_s and leptons would be local 3-quark states and currents formed from them would appear in super-Kähler action. One would obtain $N(k=1) = \sum 0 \leq q \leq 3B(4,q+1) \times B(4,q) = 56$ statesi quark multiplet. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

Option b) involving only quarks as fundamental fermions does not predict unobserved gauge bosons whereas Option a) involving both leptons and quarks as fundamental fermions does so.

1. For Option a) taking into account quarks and restricting to electro-weak bosonic states to those with $(B = L = 0)$ leads to a doubling of bosonic states at $k = 2$ level. The couplings of gauge bosons require that the states are superpositions of quark and lepton pairs with coefficients proportional to the coupling parameters. There are two orthogonal superpositions of quark and lepton pairs having orthogonal charge matrices with inner product defined by trace for the product. Ordinary gauge bosons correspond to the first combination.

   The orthogonality of charge matrices gives a condition on them. The charged matrices having vanishing trace can be chosen that they have opposite signs for opposite H-chiralities. For charge matrices involving unit matrix one must have charge matrices proportional to (-3,1) for (L,q) one must have (1,3) for second state. For gluons there is no condition if one treats color octet as Lie algebra generator with vanishing trace. The problem is that there is no experimental evidence for these bosons.

2. For Option b) leptons would be local 3-quark states and spartners of quarks. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

3. Option b) predicts that given quark with given isospin and M^4 helicity L or R), say u_L, has 5 spartners with same quantum numbers given by $u_L u_R \overline{u}_L$, $u_L d_R \overline{d}_L$, $u_L d_L \overline{d}_R$; $u_R d_L \overline{d}_L$; and $d_L d_R \overline{u}_L$. These 6 states cannot correspond to quark families and SUSY breaking due to the possibility of having different p-adic scale (ramified prime) making the mass scale of the spartners large is suggestive.

There would be two phases of matter corresponding to local and bi-local states (baryons would be 3-local states).

1. For both phases electro-weak bosons and also gluons with electro-weak charge matrix 1 to bosonic super action as states involving only single partonic 2-surface. As already mentioned, also color counterparts of $SU(2)_w$ bosons are possible. Also graviton could correspond to spartner for bosonic super-action. This would give essentially the original model for family replication. 2-surfaces would be homologically trivial in this phase analogous to Coulomb phase.

2. In the dual phase the bi-local states would correspond to non-vanishing homology charges for quarks at least. In this phase one should assign also to leptons 2 wormhole contacts. In super-conducting phase it could the second electron of Cooper pair. Massive photons in this phase would consist of homologically charged fermion pairs. Lepton could also involve screening lepton-neutrino pair at second wormhole contact.

The universality of gauge boson couplings provides a test for the model.

1. In bi-local model gauge bosons would correspond to representations of a dynamical symmetry group $SU(3)_g$ associated with the 3 genera [K1]. Bosons would correspond to octet and singlet representations and one expects that the 3 color neutral states are light. This would give 3 gauge boson generations. Only the couplings of the singlet representation of $SU(3)_g$ would be universal and higher generations would break universality both for both gluons and

electro-weak bosons. There is evidence the breaking of universality as also for second and third generation of some weak bosons and the mass scales assigned with Mersenne primes above M^{89} are correct [K3].

2. If also fermions correspond to closed flux tubes with 2 wormhole contacts, the fermion boson couplings would correspond to the gluing of two closed flux tube strings along their both "ends" defined by wormhole contacts. A pair of 3-vertices for Feynman diagrams would be in question. If fermions are associated with single wormhole contact, its is not so easy to imagine how the closed bosonic flux tube could transform to single wormhole contact in the process. The wormhole contacts that meet and have opposite fermion numbers should disappear. This is allowed in the scenario involving 6-branes if the magnetic flux is trivial as it must be. For quarks and gluons the homology charges must be opposite if wormhole contact is to disappear.

3. If gauge bosons correspond to local fermion pairs, the most natural boson states have fixed value of g apart from topological mixing giving rise to CKM mixing just like fermions and universality is not natural. One can of course assume topological mixing guaranteeing it. Ordinary gauge bosons should be totally de-localized in the space of 3 lowest genera [K1] (analogous to constant plane waves) in order to have universality. The vertices could be understood as a fusion of partonic 2-surfaces. One should however understand why the mixing is so different for fermions and bosons. SUSY would suggest identical mixings.

The simplest model corresponds to quarks as fundamental fermions. Leptons and various bosons would be local composites in perturbative phase. In monopole dominate phase hadronic quarks would have homology charges and gluons would be pairs of quark and anti-quark at opposite throats of closed monopole flux tube. Basically particle reaction vertices would correspond to gluing of 3-surfaces along partonic 2-surfaces at 3-spheres defining $t = r_n$ hyperplanes of M^4.

4 Appendix: Still about the topology of elementary particles and hadrons

In its recent form TGD allows several options for the model of elementary particles [L11]. I wrote this piece of text because I got worried about details of the definition of wormhole contact appearing as basic building brick of elementary particle.

1. Wormhole contacts in 4-D sense (having Euclidian signature of induced metric) modellable as deformed pieces of CP_2 type extremals connecting Minkowskian space-time sheets (representable as graphs of a map $M^4 \rightarrow CP_2$) are identified basic building bricks of elementary particles. 3-D light-like orbits of 2-D wormhole throats- partonic 2-surfaces - at which the signature of induced metric changes from Euclidian to Minkowskian - partonic orbits - are assumed to be carriers of elementary particle quantum numbers localized at points representing intersections of fermionics string world sheets with the partonic 2-surfaces.

2. One can identify simplest wormhole contact as topological sum: two surfaces touch each other. Remove 3-D regions from both space-time sheets and connecting the topologically identical boundaries with a cylinder $X^2 \times D^1$, where X^2 has the topology of the boundary characterized by genus. The assumption that X^2 is boundary requires that its projection to CP_2 is homologically trivial.

   This is not consistent with the assumption that the flux tube carries monopole flux. These wormhole contacts are unstable and must be distinguished from wormhole contacts mediating monopole flux. I have not however defined the notion precisely enough.

3. One can consider two situations in which homologically non-trivial wormhole contact appears.

   Option I: Assume that the 3-D time=constant sections of two Minkowskian space-time sheets are glued together along their boundaries to form a closed 2-sheeted surface and the throats of wormhole contact - partonic 2-surfaces - serve as magnetic charges creating opposite fluxes. One can say that the two throats have opposite homology charges and therefore form a homologically trivial 2-surface to which one can glue the wormhole contact along its boundaries. The flux at sheet B could be seen as return flux from sheet A and the throat could be seen as very short monopole flux tube.

Option II: Assume no gluing along boundaries for the 3-D time=constant sections of two Minkowskian space-time sheets. In this case one must assume at least two wormhole contacts to get vanishing homology charges at both sheets. At both space-time sheets the throats of the contacts with opposite homology charges would be connected by monopole fluxes flowing through the wormhole contacts identifiable as a very short monopole flux tube. This makes sese also for the Option I and might be required since is not clear whether space-time having boundaries carrying monopole flux can be glued together.

Remark: One can also consider the light-like orbit of partonic 2-surface connecting its ends (the minimal distance between partonic 2-surfaces vanishes). The homology charges of ends are opposite in ZEO.

The proper identification of the model of elementary particles remains still open [L11] [K3]. What relevance do these two options this picture have to the model of elementary particles?

1. For Option I leptons and gauge bosons could be identified as single wormhole contact carrying non-trivial homology flux. The size scale of the closed space-time sheet would correspond to the Compton wavelength of the particle. This model is the simplest one at the level of scattering diagrams and was re-considered in [L11].

   Even Euclidian regions of single space-time sheet with vanishing homology charge can be considered as a model for leptons and gauge bosons. In this case it is however not clear how to understand how the size scale of the particle as Compton length could be understood at space-time level. This model was one of the first models. I have also considered the identification of the particle as boundary component of Minkowskian space-time surface.

2. Option II was assumed in the model following the original model for leptons and gauge bosons. It was also proposed that electroweak confinement as dual description of massivation takes place in the sense that the weak charges associated with the two wormhole contacts cancel each other. The size scale of flux tube at given sheet would correspond to the Compton length assignable to the particle. In this case scattering amplitudes are more complex topologically.

What about baryons?

1. The simplest model assumes that quarks do not differ from leptons and gauge bosons in any manner. The contribution of the quarks to masses of hadrons is very small fraction of total mass, which suggests that color flux tubes carrying also homology charge are present and give the dominating contribution.

   One can also consider a structure formed by color magnetic monopole flux tubes carrying most of the hadron mass with Minkowskian signature carrying flux of 2 units branching to two flux tubes carrying 1 unit each. The flux tubes would have length given by hadronic p-adic length scale. The ends of flux tubes would be wormhole throats connected by wormhole contacts to the mirror image of this structure. One can say that homology charges 2,-1,-1 assignable to the throats of single space-time sheet sum up to zero. This brings in mind color hypercharge. Could color confinement have vanishing of homology charge as classical space-time correlate?

2. In this article I have considered two alternative identification of leptons. Leptons and quarks could correspond to the different chiralities of $M^4 \times CP_2$ spinors and lepton and baryon numbers would be separately conserved. For second option leptons would b local 3-quark composites and therefore analogous to spartners of quarks: this option is possible only in TGD framework and the reason is that color is not spin-like quantum number in TGD framework. Baryon and lepton numbers would not be separately conserved.

   One can ask what could be the simplest mechanism inducing the decay of baryon as 3-quark composite involving only 3 wormhole contacts and giving lepton as a local 3-quark composite plus something. Wormhole throats of 3 quarks carrying the quark quantum numbers should fuse together to form a leptonic wormhole throat, and the 3 quark lines representing boundaries of string world sheets should fuse to single line. If the sum of quark homology charges is vanishing, lepton must have a vanishing homology charge unless the reaction involves also a step taking care of the conservation of homology charge as a decay of the resulting wormhole contact with vanishing monopole flux to two wormhole contacts with opposite monopole fluxes. Already the first step of the decay process is quite complex, and one can hope that the rate for the reaction is slow enough.

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