

# p-Adic Physics: Physical Ideas

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February 2, 2024

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### Abstract

The most important p-adic concepts and ideas are p-adic fractality, 4-D spin glass analogy, p-adic length scale hypothesis, p-adic realization of the Slaving Principle, p-adic criticality, and the non-determinism of the p-adic differential equations justifying the interpretation of the p-adic space-time regions as cognitive representations. These ideas are discussed in this chapter in a more concrete level than in previous chapters in the hope that this might help the reader to assimilate the material more easily. Some of the considerations might be a little bit out of date since the chapter is written much earlier than the preceding chapters.

1. 2-D thermodynamical criticality is accompanied by conformal invariance. The proposed quantum criticality of quantum TGD motivated the attempt to generalize conformal invariance to the 4-dimensional context providing a motivation of the p-adic approach. After almost two decades after the emergence of the idea about extended conformal invariance the view about conformal invariance is much more detailed and is indeed associated with quantum criticality, which reflects the non-determinism of Kähler action.
2. In TGD as a generalized number theory approach p-adic space-time regions emerge completely naturally and have interpretation as cognitive representations of the real physics. If this occurs already at the level of elementary particles, one can understand p-adic physics as a model for a cognitive model about physics provided by Nature itself. The basic motivation for this assumption is the p-adic non-determinism of the p-adic field equations making them ideal for the simulation purposes. The p-adic–real phase transitions are the second basic concept allowing to understand how intention is transformed to action and vice versa: the occurrence of this process even at elementary particle level explains why p-adic length scale hypothesis works. This picture is consistent with the idea about evolution occurring already at the level of elementary particles and allowing the survival of the systems with largest cognitive resources.
3. Spin glass analogy, which was the original motivation for p-adicization before the discovery that p-adic regions of space-time emerge automatically from TGD as a generalized number theory approach, is discussed at WCW level. The basic idea is that the maximum (several of them are possible) of the exponential of the Kähler function with respect to the fiber degrees of freedom as function of zero modes is p-adic fractal. This together with spin glass analogy suggest p-adic ultra-metricity of the reduced WCW  $CH_{red}$ , the TGD counterpart of the energy landscape.
4. Slaving Principle states that there exists a hierarchy of dynamics with increasing characteristic length (time) scales and the dynamical variables of a given length scale obey dynamics, where the dynamical variables of the longer length (time) scale serve as “masters” that is effectively as external parameters or integration constants. The dynamics of the “slave” corresponds to a rapid adaptation to the conditions posed by the “master”. p-Adic length scale hypothesis allows a concrete quantification of this principle predicting a hierarchy of preferred length, time, energy and frequency scales.
5. Critical systems are fractals and the natural guess is that p-adic topology serves also as an effective topology of real space-time sheets in some length scale range and that real non-determinism of Kähler action mimics p-adic non-determinism for some value of prime  $p$ . This motivates some qualitative p-adic ideas about criticality.
6. The properties of the  $CP_2$  type extremals providing TGD based model for elementary particles and topological sum contacts, are discussed in detail.  $CP_2$  type extremals could be for TGD what black holes are for General Relativity. Black hole elementary particle analogy is discussed in detail and the generalization of the Hawking-Bekenstein formula is shown to lead to a prediction for the radius of the elementary particle horizon and to a justification for the p-adic length scale hypothesis. A deeper justification for the p-adic length scale hypothesis comes from the assumption that systems with maximal cognitive resources are winners in the fight for survival even in elementary particle length scales.
7. Quantum criticality in its simplest variants states that states Kähler coupling strength  $\alpha_K$  is analogous to critical temperature. In principle allows the dependence of the  $\alpha_K$  on on zero modes. It would be nice if  $\alpha_K$  were RG invariant in strong sense but the expression for gravitational coupling constant implies that it increases rapidly as a function of p-adic length scale in this case. This led to the hypothesis that  $G$  is RG invariant. The hypothesis fixes the p-adic evolution of  $\alpha_K$  completely and implies logarithmic dependence of  $\alpha_K$  on p-adic length scale. It has however turned out that the RG invariance might after all be possible and is actually strongly favored by different

physical arguments. The point is that  $M_{127}$  is the largest Mersenne prime for which p-adic length scale is non-super-astronomical. If gravitational interaction is mediated by space-time sheets labelled by this Mersenne prime, gravitational constant is effective RG invariant even if  $\alpha_K$  is RG invariant in strong sense. This option is also ideal concerning the p-adicization of the theory.

## 1 Introduction

p-Adic topologies form an infinite hierarchy and p-adic physics leads to a vision about many-sheeted space-time as a hierarchical structure consisting of p-adic and real space-time sheets of increasing size and increasing value of prime  $p$ . These surfaces are glued together using topological sum or join along boundaries bonds. Contrary to the original expectations, p-adic space-time regions represent “mind-stuff” rather than “matter” which is also present and represented by real and infinite-p p-adic regions. Thus p-adic provide “cognitive representations” for matter like regions and this is why their physics provides a way to understand real physics. If p-adic-to-real phase transitions are possible, one can understand why it is possible to assign p-adic prime even to real regions. In fact, the hypothesis that p-adic regions provide a cognitive model for real physics, poses very strong constraints on real physics.

There is a “holy trinity” of non-determinisms in TGD in the sense that there is the non-determinism associated with the quantum jumps, the classical non-determinism of the Kähler action and p-adic non-determinism. The non-determinism of quantum jumps can involve also a selection between various multi-furcations for various absolute minima of the Kähler action in which case it represents a genuine volitional act. p-Adic non-determinism in turn corresponds to the non-determinism of pure imagination with no material consequences. Also real space-time sheets with finite time duration are also possible and they might represent what might be called “sensory space-time sheets” as opposed to cognitive space-time sheets. Cognitive space-time sheets can be transformed to real ones in quantum jumps inducing change of control parameters of the polynomial defining space-time surface: if the change is such that the p-adic root is replaced with a real root, one can say that thought is transformed into action. The reverse of this process is the transformation of sensory input into cognition.

“Holy trinity” implies that it should be possible to determine the p-adic prime characterizing a given space-time region (or space-time sheet) by observing a large number of quantum time developments of this system. The characteristic p-adic fractality, that is the presence of time scales  $T(p, k) = p^k T_p$ , should become manifest in the statistical properties of the cognitive time developments which in should turn reflect the properties of the real physics since cognitive representations are in question. For instance, quantum jumps with especially large amplitude would tend to occur at time scales  $T(p, k) = p^k T_p$ .  $T(p, k)$  could also provide series of characteristic correlation times. Needless to say, this prediction means definite departure from the non-determinism of ordinary quantum mechanics and only at the limit of infinite  $p$  the predictions should be identical. An interesting possibility is that  $1/f$  noise [D1] is a direct manifestation of the classical non-determinism: if this is the case, it should be possible to associate a definite value of  $p$  to  $1/f$  noise. Also transformations of the p-adic cognitive space-time sheets to real space-time sheets of a finite time duration and vice versa might be involved with the  $1/f$  noise so that  $1/f$  noise would be a direct signature of cognitive consciousness.

The “physical” building blocks of p-adic TGD, as opposed to the philosophical mathematical ones briefly summarized above, and in more detail in previous chapters, are spin glass analogy leading to the general picture about how finite-p p-adicity emerges from quantum TGD, the identification of elementary particles as  $CP_2$  type extremals, and elementary particle black hole analogy. These building blocks have been present as stable pieces of theory from beginning whereas philosophical ideas and interpretations have undergone rather wild fluctuations during an almost last decade of p-adic TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at <http://tgdtheory.fi/tgdglossary.pdf> [L2].

## 2 P-Adic Numbers And Spin Glass Analogy

Spin glass phase decomposes into regions in which the direction of the magnetization varies randomly with respect to spatial coordinates but remains constant in time. What makes spin glass special is that the boundary regions between regions of different magnetization do not give rise to large surface energies. Spin glass structure emerges in two ways in TGD framework.

1. Spin glass behavior at the level of real physics is encountered in TGD framework because of the classical non-determinism of the Kähler action. The classical non-determinism of  $CP_2$  type extremals represents the manifestation of the spin glass analogy at the level of elementary particle physics. In macroscopic length scales real physics spin glass analogy makes possible “real world engineering”.
2. Spin glass behavior at the level of cognition is encountered because of the p-adic non-determinism and makes possible what might be called imagination or “cognitive engineering”. The point is that any piecewise constant function has a vanishing p-adic derivative. Therefore any function of the spatial coordinates depending on a finite number of the binary digits is a pseudo constant. The discontinuities of this kind in the field variables do not lead to infinite surface energies in the p-adic context as they would in the real context.

Spin glass energy landscape is characterized by an ultra-metric distance function. The reduced WCW  $CH_{red}$  consisting of the maxima of the Kähler function with respect to quantum fluctuating degrees of freedom and zero modes defines the TGD counterpart of the spin glass energy landscape. This notion makes sense only in real context since p-adic space-time regions do not contribute to the Kähler function and all p-adic configurations are equally probable. The original vision was that if the ultra-metric distance function in  $CH_{red}$  is induced from a p-adic norm, a connection between p-adic physics and real physics also at the level of space-time might emerge somehow. It seems however that the ultra-metricity of  $CH_{red}$  need not directly relate to the p-adicity at the space-time level which can be understood if p-adic space-time regions give rise to cognitive representations of the real regions.

Of course, it *might* be that the p-adic prime characterizing cognitive representation of a real region characterizes also the reduced WCW associated with the region in question (one must of course assume that the reduced WCW approximately decomposes into a Cartesian product of the reduced WCW s associated with real regions).

### 2.1 General View About How P-Adicity Emerges

In TGD classical theory is exact part of the quantum theory and in a well defined sense appears already at the level of the configuration space geometry: the definition of WCW Kähler metric [K9] associates a unique space-time surface to a given 3-surface. The vacuum functional of the theory (exponent of the Kähler function) is analogous to the exponent  $exp(H/T_c)$  appearing in the definition of the partition function of a critical system so that the Universe described by TGD is quantum critical system. Critical system is characterized by the presence of two phases, which can be present in arbitrary large volumes. The TGD counterpart of this seems to be the presence of two kinds of 3-surfaces for which either Kähler electric or Kähler magnetic field energy dominates. These 3-surfaces have outer boundaries for purely topological reasons and these boundaries can be of a macroscopic size. Therefore it seems that 3-space should be regarded as what could be called topological condensate with a hierarchical, fractal like structure: there are 3-surfaces (with boundaries) condensed on 3-surfaces condensed on.....

This leads to a radically new manner to see the world around us. The outer surfaces of the macroscopic bodies correspond to the boundaries of 3-surfaces in the condensate so that one can see the 3-topology in all its complexity just by opening one’s eyes! A rather compelling evidence for the basic ideas of TGD if one is willing to give up the nebulous concept of “material object in topologically trivial 3-space” and to allow nontrivial 3-topology in macroscopic length scales. A second rather radical departure from the conventional picture of the 3-space is that 3-space is not connected in TGD Universe but contains arbitrary many disjoint components. In fact the actual Universe should consist of infinitely many 3-surfaces condensed on each other.

In two-dimensional critical systems conformal transformations act as symmetries and conformal invariance implies the Universality of critical systems. This suggests that one should try to find the generalization of the conformal invariance to higher dimensional, in particular, 4-dimensional case. It finally turned out that quaternion-conformal invariance realizes quantum criticality four 4-surfaces imbedded to 8-dimensional space. As a by product an explanation for space-time and embedding space dimensions results.

In this approach the p-adic regions of the space-time surface result dynamically. Space-time surface is defined by the vanishing condition of a polynomial of two quaternion-valued variables  $q$  and  $p$ . This condition gives  $p$  as a function of  $q$ . It can however occur that some components of  $p$  become complex numbers. They must be however real so that the solution fails to exist in the real sense. It might be however possible to perform the completion of the rational space-time surface to a p-adic space-time surface and for some values of the p-adic prime the series defining the power series representing  $p = f(q)$  might converge to a number in some algebraic extension of the ordinary p-adic numbers. Even more general rational-adic topologies in which norm is power of a rational number are possible. p-Adic numbers would thus be very closely related with quaternion-conformal invariance and criticality.

p-Adic topologies form an infinite hierarchy and p-adic physics leads to a vision about many-sheeted space-time as a hierarchical structure consisting of p-adic 4-surfaces of increasing size and increasing value of prime  $p$ . These surfaces are glued together using topological sum operation. Contrary to the original expectations, this hierarchy is the hierarchy for the regions of space-time representing “mind-stuff” rather than “matter” which is also present and represented by real and infinite-p p-adic regions. p-Adic provide “cognitive representations” for matter-like regions and this is why their physics provides a way to understand real physics.

## 2.2 P-Adic Numbers And The Analogy Of TGD With Spin-Glass

The vacuum degeneracy of the Kähler action leads to precise spin glass analogy at the level of the WCW geometry and the generalization of the energy landscape concept to TGD context leads to the hypothesis about how p-adicity is realized at the level of the WCW . Also the concept of p-adic space-time surface emerges rather naturally.

### 2.2.1 Spin glass briefly

The basic characteristic of the spin glass phase [B1] is that the direction of the magnetization varies spatially, being constant inside a given spatial region, but does not depend on time. In the real context this usually leads to large surface energies on the surfaces at which the magnetization direction changes. Regions with different direction of magnetization clearly correspond non-vacuum regions separated by almost vacuum regions. Amusingly, if 3-space is effectively p-adic and if magnetization direction is p-adic pseudo constant, no surface energies are generated so that p-adics might be useful even in the context of the ordinary spin glasses.

Spin glass phase allows a great number of different ground states minimizing the free energy. For the ordinary spin glass, the partition function is the average over a probability distribution of the coupling constants for the partition function with Hamiltonian depending on the coupling constants. Free energy as a function of the coupling constants defines “energy landscape” and the set of free energy minima can be endowed with an ultra-metric distance function using a standard construction [A9].

### 2.2.2 Vacuum degeneracy of Kähler action

The Kähler action defining WCW geometry allows enormous vacuum degeneracy: any four-surface for which the induced Kähler form vanishes, is an extremal of the Kähler action. Induced Kähler form vanishes if the  $CP_2$  projection of the space-time surface is Lagrange manifold of  $CP_2$ : these manifolds are at most two-dimensional and any canonical transformation of  $CP_2$  creates a new Lagrange manifold. An explicit representation for Lagrange manifolds is obtained using some canonical coordinates  $P_i, Q_i$  for  $CP_2$ : by assuming

$$P_i = \partial_i f(Q_1, Q_2) ,$$

where  $f$  arbitrary function of its arguments. One obtains a 2-dimensional sub-manifold of  $CP_2$  for which the induced Kähler form proportional to  $dP_i \wedge dQ^i$  vanishes. The roles of  $P_i$  and  $Q_i$  can obviously be interchanged. A familiar example of Lagrange manifolds are  $p_i = \text{constant}$  surfaces of the ordinary  $(p_i, q_i)$  phase space.

Since vacuum degeneracy is removed only by classical gravitational interaction there are good reasons to expect large ground state degeneracy, when system corresponds to a small deformation of a vacuum extremal. This degeneracy is very much analogous to the ground state degeneracy of spin glass.

### 2.2.3 Vacuum degeneracy of the Kähler action and physical spin glass analogy

Quite generally, the dynamical reason for the physical spin glass degeneracy is the fact that Kähler action has a huge vacuum degeneracy. Any 4-surface with  $CP_2$  projection, which is a Legendre sub-manifold (generically two-dimensional), is vacuum extremal. This implies that space-time decomposes into non-vacuum regions characterized by non-vanishing Kähler magnetic and electric fields such that the (presumably thin) regions between the non-vacuum regions are vacuum extremals. Therefore no surface energies are generated. Also the fact that various charges and momentum and energy can flow to larger space-time sheets via wormholes is an important factor making possible strong field gradients without introducing large surfaces energies. From a preferred extremal of Kähler action one obtains a new one by adding arbitrary space-time surface which is vacuum extremal. Uniqueness of the absolute minima in the sense that real regions of space-time  $X^4(X^3)$  are unique could be achieved by requiring that vacuum regions are p-adic and represent thus cognitive regions whereas real regions carry non-vanishing induced Kähler field.

The symplectic invariance of the Kähler action for vacuum extremals allows a further understanding of the vacuum degeneracy. The presence of the classical gravitational interaction spoils the canonical group  $Can(CP_2)$  as gauge symmetries of the action and transforms it to the isometry group of  $CH$ . As a consequence, the  $U(1)$  gauge degeneracy is transformed to a spin glass type degeneracy and several, perhaps even infinite number of maxima of Kähler function for given values of the zero modes, become possible. Thus locally, the space maxima of Kähler function should look like a union of copies of the space of zero modes. Given sheet has naturally as its boundary the 3-surfaces for which two maxima of the Kähler function coalesce or are created from single maximum by a cusp catastrophe. In catastrophe regions there are several sheets and the value of the maximum Kähler function determines which give a measure for the importance of various sheets. The quantum jumps selecting one of these sheets can be regarded as phase transitions.

In TGD framework classical non-determinism forces to generalize the notion of the 3-surface by replacing it with a sequence of space like 3-surfaces having time like separations such that the sequence characterizes uniquely one branch of multi-furcation. This characterization works when non-determinism has discrete nature. For  $CP_2$  type extremals which are bosonic vacua, basic objects are essentially four-dimensional since  $M_+^4$  projection of  $CP_2$  type extremal is random light like curve. This effective four-dimensionality of the basic objects makes it possible to topologize Feynman diagrammatics of quantum field theories by replacing the lines of Feynman diagrams with  $CP_2$  type extremals.

In TGD framework spin glass analogy holds true also in the time direction, which reflects the fact that the vacuum extremals are non-deterministic. For instance, by gluing vacuum extremals with a finite space-time extension (also in time direction!) to a non-vacuum extremal and deforming slightly, one obtains good candidates for the degenerate absolute minima. This non-determinism is expected to make the absolute minima of the Kähler action highly degenerate. The construction of S-matrix at the high energy limit suggests that since a localization selecting one degenerate maximum occurs, one must accept as a fact that each choice of the parameters corresponds to a particular S-matrix and one must average over these choices to get scattering rates. This averaging for scattering rates corresponds to the averaging over the thermodynamical partition functions for spin glass. A more general is that one allows final state wave functions to depend on the zero modes which affect S-matrix elements: in the limit that wave functions are completely localized, one ends up with the simpler scenario.

The real effective action is expected to be Einstein-Yang-Mills action for the induced gauge fields. This action does not possess any vacuum degeneracy. The space-time surfaces are certainly absolute minima of the Kähler action and EYM-action could take a dynamical role only in the

sense that extremality with respect to classical part of EYM action selects one of the degenerate absolute minima of the Kähler action. On the other hand, the construction of S-matrix suggests that the choice of particular parameter values characterizing zero modes affects only the coupling constants and propagators of the effective Einstein-Yang-Mills theory, and that one must perform averaging over the predictions of these theories. Thus EYM action could at most fix a gauge.

#### 2.2.4 p-Adic non-determinism and spin glass analogy

One must carefully distinguish between cognitive and physical spin-glass analogy. Cognitive spin-glass analogy is due to the p-adic non-determinism. p-Adic pseudo constants induce a non-determinism which essentially means that p-adic extrema depend on the p-adic pseudo constants which depend on a finite number of positive binary digits of their arguments only. Thus p-adic extremals are glued from pieces for which the values of the integration constants are genuine constants. Obviously, an optimal cognitive representation is achieved if pseudo constants reduce to ordinary constants.

More precisely, any function

$$\begin{aligned} f(x) &= f(x_N) , \\ x_N &= \sum_{k \leq N} x_k p^k , \end{aligned} \quad (2.1)$$

which does not depend on the binary digits  $x_n$ ,  $n > N$  has a vanishing p-adic derivative and is thus a pseudo constant. These functions are piecewise constant below some length scale, which in principle can be arbitrary small but finite. The result means that the constants appearing in the solutions the p-adic field equations are constants functions only below some length scale. For instance, for linear differential equations integration constants are arbitrary pseudo constants. In particular, the p-adic counterparts of the absolute minima (defined by the correspondence with infinite primes) are highly degenerate because of the presence of the pseudo constants. This in turn means a characteristic randomness of the spin glass also in the time direction since the surfaces at which the pseudo constants change their values do not give rise to infinite surface energy densities as they would do in the real context.

The basic character of cognition would be spin glass like nature making possible “engineering” at the level of thoughts (planning) whereas classical non-determinism of the Kähler action would make possible “engineering” at the level of the real world.

#### 2.2.5 Localization in zero modes

The Kähler function defining WCW metric possesses infinite number of zero modes which represent non-quantum-fluctuating degrees of freedom. The requirement that physics is local at the level of zero modes implies that each quantum jump involves a localization in zero modes. This localization could be complete or in a region whose size is determined by the p-adic length scale hypothesis.

Localization would mean an enormous calculational simplification: functional integral reduces into ordinary functional integral over the quantum-fluctuating degrees of freedom and there is no need to integrate over the zero modes. The complete or partial localization in zero modes would explain why the world of conscious experience looks classical. Perhaps the complete localization is however too much to wish for: it could however be that one must use wave functionals in the zero modes only in the case that one is interested in a comparison of the transition rates associated with different values of zero modes rather than in transition rates with the condition that a localization has occurred to definite values of zero modes.

The functional integral over the fiber degrees of freedom can be approximated by a Gaussian integrals around maxima. Classical non-determinism would suggest the possibility of several maxima in fiber degrees of freedom but the symmetric space property of the fiber suggests that there is only single maximum of Kähler function. The existence of single maximum gives good hopes that the configuration space integration reduces effectively to Gaussian integration of free field theory.



### 2.3 The Notion Of The Reduced WCW

Quantum jumps occur with highest probability to those values of zero modes which correspond to the maxima of the Kähler function and a simplified description of the situation is obtained by considering the reduced WCW  $CH_{red}$  consisting of the maxima of Kähler function with respect to both zero modes and quantum fluctuating degrees of freedom.

The hypothesis that the space  $CH_{red}$  is an enumerable set is a natural first guess. In macroscopic length scales, one might indeed hope that the generation of Kähler electric fields reducing the vacuum degeneracy could imply a discrete degeneracy for the maxima of the Kähler action.

In elementary particle length scales this hypothesis fails and it is good to analyze the situation in more detail since it gives some about how complex the situation can be. For the so called  $CP_2$  type extremals the classical non-determinism gives rise to a functional continuum of degenerate maxima of the Kähler function. The degenerate maxima correspond to random zitterbewegung orbits for which the “time parameter”  $u$  is an arbitrary function of  $CP_2$  coordinates. In this case however zero modes characterizing light like random curve representing the zitterbewegung orbit behave exactly like conformal gauge degrees of freedom. The choice of the “time parameter”  $u$  however affects S-matrix elements: dependence is very weak and only through the volumes of the propagator lines determined by the selection of  $u$  (Kähler action for  $CP_2$  type extremal is proportional to its volume) occurring in quantum jump. Effectively the functional continuum is replaced with the real continuum of the volume of the propagator line varying from zero to the volume of  $CP_2$ .

A localization for the positions of the vertices of the Feynman diagrams defined by  $CP_2$  type extremals cannot however be assumed. Neither can one assume that only single Feynman diagram is selected if one wants that a generalization of ordinary Feynman diagrammatics results. There are several alternative identifications.

1. The degrees represented by Feynman diagrams with varying positions of vertices represent fiber degrees of freedom so that there would be slight dependence of the Kähler function on the positions of the vertices. Certainly the Feynman diagrams with different topologies have different value of Kähler action and must correspond to fiber degrees of freedom. The reason is that vertex regions of the Feynman diagrams must involve deformations of  $CP_2$  extremals since otherwise Feynman diagrams are singular as 4-manifolds. Note that the idea about localization in fiber degrees of freedom is not favored by this example.
2. The positions for the vertices of the Feynman diagram are excellent candidates for zero modes and localization is not possible now. The fact that these degrees of freedom correspond to center of mass degrees of freedom related to the isometries of the theory might distinguish between them and other zero modes. One can consider also a refinement for localization in the zero modes hypothesis: localization occurs only in length scale resolution defined by the p-adic length scale. In fact, the assumption that  $CP_2$  type extremals have suffered topological condensation on space-time sheets with size of order p-adic length scale characterizing the elementary particle implies this.

Whether the notion of  $CH_{red}$  makes sense for the p-adic space-time regions is not at all obvious. For the proposed construction of the WCW metric p-adic regions do not contribute to the Kähler function which is real-valued. Only in case that the p-adic contribution is rational number, it could be interpreted as a real valued contribution to the Kähler function. In case of  $CP_2$  type extremals this is not the case although the exponent of the Kähler function for a full  $CP_2$  type extremal is a rational number if the proposed model for the p-adic evolution of Kähler coupling strength is correct. If it does not make sense to distinguish between the maxima of the Kähler function in the p-adic context, one cannot define  $CH_{red}$  on basis of this criterion. From the point of view of cognition this means maximal freedom of imagination.

An interesting question is whether one must count the cognitive degeneracy as a degeneracy of physical states. If localization occurs in each quantum jump with respect to both real and p-adic zero mode degeneracy, and if all cognitive options are equally probable, then the only conclusion seems to be that space-time surfaces for which the cognitive degeneracy is highest, represent the most probable final states. This would mean that the systems with the highest cognitive resources would be winners in the struggle for survival.

### 2.3.1 Explicit definition of the ultra-metric distance function for energy landscape

The points of  $CH_{red}$  are completely analogous to the minima of the free energy and the precise analogy with spin glass suggests that  $CH_{red}$  must possess naturally an ultra-metric topology. One can quite generally construct an explicit ultra-metric distance function for the set of energy minima in a given energy landscape describing energy as a function of the coordinates of some WCW using existing recipes [B5]. The concept is useful when the energy landscape has fractal like structure. An attractive metaphor is to regard energy as a height function for a landscape with mountains.

The distance function between two energy minima should describe the difficulty of getting from a given minimum to another one. A concrete measure for this difficulty is obtained by considering all possible paths from  $x$  to  $y$ . The height for the highest point on this path, absolute maximum  $h_{max}(\gamma)$  of the height function on this path gives the measure for the difficulty for reaching  $y$  along the path  $\gamma$ . There exists some easiest path from  $x$  to  $y$ . The difficulty to reach  $y$  from  $x$  can be defined as the height of the highest point associated with the easiest path and hence the minimum of  $h_{max}(\gamma)$  in the set of all possible paths from  $x$  to  $y$ :

$$d(x, y) = \text{Min}(h_{max}(\gamma(x, y))) .$$

It is easy check that this distance function is ultra-metric:

$$d(x, z) \leq \text{Max}\{d(x, y), d(y, z)\} .$$

All what is needed is to notice that for any path  $x \rightarrow z$  going through  $y$  highest point of the path is either the highest point associated with the path from  $x \rightarrow y$  or  $y \rightarrow z$ : from this the inequality follows trivially since one can in principle find also easier paths.

### 2.3.2 Identification of the height function in the case of the reduced WCW ?

Obviously the negative for the maximum of Kähler function as function of zero modes is the counterpart of free energy. This function could well be many valued but this is an unessential complication. It is not clear whether  $K$  is negative definite (there are strong reasons to believe that this is the case). One can however consider any positive definite function of  $K$  as a height function defining an ultra-metric norm in the manner suggested. The requirement that p-adic norm results should fix the definition uniquely.

The exponential  $\exp(-K_{max})$  of the maximum of Kähler function as function of the zero modes, which is the inverse for the vacuum functional of the theory, is the first guess for the height function defining the ultra-metric norm (the wandering from 3-surface  $X^3$  to  $Y^3$  corresponds to quantum tunnelling physically.). The justification for this identification is that the integration over the fiber degrees of freedom gives Gaussian determinant cancelling the metric determinant and leaves on the exponent of Kähler function to the functional integral over zero modes. The intuitive expectation is that ultra-metric norm is p-adic for some  $p$  and that the space of zero modes decomposes into regions  $D_p$ . In order to get a power of  $p$  as required by p-adicity, one can expand  $h$  as powers of  $p$  and identify p-adic norm as  $p^n$  for the highest binary digit  $n$  with non-vanishing coefficient.

The height function can have a normalization factor and this factor could be chosen so that the ultra-metric norm is a power of  $p$  for  $CP_2$  type extremals, which are certainly very important building blocks of preferred extremals. The argument relating the gravitational coupling constant to the Kähler coupling strength and fixing the dependence of the Kähler coupling strength on the prime  $p$ , suggests that one must define the height function as

$$h_p = \frac{\exp(-K(p))}{\exp(-K(p=1))} ,$$

where the Kähler function at  $p = 1$  is formally obtained by regarding the value of the Kähler coupling strength as a function in the set of all natural numbers.

### 2.3.3 Does the proposed height function $h_p$ define p-adic topology?

The great question is whether one can obtain p-adic ultra-metricity in this manner. There is some evidence for this.

1. Criticality and spin glass analogy suggests that  $\exp(K)$  as a function of zero modes is fractal. If it is p-adic fractal then p-adic topology is expected to be a natural consequence: in this case the map of  $CH_{red}$  to its p-adic counterpart could make it possible to replaced  $CH_{red}$  with a smooth function.
2.  $CP_2$  type extremals, the counterparts of black holes and a model of elementary particle in TGD, have finite negative Kähler action. One can glue  $CP_2$  type extremals to any space-time surface to lower the Kähler action. 3-surfaces  $Z^3$  on path from  $X^3$  to  $Y^3$  containing  $CP_2$  type extremals on  $X^4(Z^3)$  are excellent candidates for “mountains” in the landscape metaphor. The height of  $Z^3$  is roughly described by the number of  $CP_2$  type extremals glued on  $X^4(Z^3)$ .
3. The argument leading to a correct prediction of gravitational constant in terms of assuming that Kähler coupling strength  $\alpha_K$  depends on zero modes only through the p-adic prime assumed to characterize a given region  $D_p$  of WCW for which the set of maxima of Kähler function as function of zero modes should obey has p-adic topology. The crucial input is the relationship

$$\exp(K_p(CP_2)) \frac{R^2}{G} = \frac{1}{p} ,$$

which is equivalent with  $G = \exp(K_p(CP_2)) L_p^2$  , where  $L_p \simeq \sqrt{p} \times R$  is the p-adic length scale and  $R \simeq 10^4 \sqrt{G}$  is  $CP_2$  size and the fundamental p-adic length scale. This formula is a dimensional estimate for gravitational coupling strength in terms of the p-adic length scale squared and the exponential of Kähler function for  $CP_2$  type extremal describing graviton. The exponent gives the probability for the appearance of one virtual graviton in a given quantum state. The probability is very small since the exponent is negative for  $CP_2$  type extremal and gravitation is consequently a very weak interaction.

4. If one makes the identification

$$\frac{R^2}{G} (\sim 10^8) = \exp(-K_{p=1}),$$

then the function

$$h_p = \frac{\exp(-K_p)}{\exp(-K_{p=1})}$$

is the n: th power of  $p$  for a vacuum extremal to which  $n$   $CP_2$  type extremals are glued. This is just the p-adic norm  $p^n$ ! If  $h_p$  were  $p^n$ -valued in the general case it would be a p-adic pseudo constant and rather tame as a fractal. Very probably, this is not true in the general case and the p-adic norm of the p-adic counterpart of  $h_p$  in the canonical identification

$$N_p \equiv |Id(h_p)|_p , \\ Id(\sum x_n p^n) = \sum_n x_n p^{-n} .$$

depending on the most significant pinary digit of  $h_p$  only, is a good candidate for a p-adically ultra-metric height function having also a correct normalization. In any case, it seems that the number of virtual  $CP_2$  type extremals (gravitons!) glued to an preferred extremal  $X^4(X^3)$  could define the height function. p-Adicity would emerge naturally and would have a direct physical meaning. Of course, this identification works for  $n \geq 0$  only: the physical interpretation of the p-adic norm in  $n < 0$  case is open.

A possible interpretation in terms of virtual graviton emission suggests the interpretation of the factor  $\frac{R^2}{G} = \exp(-K_{p=1})$  as a Gaussian determinant  $\sqrt{\det_G}$  associated with the integration over the zero modes around the maximum. The definition of Gaussian determinant in the real context is problematic and p-adicization plus adelic decomposition of the functional integral might provide a precise definition of  $\sqrt{\det_G}$ . The divergence of the Gaussian determinant in the real context

would lead to the vanishing of the gravitational constant. This picture is in accordance with the assumption that gravitational constant does not appear in quantum TGD as a fundamental constant and that the curvature scalar term in the low energy effective action essentially results from radiative corrections and hence derives from the logarithm of  $\det_G$ .

### 3 P-Adic Numbers And Quantum Criticality

TGD Universe is quantum critical in the sense that the value of Kähler coupling constant is completely analogous to critical temperature. Therefore the obvious question is how p-adicity might relate to quantum criticality.

#### 3.1 Connection With Quantum Criticality

p-Adicization of the reduced WCW relates in an interesting manner to quantum criticality. At quantum criticality the number of the absolute minima of Kähler action for a surface  $Y^3$  belonging to light cone boundary measures the cognitive resources of this surface and of its diffeomorphs.  $N_d$  is assumed to behave as  $N_d \sim \exp(-K_{cr})$ , where Kähler function is evaluated for the critical value  $\alpha_{cr}$  of the Kähler coupling strength.  $\alpha_{cr}$  is like Hagedorn temperature appearing in the thermodynamics of strings. Above  $\alpha_{cr}$  the theory might not be mathematically well defined since (at least real) the sum over the WCW integrals associated with the maxima of Kähler function would diverge exponentially at the limit when the value of Kähler function increases. In string thermodynamics this corresponds to the growth of number  $g(E)$  of the states of given energy more rapidly than the inverse of the Boltzmann factor  $\exp(-E/T_H)$ . Below  $\alpha_{cr}$  the theory is certainly well defined but in TGD framework the cognitive resources of the Universe would not be maximal since vacuum functional would differ significantly from zero for very few space-time surfaces only. At quantum criticality the situation is optimal but it is not clear whether the real theory makes sense at quantum criticality: at least in string thermodynamics the partition function diverges also at Hagedorn temperature.

The cognitive resources of p-adic space-time sheet are measured by the entropy type quantity  $\log(N_d)/\log(2)$  having lower bound  $\log(p)/\log(2)$  bits for the 3-surfaces allowed by the vacuum functional. For instance, the maximal cognitive resources of electronic space-time sheet ( $M_{127} = 2^{127} - 1$ ) would be 127 bits. In TGD one must allow even infinite primes and for these cognitive resources can be literally infinite.

#### 3.2 Geometric Description Of The Critical Phenomena?

The idea that critical systems might have a geometric description is not new. There is a lot of evidence that simple, purely geometric lattice models based on the bond concept reproduce same critical exponents as the thermal models [B4]. The probability for a bond to exist corresponds to temperature in these models. For example, in a bond percolation model it is possible to relate the critical exponents to various fractal dimensions. This provides a nice manner to reduce the problem of predicting critical temperature to that of predicting the critical probability for the bond. This problem is local and once the temperature dependence of the bond probability and critical bond probability are known one can calculate the critical temperature.

What is new that in TGD approach the concept of bond ceases to be a phenomenological concept related to the simple modelling of the critical systems. TGD predicts that the boundaries of 3-surfaces can have arbitrarily large sizes. Furthermore, the formation of the join along boundaries bonds connecting the boundaries of two disjoint 3-surfaces seems to provide the basic mechanism for the formation of macroscopic quantum systems with long range correlations. This means that phase transitions should basically correspond to changes in the connectedness of the boundary of the 3-space. The description of the super fluidity, super conductivity and Quantum Hall effect based on the join along boundaries bond concept is suggested in [K10, K1] and also other phase transitions might be describable in the same manner. In hadronic length scale flux tubes correspond to color flux tubes connecting valence quarks. In nuclear length scale the short range part of the nuclear force corresponds to the formation of join along boundaries bonds between nucleons.

p-Adic approach suggests a concrete description for the phase transition changing the connectedness of the 3-surface. Disjoint 3-surfaces are labelled by p-adic numbers, whose p-adic expansion does not contain powers  $p^n$  with  $n > N$ , where  $N$  is some finite integer: the larger the value of  $N$  the larger the degree of disjointness. This means that phase transitions (say evaporation or condensation) changing the connectedness of the 3-surface should correspond to transitions changing the value of  $N$ . In evaporation process  $N$  increases and in condensation process  $N$  decreases. Also catastrophic processes like the breaking of a solid object to pieces might correspond to increase in  $N$ . Typical self organization processes such as biological growth and healing might correspond to a gradual decrease of  $N$ .

Fractal like configurations with a discrete scale invariance are known to play important role in the description of the critical phenomena: they are the most probable configurations at the critical point. The idea that fractal corresponds to a fixed point of a discrete scaling transformation, is in accordance with the definition of the fractals as fixed points for a set of affine transformations acting on subsets of some metric space [A7]. A natural candidate for the discrete scaling transformation is the transformation of the 4-surface induced by the multiplication of the p-adic argument  $Z$  of  $H$ -coordinate  $h(Z)$  by a power of  $p$ :  $Z \rightarrow p^n Z$ . A tempting idea is that most probable 3-spaces indeed are invariant under these scalings. This even suggests that something, which might be called “Mandelbrot cosmology”, might provide a description of the Universe in all length scales as a 4-dimensional analog of Mandelbrot set. The breaking of the discrete scaling invariance is bound to occur, when one considers finite subsystem instead of the whole Universe. p-Adic cutoff might provide an elegant description for the breaking of the exact scaling invariance: 3-surface in question depends on finite number of the pinary digits of  $Z$  only.

### 3.3 Initial Value Sensitivity And P-Adic Differentiability

Initial value sensitivity is one of the basic properties of the critical systems and implies unpredictability in practice. p-Adic differentiability seems to be related to this property in a very general manner. Consider a configuration of an initial value sensitive system, which can possess very high dimension. For definiteness, assume that the dynamics is described by some differential equations, which can be reduced to equations of first order for WCW coordinates  $X$  (we do not bother to write indices):

$$\frac{dX}{dt} = J(X) . \quad (3.1)$$

Space-time coordinate is a p-adic number one can assume that time coordinate is a p-adic number, too.

The purely p-adic feature of this differential equation follows from the fact that any function depending on a finite number of pinary digits of a p-adic number possesses a vanishing p-adic derivative! This implies that the integration constants are not just ordinary constants but functions of the p-adic number  $t$  depending on finite number of pinary digits of  $t$ ! Obviously this implies classical non-determinism in long time scales! One can construct solutions of the differential equation in the form  $X(t) = X_0(t) + X_1(t)$ , where  $X_0(t)$  depends on a finite number of pinary digits of the p-adic time  $t$  and equations reduce to

$$\frac{dX_1}{dt} = J(X_0 + X_1) . \quad (3.2)$$

Of course, one must be careful in defining what “finite number of pinary digits” means, when p-adic cutoff is actually present. The simplest integration constants depend on the p-adic norm of  $t$  (or on the lowest pinary digit of  $t$ ) only.

The result is in accordance with the so called Slaving Principle [B2]. One can think that the dynamics in long time scales (low pinary digits of p-adic number  $t$ ) is given by the integration constants having arbitrary dependence on these pinary digits and the dynamics in short length scales is determined by the differential equations in the “background” given by these time dependent integration constants.

Initial value sensitivity implies effectively non-deterministic behavior and p-adic numbers perhaps provide a possibility to describe it properly. The properties of the Kähler function suggests that the classical non-determinism might be in fact actual. The point is that the classical space time surface associated with a given 3-surface need not be unique. This surface is determined as a preferred extremal of the so called Kähler action and Kähler action possesses enormous vacuum degeneracy [K3]: the most general vacuum extremal has 2- dimensional  $CP_2$  projection, which is so called Lagrange manifold possessing a vanishing induced Kähler form. Symplectic transformations and  $Diff(M^4)$  act as exact dynamical symmetries of the vacuum extremals and  $Diff(M^4)$  contains p-adically analytic transformations of  $M^4$  as subgroup. It might well happen that those absolute minima, which are obtainable as small deformations of the vacuum extremals inherit the characteristic degeneracy of the vacuum extremals.

The classical macroscopic non-determinism might be essential to the possibility of the quantum measurements. In TGD the state function reduction is described as “jump between histories” that is two deterministic time developments [K12]. In quantum measurement microscopic and macroscopic system are strongly correlated and microscopic transition induces a phase transition like phenomenon in a macroscopic critical system. The general belief is that quantum effects become unimportant in macroscopic systems. The situation need not be this if macroscopic system is critical, or even non-deterministic.

In the TGD inspired theory of “thinking systems”, conscious thoughts correspond to quantum jumps selecting one of the possible time developments in the quantum superposition of several quantum average effective space-time times allowed by the non-determinism. p-Adic pseudo constants could provide a mathematical description for this non-determinism. These “cognitive” quantum jumps are certainly involved with a realistic description of a quantum measurement modelling also the presence of the observer quantum mechanically.

It turns out that quantum non-determinism, classical non-determinism of Kähler action and p-adic non-determinism are very closely related in quantum TGD: one could even speak of a holy trinity of non-determinisms. Quantum non-determinism corresponds closely to the classical non-determinism of Kähler action: quantum jumps select between various branches of the branches of multi-furcations of classical space-time surface. The p-adic counterparts of these branches are in turn obtained by varying pseudo constants in the solution of the p-adic Euler-Lagrange equations for the Kähler action: this requirement in fact makes it possible to assign unique p-adic prime to a given, sufficiently small space-time region.

### 3.4 There Are Very Many P-Adic Critical Orbits

An interesting connection between the p-adicity and initial value sensitive systems is related to the possibility to replace also the WCW (possibly infinite dimensional) with an algebraic extension of the p-adic numbers. The underlying motivation is the need to get a proper mathematical description of the finite accuracy for the observables and p-adic cutoff provides this description.

This in turn suggests Universality in some aspects of the dynamical behavior. The dynamical equations  $dX/dt = J(X)$  define a flow that is a diffeomorphism  $X \rightarrow F(X, t)$  of WCW. This flow contains as integration constants arbitrary functions of the p-adic time coordinate  $t$  depending on a finite number of binary digits of  $t$  so that classical non-determinism is present. By p-adic conformal invariance this diffeomorphism ought to be p-adically analytic map that is representable as a power series of the algebraically extended p-adic numbers  $x$  and  $t$ .

The p-adic analyticity of the dynamic diffeomorphism gives strong constraints on the properties of the dynamic map. A particularly interesting map is in this respect Poincare map. One can ask several interesting questions. How does the Universal behavior of one- dimensional and 2-dimensional analytic iterated maps generalize to the p-adic case? What do attractors look like? What are the counterparts of Julia set and Mandelbrot set? What about routes to chaos? Could p-adic hypothesis provide deeper explanation for the fact that period doubling seems to be a rather general mechanism for the transition to turbulence. It might be possible to answer these questions since p-adic analyticity is very strong constraint on the behavior of the maps.

Already the study of the simplest p-adic complex maps reveal some surprises. The simplest map to study is the map  $Z \rightarrow Z^n$  for any extension of p-adic numbers (dimension is arbitrary!). The repeller consists of the points p-adic norm equal to one. Due to the roughness of the p-adic topology, the real counterpart of the repeller is of same dimension as WCW itself so that the

critical orbits form a set with a non-vanishing measure! For example, in the 2-dimensional case and for the 2-adic extension, the set of the critical orbits corresponds in the real plane to a square  $(1/2, 1] \times (1/2, 1]$ .

How do the small deformations of  $Z \rightarrow Z^n$  of form  $Z \rightarrow Z^n + \epsilon Z^m$  affect the set of the critical orbits? If the norm of the parameter  $\epsilon$  is sufficiently small, the previous repeller belongs to the repeller also now. Also new points can appear in repeller. These considerations suggest that the repellers/attractors of the p-adically analytic maps have rather simple structure as compared to their real and complex counterparts. An interesting possibility is that in general case these sets are fractal like objects resembling the fractals associated with p-adic order parameters.

The fact that set of critical orbits is n-dimensional rather than  $(n - 1)$  or lower-dimensional in the p-adic case suggests an interesting physical interpretation in accordance with the general idea that p-adic topology corresponds to criticality. In ordinary situation these orbits are not very interesting because a small deformation spoils their criticality. In p-adic case the situation is different since the critical orbits are meta-stable and their are very many of them. In TGD one can even identify good candidates for the set of of these meta-stable critical orbits as small deformations of the vacuum extremals of the Kähler action. Needless to emphasize, this vacuum degeneracy is a phenomenon not encountered in the standard field theories.

## 4 P-Adic Slaving Principle And Elementary Particle Mass Scales

The understanding of the elementary particle mass scales is a fundamental problem in the unified field theories. The attempts to understand the generation of the mass scales dynamically have not been successful. The basic problem is the fine tuning difficulty: the predicted mass scale hierarchy is not stable under the small changes of the model parameters. A possible explanation for the failure is that the fundamental mass scales are really fundamental and therefore cannot depend on the details of the dynamical model.

Criticality is known to imply Universality and criticality indeed is the fundamental property of Kähler action. Therefore the derivation of the elementary particle length scale(s) should be based on a proper formulation of the criticality concept. p-Adic numbers indeed provide a promising tool in this respect and the following arguments show that it is possible not only to understand some general elementary particle length scale but leptonic, hadronic and intermediate gauge boson length scales plus a small number of shorter length scales in terms of primes near prime powers of two. The most important length scales correspond to Mersenne primes: there are only sixteen Mersenne primes below electron length scale and the remaining Mersenne primes correspond to super astronomical length scales.

What is nice that the p-adic hypothesis makes possible to express these length scales as square roots of Mersenne primes and possibly Fermat primes, that is prime numbers of type  $p = 2^m \pm 1$ . What is amusing is that Mersenne primes are closely related to the so called Perfect Numbers  $n = 2^{m-1}(2^m - 1)$  representable not only as a product of their prime factors but also as a sum of their proper divisors. The ancient number mystics believed that this property makes these numbers very exceptional in the World Order!

### 4.1 P-Adic Length Scale Hypothesis

p-Adic length scale hypothesis has served as a basic hypothesis of p-adic TGD for several years. This hypothesis states that the scales  $L_p = \sqrt{pl}$ ,  $l = 1.376 \cdot 10^4 \sqrt{G}$  are fundamental length scale at p-adic condensate level  $p$ . The original interpretation of the hypothesis was following:

1. Above the length scale  $L_p$  p-adicity sets on and effective course grained space-time topology is p-adic rather than ordinary real topology.
2. The length scale  $L_p$  serves as a p-adic length scale cutoff for the field theory description of particles. This means that space-time begins to look like Minkowski space so that quantum field theory  $M^4 \rightarrow CP_2$  becomes a realistic approximation. Below this length scale string like objects and other particle like 3-surfaces dominate.

3. It is un-natural to assume that just single p-adic field would be chosen from the infinite number of possibilities. Rather, there is an infinite number of cutoff length scales. To each prime  $p$  there corresponds a cutoff length scale  $L_p$  above which p-adic quantum field theory  $M^4 \rightarrow CP_2$  makes sense and one has a hierarchy of p-adic quantum field theories. These different p-adic field theories correspond to different hierarchically levels possibly present in the topological condensate. Hierarchical ordering  $p_1 < p_2 < \dots$  means that only the surface  $p_1 < p_2$  can condense on the surface  $p_2$ . The condensed surface can in practice be regarded as a point like particle at level  $p_2$  described by the p-adic conformal field theory below length scale  $L_{p_2}$ .

The work with p-adic QFT has however demonstrated that the hypothesis a) and b) are probably wrong and the following interpretation is closer to the truth.

1. The length scale  $L_p = \sqrt{pl}$  defines an *infrared* cutoff rather than ultraviolet cutoff for a p-adic quantum field theory formulated in terms of quarks and leptons and gauge bosons. For instance, for hadrons this length scale is of order hadron size and  $L_p$  defines UV cutoff for possibly existing field theory describing hadrons as basic objects. Above  $L_p$  real topology effectively replaces the p-adic one (real continuity implies p-adic continuity) and if length scale resolution  $L_p$  is used real physics is excellent approximation.
2. p-Adic QFT is free of UV divergences with any UV cutoff and there is no need to assume that p-adicity fails below some length scale. Rather, p-adicity is completely general property of the effective quantum average space-time defined by the Quantum TGD, which is based on the real number field. The concept of the effective space-time, or topological condensate, is in turn necessary for the formulation of field theory limit of TGD. The analogy of Quantum TGD with spin glass phase gives strong support for the p-adic topological condensate consisting of p-adic regions with different  $p$  glued together along their boundaries.

p-Adic topologies form a hierarchy of increasingly coarser topologies. The p-adic norm  $N(x_p)$  defines a function of a real argument via the canonical identification of the non-negative real numbers and p-adic numbers. The p-adic norm is same as ordinary real norm for  $x = p^k$  and is constant at each interval  $[p^k, p^{k+1})$ . This means that

1. p-adic topologies are coarser than real topologies so that the functions, which are continuous in the p-adic topology need not be continuous in the real topology.
2. p-adic topologies are ordered: the larger the value of  $p$ , the coarser the topology in the long length scales. In short length scales the situation is just the opposite.

## 4.2 Slaving Principle And P-Adic Length Scale Hypothesis

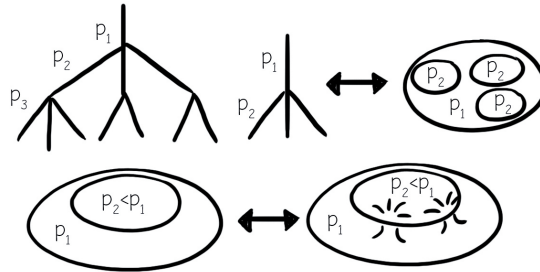
Slaving Principle states that there exists a hierarchy of dynamics with increasing characteristic length (time) scales and the dynamical variables of a given length scale obey dynamics, where the dynamical variables of the longer length (time) scale serve as “masters” that is effectively as external parameters or integration constants. The dynamics of the “slave” corresponds to a rapid adaptation to the conditions posed by the “master”.

p-Adic length scale hierarchy suggests a quantitative realization of this philosophy.

1. By the previous considerations there is an infinite hierarchy of length scales  $L_p$  such that the space-time surfaces below the length scale  $L_p$  look like Minkowski space and p-adic quantum field theory  $M^4 \rightarrow CP_2$  makes sense below the length scale  $L_p$ . These length scales are associated with the different condensation levels present in the topological condensate and define the typical size of the p-adic surface in absence of the collective quantum effects, which should correspond to the formation of the flux tubes between objects with size of order  $L_p$ . The reason why the typical size is just this is that the embedding of the p-adic coordinate space into space  $H$  has strongest discontinuities in the real topology, when coordinate values correspond to powers of  $p$  so that a typical embedding decomposes into separate pieces with size of order  $L_p$ . Of course, this kind of discontinuity is possible for all powers of  $p$  but is not observable in shorter length scales for the physically most interesting values of  $p$  due to the extreme smallness of the corresponding length scales.



2. The lowest level of the hierarchy corresponds to 2-adic dynamics and this field theory makes sense below the cutoff length scale  $L_2 = \sqrt{2}l$  defining the typical size for a 2-adic surface. Solutions of the 2-adic field equations are non-deterministic due to the possibility of the integration constants depending on finite number of binary digits. The dependence on a finite number of positive bits of the real coordinates only means that they are genuine constants below some length scale  $L_2(\text{lower}) < L_2$ , which in principle depends on the state of the system.
3. 2-adic pseudo-constants are analogous to external parameters and should be determined by the dynamics associated with the longer length and time scales. The properties of the p-adic numbers suggest that these constants in turn are p-adically differentiable functions of their argument with some value of  $p_1 > 2$  determined by the  $p_1$ -adic dynamics describing the interaction between  $p = 2$  surface condensed on  $p = p_1$  level and  $p = p_1$  background surface. The  $p_1$ -adic integration constants associated with these functions are actual constants above the length scale  $L_{p_1}(\text{lower}) \geq L_2(\text{lower})$  but also these in principle depend on a finite number of binary digits and their values are determined by the interaction of  $p_1$  level with the next level in the condensation hierarchy.
4. At the next level  $p_1$  one encounters  $p_1$ -adic dynamics and new p-adic integration constants. The net effect is that one obtains a hierarchy of p-adic numbers  $2 < p_1 < p_2 < \dots$  in correspondence with the length and time scales  $L_2 < L_{p_1} < L_{p_2} < \dots$ : the higher the boss the larger the  $p$ . In TGD it is very tempting to interpret the various levels of the slaving hierarchy as the levels of the topological condensate so that the surfaces at level  $p$  are condensed on the surfaces of level  $p_1 > p$  (see **Fig. 1** ). Not all values of  $p$  need be present in the hierarchy and it might well happen that certain values of  $p$  are in an exceptional position physically.



**Figure 1:** Two-dimensional visualization of topological condensate concept

### 4.3 Primes Near Powers Of Two And Slaving Hierarchy: Mersenne Primes

All values of  $p$  are in principle present in the Slaving Hierarchy but the assumption that all values of  $p$  are equally important physically is not realistic. The point is that the number  $N(n)$  of primes smaller than  $n$  behaves as  $N(n) \sim n/\ln(n)$  and there are just too many prime numbers. For example, for  $n = 10^{38}$  there are about one prime number per 87 natural numbers!

A natural looking assumption is that a new physically important length scale emerges, when a fixed number of powers of 2 combine to form a new length scale. The reason is that a given interval  $[2^k, 2^{k+1})$  forms an independent fractal unit (for the simplest fractals these intervals are related by a similarity, see figures in [K14] and it is therefore unnatural to cut this unit into pieces as would happen if  $p$  were far from a power of two. This breaking would indeed happen since p-adically differentiable functions have sharp gradients at points  $p^k$ . This non-breaking or “synergy”

is reached provided the allowed primes are as close as possible to powers of 2:  $p \simeq 2^m$ . It should be noticed that this condition also guarantees that the frequency peaks associated with various powers of  $p$  in good approximation correspond to period doubling frequencies characteristic to fractal and chaotic systems.

The best approximation achievable corresponds to Fermat and Mersenne primes

$$p = 2^m \pm 1 . \tag{4.1}$$

It can be shown that for Fermat primes (+) the condition  $m = 2^k$  must be satisfied and for Mersenne primes (-)  $m$  must be itself prime.

How abundant are the prime numbers of type  $p = 2^m \pm 1$ ? The great surprise was that there are very few numbers of this kind!

1. The primes of type  $2^m + 1$ , Fermat primes, are very rare: only 5 numbers in the range  $1 < n < 2^{2^{21}} \simeq 10^{10^6}$  (!) [A2] and there are good arguments suggesting that the number of the Fermat primes is finite! The known Fermat primes correspond to  $m = 2^k$ , with  $k = 0, 1, 2, 3, 4$ . The corresponding primes are  $p = 3, 5, 17, 257, 65537$ . Note that the lowest Fermat prime 3 is also a Mersenne prime. It will be later found that p-adic conformal invariance is in TGD possible for primes  $p$  satisfying the condition  $p \bmod 4 = 3$  and this condition is not satisfied by Fermat primes  $F > 3$ .
2. The primes of form  $2^m - 1$ , Mersenne primes, are also there as follows from the requirement that  $m$  is prime. The list of allowed exponents of  $m$  consists of the following numbers:

$$2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, \dots$$

One can make two observations about these numbers:

1.  $m = 127$  corresponds to the number  $10^{38}$  fundamental to Physics. The square root of this number gives the ratio of the proton length scale to Planck length scale. This suggests the possibility that fundamental physical length scales are given by square roots of Mersenne and possibly Fermat primes using some length scale of order Planck scale as a unit.
2.  $m = 61$  corresponds to the number of order  $10^{19}$ : this in turn allows the possibility that fundamental physical length scales are linearly related to Fermat and Mersenne primes. This alternative however turns out to be not the correct one.

These observations lead to following scenario for the fundamental length scales:

1. The p-adic length scale  $L_p$ , below which p-adic quantum field theory approximation makes sense, is proportional to the square root of  $p$  and these length scales are p-adically the most interesting length scales:

$$\begin{aligned} L_p &= \sqrt{pl} , \\ l &\sim k \cdot 10^4 \sqrt{G} , \\ k &\simeq 1.376 . \end{aligned} \tag{4.2}$$

Only quite recently the physical interpretation of the length scale  $l$  was found. Contrary to the original expectations,  $CP_2$  is not of order Planck length but of order  $l$ . At this length scale Euclidian regions of space-time, in particular  $CP_2$  type extremals representing elementary particles, become important. Above this length scale a field theory in Minkowski space is expected to be a good approximation to quantum physics.

2. Physically the most interesting length scales correspond to the p-adic cutoff length scales  $L_p$  associated with the Mersenne primes  $M_n$ .

3. The fact that  $l$  is of the same order of magnitude as the length scale at which the coupling constants of the standard model become approximately equal, is not probably an accident. Below  $l$  it is not anymore sensible to speak about the topological condensation of  $CP_2$  type extremals since  $CP_2$  type extremals themselves have size of order  $l$ . Hence the symmetry breaking effects caused by the topological condensation cannot be present in the string model type description applying below  $l$ .

The predictions are as follows:

1.  $m = 127$  corresponds to electron Compton length.
2.  $m = 107$  corresponds to proton Compton length  $L_P$ .
3.  $m = 89$  corresponds to length scale of order  $1/256$  times proton Compton length and is identifiable approximately as  $L_W/2\sqrt{2}$ , where  $L_W$  is intermediate boson length scale of about  $L_P/100$ .
4.  $m = 61$  corresponds to length scale of the order of  $10^{-6}L_P$  is not reachable by the present day accelerators.
5.  $m = 521$  corresponds to a completely super-astronomical length scale of order  $10^{27}$  light years!

It seems that the proposed scenario might have caught something essential in the problem of the elementary particle mass scales: it predicts correctly 3 fundamental length scales associated with leptons, hadrons and intermediate gauge bosons from number theory; there is extremely large gap in the length scale hierarchy after electron Compton length and new shorter length scales exist but unfortunately they are outside the reach of the present day experiments. The calculations of the third part of the book show that not only the mass scales can be understood but also particle masses can be predicted with errors below one per cent using the length scale hypothesis combined with the p-adic Super Virasoro invariance and p-adic thermodynamics.

#### 4.4 Length Scales Defined By Prime Powers Of Two And Finite Fields

Above  $M_{127}$  there is an extremely large gap for Mersenne primes and this suggests that there must be also other physically important primes. Certainly all primes near powers of two define physically interesting length scales by 2-adic fractality but there are too many of them. The first thing, which comes into mind is to consider the set of primes near prime powers of two containing as special case Mersenne primes. The following argument is one of the many arguments in favor of these length scales developed during last years.

TGD Universe is critical at quantum level and criticality is related closely to the scaling invariance. This suggests that unitary irreducible representations of p-adic scalings  $x \rightarrow p^m x$ ,  $m \in \mathbb{Z}$  should play central role in quantum theory. Unitarity requires that scalings are represented by a multiplication with phase factor and the reduction to a representation of a finite cyclic group  $Z_m$  requires that scalings  $x \rightarrow p^m x$ ,  $m$  some integer, act trivially. In ordinary complex case the representations in question correspond to the phase factors  $\Psi_k(x) = |x|^{(\frac{ik2\pi}{\ln(p)})} = \exp(i\ln(|x|)\frac{k2\pi}{\ln(p)})$ ,  $k \in \mathbb{Z}$  and the reduction to a representation of  $Z_m$  is also possible but there is no good reason for restricting the consideration to discrete scalings.

1. The Schrödinger amplitudes in question are p-adic counterparts of the ordinary complex functions  $\Psi_k(x) = \exp(i\ln(|x|)k\frac{ik2\pi}{\ln(p)})$ ,  $k \in \mathbb{Z}$ . They have a unit p-adic norm, they are analogous to plane waves, they depend on p-adic norm only and satisfy the scaling invariance condition

$$\begin{aligned}
 \Psi_k(p^m x|p \rightarrow p_1) &= \Psi_k(x|p \rightarrow p_1) , \\
 \Psi_k(x|p \rightarrow p_1) &= \Psi_k(|x|_p|p \rightarrow p_1) , \\
 |\Psi_k(x|p \rightarrow p_1)|_p &= 1 ,
 \end{aligned} \tag{4.3}$$

which guarantees that these functions are effectively functions on the set of the p-adic numbers with cutoff performed in  $m$ : th power.

2. The solution to the conditions is suggested by the analogy with the real case:

$$\begin{aligned}\Psi_k(x|p \rightarrow p_1) &= \exp\left(i \frac{kn(x)2\pi}{m}\right) , \\ n(x) &= \ln_p(N(x)) \in N ,\end{aligned}\tag{4.4}$$

where  $n(x)$  is integer (the exponent of the lowest power of the p-adic number) and  $k = 0, 1, \dots, m - 1$  is integer. The existence of the functions is however not obvious. It will be shortly found that the functions in question exist in  $p > 2$ -adic for all  $m$  relatively prime with respect to  $p$  but exist for all odd  $m$  and  $m = 2$  in the 2-adic case.

3. If  $m$  is prime (!) the functions  $K = \Psi_k$  form a finite field  $G(m, 1) = Z_m$  with respect to the p-adic sum defined as the p-adic product of the Schrödinger amplitudes

$$K + L = \Psi_{k+l} = \Psi_k \Psi_l ,\tag{4.5}$$

and multiplication defined as

$$KL = \Psi_{kl} .\tag{4.6}$$

Hence, if the proposed Schrödinger amplitudes possessing definite scaling invariance properties are physically important, then the length scales defined by the prime powers of two must be physically special since Schrödinger amplitudes or equivalently, the p-adic scaling momenta  $k$  labeling them, have a natural finite field structure. By the Slaving Hierarchy Hypothesis, also the p-adic length scales near prime powers of two (and perhaps of prime  $p > 2$ , too) are therefore physically interesting. p-Adic scalings correspond to p-adic translations if p-adic coordinates correspond to exponentials of the ordinary linear coordinates so that translations are represented by scalings.

The generalized plane waves exist p-adically if nontrivial  $N = p$ : th root of the quantity  $\exp(i2\pi) = 1$  exists.

1.  $N = 2$ : th roots of 1 exist trivially for all values of  $p$ .
2. In 2-adic case the roots exist always for odd values of  $N$  and especially so for prime values of  $N$ : the trick is to write  $1^{1/N} = -(-1)^{1/N} = -(1 - 2)^{1/N}$  and use the Taylor series

$$\begin{aligned}(1 + x)^{1/N} &= \sum_n \frac{A_n}{n!} x^n , \\ A_n &= \prod_{k=0}^{n-1} \left(\frac{1}{N} - k\right) (-1)^n , \\ x &= -2 .\end{aligned}\tag{4.7}$$

to show the existence of one root different from the trivial root. In 2-adic case the powers of  $x = 2$  converge to zero rapidly and compensate the powers of 2 coming from  $n!$  in the denominator. The coefficients  $A_n$  possess 2-adic norm not larger than 1.

3. For  $p > 2$  nontrivial  $N = p$ : th roots do not allow representation as plane waves for the simple reason that only the trivial  $p$ : th root of 1 exists p-adically. Roots of unity must have p-adic norm equal to one and by writing the condition modulo  $p$  one obtains a condition  $a^N \bmod p = 1$  in  $G(p, 1)$ . The roots of unity in  $G(p, 1)$  satisfy always  $a^{p-1} = 1$  and the possible orders  $N$  are factors of  $p - 1$ . In particular, prime roots with  $p_1 > p - 1$  are not possible. The number of prime factors is typically quite small. For instance, for primes of order  $p = 2^{127}$  the number of prime roots is of order 6.

The conclusion is that for  $p > 2$  only those finite fields  $G(p_1, 1)$  for which  $p_1$  is factor of  $p - 1$  are realizable as representation of phase factors whereas for  $p = 2$  all fields  $G(p_1, 1)$  allow this kind of representation. Therefore  $p = 2$ -adic numbers are clearly exceptional. In the p-adic case the functions  $\Psi_p(x, |p \rightarrow p_1)$  give irreducible representations for the group of p-adic scalings  $x \rightarrow p^m x$ ,  $m \in Z$  and the integers  $k$  can be regarded as scaling momenta. This suggests that these functions should play the role of the ordinary momentum eigenstates in the quantum theory of fractal structures. The result motivates the hypothesis that prime powers of two and also of  $p$  define physically especially interesting p-adic length scales: this hypothesis will be of utmost importance in future applications of TGD.

The ordinary (number theoretic) p-adic plane waves associated with the translations can be constructed as functions  $f_k(x) = a^{kx}$ ,  $k = 0, \dots, n$ ,  $a^n = 1$ . For  $p > 2$  these plane waves are periodic with period  $n$ , which is factor of  $p - 1$  so that wavelengths correspond to factors of  $p - 1$  and generate a finite number of physically favored length scales. The p-adic plane waves with the momenta  $k = 0, \dots, p - 2$  form finite field  $G(p, 1)$ , when p-adic arithmetics is replaced with the modulo  $p$  arithmetics, that is to accuracy  $O(p)$  (note that the definition of the arithmetic operations is *not* the same as in the previous case). The square roots of the p-adic plane waves are also well defined

The important property of the p-adic plane waves is that they are pseudo constants: this property played profound role in the earlier formulations of the p-adic QFT limit. It took a considerable time to discover that the counterparts of the ordinary real plane waves providing representations for translation group exists and satisfy the appropriate orthogonality relations. Therefore number theoretic plane waves do not play so essential role in p-adic QFT as was originally believed.

## 5 $CP_2$ Type Extremals

$CP_2$  type extremals are perhaps the most important vacuum extremals of the Kähler action. The reason is that they are vacuum extremals with a negative and finite Kähler action and hence favored both by the absolute minimization of the Kähler action and criticality (randomness of light-like projection to  $M^4$  implies criticality). It seems that also other identification of preferred extremals allow  $CP_2$  type vacuum extremals and actually all known extremals. On the other hand, maximization of Kähler function does not favor  $CP_2$  type extremals because the virtual  $CP_2$  type extremals are exponentially suppressed.  $CP_2$  type extremals seem to play the same role as black holes possess in General Relativity. p-Adic thermodynamics, leading to excellent predictions for the masses of the elementary particles, predicts that elementary particles should possess p-adic entropy and Hawking-Bekenstein law for the entropy generalizes.

In GRT based cosmology black holes populate the most probable Universe, which is of course a problem: in TGD black holes are replaced by elementary particles. The second law of thermodynamics requires that the very early Universe should have a low entropy and hence that black holes should populate the recent day Universe: in TGD the very early cosmology is dominated by cosmic strings, which is a low entropy state. The absolute minimization of the Kähler action would imply that most cosmic strings would decay to elementary particles and produce p-adic entropy. It is not clear whether also criticality implies this. To get a grasp of the orders of magnitude, it is good to notice that electron, which corresponds to  $p = M_{127} = 2^{127} - 1$ , has entropy equal to 127 bits.

The basic observation is that the  $M_+^4$  projection of the  $CP_2$  type extremal corresponds to a light like random curve and the quantization of this motion leads to Virasoro algebra and Kac Moody algebra characterizing quantized transversal motion superposed with the cm motion.  $CP_2$

type extremals allow covariantly constant right handed neutrino spinors as solutions of the Dirac equation for the induced spinors in the interior and this leads to  $N = 1$  super symmetry and a generalization of the Virasoro invariance to Super Virasoro invariance.

The previous p-adic mass calculations were based on this picture but it turned out that the Super Virasoro invariance and related Kac Moody symmetries generalize to the level of WCW geometry and in an extended form provide the basic symmetries of the quantum TGD. Although the quantization of the zitterbewegung motion of the  $CP_2$  type extremals is a phenomenological procedure only, and is not needed in the fundamental theory, it deserves to be described because of its key role in the development of quantum TGD. There were however some strange features involved: for instance,  $N = 1$  super-symmetry generated by right-handed neutrino was exact only for minimal surfaces.

The realization that super-symmetry requires Kähler-Dirac action led to the final breakthrough.  $CP_2$  type extremals allow quaternion-conformal symmetries and the super-generators associated with quark and lepton numbers are non-vanishing despite the fact that vacuum extremals are in question. Even Super-Kac-Moody generators are non-vanishing. Even more,  $CP_2$  type extremals cease to be vacua for Dirac action. Especially beautiful feature of  $CP_2$  type extremals is that they can describe also massive states and zitterbewegung is the geometric correlate of massivation.

## 5.1 Zitterbewegung Motion Classically

The  $M_+^4$  projection of a  $CP_2$  type extremal is a random light like curve. Also Dirac equation, which gives also classically rise to a motion with light velocity and this motivates the term “zitterbewegung”. Zitterbewegung occurs at the light of velocity and any given 3-velocity gives rise to the solution of light likeness condition if one fixes the time component of velocity to be

$$\frac{dm^0}{d\tau} = \sqrt{m_{ij} \frac{dm^i}{d\tau} \frac{dm^j}{d\tau}} . \quad (5.1)$$

The vanishing of  $CP_2$  part of the second fundamental form requires that velocity and acceleration are orthogonal:

$$m_{kl} \frac{dm^k}{d\tau} \frac{d^2m^l}{d\tau^2} = 0 . \quad (5.2)$$

This condition is identically satisfied.

A very general solution to the conditions is provided by the equations

$$\frac{d^2m^k}{d\tau^2} = F^{kl} \frac{dm^l}{d\tau} , \quad (5.3)$$

describing the motion the of massless charged particle in external Maxwell field.

## 5.2 Basic Properties Of $CP_2$ Type Extremals

$CP_2$  type extremal has the following explicit representation

$$m^k = f^k(u(s^k)) , \quad m_{kl} \frac{df^k}{du} \frac{df^l}{du} = 0 . \quad (5.4)$$

The function  $u(s^k)$  is an arbitrary function of  $CP_2$  coordinates and serves effectively as a time parameter in  $CP_2$  defining a slicing of  $CP_2$  to time=constant sections. The functions  $f^k$  are arbitrary apart from the restriction coming from the light likeness. When one expands the functions  $f^k$  to Fourier series with respect to the parameter  $u$ , light likeness conditions reduce to classical Virasoro conditions  $L_n = 0$ .

It is possible to write the expression for  $m^k$  in a physically more transparent form by separating the center of mass motion and by introducing p-adic length scale  $L_p$  as a normalization factor.

$$\frac{m^k}{L_p} = m_0^k + p_0^k u + \sum_n \frac{1}{\sqrt{n}} a_n^k \exp(i2\pi n u) + c.c. \quad (5.5)$$

The first term corresponds to the center of mass term responsible for rectilinear motion along geodesic line and second term corresponds to the zitterbewegung motion.  $p^k$  serves as an effective classical momentum which can be normalized as  $p_k p^k = \epsilon$ ,  $\epsilon = \pm 1$  or  $\epsilon = 0$ . What has significance is whether  $p^k$  is time like, light like, or space like. Conformal invariance corresponds to the freedom to replace  $u$  with a new “time parameter”  $f(u)$ .

The physically most natural representation of  $u$  is as a function  $f(U)$  of the fractional volume  $U$  for a 4-dimensional sub-manifold of  $CP_2$  spanned by the 3-surfaces  $X^3(U = 0)$  and  $X^3(U)$ :

$$u = f(U) \quad , \quad U = \frac{V(s^k)}{V(CP_2)} = \frac{S_K(u)}{S_K(CP_2)} \quad (5.6)$$

The range of the values for  $U$  is bounded from above:  $U \leq V_{max}/V(CP_2)$  and the value  $U = 1$  is possible only if  $CP_2$  type extremal begins and ends as a point.  $U$  represents also Kähler action using the value of the Kähler action for  $CP_2$  as a unit.

The requirement that  $CP_2$  type extremal extends over an infinite time and spatial scale implies the requirement

$$f(U_{max}) = \infty \quad (5.7)$$

For  $f(U_{max}) < \infty$   $CP_2$  type extremal can exist only in a finite temporal and spatial interval for finite values of “momentum” components  $p^k$ . This suggest a precise geometric distinction between real and virtual particles: virtual particles correspond to the functions  $f(U_{max}) < \infty$  in contrast to the incoming and outgoing particles for which one has  $f(U_{max}) = \infty$ . This hypothesis, although it looks like an ad hoc assumption, is at least worth of studying.

The mere requirement that virtual  $CP_2$  type extremal extends over a temporal or spatial distance of order  $L > L_p$  implies that for  $L < L_p$  the value of  $U$  is smaller than one. Kähler action, which is given by

$$S_K(X^4) = U \times S_K(CP_2) \quad , \quad (5.8)$$

remains small for distances much smaller than  $L$ . For  $f(U_{max}) = \infty$  this is even more true. This has an important implication: below a certain length scale the exponential of the Kähler action associated with the internal line of a Feynman diagram does not give rise to a suppression factor whereas above some characteristic length  $L$  and time scale there is an exponential suppression of the propagator by the factor  $\exp(-S_K(CP_2))$  practically hindering the propagation over distances larger than this length scale.

The presence of the exponential obviously introduces an effective infrared cutoff: this cutoff is prediction of the fundamental theory rather than ad hoc input as in quantum field theories. Of course, infrared cutoff results also from the condition  $f(U_{max}) < \infty$ . Physically the infrared cutoff results from the topological condensation of the  $CP_2$  type extremals to larger space-time sheets. These could correspond to massless extremals (MEs). p-Adic length scale  $L_p$  is an excellent candidate for the cutoff length scale in the directions transversal to ME.

The suppression factor coming from the exponent of the Kähler action implies a distance dependent renormalization of the propagators. In the long length scale limit the suppression factor approaches to a constant value

$$\exp \left[ -\frac{V_{max}}{V(CP_2)} S_K(CP_2) \right] \quad ,$$

and can be absorbed to the coupling constant so that the dependence on the maximal length of the internal lines can be interpreted as an effective coupling constant evolution. For instance, the smallness of the gravitational constant could be understood as follows. Since gravitons propagate over macroscopic distances, the virtual  $CP_2$  type extremals develops a full Kähler action and there

is huge suppression factor reducing the value of the gravitational coupling to its observed value: at short length scales the values of the gravitational coupling approaches to  $G_{short} = L_p^2$  which means strong gravitation for momentum transfers  $Q^2 > 1/L_p^2$ . The values of  $V_{max}$  and thus those of the suppression factor can vary: only at the limit when  $CP_2$  type extremal has point like contact with the lines it joins together, one has  $V_{max} = V(CP_2)$ . If the boundary component characterizing elementary particle family belongs to  $CP_2$  type extremal (it could be associated with a larger space-time sheet),  $CP_2$  type extremal contains a hole: also this reduces the maximal volume of the  $CP_2$  type extremal.

### 5.3 Quantized Zitterbewegung And Super Virasoro Algebra

Calculating various Fourier components of right left hand side of the light likeness condition  $m_{kl}p^k p^l = 0$  for  $p^k = dm^k/du$  explicitly using the general expansion for  $m^k$  separating center of mass motion from zitterbewegung, one obtains classical Virasoro conditions

$$\begin{aligned} p_0^2 &= L_0 , \\ L_n|phys\rangle &= 0 , . \end{aligned} \quad (5.9)$$

where  $L_n$  are defined by their classical expressions as bi-linears of the Fourier coefficients. Therefore interior degrees of freedom give Virasoro algebra and zitterbewegung is more or less equivalent with the classical string dynamics.

It is not however not obvious whether a quantization of this dynamics is needed. If quantization is needed (perhaps to formulate the unitarity conditions in zero modes properly), it corresponds to the construction of the bosonic wave functionals in zero modes defined by the zitterbewegung degrees of freedom. Quantization could be carried out in the same manner as in string models.

The simplest assumption motivated by the Euclidian metric of  $CP_2$  type extremal is that the commutator of  $p^k$  and  $m^k$  is proportional to a delta function as in ordinary quantization. One can Fourier expand  $m^k$  and  $p_k$  in the form

$$\begin{aligned} m^k &= m_0^k + p_0^k s + \frac{1}{K} \sum \frac{1}{n} a_n^{k,\dagger} \exp(inKs) + \sum \frac{1}{n} a_n^k \exp(-inKs) , \\ p^k &= p_0^k + i \sum a_n^{k,\dagger} \exp(inKs) - i \sum a_n^k \exp(-inKs) . \end{aligned} \quad (5.10)$$

Here cm motion has been extracted and the formula is identical with the formula expressing the motion for a fixed point of string. The parameter  $K$  is Kac Moody central charge. Note that the exponents  $\exp(iKns)$  exist provided that  $Ks$  is p-adically of order  $O(p)$  or, if algebraic extension by introducing  $\sqrt{p}$  is allowed, of order  $O(\sqrt{p})$ .

The commutator of  $p_i$  and  $m^j$  is of the standard form if the oscillator operators obey Kac-Moody algebra

$$\begin{aligned} [p_{i,0}, m_0^j] &= m_i^j , \\ Comm(a_{i,m}^\dagger, a_n^j) &= Km\delta(m,n)m_i^j . \end{aligned} \quad (5.11)$$

Here  $K$  appears Kac-Moody central charge, which must be integer in the real context at least.

Expressing the light likeness condition as quantum condition, one obtains an infinite series of conditions, which give the quantum counterparts of the Virasoro conditions

$$\begin{aligned} p_0^2 &= kL_0 , \\ L_n|phys\rangle &= 0 , \quad n < 0 . \end{aligned} \quad (5.12)$$

$k$  is some proportionality constant. One can solve these conditions by going to the transverse gauge in which physical states are created by oscillator operators orthogonal to an arbitrarily chosen light like vector. What quantization means physically is that zitterbewegung amplitudes are constrained



by a Gaussian vacuum functional. A good guess motivated by the p-adic considerations is that the width of the ground state Gaussian is given by a p-adic length scale  $L_p$ : this is achieved if  $m^k$  is replaced with  $m^k/L_p$  in the general expression for  $m^k(u)$ . The experience with string models would suggest that vacuum functionals might be crucial for the understanding of graviton emission.

## 5.4 Zitterbewegung At The Level Of The Kähler-Dirac Action

At the level of the Kähler-Dirac action zitterbewegung motion implies that the conserved momentum associated with  $CP_2$  type extremal, besides being conserved and non-vanishing, is also time like. This means that zitterbewegung creates massive particles besides massless particles as well as off-mass-shell versions of both and Super Virasoro conditions imply the quantization of the mass squared spectrum.

This means that in quantum TGD Feynman diagrammatics is topologized in the sense that the lines of Feynman diagram correspond to  $CP_2$  type extremals which in general performing zitterbewegung. The non-determinism of the  $CP_2$  type extremals means that one obtains a sum over over all possible diagrams with vertices at arbitrary space-time locations just as in quantum field theory approach. What is so nice that the time-development operator associated with an individual line of the diagram is the exponent of the Hamiltonian operator identified as the Poincare energy associated with the modified Dirac action. This operator is that associated with a free theory and contains no nonlinear terms. Interactions result from criticality property of the extremals of Kähler action. In particular, one gets rid of the divergences of the interacting quantum field theories by the topologization of the Feynman diagrammatics.

## 6 Black-Hole-Elementary Particle Analogy

String models have provided considerable insights into black hole thermodynamics by reducing it to ordinary thermodynamics for stringy black holes [B3] although one still does not understand, which is the mechanism of the thermalization. In TGD context elementary particles are regarded as thermodynamical systems in p-adic sense. This is something new since the standard theories of particle physics describe elementary particles as pure quantum states. The resulting thermal description of the the particle massivation is extremely successful. The fact that one can associate a well defined entropy to an elementary particle, suggests an analogy between black holes and elementary particles and this analogy indeed exists in a quite precise form as will be found. It also leads to a partial explanation for the p-adic length scale hypothesis serving as the corner stone of the p-adic mass calculations. The identification of the  $CP_2$  type extremal as a cognitive representation of elementary particle suggests that p-adic entropy characterizes information associated with a cognitive representation provided by  $CP_2$  type extremal.

### 6.1 Generalization Of The Hawking-Bekenstein Law Briefly

In TGD elementary particles are modelled as so called  $CP_2$  type extremals, which are surfaces with a size of order Planck length having metric with Euclidian signature. These vacuum surfaces are isometric with  $CP_2$  itself and have a one-dimensional, random light like curve as the  $M_+^4$  projection. A natural candidate for the TGD counterpart of the black hole horizon is the surface at which the Euclidian signature of the metric associated with the  $CP_2$  type extremal is changed to the Minkowskian signature of the background space-time. The radius  $r$  of this surface is the crucial length scale for the topological condensation and the simplest guess is that it is of the order of the size of the  $CP_2$  radius and hence of the fundamental p-adic length scale. The hope is that the generalization of the black hole thermodynamics, with  $r$  replacing the radius of the black hole horizon, could give this information.

p-Adic mass calculations indeed give the p-adic counterpart of the Hawking-Bekenstein formula  $S \propto GM^2$  as an identity at p-adic level:

$$S_p = -\frac{1}{T_p}(M_p^2/m_0^2) ,$$

where  $1/T_p = n$  is the integer valued inverse of the p-adic temperature and the mass scale  $m_0^2/3$  corresponds to unit p-adic number in the unit used. The peculiar looking sign of  $S_p$  does not have in the p-adic context the same significance as in real context since the real counterpart of  $S_p$  is positive. Although p-adic entropy and mass squared are linearly related, the real counterparts are not in such a simple relation. In case of massive particles the real counterpart of the entropy is in excellent approximation equal to  $S = \log(p)$  whereas the mass is of order  $1/p$  ( $p$  is of order  $10^{38}$  for electron!). For massless (or nearly massless) particles one has  $S \leq \log(p)/p$ . The large difference between fermionic and photonic entropies does not favor pair annihilation and this suggests that matter antimatter asymmetry is generated thermodynamically. For instance, via the topological condensation of fermions and anti-fermions on different space-time sheets during the early cosmology.

The generalization of the Hawking-Bekenstein formula in the form of the area law  $S = A/4G$  reads as

$$S = \frac{xA}{4l^2} ,$$

where the fundamental p-adic length scale  $l \simeq 1.376 \cdot 10^4 \sqrt{G}$  replaces Planck length  $\sqrt{G}$  and  $x$  is a numerical constant near unity. The radius of the elementary particle horizon is in an excellent approximation given by  $r(p) = \sqrt{\frac{\log(p)}{\pi x}} l$ . Particles are thus surrounded by an Euclidian region of the space-time with radius  $r$ . Thus the fundamental p-adic length scale  $l$  of order  $CP_2$  size has a direct geometric meaning. For instance, in the energy scales below  $1/l$  the induced metric of the space-time becomes Euclidian and it might be possible to describe particle physics using Euclidian field theory: essentially QFT in a small deformation of  $CP_2$  would be in question. It is encouraging, that  $l$  is also the length scale at which the standard model couplings become identical and super symmetry is expected to become manifest.

The p-adic length scale hypothesis stating that the primes  $p$  near prime powers of two are the physically most interesting p-adic primes, is the cornerstone of p-adic mass calculations but there is no really convincing argument for why should it be so. The proportionality of  $r$  to  $\sqrt{\log(p)}$  suggests an explanation for the p-adic length scale hypothesis. The point is that for  $p \simeq 2^k$ ,  $k$  prime, one has  $r \propto L(k)$  and if the numerical constant  $x$  is chosen to be  $x = \frac{\log(2)}{\pi}$ , the radius of elementary particle horizon is in excellent approximation  $r(p \simeq 2^k) = L(k)$ . Note also that the area of the elementary particle horizon becomes quantized in multiples of prime. This suggests that the precise value of  $p \simeq 2^k$  is such that this condition is satisfied optimally and that physics is  $k$ -adic below  $r$  and  $p \simeq 2^k$ -adic above  $r$ .

$M_+^4 \times CP_2$  allows the embedding of Schwarzschild metric in the region below Schwarzschild radius but the embedding fails for too small values of the radial variable [K18]. An interesting possibility is that black hole entropy is just the sum of the elementary particle entropies topologically condensed below the horizon. This would give  $S_{TGD} \propto \sum m_i^2 < S_{GRT} \propto (\sum m_i)^2$ . An interesting problem is related to the detailed definition of p-adic entropy: are the entropies of particles with same value of  $p$  additive as p-adic numbers or does the additivity hold true for the real counterparts of the p-adic entropies. A related question is whether it might be that also in case of black holes additivity holds true, not for the mass as it is usually assumed, but for the p-adic mass squared for a given  $p$  (in TGD inspired model of hadron this is true for quark masses). This could be understood as a result of strong gravitational interactions. The additivity with respect to mass squared would give an upper bound of order  $10^{-4}/\sqrt{G}$  for the contribution of a given p-adic prime to the total mass. For instance, the total contribution of electrons to the mass would be always below this mass irrespective of the number of electrons!

## 6.2 In What Sense $CP_2$ Type Extremals Behave Like Black Holes?

$CP_2$  type extremals are in some respects classically black hole like objects since their metric is Euclidian. When this kind of surface is glued to Minkowskian background there must exist a two-dimensional surface, where the signature of the induced metric changes from the Minkowskian  $(1, -1, -1, -1)$  to the Euclidian  $(-1, -1, -1, -1)$ . On this surface, which could be called elementary particle horizon, the metric is degenerate and has the signature  $(0, -1, -1, -1)$ . Physically elementary particle horizon can be visualized as the throat of the wormhole feeding the elementary

particle gauge fluxes to the background space-time. Of course, one cannot exclude the presence of several wormholes for a given space-time sheet.

This surface indeed behaves in certain respects like horizon. Time like geodesic lines cannot go through this surface. The reason is that the square of the four velocity associated with the geodesic is conserved:

$$v_\mu v^\mu = 1, 0 \text{ or } -1,$$

depending on whether the geodesic is time like, light like or space like. Clearly, a time like geodesic cannot enter from the external world to the interior of the  $CP_2$  type extremal. If a space like geodesic starts from the interior of the  $CP_2$  type extremal it can in principle continue as a space like geodesic into the exterior. These analogies should not be taken too seriously: it does not make sense to identify particles orbits as geodesics in these length scales shorter than the actual sizes of particle.

These analogies suggest that Hawking-Bekenstein formula  $S = A/4G$  relating black hole entropy to the area of the black hole horizon, might have a generalization to the elementary particle context with the radius of the elementary particle horizon replacing the black hole horizon. The unit of the area need not be determined by Planck length  $\sqrt{G}$ , it could be replaced by the fundamental p-adic length scale  $l \sim 10^4 \sqrt{G}$ : this length scale indeed replaces Planck length as a fundamental length scale in TGD.

### 6.3 Elementary Particles As P-Adically Thermal Objects?

In the p-adic mass calculations elementary particles were assumed to be thermal objects in the p-adic sense. What is new that energy is replaced with mass squared and the thermalization is believed to result from the interactions of a topologically condensed  $CP_2$  type extremal with the background space-time surface of a much larger size. The thermalization mixes massless states with Planck mass states and gives rise to particle massivation. Super Virasoro invariance – abstracted from the Virasoro invariance of the  $CP_2$  type extremals – together with the general symmetry considerations based on the symmetries of  $M_+^4 \times CP_2$ , leads to the realization of the mass squared operator essentially as the Virasoro generator  $L_0$  in certain representations of the Super Virasoro algebra constructed using the representations of various Kac Moody algebras associated with Lorentz group, electro-weak group and color group.

– $L_0$  takes thus the role of a Hamiltonian in the partition function:

$$\exp(-H/T) \rightarrow p^{L_0/T_p},$$

where  $T_p$  is the p-adic temperature, which by number theoretic reasons is quantized to  $1/T_p = n$ ,  $n$  a positive integer. Mass squared is essentially the thermal expectation of  $L_0$ . The real mass squared is the real counterpart of the p-adic mass squared in the canonical identification  $x = \sum x_n p^n \rightarrow \sum x_n p^{-n} \equiv x_R$  mapping p-adics to reals. Assuming that elementary particles correspond to p-adic primes near prime powers of two, one obtains excellent predictions, not only for the mass scales of elementary particles but also for the particle mass ratios. For instance, electron corresponds to the Mersenne prime  $M_{127} = 2^{127} - 1$ .

It should be noticed that the real counterpart of the p-adic inverse temperature  $1/T_p$  is naturally defined as

$$\left(\frac{1}{T_p}\right)_r = \left(\frac{1}{T_p}\right)_R \log(p),$$

where  $\log(p)$  factor results from the definition of Boltzmann weights as powers of  $p$  rather than power of  $e$ . The real counterpart  $T_r$  of  $T_p$  can be identified as

$$T_r = \frac{1}{n \log(p)}. \quad (6.1)$$

One might wonder about whether the sign of  $T_p$  should be taken as negative since positive exponent of  $L^0$  appears in the Boltzmann weights. The sign is correct; for the opposite sign  $T_r$  would be

in good approximation equal to  $\frac{1}{(p-n)\log(p)}$ , which is not consistent with the fact that physically temperature decreases when  $n$  increases.

As already explained, the new vision about p-adics and cognition forces to modify this early vision by interpreting  $CP_2$  type extremals as cognitive representations of elementary particles rather than genuine elementary particles.

### 6.3.1 p-Adic mass squared

The thermal expectation of the p-adic mass squared operator is proportional to the thermal expectation of the Virasoro generator  $L_0$ :

$$\begin{aligned} M_p^2 &= k \langle L_0 \rangle , \\ k &= 1 . \end{aligned} \quad (6.2)$$

The correct choice for the value of the rational number  $k$  is  $k = 1$  as became clear in the recent reconstruction of the quantum TGD [K11].

The real mass squared  $M^2$  is identified as

$$\begin{aligned} M^2 &= \frac{M_R^2 \pi^2}{l^2} , \\ l &\simeq 1.376 \cdot 10^4 \sqrt{G} , \end{aligned} \quad (6.3)$$

where  $l$  is the fundamental p-adic length scale and  $M_R^2$  is the real counterpart of  $M_p^2$  in the canonical identification.  $\sqrt{G}$  is Planck length scale.

### 6.3.2 p-Adic entropy is proportional to p-adic mass squared

The definition of the p-adic entropy involves some number theory. The general definition

$$S = -p_n \log(p_n) ,$$

in terms of the probabilities  $p_n$  of various states does not work as such since the e-based logarithm  $\log(p_n)$  does not exist p-adically. Since p-adic Boltzmann weights are integer powers of  $p$  it is natural to modify somehow the p-based logarithm  $\log_p(x)$  so that the resulting logarithm  $Log_p(x)$  exists for any p-adic number and has the basic property

$$Log_p(xy) = Log_p(x) + Log_p(y) ,$$

guaranteeing the additivity of the p-adic entropy for non-interacting systems. The definition satisfying these constraints is

$$Log_p(x = \sum_{n \geq n_0} x_n p^n) \equiv n_0 . \quad (6.4)$$

The lowest power in the expansion of  $x$  in powers of  $p$  fixes the value of the logarithm in the same way as it determines also the norm of the p-adic number. This leads to the definition of p-adic entropy as

$$S_p = - \sum_p p_n Log_p(p_n) . \quad (6.5)$$

In p-adic thermodynamics the p-adic probabilities have the general form

$$p_n = \frac{p^{L_0(n)/T_p}}{Z} .$$

Here  $L_0(n)$  denotes the eigenvalue of the Virasoro generator  $L_0$ , which is integer. The partition function  $Z = \text{trace}(p^{L_0/T_p})$  has unit p-adic norm if the ground state is massless, so that its p-adic logarithm vanishes in this case:  $\text{Log}_p(Z) = 0$ . This implies  $\text{Log}_p(p_n) = \text{Log}_p(p^{L_0(n)/T_p}) = L_0(n)/T_p$  so that the p-adic entropy reduces to

$$S_p = \frac{1}{T_p} \langle L_0 \rangle , \quad (6.6)$$

and hence that the p-adic mass squared and p-adic entropy are proportional to each other

$$S_p = -\frac{1}{kT_p} M_p^2 . \quad (6.7)$$

By noticing that the entropy for Schwarzschild black hole is given by

$$S = 4\pi GM^2 , \quad (6.8)$$

one finds that in the p-adic context the analog of the Hawking-Bekenstein formula indeed holds as an identity.

The proposed identification of the entropy is in accordance with the formula  $dE = TdS$ . In the p-adic context  $E$  should clearly be replaced by  $\langle -L_0 \rangle$  and  $T$  by  $T_p$ . The differentials do not however make sense since the thermodynamical quantities are now discrete. Since only  $\langle -L_0 \rangle$  and  $T_p$  appear as variables one could define

$$\langle -L_0 \rangle = T_p S_p .$$

This definition gives  $S_p = -\frac{1}{kT_p} M_p^2$  and is in accordance with the standard definition of the Shannon entropy. The definition for the real counterpart of the p-adic entropy is

$$S = \log(p) S_R .$$

The inclusion of  $\log(p)$ -factor maximizes the resemblance with the usual Shannon entropy defined in terms of the e-based logarithm and makes it possible to compare the real counterpart of entropy with other kind of entropies.

### 6.3.3 The real counterparts of entropy and mass squared are not linearly related

Due to the delicacies related to the canonical identification, the real counterparts of entropy and mass squared differ drastically from each other and there is no simple relationship between the two quantities. The reason is that the vacuum expectation of  $-L_0$  is of order  $-np$  for particles having  $T_p = 1$  and, essentially due to the presence of minus sign, one has  $S_R(p) = 1$  in an excellent approximation, whereas the real counterpart of  $M_p^2$  is of order  $n/p$ . For photon and other (nearly) massless bosons the entropy vanishes or is very small.

The fundamental difference in the thermal properties of fermions and massless bosons should have observable consequences. For instance, the annihilation of fermion-anti-fermion pair to massless particles means a considerable reduction of the p-adic entropy and would not be a favorable process thermodynamically. Thus the second law of thermodynamics would favor the presence of net fermion and anti-fermion number densities. For instance, fermions and anti-fermions could suffer a topological condensation on different space-time sheets to avoid annihilation during early cosmology or anti-fermions could even suffer topological evaporation as suggested in [?, ?]. This in turn would lead to the generation of matter-antimatter asymmetry. It should be noticed that large entropies are in accordance with the second law of thermodynamics.

### 6.3.4 Hawking-Bekenstein area formula in elementary particle context

Hawking-Bekenstein formula in the p-adic form  $S_p \propto M_p^2$  holds true on basis of the previous considerations although there are no hopes of deriving the area law from the first principles at this stage. Hawking-Bekenstein formula can be also written in the form

$$S = \frac{A}{4G} ,$$

relating black hole entropy to the area of the black hole horizon. One might hope that in the real context a generalization of the area law to the form

$$S = x \frac{A}{4L^2} ,$$

where  $L$  is some fundamental length scale analogous to the gravitational constant  $G$  and  $x$  is some numerical constant near unity, would hold true. Since the size of  $CP_2$  defines the fundamental p-adic length scale and replaces  $\sqrt{G}$  as a fundamental length scale in TGD, it is conceivable that  $L$  is of the order of the  $CP_2$  size  $l \sim 10^4 \sqrt{G}$ . The area in question would be most naturally the area of the elementary particle horizon, where the signature of the induced metric for the topologically condensed  $CP_2$  type extremal changes from Euclidian to Minkowskian. It is well known that  $l$  is also the length scale at which the couplings of the standard model become identical and supersymmetry is expected to become manifest. This is what is expected since above cm energy  $1/l$  one would have an Euclidian quantum field theory in  $CP_2$ .

The radius  $r$  of the elementary particle horizon is of order

$$r \simeq \sqrt{\log(p)} L . \quad (6.9)$$

This means that the # contacts connecting the  $CP_2$  type extremal to the background space-time are surrounded by an Euclidian region with a size of order  $L$ .

It is interesting to look for the detailed form of the Hawking-Bekenstein law for elementary particles. One obtains the following general relationship

$$\begin{aligned} S &\equiv \log(p) S_R = \log(p) \left( \left\langle \frac{-L_0}{T_p} \right\rangle \right)_R = X \log(p) M_R^2 = X \times \log(p) \frac{l^2}{\pi^2} M^2 , \\ X &\equiv \frac{M_R^2}{S_R} . \end{aligned} \quad (6.10)$$

For massive particles  $X \sim p$  holds true. Hence the entropy is related by a factor  $p \cdot 10^8$  to the corresponding black hole entropy:

$$\begin{aligned} S &= a^2 S_{BH} , \\ S_{BH} &= 4\pi G M^2 \\ a &= \sqrt{\frac{\log(p) X}{4\pi^3}} \frac{l}{\sqrt{G}} \sim 10^4 , \\ l &\simeq 1.376 \cdot 10^4 \sqrt{G} . \end{aligned} \quad (6.11)$$

## 6.4 P-Adic Length Scale Hypothesis And P-Adic Thermodynamics

The basic assumption of p-adic mass calculations is that physically interesting p-adic primes correspond to prime powers of two:

$$p \simeq 2^k , \quad k \text{ prime} .$$

There are several arguments in favor of this hypothesis but no really convincing argument. The area law however leads to a very attractive, if not even convincing, explanation of the p-adic length scale hypothesis.

The proportionality of the elementary particle horizon radius to  $\sqrt{\log(p)}$  suggests quite attractive partial explanation for the p-adic length scale hypothesis. The point is that for  $p \simeq 2^k$ ,  $k$  prime one has  $r \propto L(k)$ . Thus, if the numerical constant  $x$  is chosen suitably, it is possible to obtain very precisely

$$r(p \simeq 2^k) = L(k) .$$

The reason is that the p-adic entropy is in thermal equilibrium very near to its maximum value. The required value of the coefficient  $x$  is

$$x = \frac{\log(2)}{\pi} . \quad (6.12)$$

The requirement that  $r_F$  ( $r_B$ ) is as near as possible to the appropriate p-adic length scale  $L(k)$  ( $L(k)\sqrt{p}$ ) fixes also the precise value of the p-adic prime  $p \simeq 2^k$ .

This hypothesis means that the area of the elementary particle horizon is quantized in the multiples of prime  $k$ :

$$A = kA_1 . \quad (6.13)$$

The quantization law for the area has been proposed also in the context of the non-perturbative quantum gravity. A suggestive possibility is that physics is  $k$ -adic below the elementary particle horizon and  $p \simeq 2^k$ -adic above it. The appearance of an additional  $k$ -adic length scale suggests that for  $p \simeq 2^k$  the degeneracy of the effective space-time surfaces is especially large due to the additional  $k$ -adic degeneracy and that the p-adic scattering amplitudes are be especially large for this reason. Hence the favored p-adic primes would emerge purely dynamically.

It must be noticed that k-adic fractality allows also more general primes of type  $p \simeq 2^{k^n}$ , where  $k$  is prime and  $n$  is integer. For these primes the radius of the elementary particle horizon is  $\sqrt{k^{n-1}}L(k)$  and hence also a natural k-adic length scale. There are very few physically interesting length scales of this type. As the p-adic mass calculations show, the best fit to the neutrino mass squared differences is obtained for  $p_\nu \simeq 2^{132=169}$  rather than  $p \simeq 2^{167}$ . The length scale  $L(p_\nu)$  is also the natural length scale associated with the double cell layers appearing very frequently in bio-systems ( $k = 167$  corresponds to the typical size of a cell)!

## 6.5 Black Hole Entropy As Elementary Particle Entropy?

In TGD Schwartzchild metric does not allow a global embedding as a surface in  $M_+^4 \times CP_2$ . One can however find embeddings, which extend also below the Schwartzchild radius. This suggests that particles in the interior of the black hole are topologically condensed below the radius  $r_s$ . The problem is whether the single particle entropies are additive as real numbers or as p-adic numbers.

### 6.5.1 Additivity of real entropies?

Consider first the additivity as real numbers. With this assumption the sum for the real counterparts of the p-adic entropies of various particles gives a lower bound for the black hole entropy:

$$S = \sum_i S(i) = \sum_i km_i^2 .$$

This entropy is by a factor is  $10^8 \cdot p$  larger than the corresponding black hole entropy so that black hole-elementary particle analogy does not work at quantitative level. For sufficiently large particle numbers elementary particle entropy becomes smaller than the black hole entropy, which behaves as  $(\sum m_i)^2$ . In case of protons  $p = M_{107} = 2^{107} - 1$  the critical value of  $N$  would be roughly  $N \sim 10^{32}$ , which would mean black hole with a mass of order 100 kilograms.

### 6.5.2 Additivity of the p-adic entropies?

One can consider also a different definition of the black hole entropy. In p-adic thermodynamics the natural additive quantity for many particle systems is the Virasoro generator  $L_0$  (mass squared essentially) rather than energy. The additivity works quite nicely for the TGD based model of a hadron as a bound state of quarks. Therefore one could consider the possibility that also for black holes the mass squared of elementary particles with same value of p-adic prime  $p$  is p-adically additive

$$(m_p^2)_R = \left(\sum_i m_p^2(i)\right)_R \text{ rather than } m = \sum m_i .$$

Therefore for a black hole containing only particles with single value of the p-adic prime  $p$ , the Hawking-Bekenstein formula in the form

$$S_p \propto M_p^2$$

would hold true. For the real counterparts this proportionality does not hold.

When the particle number  $N$  exceeds  $p/n$ , the mass squared of the system reduces from its upper bound  $10^{-4}/\sqrt{G}$  by a factor of order  $1/\sqrt{p}$ . Thus the mass of, say, the electrons inside black hole, is always below this upper bound irrespective of the number of the electrons!

If particles with several p-adic primes are present inside the black hole then the formula for the black hole entropy reads as

$$S = \sum_p S(p) = \sum_p k(p) M^2(p) ,$$

so that the proportionality to the total mass squared does not hold true except approximately (in the case that the mass is in good approximation given by the total mass of a particular particle species).

## 6.6 Why Primes Near Prime Powers Of Two?

The great challenge of TGD is to predict the p-adic prime associated with a given elementary particle. The problem decomposes into the following subproblems.

1. One must understand why there is a definite value of the p-adic prime associated with a given real region of space-time surface (in particular, the space-time surface describing elementary particle) and how this prime is determined. The new view about p-adicity allows to understand the possibility to label elementary particles by p-adic primes if p-adic-real phase transitions occur already at elementary particle level or if real elementary particle regions are accompanied by p-adic space-time sheets possible providing some kind of a cognitive model of particle. The latter alternative has turned out to be the correct one.

The great question mark is the correlation of the p-adic prime characterizing the particle with the quantum numbers of the particle: is this correlation due to the intrinsic properties of the particle or perhaps a result of some kind of adaptation at elementary particle length scales. In the latter case sub-cosmologies with quite different elementary particle mass spectra are possible. On the other and, quantum self-organization does not allow too many final state patterns, so that elementary particle mass spectrum could be more or less a constant of Nature.

2. One must understand why quantum evolution by quantum jumps has led to a situation in which elementary particle like surfaces correspond to some preferred primes. It indeed seems that an evolution at elementary particle level is in question (how p-adic evolution follows from simple number theoretic consistency conditions is discussed in the [K8]). It seems that the degeneracy due to the p-adic space-time regions associated with the system must be counted as giving rise to different final states in a quantum jump between quantum histories. If the number  $N_d(X^3)$  of the physically equivalent cognitive variants of the space-time surface is especially high, this particular physical state dominates over the other final states of the quantum jump. Highly cognitive systems are winners in the fight for survival. Thus in TGD framework evolution is also, and perhaps basically, evolution of cognition.



3. One should also understand why the primes  $p \simeq 2^k$  near prime powers of two are favored physically and to predict the value of  $k$  for an elementary particle with given quantum numbers. The analogy between elementary particles and black holes suggests only a partial explanation for the prime powers of 2 and the real explanation should probably involve enhanced cognitive resources for these primes.

In order to formulate the argument supporting p-adic length scale hypothesis one must first describe the general conceptual background.

1. WCW of the 3-surfaces decomposes into regions  $D_P$  labelled by infinite p-adic primes. In each quantum jump localization of  $CH$  spinor field to single sector  $D_P$  must occur if localization in zero modes occurs. Quantum time development corresponds to a sequence of quantum jumps between quantum histories and the value of the infinite-p p-adic prime  $P$  characterizing the 3-surface associated with the entire universe increases in a statistical sense. This has natural interpretation as evolution. In a well defined sense the infinite prime characterizing infinitely large universe is a composite of finite p-adic primes characterizing various real regions (space-time sheets) of the space-time. The effective infinite-p p-adic topology associated with this infinite prime is very much like real topology since canonical identification mapping infinite number to its real counterpart just drops the infinitesimals of infinite-p p-adic number. Therefore real physics is an excellent approximation at this level. If the S-matrix is complex rational, the approximation is in fact exact. Note that real topology is quite possible also at the level of WCW and WCW might consist of both real and infinite-P p-adic regions.
2. The requirement that quantum jumps correspond to quantum measurements in the sense of QFT, implies that also localization in zero modes occurs in each quantum jump: localization could occur also in the length scale resolution defined by the p-adic length scale  $L_p$ . The strongest hypothesis suggested by the properties of thermodynamical spin glasses is that quantum jump occurs to a state localized around single maximum of the Kähler function.
3. This picture suggests that evolution has occurred already at the elementary particle level and selected preferred p-adic primes characterizing the space-time regions associated with the elementary particles. A crucial question is whether this evolution could have occurred for isolated elementary particles or whether the interaction of the elementary like space-time regions with the surrounding space-time has served as a selective pressure. It might well be that the latter option is the correct one. If this is the case, one can say that the winners in the fight for survival correspond to infinite primes, which are composites of preferred finite primes, perhaps the finite primes given by the p-adic length scale hypothesis.
4. In TGD framework evolution is also evolution of cognition and the most plausible guess is that p-adic non-determinism is what makes cognition possible. Of course, also the classical non-determinism of Kähler action is also present and also important. Perhaps one should call the space-time sheets of finite time duration made possible by this non-determinism as “sensory space-time sheets” as opposed to p-adic space-time sheets. Certainly this non-determinism should be responsible for volition. In any case, the degenerate space-time sheets are not physically equivalent in this case as they are in case of the p-adic non-determinism. The number  $N_d(X^3)$  of the p-adically degenerate and physically equivalent absolute minima  $X^4(X^3)$  of Kähler action is the measure for the cognitive resources of the 3-surface. The basic idea is simple: if  $N_d(X^3)$  is very large then quantum jumps lead with high probability to some degenerate physically equivalent maximum of the Kähler function associated with given value of  $p$ . One can see this also from the point of view of an elementary particle: the high cognitive degeneracy could mean that the particle can adapt to the environment - the surviving elementary particles would be the most intelligent ones! What one should be able to show is that cognitive degeneracy is especially large for some preferred primes so that evolution selects these primes as the most intelligent ones.

About two decades after writing the above lines I would formulate the same idea in terms of hierarchy of Planck constants  $h_{eff} = n \times h$ . Large  $h_{eff}$  assignable to the magnetic body of the particle means high intelligence and large negentropic resources. This can be made

concrete by using concrete model for elementary particles as wormhole contacts connected by Kähler magnetic flux tubes carrying monopole flux at both space-time sheets involved. The magnetic flux tubes would correspond to large  $h_{eff}$ .

In this conceptual framework one can develop more precise variants for arguments supporting the p-adic length scales hypothesis.

1. The simplest possibility is that single maximum of Kähler function is selected in the quantum jump. In this case the relative rate for quantum jumps to a given physical final state with fixed physical configuration is proportional to the p-adic cognitive degeneracy  $N_d(N)$ , where  $N$  denotes the infinite primes characterizing the interacting space-time surface associated with the final state.  $N$  decomposes into a product of infinite primes  $p$  and  $N_d(N)$  decomposes into a product  $N = \prod_p N_d(p)$ .  $N_d(N)$  is maximized if  $N_d(p)$  is maximized. The elementary systems for which  $N_d(p)$  is especially large are winners.
2. The situation reduces to the level of finite p-adic primes if takes seriously the argument allowing to estimate the value of the gravitational constant. The argument was based on the assumption that  $P$  decomposes in a well defined sense into passive primes  $p_i$  and active prime  $p$  characterizing elementary particle: thus there would be the correspondence  $P \leftrightarrow p$ . This suggests that it is possible to understand the finite p-adic prime  $p$  associated with the elementary particle by restricting the consideration to the 3-surfaces describing topologically condensed elementary particles: that is,  $CP_2$  type extremals glued to a space-time sheet with size of order Compton length. p-Adic cognitive degeneracy  $N_d(p)$  should be especially high for p-adic primes predicted by the p-adic length scale hypothesis.
3. The interpretation of p-adic regions as cognitive regions suggests a more concrete explanation for the p-adic length scale hypothesis. The degeneracy due to p-adic non-determinism for the p-adic  $CP_2$  type extremals presumably depends on the value of the p-adic prime characterizing the cognitive version of elementary particle. If real and p-adic space-time sheets accompany each other as adelic vision assumes, one might understand the origin of the p-adic length scale hypothesis. p-Adic primes near prime powers of two are winners because the the degeneracy due to p-adic non-determinism is especially larger for them and allows cognitive representations with high information content. The observed elementary particles would thus dominate in the Universe simply because the thoughts about them are winners in the fight for survival.

The question why particle correspond to preferred p-adic primes(s) must be also answered and here very general number theoretic explanation has emerged [K19]. Therefore the remaining question is why p-adic length scale hypothesis and why the correlation of p-adic prime with the quantum numbers and topology of the partonic 2-surface. Here the answer about two decades later would be that the value of Planck constant  $h_{eff} = n \times h$  measuring the negentropy resources is very large for survivors.

4. The black hole-elementary particle analogy suggests that the primes  $p \simeq 2^k$ ,  $k$  prime, are especially interesting since the radius of the elementary particle horizon is the p-adic length scale  $L(k)$ . This could be understood since k-adicity provides an additional cognitive degeneracy for the absolute minima of Kähler function coming from the region of size  $L(k)$  surrounding a topologically condensed elementary particle and any # contact. This enhances the value of  $N_d(p)$  further by a multiplicative factor  $N_d(k)$  so that  $N_d(p)$  becomes especially large.
5. These arguments do not yet tell how to deduce the prime  $k$  associated with a given elementary particle. Cognitive resources are measured by a negative on an negentropy type quantity proportional to  $N_c = \log(N_d(p))$ . A natural guess is that  $N_c$  is dominated by a term proportional to  $\log(p)$ :  $N_c = A(p) + \log(p)$ . For  $p \simeq 2^k$  one has an additional source of cognitive degeneracy which gives  $N_c = \log(k) + \log(p)$  instead of  $N_c = \log(p)$  and these primes thus correspond to the local maxima of cognitive resources as a function of  $p$ . Quite generally, the larger the  $p$ , the more probable is its appearance as elementary particle prime (neglecting the constraints coming from, say, the cosmic temperature). Hence it seems that the p-adic

evolution of a given elementary particle is frozen to some local maximum of  $N_d(p(k))$ , with  $p(k)$  given by the p-adic length scale hypothesis.

6. Freezing can be understood if the transition probabilities  $P(k \rightarrow k_1)$  are so small that further evolution by quantum jumps is impossible. A possible interpretation of the transition  $k_i \rightarrow k_j$  is a p-adic phase transition changing the elementary particle horizon from radius  $L_{k_i}$  to  $L_{k_j}$  so that  $P(k_i \rightarrow k_j)$  would describe the probability of this phase transition. For neutrinos the transition probabilities  $P(k_i \rightarrow k_j)$  between different sectors allowed by the p-adic length scale hypothesis seem to be largest whereas for higher quark generations they seem to be smallest. Furthermore,  $k$  is smaller for higher generations. In particular,  $P(k_i \rightarrow k_j)$  seems to be largest for spherical boundary topology. This suggests that the (phase) transition probabilities  $P(k_i \rightarrow k_j)$  decrease as a function of the strength of the dominating particle interaction and of the genus of the particle (reflecting itself via the modular contribution to the particle mass increasing as a function of genus).

To sum up, the proposed formula would dictate the evolution of  $\alpha_s$  from the evolution of the electro-weak parameters without any need for perturbative computations. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

## 7 General Vision About Coupling Constant Evolution

**Addition:** The view about coupling constant evolution has changed radically during 2016-2017 [K5, K7, K2, K16] as the number theoretic vision about TGD as adelic physics and the vision about twistor lift of TGD have co-evolved. Number theoretic vision has extremely powerful consequences and has led to amazingly simple proposals for the scattering amplitudes and coupling constant evolution. The following rather old arguments making guesses about the values of coupling constants are in the light of the new vision obsolete so that this section can be regarded more or less as a curiosity. I have however decided to keep the section.

Zero energy ontology, the construction of  $M$ -matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type  $II_1$ , the realization that symplectic invariance of  $N$ -point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination making possible a rather concrete vision about coupling constant evolution in TGD Universe and even a rudimentary form of generalized Feynman rules.

p-Adic coupling constant evolution is discrete by p-adic length scale hypothesis justified by zero energy ontology. Discreteness means that continuous mass scale is replaced by mass scales coming as half octaves of  $CP_2$  mass. One key question has been whether it is Kähler coupling strength  $\alpha_K$  or gravitational coupling constant, which remains invariant under p-adic coupling constant evolution. Second problem relates to the value of  $\alpha_K$ .

The realization that Kähler-Dirac action could be the fundamental variational principle initiated the process, which led to an answer to these and many other questions. The idea that some kind of Dirac determinant gives the vacuum functional identifiable as exponent of Kähler function in turn identifiable as Kähler action  $S_K$  for a preferred extremal came first. The basic challenges are to understand the conditions fixing the preferred extremal of Kähler action and how to define the Dirac determinant. After experimentation with several alternatives it became clear that the Kähler-Dirac action contains besides the term defined by Kähler action also a measurement interaction term guaranteeing quantum classical correspondence. An alternative idea inspired by TGD as almost topological QFT vision and quantum holography was that 3-D Chern-Simons action for light-like 3-surfaces at which the induced metric of the space-time surface changes its signature could be enough. This turned out to be not the case.

The most important outcome is a formula for Kähler coupling strength in terms of a calculable and manifestly finite Dirac determinant without any need for zeta function regularization. The formula fixes completely the number theoretic anatomy of Kähler coupling strength and of other

gauge coupling strengths. When the formula for the gravitational constant involving Kähler coupling strength and the exponent of Kähler action for  $CP_2$  type vacuum extremal - which remains still a conjecture - is combined with the number theoretical results and with the constraints from the predictions of p-adic mass calculations, one ends up to an identification of Kähler coupling strength as fine structure constant at electron length scale characterized by p-adic prime  $M_{127}$ . Also the number theoretic anatomy of the ratio  $R^2/\hbar G$ , where  $R$  is  $CP_2$  size, can be understood to high degree and a relationship between the p-adic evolutions of electromagnetic and color coupling strengths emerges.

## 7.1 General Ideas About Coupling Constant Evolution

### 7.1.1 Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. “Any physical state is creatable from vacuum” becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. The proposed realization of Equivalence Principle at quantum level is based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to  $M^4$  with effective metric satisfying Einstein’s equations as a reflection of the underlying Poincare invariance.

### 7.1.2 Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K4] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the “world of classical worlds” represents a von Neumann algebra [A6] known as hyperfinite factor of type  $II_1$  (HFF) [K4, K20, K6]. HFF [A3, A5] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [A1]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A10], anyons [D2], quantum groups and conformal field theories [A11], and knots and topological quantum field theories [A8, A4].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

One can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale  $T_{p,2} = \sqrt{p}T_p$  by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship  $T_p = L_p^2/Rc$ , where  $R$  is  $CP_2$  size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as  $T_n = 2^{-n}T$  since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K4]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics.

### 7.1.3 How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

In zero energy ontology zero energy states have as embedding space correlates causal diamonds for which the distance between the tips of the intersecting future and past directed light-cones comes as integer multiples of a fundamental time scale:  $T_n = n \times T_0$ . p-Adic length scale hypothesis allows to consider a stronger hypothesis  $T_n = 2^n T_0$  and its generalization a slightly more general hypothesis  $T_n = p^n T_0$ ,  $p$  prime. It however seems that these scales are dynamically favored but that also other scales are possible.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy  $T_n = 2^n T_0$  induce p-adic coupling constant evolution and explain why p-adic length scales correspond to  $L_p \propto \sqrt{p}R$ ,  $p \simeq 2^k$ ,  $R$   $CP_2$  length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of  $\sqrt{2}$  rather than 2 and the strongly favored values of  $k$  are primes and thus odd so that  $n = k/2$  would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time  $t$  satisfies  $r^2 = Dt$  suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces  $X^2$  are as 2-D dynamical systems random apart from light-likeness of their orbit. For  $CP_2$  type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in  $M^4$ . The orbits of Brownian particle would now correspond to light-like geodesics  $\gamma_3$  at  $X^3$ . The projection of  $\gamma_3$  to a time=constant section  $X^2 \subset X^3$  would define the 2-D path  $\gamma_2$  of the Brownian particle. The  $M^4$  distance  $r$  between the end points of  $\gamma_2$  would be given  $r^2 = Dt$ . The favored values of  $t$  would correspond to  $T_n = 2^n T_0$  (the full light-like geodesic). p-Adic length scales would result as  $L^2(k) = DT(k) = D2^k T_0$  for  $D = R^2/T_0$ . Since only  $CP_2$  scale is available as a fundamental scale, one would have  $T_0 = R$  and  $D = R$  and  $L^2(k) = T(k)R$ .
2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via  $T_p = L_p/c$  as assumed implicitly earlier but via  $T_p = L_p^2/R_0 = \sqrt{p}L_p$ , which corresponds to secondary p-adic length scale. For instance, in the case of electron with  $p = M_{127}$  one would have  $T_{127} = .1$  second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to  $L(169) \simeq 5 \mu\text{m}$  (size of a small cell) and  $T(169) \simeq 1. \times 10^4$  years. A deep connection between elementary particle physics and biology becomes highly suggestive.
3. In the proposed picture the p-adic prime  $p \simeq 2^k$  would characterize the thermodynamics of the random motion of light-like geodesics of  $X^3$  so that p-adic prime  $p$  would indeed be an inherent property of  $X^3$ . For  $T_p = pT_0$  the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case,  $p$  would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW .

## 7.2 The Bosonic Action Defining Kähler Function As The Effective Action Associated With The Induced Spinor Fields

One could *define* the classical action defining Kähler function as the bosonic action giving rise to the divergences of the isometry currents. In this manner bosonic action, especially the value of the Kähler coupling strength, would come out as prediction of the theory containing no free parameters.

Thus the Kähler action  $S_B$  of preferred extremal of Kähler action defining Kähler function could be *defined* by the functional integral over the Grassmann variables for the exponent of the massless Dirac action. Formally the functional integral is defined as

$$\begin{aligned} \exp(S_B(X^4)) &= \int \exp(S_F) D\Psi D\bar{\Psi} , \\ S_F &= \bar{\Psi} \left[ \hat{\Gamma}^\alpha D_\alpha^\rightarrow - D_\alpha^\leftarrow \hat{\Gamma}^\alpha \right] \Psi \sqrt{g} . \end{aligned} \tag{7.1}$$

Formally the bosonic effective action is expressible as a logarithm of the fermionic functional determinant resulting from the functional integral over the Grassmann variables

$$\begin{aligned} S_B(X^4) &= \log(\det(D)) , \\ D &= \hat{\Gamma}^\alpha D_\alpha^\rightarrow . \end{aligned} \tag{7.2}$$

### 7.2.1 Formula for the Kähler coupling strength

The identification of exponent of Kähler function as Dirac determinant leads to a formula relating Kähler action for the preferred extremal to the Dirac determinant. The eigenvalues are proportional to  $1/\alpha_K$  since the matrices  $\hat{\Gamma}^\alpha$  have this proportionality. This gives the formula

$$\exp\left(\frac{S_K(X^4(X^3))}{8\pi\alpha_K}\right) = \prod_i \lambda_i = \frac{\prod_i \lambda_{0,i}}{\alpha_K^N} . \tag{7.3}$$

Here  $\lambda_{0,i}$  corresponds to  $\alpha_K = 1$ .  $S_K = \int J^* J$  is the reduced Kähler action.

For  $S_K = 0$ , which might correspond to so called massless extremals [K3] one obtains the formula

$$\alpha_K = \left(\prod_i \lambda_{0,i}\right)^{1/N} . \tag{7.4}$$

Thus for  $S_K = 0$  extremals one has an explicit formula for  $\alpha_K$  having interpretation as the geometric mean of the eigenvalues  $\lambda_{0,i}$ . Several values of  $\alpha_K$  are in principle possible.

p-Adicization suggests that  $\lambda_{0,i}$  are rational or at most algebraic numbers. This would mean that  $\alpha_K$  is  $N$ : th root of this kind of number.  $S_K$  in turn would be

$$S_K = 8\pi\alpha_K \log\left(\frac{\prod_i \lambda_{0,i}}{\alpha_K^N}\right) . \tag{7.5}$$

so that  $S_K$  would be expressible as a product of the transcendental  $\pi$ ,  $N$ : th root of rational, and logarithm of rational. This result would provide a general answer to the question about number theoretical anatomy of Kähler coupling strength and  $S_K$ . Note that  $S_K$  makes sense p-adically only if one adds  $\pi$  and its all powers to the extension of p-adic numbers. The exponent of Kähler function however makes sense also p-adically.

### 7.3 A Revised View About Coupling Constant Evolution

The development of the ideas related to number theoretic aspects has been rather tortuous and based on guess work since basic theory has been lacking.

1. The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. Second first guess was that Kähler coupling strength equals to the value of fine structure constant at electron length scale corresponding to Mersenne prime  $M_{127}$ . Later I replaced fine structure constant with electro-weak U(1) coupling strength at this length scale. The recent discussion returns back to the roots in both aspects.
2. The recent discussion relies on the progress made in the understanding of quantum TGD at partonic level [K21]. What comes out is an explicit formula for Kähler couplings strength in terms of Dirac determinant involving only a finite number of eigenvalues of the Kähler-Dirac operator. This formula dictates the number theoretical anatomy of  $g_K^2$  and also of other coupling constants: the most general option is that  $\alpha_K$  is a root of rational. The requirement that the rationals involved are simple combined with simple experimental inputs leads to very powerful predictions for the coupling parameters.
3. A further simplification is due to the discreteness of p-adic coupling constant evolution allowing to consider only length scales coming as powers of  $\sqrt{2}$ . This kind of discretization is necessary also number theoretically since logarithms can be replaced with 2-adic logarithms for powers of 2 giving integers. This raises the question whether  $p \simeq 2^k$  should be replaced with  $2^k$  in all formulas as the recent view about quantum TGD suggests.
4. The prediction is that Kähler coupling strength  $\alpha_K$  is invariant under p-adic coupling constant evolution and from the constraint coming from electron and top quark masses very near to fine structure constant so that the identification as fine structure constant is natural. Gravitational constant is predicted to be proportional to p-adic length scale squared and corresponds to the largest Mersenne prime ( $M_{127}$ ), which does not correspond to a completely super-astronomical p-adic length scale. For the parameter  $R^2/G$  p-adicization program allows to consider two options: either this constant is of form  $e^q$  or  $2^q$ : in both cases  $q$  is rational number.  $R^2/G = \exp(q)$  allows only  $M_{127}$  gravitons if number theory is taken completely seriously.  $R^2/G = 2^q$  allows all p-adic length scales for gravitons and thus both strong and weak variants of ordinary gravitation.
5. A relationship between electromagnetic and color coupling constant evolutions based on the formula  $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$  is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of  $\alpha_s$  at intermediate boson length scale is correct.

It seems fair to conclude that the attempts to understand the implications of p-adicization for coupling constant evolution have begun to bear fruits.

#### 7.3.1 Identifications of Kähler coupling strength and gravitational coupling strength

To construct an expression for gravitational constant one can use the following ingredients.

1. The exponent  $\exp(2S_K(CP_2))$  defining the value of Kähler function in terms of the Kähler action  $S_K(CP_2)$  of  $CP_2$  type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{S_{K,R}(CP_2)}{8\pi\alpha_K} = \frac{\pi}{8\alpha_K} . \quad (7.6)$$

Since  $CP_2$  type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) . \quad (7.7)$$

$a < 1$  conforms with the idea that a piece of  $CP_2$  type extremal defining a wormhole contact is in question. One must however keep mind open in this respect.

2. The p-adic length scale  $L_p$  assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime  $M_{127} = 2^{127} - 1$  defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that  $M_{127}$  characterizes these space-time sheets.

1. *The formula for the gravitational constant*

A long standing basic conjecture has been that gravitational constant satisfies the following formula

$$\begin{aligned} \hbar G &\equiv r \hbar_0 G = L_p^2 \times \exp(-2a S_K(CP_2)) , \\ L_p &= \sqrt{p} R . \end{aligned} \quad (7.8)$$

Here  $R$  is  $CP_2$  radius defined by the length  $2\pi R$  of the geodesic circle. What was noticed before is that this relationship allows even constant value of  $G$  if  $a$  has appropriate dependence on  $p$ .

This formula seems to be correct but the argument leading to it was based on two erratic assumptions compensating each other.

1. I assumed that modulus squared for vacuum functional is in question: hence the factor  $2a$  in the exponent. The interpretation of zero energy state as a generalized Feynman diagram requires the use of vacuum functional so that the replacement  $2a \rightarrow a$  is necessary.
2. Second wrong assumption was that graviton corresponds to  $CP_2$  type vacuum extremal—that is wormhole contact in the recent picture. This does allow graviton to have spin 2. Rather, two wormhole contacts represented by  $CP_2$  vacuum extremals and connected by fluxes associated with various charges at their throats are needed so that graviton is string like object. This saves the factor  $2a$  in the exponent.

The highly non-trivial implication to be discussed later is that ordinary coupling constant strengths should be proportional to  $\exp(-a S_K(CP_2))$ .

The basic constraint to the coupling constant evolution comes for the invariance of  $g_K^2$  in p-adic coupling constant evolution:

$$\begin{aligned} g_K^2 &= \frac{a(p, r) \pi^2}{\log(pK)} , \\ K &= \frac{R^2}{\hbar G(p)} = \frac{1}{r} \frac{R^2}{\hbar_0 G(p)} \equiv \frac{K_0(p)}{r} . \end{aligned} \quad (7.9)$$

2. *How to guarantee that  $g_K^2$  is RG invariant and  $N$ : th root of rational?*

Suppose that  $g_K^2$  is  $N$ : th root of rational number and invariant under p-adic coupling constant evolution.

1. The most general manner to guarantee the expressibility of  $g_K^2$  as  $N$ : th root of rational is guaranteed for both options by the condition

$$a(p, r) = \frac{g_K^2}{\pi^2} \log\left(\frac{pK_0}{r}\right) . \quad (7.10)$$

That  $a$  would depend logarithmically on  $p$  and  $r = \hbar/\hbar_0$  looks rather natural. Even the invariance of  $G$  under p-adic coupling constant evolution can be considered.



2. The condition

$$\frac{r}{p} < K_0(p) . \quad (7.11)$$

must hold true to guarantee the condition  $a > 0$ . Since the value of gravitational Planck constant is very large, also the value of corresponding p-adic prime must very large to guarantee this condition. The condition  $a < 1$  is guaranteed by the condition

$$\frac{r}{p} > \exp\left(-\frac{\pi^2}{g_K^2}\right) \times K_0(p) . \quad (7.12)$$

The condition implies that for very large values of  $p$  the value of Planck constant must be larger than  $\hbar_0$ .

3. The two conditions are summarized by the formula

$$K_0(p) \times \exp\left(-\frac{\pi^2}{g_K^2}\right) < \frac{r}{p} < K_0(p) \quad (7.13)$$

characterizing the allowed interval for  $r/p$ . If  $G$  does not depend on  $p$ , the minimum value for  $r/p$  is constant. The factor  $\exp\left(-\frac{\pi^2}{g_K^2}\right)$  equals to  $1.8 \times 10^{-47}$  for  $\alpha_K = \alpha_{em}$  so that  $r > 1$  is required for  $p \geq 4.2 \times 10^{-40}$ .  $M_{127} \sim 10^{38}$  is near the upper bound for  $p$  allowing  $r = 1$ . The constraint on  $r$  would be roughly  $r \geq 2^{k-131}$  and  $p \simeq 2^{131}$  is the first p-adic prime for which  $\hbar > 1$  is necessarily. The corresponding p-adic length scale is 1 Angstroms.

This conclusion need not apply to elementary particles such as neutrinos but only to the space-time sheets mediating gravitational interaction so that in the minimal scenario it would be gravitons which must become dark above this scale. This would bring a new aspect to vision about the role of gravitation in quantum biology and consciousness.

The upper bound for  $r$  behaves roughly as  $r < 2.3 \times 10^7 p$ . This condition becomes relevant for gravitational Planck constant  $GM_1 M_2 / v_0$  having gigantic values. For Earth-Sun system and for  $v_0 = 2^{-11}$  the condition gives the rough estimate  $p > 6 \times 10^{63}$ . The corresponding p-adic length scale would be of around  $L(215) \sim 40$  meters.

4. p-Adic mass calculations predict the mass of electron as  $m_e^2 = (5 + Y_e)2^{-127}/R^2$  where  $Y_e \in [0, 1)$  parameterizes the not completely known second order contribution. Top quark mass favors a small value of  $Y_e$  (the original experimental estimates for  $m_t$  were above the range allowed by TGD but the recent estimates are consistent with small value  $Y_e$  [K15]). The range  $[0, 1)$  for  $Y_e$  restricts  $K_0 = R^2/\hbar_0 G$  to the range  $[2.3683, 2.5262] \times 10^7$ .
5. The best value for the inverse of the fine structure constant is  $1/\alpha_{em} = 137.035999070(98)$  and would correspond to  $1/g_K^2 = 10.9050$  and to the range  $(0.9757, 0.9763)$  for  $a$  for  $\hbar = \hbar_0$  and  $p = M_{127}$ . Hence one can seriously consider the possibility that  $\alpha_K = \alpha_{em}(M_{127})$  holds true. As a matter fact, this was the original hypothesis but was replaced later with the hypothesis that  $\alpha_K$  corresponds to electro-weak  $U(1)$  coupling strength in this length scale. The fact that  $M_{127}$  defines the largest Mersenne prime, which does not correspond to super-astrophysical length scale might relate to this co-incidence.

To sum up, the recent view about coupling constant evolution differs strongly from previous much more speculative scenarios. It implies that  $g_K^2$  is root of rational number, possibly even rational, and can be assumed to be equal to  $e^2$ . Also  $R^2/\hbar G$  could be rational. The new element is that  $G$  need not be proportional to  $p$  and can be even invariant under coupling constant evolution since the parameter  $a$  can depend on both  $p$  and  $r$ . An unexpected constraint relating  $p$  and  $r$  for space-time sheets mediating gravitation emerges.

### 7.3.2 Are the color and electromagnetic coupling constant evolutions related?

Classical theory should be also able to say something non-trivial about color coupling strength  $\alpha_s$  too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.
2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that  $\alpha_K$  is a strict RG invariant. One can consider two options.

1. The original idea was that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for  $\alpha_{U(1)} = \alpha_s = 2\alpha_K$ . Asymptotically the couplings would approach to a fixed point defined by  $2\alpha_K$  rather than to zero as in asymptotically free gauge theories.

Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \quad (7.14)$$

The relationship between U(1) and em coupling strengths is

$$\begin{aligned} \alpha_{U(1)} &= \frac{\alpha_{em}}{\cos^2(\theta_W)} \simeq \frac{1}{104.1867} , \\ \sin^2(\theta_W)|_{10 \text{ MeV}} &\simeq 0.2397(13) , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 . \end{aligned} \quad (7.15)$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value  $\sin^2(\theta_W) = 0.2397(13)$  corresponding to 10 MeV mass scale [E1] is used. Note however that the previous argument implying  $\alpha_K = \alpha_{em}(M_{127})$  excludes  $\alpha = \alpha_{U(1)}(M_{127})$  option.

2. Second option is obtained by replacing U(1) with electromagnetic gauge  $U(1)_{em}$ .

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \quad (7.16)$$

Possible justifications for this assumption are following. The notion of induced gauge field makes it possible to characterize the dynamics of classical electro-weak gauge fields using only the Kähler part of electro-weak action, and the induced Kähler form appears only in the electromagnetic part of the induced classical gauge field. A further justification is that em and color interactions correspond to unbroken gauge symmetries.

The following arguments are consistent with this conclusion.

1. In TGD framework coupling constant is discrete and comes as powers of  $\sqrt{2}$  corresponding to p-adic primes  $p \simeq 2^k$ . Number theoretic considerations suggest that coupling constants  $g_i^2$  are algebraic or perhaps even rational numbers, and that the logarithm of mass scale appearing as argument of the renormalized coupling constant is replaced with 2-based logarithm of the p-adic length scale so that one would have  $g_i^2 = g_i^2(k)$ .  $g_K^2$  is predicted to be  $N$ : th root of rational but could also reduce to a rational. This would allow rational values for other coupling strengths too. This is possible if  $\sin(\theta_W)$  and  $\cos(\theta_W)$  are rational numbers which would mean that Weinberg angle corresponds to a Pythagorean triangle as proposed already earlier. This would mean the formulas  $\sin(\theta_W) = (r^2 - s^2)/(r^2 + s^2)$  and  $\cos(\theta_W) = 2rs/(r^2 + s^2)$ .

2. A very strong prediction is that the beta functions for color and  $U(1)$  degrees of freedom are apart from sign identical and the increase of  $U(1)$  coupling compensates the decrease of the color coupling. This allows to predict the hard-to-calculate evolution of QCD coupling constant strength completely.
3.  $\alpha(M_{127}) = \alpha_K$  implies that  $M_{127}$  defines the confinement length scale in which the sign of  $\alpha_s$  becomes negative. TGD predicts that also  $M_{127}$  copy of QCD should exist and that  $M_{127}$  quarks should play a key role in nuclear physics [K17, L1], [L1]. Hence one can argue that color coupling strength indeed diverges at  $M_{127}$  (the largest not completely super-astrophysical Mersenne prime) so that one would have  $\alpha_K = \alpha(M_{127})$ . Therefore the precise knowledge of  $\alpha(M_{127})$  in principle fixes the value of parameter  $K = R^2/G$  and thus also the second order contribution to the mass of electron.
4.  $\alpha_s(M_{89})$  is predicted to be  $1/\alpha_s(M_{89}) = 1/\alpha_K - 1/\alpha(M_{89})$ .  $\sin^2(\theta_W) = .23120$ ,  $\alpha_{em}(M_{89}) \simeq 1/127$ , and  $\alpha_{U(1)} = \alpha_{em}/\cos^2(\theta_W)$  give  $1/\alpha_{U(1)}(M_{89}) = 97.6374$ .  $\alpha = \alpha_{em}$  option gives  $1/\alpha_s(M_{89}) \simeq 10$ , which is consistent with experimental facts.  $\alpha = \alpha_{U(1)}$  option gives  $\alpha_s(M_{89}) = 0.1572$ , which is larger than QCD value. Hence  $\alpha = \alpha_{em}$  option is favored.

### 7.3.3 Can one deduce formulae for gauge couplings?

The improved physical picture behind gravitational constant allows also to consider a general formula for gauge couplings.

1. The natural guess for the general formula would be as

$$g^2(p, r) = kg_K^2 \times \exp[-a_g(p, r) \times S_K(CP_2)] . \quad (7.17)$$

here  $k$  is a numerical constant.

2. The condition

$g_K^2 = e^2(M_{127})$  fixes the value of  $k$  if it's value does not depend on the character of gauge interaction:

$$k = \exp[a_{gr}(M_{127}, r = 1) \times S_K(CP_2)] . \quad (7.18)$$

Hence the general formula reads as

$$g^2(p, r) = g_K^2 \times \exp[(-a_g(p, r) + a_{gr}(M_{127}, r = 1)) \times S_K(CP_2)] . \quad (7.19)$$

The value of  $a(M_{127}, r = 1)$  is near to its maximum value so that the exponential factor tends to increase the value of  $g^2$  from  $e^2$ . The formula can reproduce  $\alpha_s$  and various electro-weak couplings although it is quite possible that Weinberg angle corresponds to a group theoretic factor not representable in terms of  $a_g(p, r)$ . The volume of the  $CP_2$  type vacuum extremal would characterize gauge bosons. Analogous formula should apply also in the case of Higgs.

3.  $\alpha_{em}$  in very long length scales would correspond to

$$e^2(p \rightarrow \infty, r = 1) = e^2 \times \exp[(-1 + a(M_{127}, r = 1)) \times S_K(CP_2)] = e^2 x , \quad (7.20)$$

where  $x$  is in the range  $[0.6549, 0.6609]$ .

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