Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory

M. Pitkänen,

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Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland. ORCID: 0000-0002-8051-4364.

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Abstract

The understanding of the unitarity of the S-matrix has remained a major challenge of Topological Geometrodynamics (TGD) for 4 decades. It has become clear that some basic principle is still lacking. Assigning S-matrix to a unitary evolution works in non-relativistic theory but fails already in the generic quantum field theory (QFT). The solution of the problem turned out to be extremely simple. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space would replace unitary S-matrix.

An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of Teichmueller matrix. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong from of holography (SH), the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmueller condition guaranteeing non-negative probabilities.

Equivalence Principle generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

1 Introduction

I have worked with the problem of understanding the construction of scattering amplitudes in the framework provided by Topological Geometrodynamics (TGD) for about four decades. It soon became clear that the naïve generalization of the path integral approach to a path integral over space-time surfaces did not work because of the horrible non-linearities involved. Around 1985 I started to work with the notion that I later called the "world of classical worlds" (WCW). Eventually I apprehended that the realization of general coordinate invariance (GCI) forces to assign to a 3-surface possibly unique space-time surface (X^4) at which the general coordinate transformations act [K5, K3]. Holography would reduce to GCI. The intuitive expectation is that either space-like 3-surfaces or light-like partonic orbits defining boundaries between Minkowskian and Euclidian space-time regions should be enough to determine X^4 as an analog of Bohr orbit. This leads to strong form of holography (SH) stating that data at partonic 2-surfaces and string world sheets code for X^4.

It should be possible to geometrize the entire quantum physics in terms of WCW geometry and associated spinor structure identifying WCW spinors as fermionic Fock states. A geometrization of the hermitian conjugation essential in quantum theory is needed. This fixed the WCW geometry to be Kähler geometry determined by Kähler function and defining Kähler form providing a realization of the imaginary unit as an antisymmetric tensor [K5]. The existence of Riemann connection fixes the Kähler geometry uniquely already in the case of loop spaces [A5]: maximal isometry group is required. In TGD framework it would correspond supersymplectic transformations of $\delta M^4_\pm \times CP_2$, where $\delta M^4_\pm$,denotes future or past light-cone [K3].

Classical physics becomes an exact part of quantum physics if the space-time surfaces are preferred extremals for some action and therefore analogous to Bohr orbits. Spinor fields should obey the modified Dirac equation (MDE). Modified Dirac action (MDA) is determined by the bosonic action via supersymmetry condition. Kähler function identified as the action for the preferred extremal associated with the 3-surface defines in complex coordinates the Kähler metric and Kähler form via its second derivatives of type $(1,1)$.

The natural looking identification of the action was as Kähler action - a non-linear generalization of Maxwell action replacing Maxwell field and metric with induced Kähler form and metric. It possessed a huge vacuum degeneracy interpreted as spin glass degeneracy and for a long time I looked this feature as something positive despite the fact that the WCW metric becomes degenerate at the vacuum extremals and classical determinism is lost. The addition of volume term having interpretation in terms of cosmological constant would have been a possible cure but would have broken conformal invariance bringing in an *ad hoc* dimensional coupling.

Decades later the proposal for a twistor lift of TGD led to the identification of fundamental action as an analog of Kähler action for 6-D twistor spaces having X^4 as base space and S^2 as fiber [L25]. The induction of the twistor structure from that for the 6+6-D product of twistor spaces of M^4 and CP_2 (these spaces are the only 4-spaces allowing twistor space with Kähler structure [A7] so that TGD is unique) to the 6-surface forces a dimensional reduction reducing 6-D Kähler action to a sum of 4-D Kähler action and volume them. The counterpart of the cosmological constant emerges dynamically. Λ depends on the p-adic length scale characterizing space-time surfaces and approaches to zero in long length scales [L25].

The ontology of standard quantum theory in which 3-D $t = constant$ slice of space-time contains the quantum states, does not fit nicely with TGD framework. Space-time surfaces in 1-1 correspondence with 3-surfaces are more natural objects to consider. This conforms also with the notion of holography implied by GCI: actually SH is highly suggestive and means that 2-D data at partonic 2-surfaces and string world sheets determined the X^4 as a preferred extremal. In particular, various anomalies suggest that the arrow of time need not be fixed.

Eventually this led to zero energy ontology (ZEO) [L23] in which quantum states are essentially superpositions of preferred extremals inside causal diamond (CD): space-time surfaces have ends at the boundaries of CD and these pairs of 3-surfaces or equivalently the 4-surfaces are the basic objects. CDs form a hierarchy: there are CDs with CDs and CDs can also intersect. They would form an analog of atlas of coordinate charts. Each CD would serve as a correlate for a conscious entity so that the charts can be said to be conscious.

ZEO leads to a quantum measurement theory and allows avoiding the basic problems of the standard quantum measurement theory. Zero energy states correspond to state pairs at opposite boundaries of CD or equivalently, superpositions of deterministic time evolutions. In state function reduction (SFR) as a superposition of classical deterministic time evolutions is replaced with a new one.

"Big" and "small" state function reduction - BSFR and SSFR - are the basic notions. In SSFRs as analogs of "weak" measurements following a unitary time evolution, the size of CD increases in statistical sense. The members of the state pairs associated with the passive boundary of CD do not change during SSFRs: this gives rise to the analog of Zeno effect. The active boundary and the states at it change. Active boundary also shifts farther from the passive one. BSFRs correspond to ordinary state function reductions and in BSFRs the arrow of time changes. One could speak of a death of a conscious entity in universal sense and reincarnation with an opposite arrow of time. For instance, the findings of Minev *et al* [L21] provide support for the time reversal [L21].

1.1 How to construct the TGD counterpart of unitary S-matrix?

The concrete construction of scattering amplitudes remained a challenge from very beginning. During years I have proposed several proposals and many important aspects of the problem are understood but simple rules are still lacking.

1. The time evolutions assignable to SSFRs should be describable by a unitary S-matrix or its

analog.

2. The counterpart of S-matrix should have the huge super-symplectic algebra (SSA) and Kac-Moody algebras related to isometries of H as symmetries. These symmetries, extended further to Yangian symmetries and quantum groups with both algebra and co-algebra structure, are expected to be a key element in the construction of the counterpart of S-matrix. In particular, product and co-product in the super-symplectic algebra define excellent candidates for vertices. What has been missing was a concrete guiding principle.

3. Feynman (or twistor) diagrammatics should generalize. Point-like particles are replaced with 3-surfaces and topologically incoming and outgoing many-particle states correspond to disjoint unions of 3-surfaces at the boundaries of CD. The first guess is that the vertices correspond to 3-surfaces at which 4-D lines of the analog of Feynman diagram meet. SH and $M^8 - H$ duality [L22] however suggest that the lines of the diagrams should correspond to 3-D light-like orbits of partonic 2-surfaces defining boundaries between space-time regions with Euclidian and Minkowskian signature of the induced metric. Also string world sheets connecting them and also serving as carriers of information in SH should be considered. The 1-D light-like intersections of strings world sheets with partonic orbits would define carriers of fermion number.

4. The identification of fermionic anti-commutation relations was a longstanding challenge. It turned out that the induction of second quantized free fermion fields from H to X^4 fixes the anti-commutations of the induced spinor fields and allows to calculate fermionic propagators. Therefore quantum algebra would give what is needed to calculate scattering amplitudes: the interaction vertices assignable to partonic 2-surfaces and fermionic propagators would result from the induction procedure. 8-D fermions have however 7-D delta functions as anti-commutators and normal ordering of fermions can produce divergences already at the level of the MDA.

   The problem disappears if the MDA is made bilocal [L35]: in this article a more detailed discussion is given and leads to a rather detailed picture about MDA.

5. $M^8 - H$ duality [L22, L14, L15, L16] allows to concretize this picture. One can regard X^4 either as a surface in the complexified M^8 or in H. $M^8 - H$ duality maps space-time surfaces from M^8 to H. Space-time surfaces in the complexified M^8 correspond to algebraic 4-surfaces determined by real polynomials with real (rational if one requires p-adicization) coefficients. Also rational and even analytic functions can be considered, in which case polynomials could be seen as approximations. The roots of the real polynomial dictate the space-time surfaces as quaternionic/associative 4-surfaces in complexified octonionic M^8. Holography becomes discrete.

   The algebraic equations defining space-time surfaces also have special solutions, in particular 6-spheres. These analogs of 6-branes have as M^4 projections in both M^8 and $H = M^4 \times CP_2 t = r_n$ hyperplanes, where r_n corresponds to a root of a real polynomial defining X^4 in complexified M^8 The interpretation of these hyper-planes is in TGD inspired consciousness is as "very special moments in the life of self".

   The solutions of the analog of Dirac equation in M^8 as algebraic equation [L36] are localized to 3-D light-like surfaces and mapped to light-like 3-surfaces in H identifiable as orbits of partonic 2-surfaces. Partonic 2-surfaces serving as vertices of topological analogs of Feynman diagrams would reside at the above described $t = r_n$ hyperplanes of $H = M^4 \times CP_2$. Scattering amplitudes would have partonic 2-surfaces as vertices and their 3-D light-like orbits as lines. The intersections of string world sheets with the partonic orbits would be 1-D lines and could be interpreted as fermion lines so that also the point particle description would be part of the picture.

   CDs inside CD would define the regions inside which particle reactions occur and this suggests a fractal hierarchy of CDs within CDs as a counterpart for the hierarchy of radiative corrections.

What is still missing is the general principle allowing a bird's eye of view about the counterpart of S-matrix. Wheeler was the first to introduce the notion of unitary S-matrix, which generalizes probability conservation to an infinite number of conditions. Could one challenge the unitary principle and consider something else instead of it?

1. Unitary time evolution is natural in non-relativistic quantum mechanics but is already problematic in quantum field theory (QFT), in particular in twistor Grassmannian approach [B5]. The idea about the reduction of physics to Kähler geometry inspires the question whether Kähler geometry of WCW could provide a general principle for the construction of the scattering amplitudes and perhaps even an explicit formulas for them.

   Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurements. Could the Kähler metric of state space code the counterpart of S-matrix and even unitary S-matrix? Also the Kähler metric satisfies conditions analogous to unitarity conditions.

   An amazingly simple argument demonstrates that one could construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric the analog of a unitary S-matrix by assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied.

   (a) If the probabilities are identified as the real parts of complex analogs $p^c_{i,j} = g_{i,\overline{j}}g^{\overline{j},i}$ of probabilities, it is enough to require $Re(p^c_{i,j}) \geq 0$. The complex analogs of $ip^c_{i,j})$ would define the analog of Teichmueller matrix [A3, A6, A4] (https://en.wikipedia.org/wiki/Teichm\unhbox\voidb@x\bgroup\let\unhbox\voidb@x\setbox\@tempboxa\hbox{u\global\mathchardef\accent@spacefactor\spacefactor}\let\begingroup\endgroup\relax\let\ignorespaces\relax\accent127u\egroup\spacefactor\accent@spacefactorller_space) for which imaginary parts of matrix elements are non-negative. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By SH, the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

   (b) By positing the condition that $g_{i,\overline{j}}$ and $g^{\overline{j},i}$ have opposite phases, one can assign to Kähler metric a unitary S-matrix but this does not seem to be necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique. These solutions would be special case of Teichmueller solutions: Teichmueller matrix would be purely imaginary. The condition looks too restrictive. For instance, for torus, this would correspond to a metric conformally equivalent with a flat metric.

2. This inspires the idea that quantum physics could be geometrized by the same way as Einstein geometrized gravitation. Take a flat Hilbert space bundle (in the case of TGD) and replace its flat Kähler metric both base space and fiber with a non-flat Kähler metric. The replacement of flat metric with a curved one would lead from a non-interacting quantum theory to an interacting one. Quantum theory would be gravitation at the level of this Hilbert bundle! This replacement is completely universal.

   In the TGD framework the world of classical worlds (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at X^4 and induced from second quantized spinors of the embedding space. Scattering amplitudes would be determined by the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in ZEO and satisfying Teichmueller condition guaranteeing non-negative probabilities.

   WCW geometry is also characterized by zero modes corresponding to non-complex coordinates for WCW giving no contribution to WCW metric. This is self-evident from SH. The zero modes would be in 1-1 correspondence with Teichmueller parameters and WCW Kähler metrics.

   Equivalence Principle (EP) generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give a possible interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic QFT [K7].

There is also challenge of constructing the Kähler metric and associated spinor structure for the spinor bundle of WCW. This would mean a specification of the analogs of Feynman rules so that instead of two problems one would have only one problem.

1. WCW gamma matrices can be identified as superpositions of fermionic oscillator operators associated with quark spinors [L24]. One can consider two approaches to the quantization of these spinors: one studies induced spinor fields obeying MDE and quantizes this or one generalizes the induction of spinors from H to the induction of second quantized spinor fields in H: this would mean simply projecting the spinor fields to X^4. The latter option is extremely simple. It seems possible to avoid divergence problems if the anti-commutators are assigned to different 3-surfaces at different boundaries of CD. This would allow the identification of the Dirac propagator. As a matter of fact, the two approaches are equivalent.

2. WCW gamma matrices would allow the identification as super generators of SSA identified as contractions of gamma matrices SSA with Killing vectors. Quantum states would be created by bosonic and fermionic SSA generators.

3. I have proposed a further supersymmetrization of both H coordinates and spinors by replacing them with expansions in powers of local composites of oscillator operators for quarks and antiquarks [L24]. This however requires Kronecker delta type anti-commutators natural for cognitive representations defining unique discretization of X^4: this allows to avoid normal ordering divergences. Induction of the H spinor fields would lead to 8-D delta function type divergences. This suggests that local composites are not quite local but states consisting of quarks and antiquarks at opposite throats of wormhole contacts identifiable as partonic 2-surfaces. One would obtain leptons as 3-quark states with quarks at the same partonic 2-surface but not at the same point anymore as in the proposal of [L24].

4. The matrix elements of the Kähler metric of WCW Hilbert bundle correspond to scattering amplitudes analogous to Feynman diagrams. What are the Feynman rules? Partonic two surfaces and their orbits correspond to vertices and propagators topologically. The TGD counterpart for $F\overline{F}B$ vertex would correspond to a bosonic wormhole contact with a fermion and antifermion at opposite wormhole throats and representing SCA generator which decomposes to two partonic 2-surfaces carrying fermions at opposite throats representing fermionic SCA generators. This allows avoiding of normal ordering divergences.

   The vertex would correspond to a product or co-product, which can be said to be time reversals of each other. The structure constants of SCA extended to quantum algebra would fix the vertices and thus the analogs of Feynman diagrams completely. Their number is presumably finite for a X^4 with fixed 3-surfaces at its ends and summation over Feynman diagrams would correspond to integration in WCW.

Before discussing them current proposal in detail, the complementary way to overview TGD as either WCW geometry or as number theory are discussed below. Readers might skip these sections at their first reading and choose to read the section discussing the basic idea in more detail.

In the sequel the basic idea about representation of scattering amplitudes as elements of Kähler metric satisfying what I call "Teichmueller condition", is discussed in TGD framework.

The detailed formulation allows a formulation of conditions for the cancellation of normal ordering divergences and also other divergences. The induction of the second quantized free spinor field from H to space-time surface fixes the propagators at the space-time level. If the creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts, divergences are avoided. ZEO suggests an alternative but not exclusive option that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

The fact that the Dirac propagators for massive particles in the TGD sense reduce in a good approximation to massless propagators when the propagation takes place along light-like distances, allows to considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

Of course, one must not forget that this proposal is only an interesting thought game. It is quite possible that zero energy ontology allows to define a natural way a unitary S-matrix or a more general isometric map between the states spaces associated with the extensions of rationals with different algebraic dimensions assignable naturally to to space-time regions inside causal diamonds. The huge symmetries of WCW generalized to Yangian symmetries could lead to a unique S-matrix and number theoretic conditions pose extremely powerful constraints. In [L39], a proposal along these lines was developed 3 years after writing this.

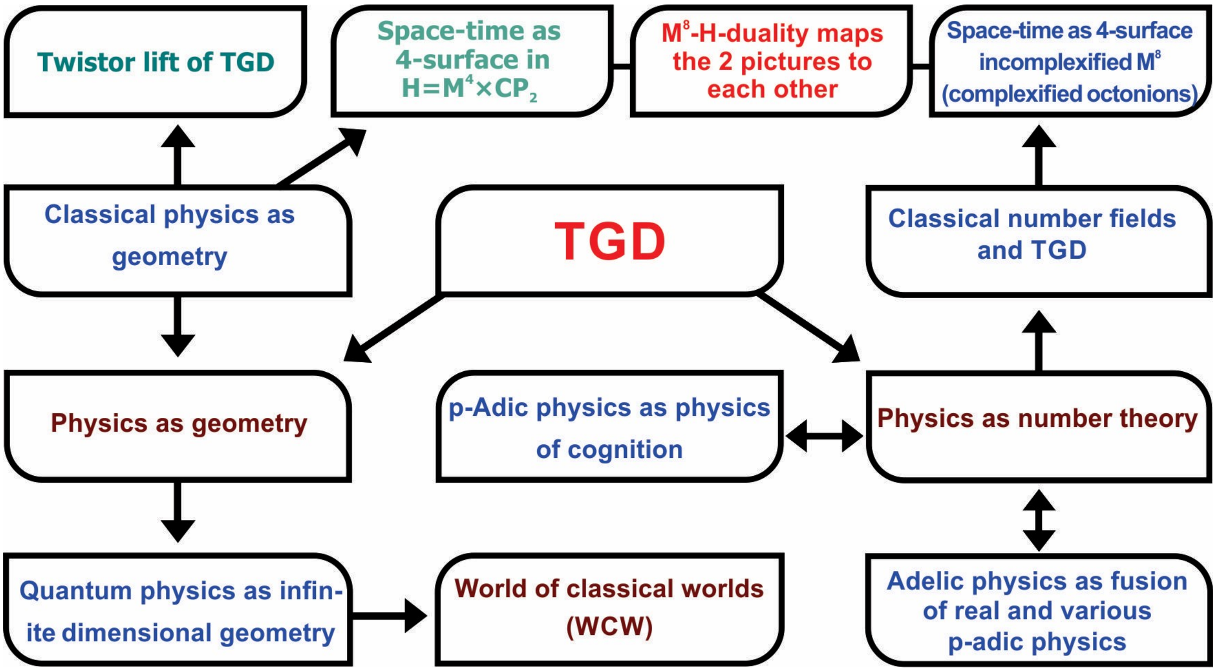

Figure 1: TGD is based on two complementary visions: physcs as geometry and physics as number theory.

2 Physics as geometry

One can end up with TGD in two ways (see **Fig. 2**). Either as a solution of energy problem of GRT realizing Einstein's dream about geometrization of classical physicsor as a generalization of hadronic string model or of superstring theory [B9]. In case of hadronic string model the generalization of string to 3-surface would allow to get rid of spontaneous compactification and the landscape catastrophe implied by it.

At fundamental level TGD could be seen as a hybrid of GRT and SRT: the notion of force does not disappear and can be defined as rate for an exchange of conserved quantity which can be Poincare or color charge. This connection with Newtonian limit is more clear than in GRT, where the conservation laws are lost.

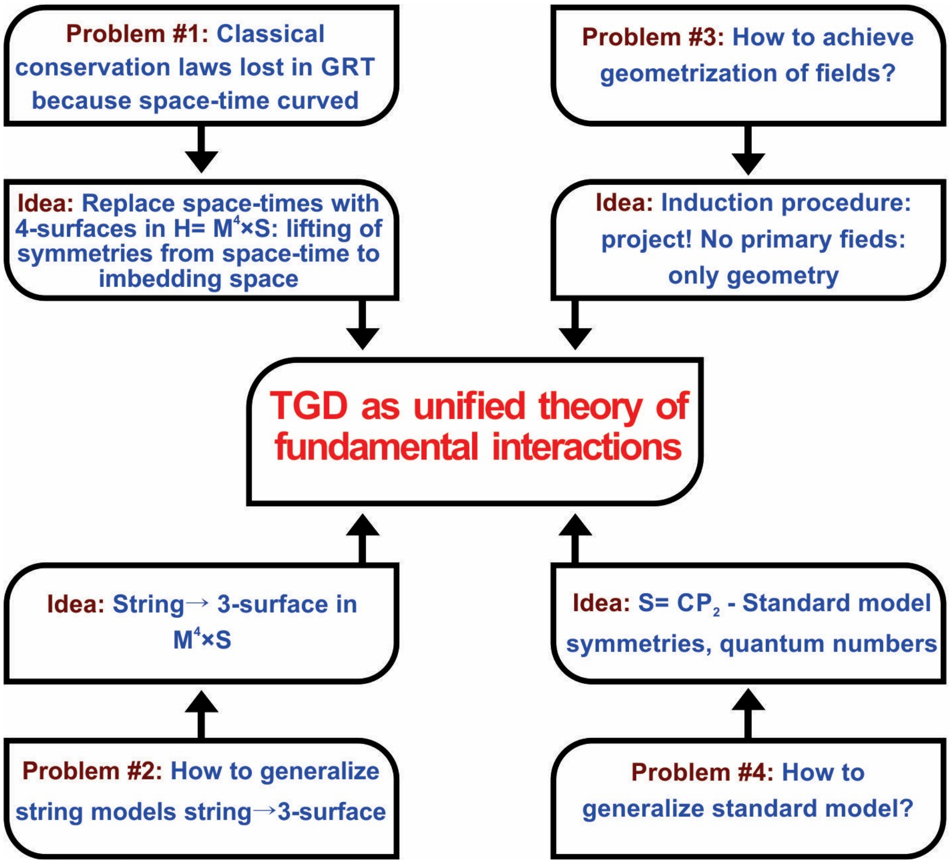

Figure 2: The problems leading to TGD as their solution.

2.1 Classical physics as sub-manifold geometry

The new elements are many-sheeted space-time topologically non-trivial in all scales, and topological field quantization implying that physical systems have field identity, field body, in particular magnetic body (MB) central in applications [L2, L1] (see **Fig. 3**).

2.1.1 Induction procedure

One ends up to a geometrization of gravitational field and gauge fields of the standard model as induced fields. Induction means induction of bundle structure is in question. Parallel translation at X^4 is carried out by using spinor connection of H and distances are measured using the metric of H. The components of induced gauge potentials and metric are projections to X^4. Color gauge potentials are identified as projections of Killing vector fields of CP_2 and one can define for them gauge algebra structure. The components of the induced color field are proportional to $H_A J$, where H_A is the Hamiltonian of color isometry and J induced Kähler form. For details see [L3] or the material at my homepage.

The induction of spinor structure allows to avoid the problems related to the definition of spinor structure for general 4-geometry encountered in GRT. For the induced spinor structure induction means projection of gamma matrices to X^4. The definition of gamma matrices is modified when classical action defining the space-time dynamics contains besides volume term also Kähler action with the projection of CP_2 Kähler form defining the analog of Maxwell field. Modified gamma matrices are contractions $T^{\alpha k}\gamma_k$ of the embedding space gamma matrices γ_k with canonical momentum currents $T^{\alpha k}$ associated with the action: this is required by the hermiticity of the modified Dirac action and means existence of infinite number of super currents labelled by the modes of the modified Dirac action.

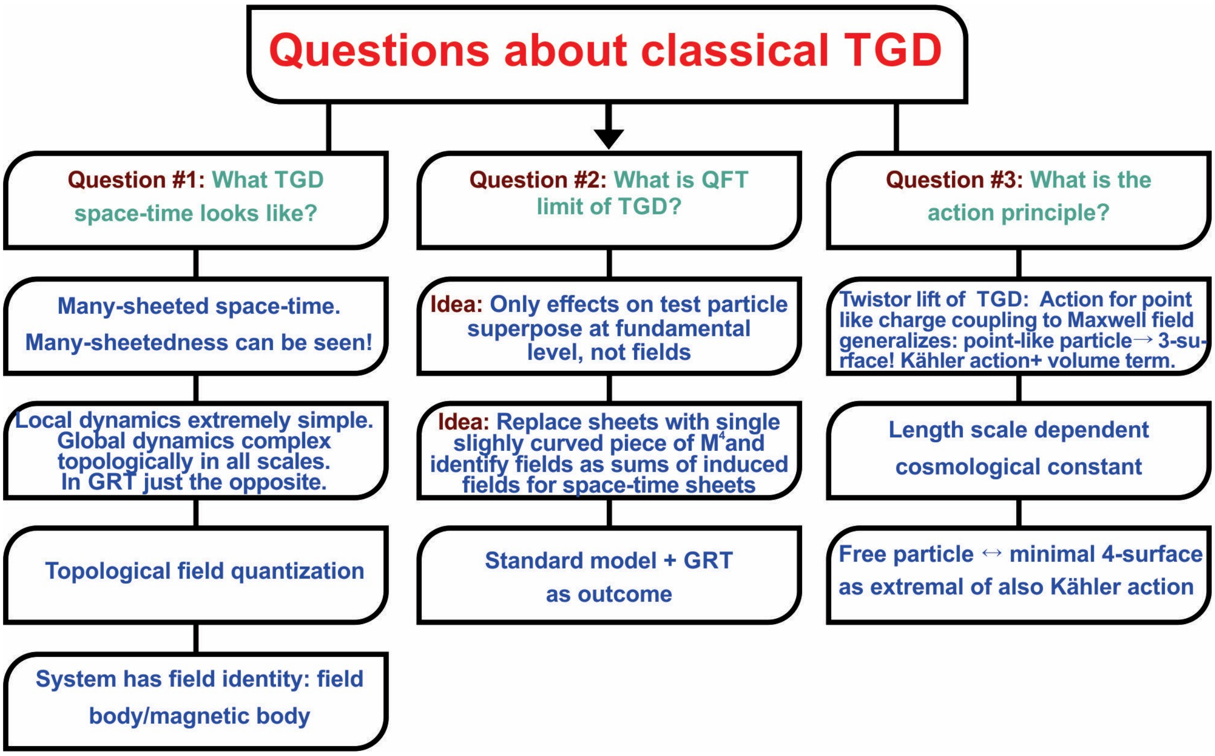

Figure 3: Questions about classical TGD.

2.1.2 Spacetime is topologically complex

Locally the theory is extremely simple: by GCI there are only 4 field-like variables corresponding to a suitable identification of embedding space coordinates as space-time coordinates. The possibility to choose the coordinates in this manner means enormous simplification since the problems caused by GCI in GRT disappear. It is however obvious that 4 field-like variables does not conform with standard model and GRT. This simplicity is compensated by topological complexity in all scales implied by the many-sheeted space-time. The QFT-GRT limit explained in introduction gives the space-time of gauge theories and GRT.

Geometrically the QFT limit for space-time surfaces having 4-D M^4 projection is obtained by replacing the sheets of many-sheeted space-time with slightly curved region of M^4 and identifying gauge potentials and gravitational field (deviation of the metric from M^4 metric) as superpositions of induced fields at various space-time sheets. Einstein's equations hold true as a remnant of the Poincare invariance.

The presence of space-time regions with M^4 projection of dimension $D < 4$ must be described at QFT limit as particle- or string-like entities. Particle-like entities correspond to CP_2 type extremals having Euclidian signature of induced metric and light-like M^4 projection. 3-D light-like surfaces serve as boundaries between them and Minkowskian space-time regions: the identification is as partonic orbits carrying fermion number serving as building bricks of elementary particles [L18].

The topology of partonic 2-surface is characterized by its genus (number of handless attached to sphere) and is propose to explain family replication for fermions. Also for bosons 3 families are predicted. The existence of 3 light fermion families is understood in terms of the fact that only 3 lowest genera have global Z_2 as conformal symmetry making possible bound state of 2 handles. For the higher genera handles would behave like particles and mass spectrum would be continuum.

Cosmic strings are fundamental objects of this kind and appear as two different species. Those carrying monopole flux mean deviation from Maxwell's theory. They are unstable against perturbations making their M^4 projection 4-D and transforming them to magnetic flux tubes playing a key role in TGD inspired cosmology.

2.1.3 Twistor lift

One could end up with the twistor lift of TGD from problems of twistor Grassmannian approach originally due to Penrose [B12] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B3, B2, B6, B1, B4].

Twistor lift of TGD [L11, L31, L32] generalizes the ordinary twistor approach [L19, L20] (see **Fig. 4**). The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense.

The basic recipe is simple: replaced fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

The pleasant surprise is that twistor space has Kähler structure only for M^4 and CP_2 [A7]: this had been discovered already when started to develop TGD! Since Kähler structure is necessary for the twistor lift of TGD, TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

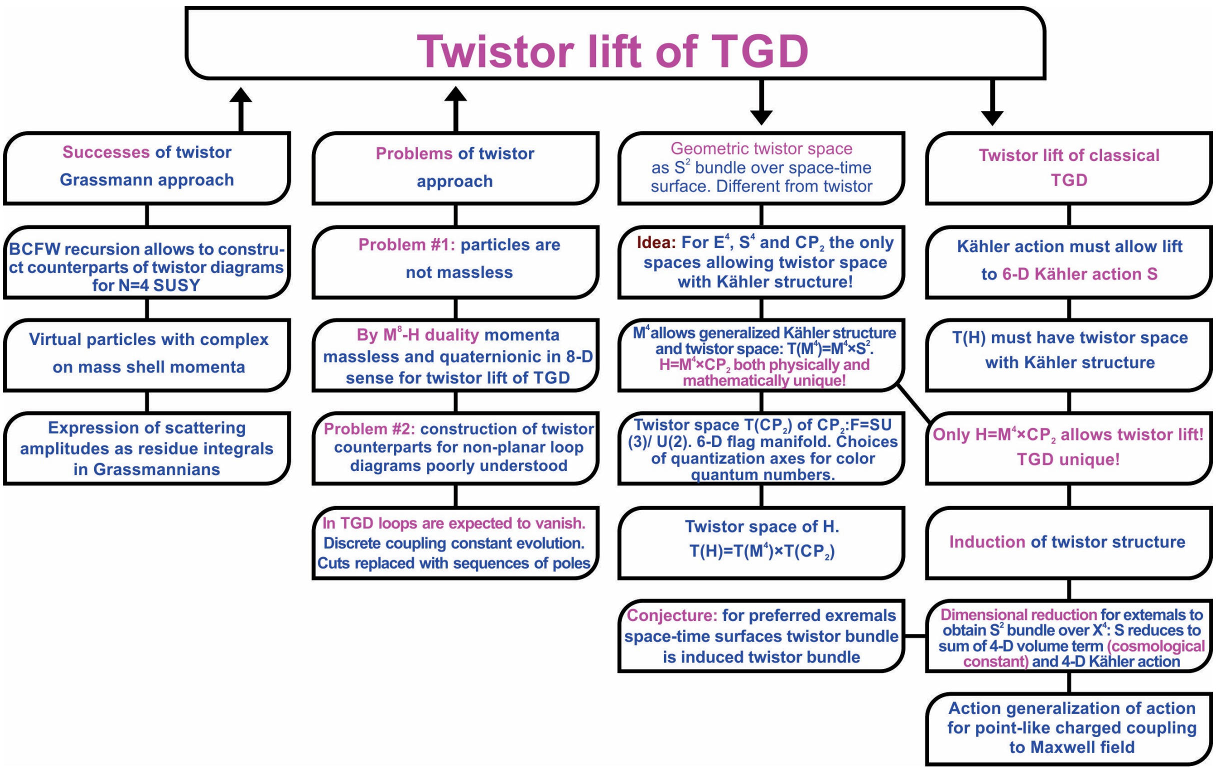

Figure 4: Twistor lift

2.2 Quantum physics as WCW geometry

2.2.1 WCW as an analog of Wheeler's superspace

Quantum TGD replaces Wheeler's superspace of 3-geometries with the "World of Classical Worlds" (WCW) as the space of 3-surfaces (see **Fig. 5**). The holography forced by general coordinate invariance (GCI) implies their 1-1 correspondence with space-time surfaces identified as preferred extremals (PEs) of the basic variational principle analogous to Bohr orbits. Classical physics becomes an exact part of quantum physics [L5, L4]. Einstein's geometrization of classical physics extends to that of quantum physics.

The geometry of infinite-D WCW (see **Fig. 5**) and physics is highly unique from its mere existence requiring maximal group of isometries: a result proved by Freed for loop spaces [A5]. The group of WCW isometries is identified as the group of symplectic (contact) transformations of $\delta M^3_+ \times CP_2$ having the light-like radial coordinate in the role of complex variable z in conformal field theories

Remark: The geometric properties of boundary of 4-D light-cone are unique by its metric 2-dimensionality. In particular, the ordinary 2-D conformal symmetries involving local scaling of the radial light-like coordinate give rise to isometries).

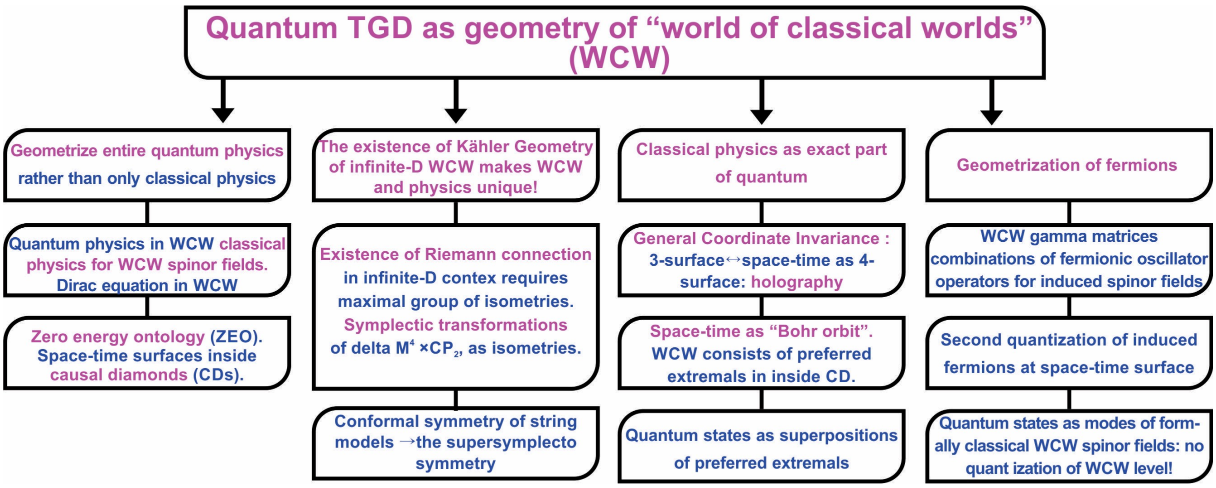

Figure 5: Geometrization of quantum physics in terms of WCW

The assumption that space-time surfaces as preferred extremals (PEs) are fundamental entities leads to zero energy ontology (ZEO) in which quantum superpositions of pairs (X^3_1, X^3_2) of 3-surfaces at opposite boundaries of causal diamond (CD) and connected by PE represent quantum states [L38]. This leads to a solution of the basic problem of quantum measurement theory due to the conflict between the determinism of field equations and non-determinism of state function reduction (SFR) and quantum measurement theory extends to a theory of consciousness bringing observer a part of the physisal system.

Quantum states are identified as modes of classical WCW spinor fields so that apart from quantum jump the theory is formally classical. WCW spinor structure involves complexified gamma matrices expressible as superpositions of second quantized oscillator operators of the induced spinor fields at space-time so that a geometrization of fermionic statistics is achieved [L9, L33, L35]. The simplest formulation assumes only quark spinors and would predict that lepton are local composites of 3 quarks.

2.3 Super-symplectic group as isometries of WCW

The work of Freed related to the geometrization of loop spaces [A5] demonstrated that the Kähler metric allows awell-defined Riemann connection onlyif it has a maximal group of isometries. This fixes the metric completely. The natural conjecture isthat this is true also in 3-D case and that the group consists of symplectic (contact) transformations at $\delta M^4_\pm \times CP_2$. Here $\delta M^4_\pm$ is future/past directed lightcone boundary containing the "upper"/"lower" boundary of a causal diamond of M^4.

WCW allows as infinitesimal isometries huge super-symplectic algebra (SSA) [K5, K3] acting on space-like 3-surfaces at the ends of space-time surfaces inside causal diamond (CD) and also generalization of Kac-Moody and conformal symmetries acting on the 3-D light-like orbits of partonic 2-surfaces (partonic super-conformal algebra (PSCA)). These symmetry algebras have a fractal structure containing a hierarchy of sub-algebras isomorphic to the full algebra. Even ordinary conformal algebras with non-negative conformal weights have similar fractal structure as also Yangian. In fact, quantum algebras are formulated in terms of these half algebras.

The proposal is that physical states are annihilated by a sub-algebra SSA$_n$ of SSA (with non-negative conformal weights), $n = 1, 2, ...,$ with conformal weights coming as n-multiples of those for SSA and thus isomorphic to the entire SSA, and by the commutator $[SSA_n, SSA]$. What remains seems to be a finite-D Kac-Moody algebra as an effective "coset" algebra obtained. Note that the resulting analog of a normal sub-group could actually be a quantum group.s There is a direct analogy with the decomposition of the Galois group Gal to a product of sub-group and normal subgroup H. If the normal subgroup H acts trivially on the representation of Gal reduces to that of the group Gal/H. Now one works at Lie algebra level: Gal is replaced with SSA and H with its sub-algebra with conformal weights multiples of those for SSA. These two hierarchies of subgroups could correspond to each other and to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs) [K8, K4]. These conditions would guarantee preferred extremal property of the space-time surface and holography or even its strong form.

2.3.1 Holography from GCI

Gravitational holography has been one of the dominating themes in recent day theoretical physics. It was originally proposed by Susskind [B11], and formulated by Maldacena as AdS/CFT correspondence [B10]. One application is by Preskill *et al* to quantum error correcting codes [B8].

By holography implied by GCI the basic variational problem can be seen either as boundary value problem with 3-surfaces at opposite boundaries of CD or as initial value problem caused by PE property. Ordinary 3-D holography is thus forced by general coordinate invariance (GCI) and loosely states that the data at 3-surface at either boundary of CD allows to determine $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

Holography poses additional strong conditions on X^4.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SSA_n with radial conformal weights coming as n-multiples of those for the entire algebra SSA and its commutator $[SSA_n, SSSA]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations. An open question is whether this hierarchy corresponds to the hierarchy of EQs with n identifiable as dimension of the extension.

2. Second conjecture is that PEs are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor

structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A7] so that TGD is unique.

2.3.2 Strong form of holography

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated

1. either with light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian,
2. or the space-like 3-surfaces at the ends of CD to determine X^4 as PE (in case that it exists),

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken the strong form of M^8-H duality [L26] mapping $X^4\subset M^8$ to $X4\subset H=M^4\times CP_2$ that it allows to map only certain 2-D sub-manifolds $X^2\subset X^4\subset M^8$: SH allows to determine $X^4\subset H$ from this 2-D data.

2.3.3 Further generalizations

This picture about WCW is not general enough.

1. M^8-H duality [L26] suggests that the notion of WCW applies also M^8 picture. The parameters of polynomials defining $X^4\subset M^8$ are assumed to be rational. The points of M^8 counterpart of WCW have the rational coefficients of these polynomials as coordinates so that WCW should be discrete in real topology. This should be the case also for H counterpart of WCW. Could one see real and p-adic variants of WCW as completions of this discrete WCW.
2. Adelic physics inspires the question whether p-adic and adelic variants of WCW make sense or is it enough to have number theoretically universal cognitive representations to define unique discretized variants of X^4 and correspondingly discretized WCW.
3. For TGD variant of SUSY [L30, L29] super coordinates for H correspond to hermitian local composites of quark oscillator operators. For super-quarks they correspond to local components with fixed quark number. Leptons can be understand as local composites of quarks - super field components [L35]. SUSY replaces modes of super-field with super-surfaces so that the components of super-field correspond to sets of disjoint 4-surfaces. This is true also for the points of super WCW.

3 Physics as number theory

Number theoretical vision is second thread of TGD. It decomposes to 3 threads corresponding to various p-adic physics [L6] fusing to adelic physics [L18], classical number fields [L7], and infinite primes [L8] (not discussed in the sequel).

3.1 p-Adic and adelic physics and extensions of rationals (EQs)

p-Adic number fields would serve as correlates of cognition and imagination (see **Fig. 6**) . Space-time is replaced with a book like structure having both real and various p-adic space-time sheets as pages. The outcome is adelic physics as fusion of various p-adic physics [L17, L18] (see `http://tinyurl.com/ycbhse5c`). The EQ induces extensions of p-adic numbers fields and of adele giving rise to a hierarchy of physics having interpretation in terms of evolution induced by the increase of the complexity of the EQ.

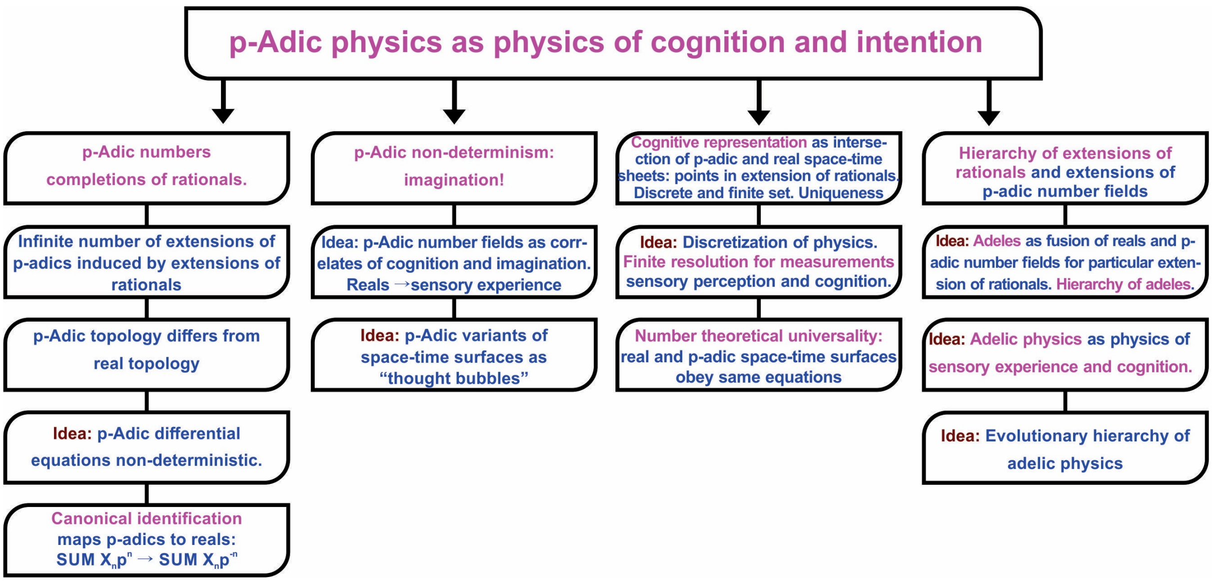

Figure 6: p-Adic physics as physics of cognition and imagination.

Adelic physics leads also the hierarchy of Planck constants $h_{eff}/h_0 = n$ with n identified as dimension of EQ labelling phases of ordinary matter behaving like dark mater, and making possible quantum coherence in arbitrarily long time scales essential for understanding living matter.

EQs are characterized by discriminant D assignable to a polynomial giving rise to the extension (for second order polynomials D has expression familiar from school days). Now polynomials with rationals (equivalently integer) valued coefficients are interesting. The primes dividing the discriminant are known as ramified primes and they have a property that for p-adic variant of polynomial degenerate roots appear in $O(p) = 0$ approximation [L28]. The interpretation could be in terms of quantum criticality and physically preferred p-adic primes are identified as ramified primes of extension [L34].

Remark: One can also consider polynomials with algebraic coefficients. The notion of Galois group make sense also for real coefficients.

The hierarchy of EQs labelling levels of dark matter hierarchy and of hierarchy of adelic physics follows from $M^8 - H$ duality allowing to identify $X^4 \subset M^8$ as a projection of $X^4_c \subset M^8_c$ - identified as complexified octonions O_c - and satisfying algebraic equations associated with a polynomial of degree n.

Real and p-adic physics are strongly correlated and mass calculations represent the most important application of p-adic physics [K6]. Elementary particles seem to correspond to p-adic primes near powers 2^k (there are also indicatons for powers of 3). Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p = 2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p = 3$ (period tripling as approach to chaos) [I1, I2]. Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2 and the challenge is to understand this [L12].

3.2 Classical number fields

Second aspect of number theoretical vision are classical number fields: reals, complex numbers, quaternions and octonions and their complexifications by a commuting imaginary unit i (see **Fig. 7**).

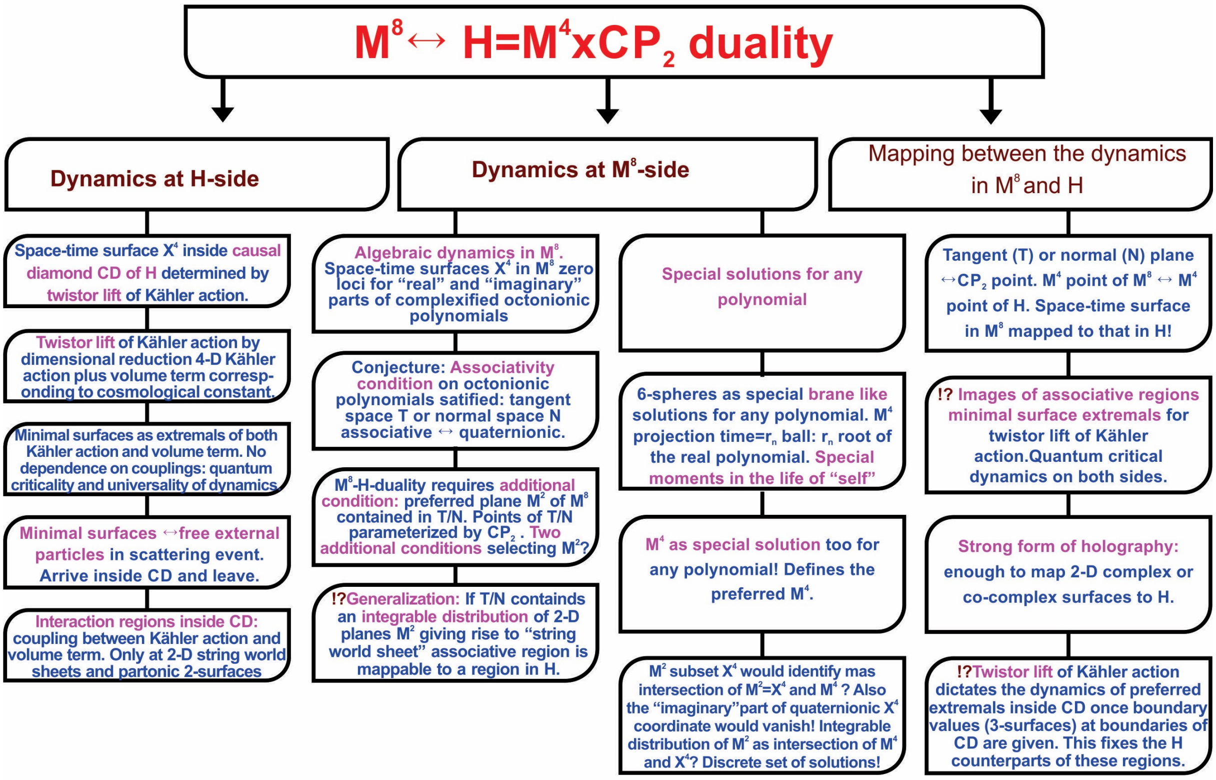

Figure 7: $M^8 - H$ duality

3.2.1 Space-time as 4-surface in $M^8_c = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8–-projection of $X^4_c \subset M^8_c = O_c$. M^4_c is identified as complexified quaternions H_c [L26, L34]. The dynamics is purely algebraic and therefore local.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X^4_c as a root for the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.

   The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X^4_c with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [?] "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

   The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L34]. For ramified primes the p-adic variants of polynomials have degenerate zeros in $O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If X^4 corresponds to the projection $O_c \to M^8 \to M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

3.2.2 How to realize $M^8 - H$ duality?

$M^8 - H$ duality (see **Fig. 7**) allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible $M^8 - H$ duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [L33].

Twistor lift allows several variants of this basic duality [L31, L32]. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L26].

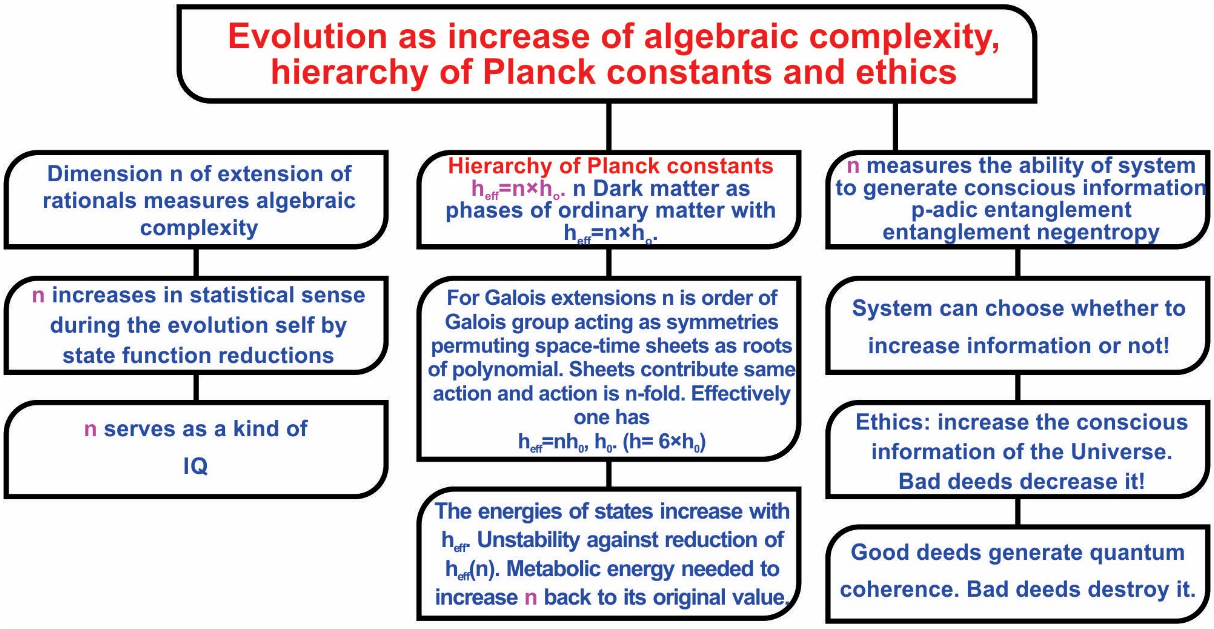

Figure 8: Number theoretic view about evolution

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of M^8-H duality and means that tangent - or normal space is associative/quaternionic.

2. Each tangent space of X^4 at x must contain a preferred $M_c^2(x) \subset M_c^4$ such that $M_c^2(x)$ define an integrable distribution and therefore complexified string world sheet in M_c^4. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X_c^2 \subset X_c^4$ in R_c sense. $E_c^2(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space.

One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M_c^2(x)$ and $E_c^2(x)$ define a slicing of X_c^4.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X_c^2 can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M_c^2 would be the simplest solution but also more general complex sub-manifolds $X_c^2 \subset M_c^4$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

   These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$

curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. SH makes possible $M^8 - H$ duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.

   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X^2_c have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.

3. There is however a delicacy involved: the X^2 might be only metrically 2-D but not topologically. The partonic orbits are 3-D light-like surfaces with metric dimension $D = 2$. The 4-metric degenerates to 2-D metric at them. Therefore their pre-images would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to 2 [L27] . If this happens, SH would not be quite so strong as expected. The study of fermionic variant of $M^8 - H$ correspondence indeed leads to this conclusion.

One can generalize the condition selecting X^2_c so that it selects 1-D surface inside X^2_c. By assuming that R_c-valued "real" or "imaginary" part of complex part of P at this 2-surface vanishes. One obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \to C_c \to H_c \to O_c$ realized as surfaces.

3.2.3 What about $M^8 - H$ duality in the fermionic sector?

During the preparation of this article I became aware of the fact that the realization $M^8 - H$ duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about $M^8 - H$ duality also in the bosonic sector and the existing phenomenological picture follows now from $M^8 - H$ duality. There are powerful mathematical guidelines available.

1. Octonionic spinors

By supersymmetry, octonionicity should have also a fermionic counterpart.

1. The interpretation of M^8_c as complexified octonions suggests that one should use complexified octonionic spinors in M^8_c. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L10]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and spinors is replaced with the non-associative octonionic product.

2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L30, L29].

3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have an analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!

   One can define for the octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M^8_c$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal spaces of X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.

4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as a complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about the sigma matrices associated with the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matrices and those of E^4 sigma matrices close to the same $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalized spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2 \otimes 2 = 3 \oplus 1$ so that one must have $1 \oplus 3 \oplus 1 \oplus 3$. The octonionic spinors indeed decompose like $1 + 1 + 3 + \overline{3}$ under $SU(3)$ representing automorphisms of the octonions. $SO(3)$ could be interpreted as $SO(3) \subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

2. Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of the dynamics should be supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for $M^8 - H$ duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corresponds to the embedding space coordinates.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4 \subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining X^4 can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity holds true if also Ψ satisfies associativity/co-associativity condition as proposed above.

3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl} o^k \overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c \rightarrow M^8 \rightarrow M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P\Psi = 0$ gives $m_{kl} P^k \overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases. $PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X_c^4 \rightarrow M^8 \rightarrow M^4$ in terms of Uncertainty Principle.

   There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in the M^8 picture?

For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3 \subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

The variation of P at the mass shell of M^8_c (to be precise) could be interpreted in terms of the width of the wave packet associated with a particle. Since the light-like curve at partonic 2-surface for fermion at X^3_L is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K6].

4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L26] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_k P^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

5. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L26] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_k P^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

6. The variant Dirac equation would hold true also at 2-surfaces $X^2 \subset X^4$ and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ defines a 2-D variant of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X^1_L. Its H-image would naturally contain the boundary of the string word sheet at X^3_L assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of the string world sheet in H would not carry an induced spinor field.

7. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this is in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

3. The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been a long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of $M^8 - H$-duality for fermions.

1. SH and the expected analogy with the bosonic variant of $M^8 - H$ duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by $M^8 - H$-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.

   The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_k P^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in the twistor Grassmann approach!

   For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler

action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.

   This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L30, L29]. S_{C-S} could emerge as a limit of 4-D action.

   The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^\alpha_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^\alpha_{B,A} = T^\alpha_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.

   Fermionic Noether current would be $J_{F,A} = \overline{\Psi} J^\alpha \Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma}$ defining magnetic flux. The modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what the twistor Grassmannian approach led to [L19, L20].

4 Could Kähler metric of state space replace S-matrix?

In the sequel a more detailed view about the reduction of S-matrix to a non-flat Kähler geometry of Hilbert space consisting of WCW spinor fields is considered. The proposal is novel in the sense that the state space would codes interactions to its geometry just like space-time geometry codes gravitational interaction in general relativity.

4.1 About WCW spinor fields

4.1.1 Induction of second quantized spinor fields from H

There are two approaches to the quantization of induced spinors at space-time surfaces, and these approaches are equivalent.

1. Induction means that gamma matrices are determined by Kähler action as analogs for projections of embedding space gamma matrices and space-time spinor field χ is simply the restriction of H spinor field Ψ. For a given action determining X^4, supersymmetry allows the identification of the modified Dirac operator D and finding of the modes of the induced H spinor field as solutions of the modified Dirac equation (MDE) $D\chi = 0$. Second quantization would replace their coefficients with oscillator operators. However, it is not clear what the anti-commutation relations for the oscillator operators are.

2. One can generalize the classical induction of spinors Ψ to an induction of second quantized spinor fields in H as a restriction of the second quantized Ψ in H to the X^4. One must however get rid of normal ordering diverges due the fact that the anti-commutators for coinciding points give 7-D delta functions. One gets rid of them, if the Ψ and $\overline{\Psi}$ are assigned to disjoin space-time regions. This leads to bi-local modified Dirac action (MDA), implying automatically the classical field equations for the action determining D.

   What does $D\Psi = 0$ really mean when Ψ is quantum field? One can develop the restrictions of the c-valued modes of Ψ in terms of modes of χ satisfying $D\chi = 0$, and obtain an expression for Ψ at X^4 in terms of these modes each satisfying MDE. The operator valued coefficients of Ψ modes contributing to a given mode of χ would define the corresponding oscillator value fermionic oscillator operators at X^4.

   Also the generalizations of the variants of MDE restricted to sub-manifolds of X^4 make sense and are needed. The beauty is that there is no need to introduce spinor fields at lower-D surfaces as independent dynamical degrees of freedom. For instance, one only a variant of a modified Dirac action defined by Cherns-Simons analog of Kähler action makes sense at light-like partonic orbits so that one has an analog of a topological quantum field theory (TQFT).

4.1.2 How to avoid normal ordering divergences from fermionic oscillator operators?

The normal ordering divergences due to the anti-commutators of fermionic fields at the same point are really serious since induce spinor fields of 8-D $H = M^4 \times CP_2$ so that normal ordering singularities are proportional to 7-D delta function $\delta^7(0)$. They are encountered already for the ordinary MDA giving rise to bosonic SCA charges as Noether charges, which also are plagued by these divergences. Normal ordering for the oscillators in the Noether charges associated with MDA would allow to get rid of the divergences but is a mere trick. The proposal considered in [L35] is to make MDA bi-local at the space-time level.

Consider the general constraints on bi-locality coming from the cancellation of the normal ordering divergences.

1. Consider first 4-D variant of MDA. The most general option for MDA is that there is an integral over the entire X^4 for both Ψ and $\overline{\Psi}$ separately sothat one has 2 4-D integrations. One obtains potential normal ordering divergences proportional to $\delta^7(0)d^8x$. If one has two space-time sheets which in the generic case intersect transversally at discrete set of points, one obtains a vanishing result. However, the self-pairing of a given space-time sheet gives a divergence as a 4-D volume integral of $\delta^3(0)$. The definition of the self-pairing as a limit of separate space-time sheets approaching each other to get rid of the divergences looks like a trick.

   This suggests that the pairing can occur only between disjoint space-time regions, most naturally space-time sheets. For instance, parallel space-time sheets with overlapping M^4 projections. Allowing pairing only between disjoint regions eliminates also the divergences associated with the bosonic Noether charges deduced from MDA and involving 3+4-D integral instead of 3-D integral.

   What could be the precise definition for these disjoint regions? $M^8 - H$ duality suggests that they correspond to different roots of the octonionic polynomial defined by real polynomials. When 2 roots coincide, one obtains a term of type $\delta^7(0)d^7x$ giving a finite result. What if the number of coinciding roots is higher than 2? This case will be discussed later in number theoretic context.

What about space-like regions, in particular the wormhole contacts expected to be small deformations of a warped embedding of CP_2 having light-like M^4 projection but having same Kähler metric and Kähler form as CP_2 [K1]? There is no pairing with a parallel space-time sheet now. It seems that the pairing must be between different wormhole contacts. This pairing could be essential for the understanding of string like entities as paired wormhole contacts providing a model for elementary particles.

2. For the bilinear MDA, the variation of the 4+4-D modified Dirac action with respect to $\overline{\Psi}$ and Ψ yields both the modified Dirac equation $D\Psi = 0$ plus the field equations for the preferred extremal. This gives the modes of the induced spinor field. In the standard picture the hermiticity condition for the Dirac action yields the same outcome and has interpretation as a supersymmetry between classical and fermionic degrees of freedom.

3. Both the phenomenological picture developed during years and $M^8 - H$ duality strongly suggest that spinors can be restricted also to lower-D surfaces. For the lower-D variants of MDA the normal ordering divergences appear already for tranversal intersections. For instance, for 3-D variant of MDA one has $\delta^7(0)d^7x$ type divergences. The only possible manner to avoid them is to require that paired regions are disjoint. For the 3-D Chern-Simons-Kähler action associated with the light-like partonic orbits the paired space-time sheets are very naturally the opposite wormhole throats so that fermions and antifermions would reside at opposite wormhole throats.

   Physical picture also suggests the assignment of actions to 2-D string world sheets and 1-D light-boundaries defining their intersections with partonic orbits.

4. Also 6-D brane-like solutions having the topology of S^6 and $t = r_n$ hyper-plane as intersection with M^4 are of physical interest. Different 4-D space-time surfaces could be glued together along 3-surfaces or 2-D partonic 2-surfaces at S^6. Arguments similar to those already discussed exclude the pairing of various objects with these 6-branes as also their self-pairing.

   Also M^4 and CP_2 define special solutions to the algebraic equations in M^8. MDA reduces to ordinary massless Dirac equation in M^4. In the case of CP_2 one has a massless Dirac equation in CP_2 and only the right-handed neutrino ν_R is possible as a solution. If only quarks are allowed, this solution is excluded. What happens for the deformations of CP_2? Could it be that quarks cannot reside inside wormhole contacts as 4-D entities? Or could one allow solutions of $D\Psi = 0$ as analytic functions of CP_2 coordinates finite in the region in which they are defined - wormhole contact does not span the entire CP_2?

Cognitive representations provide additional insights to the problem of normal ordering divergences, and it could be even argued that they are the only possible manner to define scattering amplitudes as a sequences of improving approximation natural in the approach based on hyper-finite factors of type II_1 (HFFs).

1. For a given extension of rationals determined by the polynomial defining the space-time region in M^8, the space-time surfaces inside CD are replaced with their discretizations consisting of points of M^8 in the extension considered. This surface and cognitive representation are mapped to H by $M^8 - H$ correspondence [L22]. For cognitive representations one can perform discretization by replacing the integrals defining SCA generators with discrete sums over points of the cognitive representation. This replacement is very natural since in the p-adic context the counterpart of the Riemann integral does not exist.

2. The Galois group of extension serves as a symmetry group and one can form analogs of group algebra elements - wave functions in discrete Galois group - acting on the cognitive representation and giving rise to discrete representation of quantum states. This state space has as its dimension the dimension n of the Galois group which for Galois extensions coincides with the dimension of extension [L13, L37]. This group algebra-like structure can be given Kähler metric and also spinor structure and this spinor structure could discretize the spinor structure of WCW if gamma matrices are identified as fermionic oscillator operators.

3. Also now one can avoid divergences if the paired space-time regions, say space-time sheets, in MDA are disjoint. It can however happen that n separate points at the orbit of the Galois

group approach each other and coincide: this would correspond to the touching of space-time sheets meaning coinciding roots of the octonionic polynomial. In this situation a subgroup of the Galois group would leave the intersection point invariant.

The possible normal ordering divergence comes from different pairs of the m points, which coincide. In 4-D case, the situation corresponds to transversal space-time sheets so that the divergence vanishes. For lower-dimensional surfaces, say partonic orbits, the intersections do not occur in the generic situation but if they occur, the divergence is multiplied by a sum over the values of wave function at coinciding branches and vanishes if the representation is *non-singlet*. It would thus seem that the non-singlet character of Galois representations must be posed as an additional condition.

4. This cancellation mechanism works even without discretization since the notions of Galois group and its representations make sense for arbitrary polynomial surfaces without a restriction to rational or algebraic polynomial coefficients so that the cancellation occurs for non-singlet representations when the space-time sheets intersect.

4.1.3 Are fermions 4-D in H but 3-D in M^8?

M^8-H duality suggests the restriction of the induced spinor fields to light-like 3-surfaces having 2-D partonic surfaces as ends. M^8-H duality reduces space-time surfaces in M^8 to algebraic surfaces defined by polynomials of real variable. The coefficients can be complex. Concerning p-adicization real rationals defines the most attractive option. This leads to a picture in which a hierarchy of extensions of rationals defines evolutionary and cognitive hierarchies. The extensions provide cognitive representations as unique discretizations of the X^4 with embedding space coordinates in extension of rationals and the one can formulate quantum TGD in finite measurement resolution at least using these representations.

The fermionic variant of M^8-H duality [L36] leads to the conclusion that spinor modes in M^8 are restricted at 3-D light-like surfaces obeying an algebraic equations analogs to the momentum space variant of massless Dirac equation. Are H fermions also always restricted to the 3-D light-like orbits of the partonic 2-surfaces at which the signature of the induced metric changes?

On the other hand, the picture deduced at the level of H from the cancellation of the normal ordering divergences allows 4-D fermions, and also implies field equations for X^4 itself. Can one say that free fermions can reside in 4-D space-time but reside only at the 3-D mass shell in momentum space. M^8-H duality would be analogous to the duality between space-time and momentum space descriptions of particles.

Even more, string world sheets have light-like boundaries at the parton orbits. Also fermions in H would be naturally located at string boundaries and behave like point-like particles. One would obtain a picture resembling that provided by twistor Grassmannian approach. Also the cancellation of normal ordering divergences supports this picture and leads to a detailed form of bi-linear modified Dirac action. Also strong form of holography (SH) stating that 2-surfaces carry all information needed to construct the X^4 supports this view. This is actually the same as the phenomenological picture that has been applied.

M^8-H duality predicts also "very special moments in the life self" to have as correlates 6-branes with M^4 time defining in M^8 octonionic real axis (unique rest system) having as values roots of the polynomial defining the space-time surfaces. These surfaces should contain the partonic 2-surfaces defining the reaction vertices. If there is a non-determinism associated with these surfaces it should preserve classical charges and also SSA charge.

4.1.4 Is the proposed counterpart of QFT supersymmetry only an approximate symmetry?

The proposal for the cancellation of the normal ordering divergences allows overviewing leptons as three quark composites with 3 quarks at the same wormhole throat. This option is strongly suggested by the conceptual economy since quarks are enough for WCW spinor structure.

An interesting question is whether TGD allows a counterpart of QFT supersymmetry (SUSY). This was proposed in [L24]. The idea was that both embedding space coordinates and spinors can be expanded as polynomials in the local compositesof quark and antiquark oscillator operators -

rather than anticommuting hermitian theta parameters leading to problems with fermion number conservation - with a well-defined quark number.

The proposal was that leptons are purely local 3-quark-composite analogous to a superpartner of quark: note however that quark superspinor would have quark number one so that precise spartner interpretation fails. This option and only its slightly local variant is possible only for the TGD view about color as angular momentum rather than spin-like quantum number.

This proposal was based on discrete cognitive representations as unique discretizations of the X^4 and on the crucial assumption that fermionic oscillator operators obey Kronecker delta type anticommutations rather than the 8-D anticommutations giving $\delta^7(0)$ anti-commutator singularities for the induced second quantized quark field in H. Can the notion of super-field based on local composites of quarks and antiquarks with a definite fermion number avoid normal ordering divergences for the induced anticommutation relations? One can of course think of a normal ordering of monomials but one expects problems with vertices.

This suggests that the super coordinates of H and superspinors can be only approximate notions. Superfield components would correspond to states with a fixed quark number but quarks and antiquarks would reside at opposite wormhole throats rather than forming exactly local composites. Since the throat is expected to have CP_2 size, these states would be for all practical purposes strictly local composites.

4.2 Kähler metric as the analog of S-matrix

Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurement. Could the Kähler metric define unitary S-matrix? Under simple additional conditions this is true!

4.2.1 The analogs of unitarity conditions

The following little arguments show that given Kähler metric defines an analog of unitary S-matrix giving rise positive transition probabilities, and under additional conditions also a unitary S-matrix between states with quantum numbers labeling basis of complex vectors or of complexified gamma matrices. This defines an S-matrix like entity and under some additional conditions even an unitary S-matrix.

1. The defining conditions for unitary S-matrix and Kähler metric are very similar. S and $S^\dagger$ would correspond to the covariant metric $g_{m\overline{n}}$ and contravariant metric $g_{\overline{m}n}$. Unitary for S-matrix corresponds to the conditions

$$S_{mr}S^\dagger_{rn} = S_{mr}S_{nr} = \delta_{m,n} \quad .$$

(there is summation over repeated indices). The rows of S-matrix are orthonormalized. The definition of the contravariant metriccorresponds the conditions

$$g_{m\overline{r}}g^{\overline{r}n} = \delta_{m,n} \quad .$$

The complex rows of metric tensor and contravariant metric are orthonormalized also now and rows are orthonormal

2. For S-matrix the probabilities are given by $p_{mn} = S_{mn}S^\dagger_{nm} = S_{mn}S_{mn}*$ and are real and non-negative and their sum is equal to one. Also for the Kähler metric the complex analogs of probabilities defined by

$$p^c_{mn} = g_{m\overline{r}}g^{\overline{r}n}$$

sum up to unity. Hence the real parts $Re(p^c_{mn})$ of p^c_{mn} sum up to unity whereas the imaginary parts sum up to zero.

3. p^c_{mn} are not however automatically real and non-negative and it is not clear how to interpret complex or even real but negative probabilities physically. One can however pose the positivity of the real parts of p^c_{mn} as an additional condition on the phase factors $U_{m\overline{n}} = exp(\Phi_{m\overline{n}})$ and $V_{m\overline{n}} = exp(\Psi_{m\overline{n}})$ associated with $g_{m\overline{n}} = R_{mn}U_{m\overline{n}}$ and $g^{\overline{n}m} = S_{nm}V_{\overline{n}m}$. The condition for positivity is

$$U_{m\overline{n}}V_{\overline{n}m} = cos(\Phi_{\overline{n}m} - \Psi_{\overline{n}m} \geq 0$$

and is rather mild requiring the angle difference to be in the range $(-\pi/2, \pi/2)$. This is true of the angles are in the range $(\pi/4, \pi/4)$. The condition $Re(p^c_{mn}) \geq 0$ is equivalent with the condition $Im(ip^c_{mn}) \geq 0$, and characterizes the coefficients of Teichmueller matrices [A3, A6, A4] [K2]: the meaning of this condition will be discussed below.

4. Under what conditions p^c_{mn} reduce to non-negative real numbers? One can express the probabilities as $p_{mn} = g_{m\overline{n}} \times cof(g_{m\overline{n}})/det(g)$. Note that $Z = det(g)$ is constant depending only on the point of the Kähler manifold. One can express g_{mn} as $g_{m\overline{n}} = A_{mn}U_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ as $cof(g_{m\overline{n}}) = B_{mn}V_{m\overline{n}}$. The reality condition implies

$$U_{m\overline{n}} = \overline{V_{m\overline{n}}} \ .$$

The phases of $g_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ are opposite.

This gives additional conditions. Kähler metric involves $N_{tot} = 2N^2$ real parameters There are $(N^2-N)/2$ elements in say upper diagonal and by hermiticity they are complex conjugates of the lower diagonal. This is the number N_{cond} of conditions coming from the reality. There is also one additional condition due to the fact that the probabilities do not depend on the normalization of g. The total number of real parameters is

$$N_{param} = N_{tot} - N_{cond} - 1 = N(N-1) - 1 \ .$$

For instance, for $N \in \{2, 3, 4\}$ one has $N_{param} =\in \{1, 5, 11\}$. Unitary matrix allows $N_{unit} = N^2$ real parameters and the ratio $N_{param}/N_{unit} = (N(N-1)-1)/N^2$ approaches unity for large values of N. Note that a unitary matrix with real diagonals has $N^2 - N$ parameters so that the number of parameters is the same as for a hermitian metric with unit determinant.

5. Could one transform the metric defining non-negative probabilities to a unitary matrix by a suitable scaling? One can indeed define a matrix S as a matrix $S_{mn} = \sqrt{A_{mn}B_{mn}/Z}U_{mn}$. One has $S_{mn}S_{mn}* = A_{mn}B_{mn}/Z$ given also by the product of $g_{m\overline{n}}g^{\overline{n}m}$ so that the probabilities are the same. The unitarity conditions reduce to $g_{m\overline{r}}g^{\overline{r}n} = \delta^n_m$.

In infinite dimensional case problems might be produced by the appearance of the square root of determinant expected to be infinite. However, also the cofactors are expected to diverge, and one can express them as partial derivatives of the metric determinant with respect to the corresponding element of the metric. This is expected to give a finite value for the elements of the contravariant metric. Note that the ratios of the probabilities do not depend on the metric determinant.

4.2.2 Can one distinguish between the descriptions based on Kähler metric and S-matrix?

For the Teichmueller option the proposed analog for S-matrix involves imaginary part. Does it have some physically observable consequences?

Could one imagine a physical situation allowing to test whether the S-matrix description or its TGD variant is nearer to truth? One can indeed imagine an analog of a Markov process characterized by a matrix p of transition probabilities p_{mn} at a given step. For a two-step process the transition matrix would be p^2_{mn}.

In the TGD context one would have $p_{mn} = Re(p^c_{mn})$. What happens in a two-step process? Should one use use p^2_{mn} or $Re((p^c)^2)_{mn} = Re((p^c)2)_{mn} - Im(p^c)^2_{mn}$? If both options are possible, what could distinguish physically between them?

Could the correct interpretation be that p^2_{mn} describes the process when the outcome is measured in both steps, and $Re((p^c)^2)_{mn}$ the process in which only initial and final states are measured? This picture would generalize to n-step processes and predict a deviations from the ordinary Markov process and perhaps allow to compare the predictions of the TGD view and standard view and deduce $Im(p^c)$.

S-matrix and its Hermitian conjugate correspond in standard physics to situations related by CPT symmetry defined as the product of charge conjugation C, spatial reflection P and time reversal T. The transition probabilities would remain invariant in this transformation although transition amplitudes are replaced with their complex conjugates.

What happens to CPT in TGD framework? In TGD framework CPT induces a hermitian conjugation $g_{m\overline{n}} \rightarrow g_{\overline{n}m} =$

5 The role of fermions

In this section the role of fermions (quarks as it seems) is discussed in more detail. In particular, the conditions on the scattering amplitudes from the cancellation of normal ordering divergences and co-associative octonionic spinors at the level of M^8 are discussed. Also the formulation of scattering amplitudes the level of M^8 is briefly considered.

5.1 Some observations about Feynman propagator for fundamental quark field

In the sequel the divergence cancellation mechanism and the properties of Dirac propagator are discussed in detail. The surprise is that the massive propagators with CP_2 mass scale reduce essentially to massless propagators for light-like separations. This allows understanding of why quarks can give rise to light elementary particles.

The second quantized free quark field Ψ in H defines fundamental fermions appearing as a building brick of elementary particles. The Feynman propagator for Ψ appears in the analogs of Feynman diagrams. Apart from the right handed neutrino (present only as a 3 quark composite at partonic 2-surface if only quarks are involved) the modes of Ψ are extremely massive. Elementary particles are light. How can one understand this?

In p-adic thermodynamics the generation of small mass was assumed to involve a generation of a negative, "tachyonic", ground state conformal weight encountered also in string models. M^8-H correspondence allows a more sophisticated description based on the choice of $M^4 \subset M^8$ mapped to $M^4 \subset H$. By 8-D Lorentz invariance the 4-D mass squared of ground state massless in 8-D sense, depends on the choice of $M^4 \subset H$, and with a proper re-choice of M^4 the particle having large M^4 mass becomes massless.

The action of the generators of super-conformal algebra creates states with a well-defined conformal weight, which are massless for a proper choice of $M^4 \subset M^8$. In p-adic thermodynamics the choice of $M^4 \subset M^8$ would correspond to a generation of negative ground state conformal weight.

The states can however mix slightly with states having higher value of conformal weight, and since one cannot choose M^4 separately for these states, a small mass is generated and described by p-adic thermodynamics. The classical space-time correlate for the almost masslessness is minimal surface property, which provides a non-linear geometrization for massless fields as surfaces. The non-linearity at the classical level leads to a generation of small mass in 4-D sense for which p-adic thermodynamics provides a model.

The propagators for the fundamental quarks in H correspond to CP_2 mass scale. Can this be consistent with the proposed picture? The following simple observations about the properties of predicted fermion propagator and anticommutator for the induced spinor fields lead to a result, which was a surprise to me. The propagators and anti-commutators of massive quarks at light cone boundary are in excellent approximation massless for light-like distances. This makes it possible to understand why elementary fermions are light.

This mechanism does not work in QFT defined in M^4 since inverse propagator is $\gamma^k p_k + m$ so that M^4 chiralities mix for massive states. In TGD picture H-chirality is fixed by 8-D masslessnes and the product of M^4 and CP_2 chiralities for spinors equals to the H chirality. The inverse progator is proportional to the operator $p^k\gamma + D_{CP_2}$, where D_{CP2} is CP_2 part of Dirac operator.

5.1.1 General form of the Dirac propagator in H

Second quantized quark field Ψ restricted to the space-time surface determines the Feynman propagator fundamental quark. The propagator can be expressed as a sum of left- and right-handed propagators as

$$S_F = S_{F,L} + S_{F,R} = D_L G_{F,L} + D_R G_{F,R} \ .$$

Here D_L and D_R are the left- and right-handed parts of a massless (in 8-D sense) Dirac operator D in H involving couplings to CP_2 spinor connection depending on CP_2 chirality in accordance with electroweak parity breaking. $G_{F,L}$ *resp.* $G_{F,R}$ is the propagator for a massless (in 8-D sense) scalar Laplacian in H coupling to the spinor connection assignable to left *resp.* right handed modes. G_F can be expressed by generalizing the formula from 4-D case

$$G_{F,I} \sum_n \int d^4p \frac{1}{p^2 - M^2_{n,I}} exp(ip\cdot(m_1 - m_2))\Phi^*_{n,I}(s_1)\Phi_{n,I}(s_2) \ .$$

Here one has $I \in \{L, R\}$ and the mass spectra are different for these modes. Here m_i denote points of M^4 and s_i points of CP_2. n, I, $I \in \{L, R\}$, labels the modes $\Phi_{n,I}$ of a scalar field in CP_2 associated with right and left handed modes having mass squared $M_{n,R}$. Since H-chirality is fixed to be quark chirality, there is a correlation between M^4 - and CP_2 chiralities. Apart from ν_R all modes are massive (ν_R is need not be present as a fundamental fermion) and the mass M_n, which is of order CP_2 mass about 10^{-4} Planck masses, is determined by the CP_2 length scale and depends on CP_2 chirality.

$G_{F,I}$ reduces to a superposition over massive propagators with mass $M_{n,I}$:

$$G_{F,I} = \sum_n G_F(m_1 - m_2|M_n)\Phi^*_{n,I}(s_1)\Phi_{n,I}(s_2)P_I \ .$$

Here P_I, $I \in \{L, R\}$ is a projector to the left/right handed spinors. One can express $S_{F,I}$ as a sum of the free M^4 part and interaction term proportional to the left - or right-handed part of CP_2 spinor connection:

$$S_{F,I} = D(M^4)G_{F,I} + A_I G_{F,I} \ .$$

A_I, $I \inf\{L, R\}$ acts either on s_1 or s_2 but the outcome should be the same. The first term gives sum over terms proportional to massive free Dirac propagator in M^4 allowing to get a good idea about the behavior of the propagator.

5.1.2 About the behavior of the quark propagator

The quark propagator reduces to left- and right-handed contributions corresponding to various mass values $M_{n,I}$. To get view about the behaviour of the quark propagator it is useful to study the behavior of $G_F(x, y|M)$ for a given mass as well as the behaviors of free and interacting parts of S_F its free part

From the explicit expression of $G_F(m_1 - m_2|M_n)$ one can deduce the behavior of the corresponding contribution to the Feynman propagator. Only ν_R could give a massless contribution to the progator. Explicit formula for G_F can be found from Wikipedia [A2] (`https://en.wikipedia.org/wiki/Propagator#Feynman_propagator`):

$$G_F(x, y|m) = \left\{ \begin{array}{ll} -\frac{1}{4\pi}\delta(s) + \frac{m}{8\pi\sqrt{s}}H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{im}{4\pi^2\sqrt{-s}}K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \ . \end{array} \right.$$

Here $H_1^{(1)}(x)$ is Hankel function of first kind and $K_1^{(1)}$ is modified Bessel function [A1](`https://en.wikipedia.org/wiki/Bessel_function`). Note that for massless case the Hankel term vanishes.

Consider first Hankel function.

1. Hankel function $H_\alpha^{(1)}(x)$ [A1, A2] obeys the defining formula

$$H_\alpha^{(1)}(x) = \frac{J_{-\alpha}(x) - exp(i\alpha\pi i)J_\alpha(x)}{isin(\alpha\pi)} \quad .$$

   For integer values of α one has $J_{-n}(x) = (-1)^n J_n(x)$ so that $\alpha = n$ case gives formally 0/0 and the limit must be obtained using Hospital's rule.

2. Hankel function $H_1^{(1)}(x)$ can be expressed as sum of Bessel functions of first and second kind

$$H_1^{(1)}(x) = J_1(x) + iY_1(x) \quad .$$

   J_1 vanishes at origin whereas Y_1 diverges like $1/x$ at origin.

3. The behaviors of Bessel functions and their variants near origin and asymptotically are easy to understand by utilizing Schrödinger equation inside a cylinder as a physical analogy. The asymptotic behaviour of Hankel function for large values of x is

$$H_\alpha^{(1)}(x) = \frac{2}{\pi x} exp(i(x - 3\pi/4)) \quad ,$$

4. The asymptotic behavior of Hankel function implies that the massive Feynman propagator an oscillatory behavior as a function of $m\sqrt{s}$. Modulus decreases like $1/\sqrt{m\sqrt{s}}$. The asymptotic behavior for the real and imaginary parts corresponds to that for Bessel functions of first kind (J_1) and second kind (Y_1). At origin $H_\alpha^{(1)}(x)$ diverges like $Y_1(x) \sim \frac{(x/2)^{-n}}{\pi}$ near origin. For large values of x $K_1(x)$ decreases exponentially like $exp(-x)\frac{\sqrt{\pi}}{2x}$. At origin $K_1(x)$ diverges.

5. In the recent case the quark propagator would oscillate extremely rapidly leaving only the $\delta(s)$ part so that the propagator behaves like massless propagator!

   The localization of quarks to the partonic surfaces with a size scale of CP_2 radius implies that that the oscillation does not lead to a vanishing of the Hankel contribution to the scattering amplitudes. For induced spinor fields in the interior of space-time surfaces destructive interference is however expected to occur so that behavior is like that for a massless particle. This should explain why the observed particles are light although the fundamental fermions are extremely massive. The classical propagation would be essentially along light-like rays.

   The long range correlations between quarks would come from the $\delta(s)$ part of the propagator, and would not depend on quark mass so that it would effectively behave like a massless particle. Also the action of Dirac operator on $G_F(x, y)$ in M^4 degrees of freedom is that of a massless Dirac propagator coupling to induced gauge potentials. The quarks inside hadrons and also elementary particles associated with the wormhole throats of flux tubes could be understood as quarks at different partonic 2-surfaces at the boundary of CD having light-like distance in an excellent approximation.

6. The above argument is for the Feynman propagator but should generalize also for anti-commutator. The anticommutator for Dirac operator D in M^4 can be expressed as $D\Delta(x, y)$, where D is a scalar field propagator.

$$\Delta(x, y|m) \propto \left\{ \begin{array}{ll} \frac{m}{8\pi\sqrt{s}} H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{m}{\sqrt{-s}} K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \end{array} \right. .$$

   Apart from possible proportionality constants the behavior is very similar to that for Feynman propagator except that the crucial $\delta(s)$ term making possible effectively massless propagation is absent. At light-cone boundary however $\sqrt{s}$ is zero along light rays, and this gives long range correlations between fermions at different partonic 2-surfaces intersected by light rays from the origin. Hence one could have a non-vanishing Hermitian inner product for 3-D states at boundaries of CD.

   Rather remarkably, these results provide a justification for twistor-diagrams identified as polygons consisting of light-like segments.

5.1.3 Possible normal ordering divergences

Concerning the cancellation of normal ordering divergences the singularities of the propagators G_F are crucial. The bi-linearity of the modified Dirac action forcing anticommuting quark and antiquark oscillator operators at different throats of wormhole contacts but this need not guarantee the absence of the divergence since the free quark propagator in M^4 contains mass independent $\delta(s)$ part plus the divergent part from Hankel function behaving like $1/\sqrt{sm}$. For the massless propagator assignable to ν_R the propagator would reduce to M^4 propagator and only the $\delta(s)$ would contribute.

$s = 0$ condition tells that the distance between fermion and anti-fermion is light-like and is possible to satisfy at the light-like boundary of CD. Paired quark and antiquark at the wormhole throats must reside at the same light-like radial ray from the tip of cd (cd corresponds to causal diamond in M^4). Since partonic surfaces are 2-D this condition selects discrete pairs of points at the pair of the partonic surfaces. The integration over the position of the end of the propagator line over paired partonic 2-surfaces should smooth out the divergences and yield a finite result. This would be crucial for having an inner product for states at the boundary of the light-cone.

This applies also to the point pairs at opposite throats of wormhole contact. Time-ordered product vanishing for $t_1 = t_2$ so that the points must have different values of t and this is possible. The two 2-D integrations are expected to smooth out the singularities and eliminate divergences also now.

6 Conclusions

TGD predicts revolution in quantum theory based on three new principles.

1. ZEO solving the basic paradox of quantum measurement theory. Ordinary ("big") state function reduction involves time reversal forcing a generalization of thermodynamics and leading to a theory of quantum self-organization and self-organized quantum criticality (homeostasis in living matter).

2. Phases of ordinary matter labelled by effective Planck constant $h_{eff} = nh_0$ identified as dark matter and explaining the coherence of living matter in terms of dark matter at magnetic body serving as a master, and predicting quantum coherence in all scales at the level of magnetic bodies. $h_{eff}/h_0 = n$ has interpretation as the dimension for an extension of rationals and is a measure of algebraic complexity. Evolution corresponds to the increase of n.

   Extensions of rationals are associated with adelic physics providing description of sensory experience in terms of real physics and of cognition in terms of p-adic physics. Central notion is cognition representation providing unique discretization of X^4 in terms of points with embedding space coordinates in the extension of rationals considered $M^8 - H$ duality realizes the hierarchy of rational extensions and assigns them to polynomials defining space-time regions at the level of M^8 and mapped to minimal surfaces in H by $M8 - H$ duality.

3. The replacement of the unitary S-matrix with the Kähler metric of the Kähler space defined by WCW spinor fields satisfying the analog of unitarity and predicting positive definite transition probabilities defining matrix in Teichmueller space. Einstein's geometrization of classical physics extends to the level of state space, Equivalence Principle generalizes, and interactions are coded by the geometry of the state space rather than by an *ad hoc* unitary matrix. Kähler geometry for the spinor bundle of WCW has Riemann connection only for a maximal group of isometries identified as super-symplectic transformations (SS). This makes the theory unique and leads to explicit analogs of Feynman rules and to a proof that theory is free of divergences.

In this work the third principle, which is new, is formulated and some of its consequences are discussed. The detailed formulation allows understanding of how normal ordering divergences and other divergences cancel. The key idea is to induce the second quantized free spinor field from H to space-time surface. This determines the propagators at the space-time level. The condition that creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts is enough. An alternative but not exclusive option suggested by ZEO is that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

A further observation is that the Dirac propagators for particles reduce in a good approximation to massless propagators when the propagation takes place along light-like distances: this provides a considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

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