Introduction to "Towards M-Matrix"

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1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

   Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A7]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes

for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of electromagnetic fields are nonvanishing. The correlations functions for weak fields are nonvanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement

theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

   TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

   TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A1] [**?**, **?**, **?**]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced

with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [?]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

1.2.1 TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A3, A6, A2, A5].

The identification of the space-time as a sub-manifold [A4, A9] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

1.2.2 TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

1.2.3 Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle

(EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

1.3.1 Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

1.4.1 World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

[1] There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions

1.4.2 Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

1.4.3 WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H

or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

1.4.4 The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.5 Construction of scattering amplitudes

1.5.1 Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A8, A10, A11]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

1.5.2 Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

1.5.3 The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K9]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

1.6.1 The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

1.6.2 Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are

defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p = 3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

1.6.3 p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K8].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to "mind stuff", the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.6.4 Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

1.7.1 Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \to f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

1.7.2 Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} \quad . \tag{1.1}$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \to Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \to 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

1.7.3 Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \to SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique

and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \to CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

1.7.4 What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same. The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

1.7.5 Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of $M^8 - H$ duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary M^8-H duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M^8_c$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of M^8-H duality as something more fundamental than the ordinary M^8-H duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3 \oplus \overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

1.8.1 Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K18].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

1.8.2 Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K14, K15, K13]) support the view that dark matter might be a key player in living matter.

1.8.3 Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

1.9.1 Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K25]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A7]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.
2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).
3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L3].

1.9.2 Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L1]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

2 Bird's Eye of View about the Topics of "Towards M-matrix"

This book is devoted to a detailed representation of quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory.

Number theoretic vision leads to the notion of adelic physics fusing real physics with p-adic physics as physics of cognition. It also leads to M^8-H duality raising classical number fields in central role and reducing the dynamics of space-time surfaces in $M^4 \times CP_2$ determined by action principle and subject to infinite number of analogs of gauge conditions to purely algebraic dynamics in M^8. Twistor lift of TGD is a further central notion.

The most important guiding principle is quantum classical correspondence, whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type II_1 about which the spinors of the world of classical worlds represent a canonical example.

2.1 Zero energy ontology

1. The new view about energy and time finding a justification in the framework of zero energy ontology (ZEO) means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action. ZEO has led to a new view about quantum measurement theory extending it to a theory of consciousness solving the basic paradox of quantum measurement theory in its standard form.

2. Classical theory is in a well-defined sense exact part of quantum TGD. Action principle should assign to a given 3-surface unique space-time surface analogous to Bohr orbit. In zero energy ontology (ZEO) 3-surface is identified as a disjoint pair of 3-surfaces with members located at the opposite boundaries of causal diamond (CD) being analogous to initial and final states of a unique classical time evolution represented by preferred extremals. What the action principle is and what *preferred* does mean? During years I have considered several answers to these questions.

   For a long time action was identified as 4-D Kähler action but the emergence of the twistor lift of TGD changed this view. 4-D space-time surface is replaced with the analog of its 6-D twistor-space represented as 6-D surfac having the structure of S^2 bundle with base space identifiable as 4-D space-time surface. Twistor structure of this 6-surface is induced from the 12-D Cartesian product of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having Kähler structure only for M^4 and CP_2. This allows to define 6-D Kähler action whose dimensionally reduced extremals induce of twistor structure to the 6-D surface. Quantum criticality suggests that all preferred extremals are minimal surfaces apart from 2-D singular surfaces identifiable as string world sheets and partonic 2-surfaces. The reason is that the dynamics in this case is independent of coupling parameters (Kähler coupling strength).

   The dimensionally reduced action is sum of Kähler action and volume term having interpretation in terms of cosmological constant. Minimal surfaces are extremals of both volume term and Kähler action separately. Therefore all extremals of Kähler action with non-vanishing Kähler form are also minimal surfaces so that no changes emerge. Therefore I have kept the old chapters studying extremals of Kähler action as such.

3. The differences between the Kähler action with volume term and mere Kähler action emerge only in the vacuum sector. For non-vanishing value of cosmological constant the vacuum extremals with vanishing induced Kähler form are not possible but one can consider the possibility that the dynamically determined cosmological constant [L3] can vanish at the limiting situation when the space-time surfaces have infinite size. The emerging huge vacuum degeneracy and the failure of the classical determinism in the conventional sense, would have strong implications.

   One would have near vacuum extremals of Kähler action a strongly interacting theory defined by volume action with a small cosmological constant with large quantum fluctuations characterizing quantum criticality playing a key role. Vacuum degeneracy implies spin glass degeneracy in 4-D sense. Whether this nearly vacuum degeneracy is a fundamental characteristic of TGD Universe in long length scales, remains an open question.

2.2 Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

   The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.

3. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

   Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

2.3 Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. WCW must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

2.4 Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics. This led eventually to what I call adelic physics [L1, L2]. One of the outcomes was a proposal for a number theoretical interpretation for the hierarchy of Planck constants: the integer defining effective Planck constant $h_{eff} = n \times h_0$ would correspond to the dimension of the extension of rationals defining the adele.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I christened quaternionic sub-manifolds of complexified octonionic imbedding space M^8_c.

M^8-H duality leads to a concrete proposal stating that space-time surfaces in 16-D M^8_c consist of regions for which either real or imaginary part of a complexified-octonion valued polynomial (additional imaginary unit i commutes with octonion units) vanishes. Imaginary and real part refer now to complexified quaternions $o_c = q_{1,c} + J_4 q_{2,c}$ so that 2×4 conditions give 8-D complexified space-time surface. 4-D space-time surfaces in M^8 could correspond to projections of these with respect to M^8, that is time coordinate would be real and remaining 7 coordinates imaginary.

The development of ideas involved a rather strange quirk, which I noticed while doing the updating in 2019.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

   Calibrated k-manifolds are minimal surfaces in their homology class. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and one expects that they are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces play key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality would be realized as decoupling of the two parts of action. Could all preferred extremals be regarded as calibrated in some generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

2.5 Towards M-matrix or towards S-matrix?

S-matrix codes the predictions of quantum field theory and the challenge is to construct the analogy or generalization of S-matrix.

1. In ZEO one is forced to challenge the usual notion of S-matrix. Ordinary S-matrix is between ordinary quantum states associated with time=constant snapshot of time evolution S-matrix. Now these states are replaced by zero energy states formed by these pairs with members at boundaries of CD.

   The first proposal was that S-matrix is replaced with M-matrix between zero energy states and identifiable as time-like entanglement coefficients between positive and negative energy parts of zero energy states assignable to the past and future boundaries of 4-surfaces inside causal diamond defined as intersection of future and past directed light-cones.

   M-matrix would be a product of diagonal density matrix and unitary S-matrix and there are reasons to believe that S-matrix is universal. Generalized Feynman rules based on the generalization of Feynman diagrams obtained by replacing lines with light-like 3-surfaces and vertices with 2-D surfaces at which the lines meet.

   In M-matrix approach without any constraints the state would be superposition of pairs of states with S-matrix defining entanglement coefficients. This zero energy state with sum over states associated with all CDs. The square root of density matrix could take care of the normalization: without it the state has infinite norm. For hyper-finite factors this state could be normalized to unity and one could also require that the normal unitary conditions hold true when one fixes the boundaries of CD and looks for the scattering rates for fixed states at the passive boundary of CD. This should give S-matrix components from given initial state at passive boundary of CD to states and the active boundary of CD.

   It is however far from clear what unitary time evolution following preparation of initial state could mean in this picture. It seems that the standard view about quantum measurement requires that the second boundary of CD - the passive bound - and states at it must be regarded as fixed and that unitary evolution affects only the active boundary and states at it.

   Remark: After the emergence of ZEO the name of this chapter has fluctuated between "T"owards S-matrix and "T"owards M-matrix. This reflects my fluctuating views about what the counterpart of S-matrix could be in ZEO.

2. Later it turned out that the generalization of quantum measurement theory to a theory of consciousness indeed requires a more conservative view. Observer, conscious entity, or self corresponds to a sequence of unitary time evolutions followed by state function reductions for which the active boundary of CD shifts farther away from the passive boundary, which remains unchanged.

The states at active boundary are changed by unitary time evolution implying also time delocalization of the active boundary in the moduli space of CDs with fixed passive boundary. The state function reduction induces localization in this moduli space and is analogous to weak measurement. The localization means also time localization since the temporal distance between the tips of CD is fixed. Eventually all observables are measured in the sense that there are no state function reductions not affective the states at passive boundary. The roles of passive and active boundary are changed. One can say that self dies and reincarnates as self living in opposite direction of time since its is the former passive boundary which shifts farther away from former active boundary. The distance between the tips can also increase in statistical sense only.

S-matrix would be associated with the unitary evolution assignable to the active boundary of CD and involving shift of this boundary farther away from the passive boundary.

2.6 Organization of "Towards M-matrix"

The book is divided into 5 parts.

1. The 1st part of the book has title "The recent view about field equations" and describes what is known about preferred extremals and the vision about physics as spinor geometry of "world of classical worlds" (WCW).
   (a) In the first two chapters the preferred extremals of Kähler action are studied. The twistor lift adds to this action a volume term but since the non-vacuum extremals of Kähler action are minimal surfaces, I have kept the old chapters talking about extremals of Kähler action as such.
   (b) In two chapters quantum TGD as WCW geometry and spinor structure are discussed. General coordinate invariance and generalized super-conformal symmetries - the latter present only for 4-dimensional space-time surfaces and for 4-D Minkowski space - define the basic symmetries of quantum TGD.
   (c) Occam's razor is the basic weapon used in the attempts to debunk a new theory and this argument is discussed in the last chapter of the first part of the book.
2. The 2nd part of the book is devoted to the challenge of constructing the scattering amplitudes. The dream of finding analogs of Feynman rules has turned out to be unreachable with my analytic skills, and the work is concentrated on identifying the basic principles.
   (a) Symmetries determine the predictions of quantum theory to a very high degree. General Coordinate Invariance and huge super-symplectic symmetries making possible the existence of the Kähler geometry of WCW are considered in the first chapter.
   (b) The construction of M-matrix and S-matrix in ZEO are considered in the second chapter. The third chapter is devoted to the attempts to guess what scattering amplitudes should look like.
   (c) Fourth chapter is devoted to the TGD view about coupling constant evolution. I have considered several scenarios during years. Quantum criticality reducing the evolution to discrete evolution has been in central role since the emergence of p-adic physics. The recent considerations correspond to the most recent layer in the evolution of TGD based on the proposal that coupling constant evolution reduces to that for the dynamical cosmological constant emerging in the twistor lift of TGD. The dimensional cutoff parameter of standard approach would be replaced with cosmological constant, which depends on the length scale of space-time surface. Also a connection with zeros of Riemann zeta emerges.
3. The 5 chapters in the 3rd part of the book are devoted to twistor lift of TGD. The motivation for twistor lift came from the twistor Grassmannian approach. The basic idea was that light-likeness in 8-D sense natural in M^8 and $H = M^4 \times CP_2$ pictures allows to overcome the basic problem of 4-D twistor approach since particles massless in 8-D sense can be massive in 4-D sense. In M^8 picture the 8-momenta would be quaternionic and light-like.

   Second key idea was that M^4 (and E^4) and CP_2 are completely unique as the only 4-D spaces allowing twistor space with Kähler structure. Here twistor space is identified as its geometric variant which is $M^4 \times S^2$ for M^4. The existence of 6-D Kähler action for the 6-D surface representing the counterpart of twistor bundle of space-time surface would completely fix TGD.

4. The five chapters of the 4th part of the book are devoted to various ideas inspired by category theory. I hope that the reader forgives the fact that in these chapters I am moving at the outer boundaries of my mathematical skill profile.

5. The 5th part with title "M"iscellaneous topics contains two chapters about the question whether space-time supersymmetry has TGD counterpart or not. The question remains still unsettled. The third chapter is taken as an example about side track and is about possibility of assigning coupling constant evolution to the zeros of Riemann zeta. Zeros of zeta appear in more convincing manner in the recent view about coupling constant evolution reducing to that for cosmological constant.

3 Sources

The eight online books about TGD [K26, K16, K17, K21, K11, K10, K5, K20] and nine online books about TGD inspired theory of consciousness and quantum biology [K23, K3, K12, K2, K4, K6, K7, K19, K22] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

4 The contents of the book

4.1 PART I: THE RECENT VIEW ABOUT FIELD EQUATIONS

4.1.1 Basic Extremals of the Kähler Action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the "world of classical worlds" (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?] It is not clear whether criticality and associativeity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

   Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute "preferred" is not relevant for them.

3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.

4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.

5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

4.1.2 About Identification of the Preferred extremals of Kähler Action

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [?]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

4.1.3 WCW Spinor Structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain

the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A,\gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger,\gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

   The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

awcwspin

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of

WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

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4.1.4 Recent View about Kähler Geometry and Spin Structure of World of Classical Worlds

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the Kähler-Dirac action.

4.1.5 Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent embedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At embedding space level the modes of embedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at embedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

4.2 PART II: GENERAL THEORY

4.2.1 Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. In fact, the general definition of geometry is as a structure characterized by given symmetries. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" (WCW) identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

1. Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. Kähler-Dirac equation (or Kähler-Dirac equation) gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the Kähler-Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

   A crucial feature of the Kähler-Dirac equation is the localization of the modes to 2-D surfaces with vanishing induced W fields (this in generic situation and for all modes but covariantly constant right-handed neutrino): this is needed in order to have modes with well-defined em charge. Also Z^0 fields can be vanish and is expected to do so - at least above weak scale. This implies that all elementary particles are string like objects in very concrete sense.

2. p-adic physics and p-adic variants of basic symmetries

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

3. Hierarchy of Planck constants and dark matter hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzle leads to a profound generalization of quantum TGD through a generalization of the notion of embedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the embedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups. The extension of embedding space can be seen as a formal tool allowing an elegant description of the multi-sheetednes due to the non-determinism of Kähler action. At the space-like ends the sheets fuse together so that a singular covering is in question.

The original intuition was the the space-time would be n-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies $.. \subset C(n_1) \subset C(n_3)...$ such that n_{i+1} divides n_i. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

4. Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids - ends of string world sheets - can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.
2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the Kähler-Dirac gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

4.2.2 Zero Energy Ontology and Matrices

During years the basic mathematical and conceptual building bricks of quantum TGD have become rather obvious. The basic goal is the construction of scattering amplitudes.

1. Zero Energy Ontology (ZEO) forces to generalize the notion of S-matrix by introducing M-matrix as a matrix characterizing the entanglement between pairs of states forming zero energy states.
2. Second building brick consists of various hierarchies and connections between them. There is the hierarchy of quantum criticalities for super-symplectic algebra and its Yangian extension acting as a spectrum generating algebra. This hierarchy is closely related to the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchies of criticalities correspond also to fractal hierarchies of breakings of super-symplectic gauge conformal symmetry: only the sub-algebra isomorphic to the original gauge algebra acts as gauge algebra after the breaking. At each step one criticality is reduced and the number of physical degrees of freedom increases.

   There is a natural connection between these hierarchies with the hierarchies of hyperfinite factors of type II_1 (HFFs) and their inclusions providing a description for the notion of measurement resolution.
3. Number theoretic realized as adelic physics fusing real number based physics as a correlate of sensory experience and p-adic physics as correlate of cognition involves several elements: $M^8 - H$ duality, hierarchy of effective Planck constants $h_{eff} = nh_0$ with n identified as a dimension of extension of rationals, cognitive representations characterized by extensions of rationals, and p-adic length scale hypothesis.

The identification of the TGD counterpart of S-matrix is the key topic of this chapter. What this matrix actually means is far from obvious.

1. One can characterize zero energy state by a "square root" of density matrix which is product of hermitian matrix and unitary matrix: I have called this matrix M-matrix. The unitary matrix related to the M-matrix could relate closely to the S-matrix assigned with particle reactions.

2. One can assign the analog of unitary S-matrix to "small" state function reductions (SSFRs) defining the TGD counterparts of "weak" measurements. The states at the passive boundary PB are unaffected, which has interpretation as the TGD counterpart of Zeno effect. This S-matrix could relate to the evolution of self as a conscious entity and to its cognitive time evolution.
3. One can also assign an S-matrix like entity to "big" SFRs (BSFRs) in which the arrow of time changes. This S-matrix would be the counterpart of the ordinary S-matrix and should closely relate to the M-matrix.
4. I have also introduced the notion of U-matrix, which would be defined between zero energy states without fixing states at the passive boundary essential for fixing the arrow of time. This notion has remained somewhat misty and it seems that it is not needed since the matrices assigned SSFRs and BSFRs indeed are between zero energy states.

The construction of these matrices is discussed at the general level.

4.2.3 Philosophy of Adelic Physics

The p-adic aspects of Topological Geometrodynamics (TGD) will be discussed. Introduction gives a short summary about classical and quantum TGD. This is needed since the p-adic ideas are inspired by TGD based view about physics.

p-Adic mass calculations relying on p-adic generalization of thermodynamics and super-symplectic and super-conformal symmetries are summarized. Number theoretical existence constrains lead to highly non-trivial and successful physical predictions. The notion of canonical identification mapping p-adic mass squared to real mass squared emerges, and is expected to be a key player of adelic physics allowing to map various invariants from p-adics to reals and vice versa.

A view about p-adicization and adelization of real number based physics is proposed. The proposal is a fusion of real physics and various p-adic physics to single coherent whole achieved by a generalization of number concept by fusing reals and extensions of p-adic numbers induced by given extension of rationals to a larger structure and having the extension of rationals as their intersection.

The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far from obvious and various constraints lead to the idea of number theoretic universality (NTU) and finite measurement resolution realized in terms of number theory. An attractive manner to overcome the problems in case of symmetric spaces relies on the replacement of angle variables and their hyperbolic analogs with their exponentials identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

Also the understanding of the correspondence between real and p-adic physics at various levels - space-time level, embedding space level, and level of “world of classical worlds” (WCW) - is a challenge. The gigantic isometry group of WCW and the maximal isometry group of embedding space give hopes about a resolution of the problems. Strong form of holography (SH) allows a non-local correspondence between real and p-adic space-time surfaces induced by algebraic continuation from common string world sheets and partonic 2-surfaces. Also local correspondence seems intuitively plausible and is based on number theoretic discretization as intersection of real and p-adic surfaces providing automatically finite “cognitive” resolution. he existence p-adic variants of Kähler geometry of WCW is a challenge, and NTU might allow to realize it.

I will also sum up the role of p-adic physics in TGD inspired theory of consciousness. Negentropic entanglement (NE) characterized by number theoretical entanglement negentropy (NEN) plays a key role. Negentropy Maximization Principle (NMP) forces the generation of NE. The interpretation is in terms of evolution as increase of negentropy resources.

4.2.4 Negentropy Maximization Principle

In TGD Universe the moments of consciousness are associated with quantum jumps between quantum histories. The proposal is that the dynamics of consciousness is governed by Negentropy

Maximization Principle (NMP), which states the information content of conscious experience is maximal. The formulation of NMP is the basic topic of this chapter.

NMP codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to a maximal reduction of entanglement entropy at each step. In the generic case this implies at each step a decomposition of the system to unique unentangled subsystems and the process repeats itself for these subsystems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states. The natural assumption is that self loses consciousness when it entangles via bound state entanglement.

There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement (NE), which is not bound state entanglement in standard sense since the condition that state function reduction leads to an eigenstate of density matrix requires the final state density matrix to be a projection operator.

NE might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble. For the generation of NE the outcome of the reduction is not random: the prediction is that second law is not a universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism. A natural assumption is that self experiences expansion of consciousness as it entangles in this manner. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

There are two options to consider. Strong form of NMP, which would demand maximal negentropy gain: this would not allow morally responsible free will if ethics is defined in terms of evolution as increase of NE resources. Weak form of NMP would allow self to choose also lower-dimensional sub-space of the projector defining the final state sub-space for strong form of NMP. Weak form turns out to have several highly desirable consequences: it favours dimensions of final state space coming as powers of prime, and in particular dimensions which are primes near powers of prime: as a special case, p-adic length scale hypothesis follows. Weak form of NMP allows also quantum computations, which halt unlike strong form of NMP.

Besides number theoretic negentropies there are also other new elements as compared to the earlier formulation of NMP.

1. ZEO modifies dramatically the formulation of NMP since U-matrix acts between zero energy states and can be regarded as a collection of orthonormal M-matrices, which generalize the ordinary S-matrix and define what might be called a complex square root of density matrix so that kind of a square root of thermodynamics at single particle level justifying also p-adic mass calculations based on p-adic thermodynamics is in question.
2. The hierarchy of Planck constants labelling a hierarchy of quantum criticalities is a further new element having important implications for conciousness and biology.
3. Hyper-finite factors of type II_1 represent an additional technical complication requiring separate treatment of NMP taking into account finite measurement resolution realized in terms of inclusions of these factors.

NMP has wide range of important implications.

1. In particular, one must give up the standard view about second law and replace it with NMP taking into account the hierarchy of CDs assigned with ZEO and dark matter hierarchy labelled by the values of Planck constants, as well as the effects due to NE. The breaking of second law in standard sense is expected to take place and be crucial for the understanding of evolution.
2. Self hierarchy having the hierarchy of CDs as embedding space correlate leads naturally to a description of the contents of consciousness analogous to thermodynamics except that the entropy is replaced with negentropy.

3. In the case of living matter NMP allows to understand the origin of metabolism. NMP demands that self generates somehow negentropy: otherwise a state function reduction to tjhe opposite boundary of CD takes place and means death and re-incarnation of self. Metabolism as gathering of nutrients, which by definition carry NE is the manner to avoid this fate. This leads to a vision about the role of NE in the generation of sensory qualia and a connection with metabolism. Metabolites would carry NE and each metabolite would correspond to a particular qualia (not only energy but also other quantum numbers would correspond to metabolites). That primary qualia would be associated with nutrient flow is not actually surprising!
4. NE leads to a vision about cognition. Negentropically entangled state consisting of a superposition of pairs can be interpreted as a conscious abstraction or rule: negentropically entangled Schrödinger cat knows that it is better to keep the bottle closed.
5. NMP implies continual generation of NE. One might refer to this ever expanding universal library as "Akaschic records". NE could be experienced directly during the repeated state function reductions to the passive boundary of CD - that is during the life cycle of sub-self defining the mental image. Another, less feasible option is that interaction free measurement is required to assign to NE conscious experience. As mentioned, qualia characterizing the metabolite carrying the NE could characterize this conscious experience.
6. A connection with fuzzy qubits and quantum groups with NE is highly suggestive. The implications are highly non-trivial also for quantum computation allowed by weak form of NMP since NE is by definition stable and lasts the lifetime of self in question.

4.2.5 What Scattering Amplitudes Should Look Like?

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the ZEO applied already earlier in TGD inspired cosmology. ZEO motivated the replacement of the term "S-matrix" with "M-matrix".

The general mathematical concepts are not enough to get to the level of concrete scattering amplitudes. The notion of preferred extremal inspiring the notion of generalized Feynman diagram is central in bringing in this concretia. The very notion of preferred extremals means that ordinary Feynman diagrams providing a visualization of path integral are not in question. Generalized Feynman diagrams have 4-D Euclidian space-time regions (wormhole contacts) as lines, and light-like partonic orbits of 2-surfaces as 3-D lines. String world sheets carrying fermions are also present and have 1-D boundaries at the light-like orbits of partonic 2-surfaces carrying fermion number and light-like 8-momenta suggesting strongly 8-D generalization of twistor approach.

The resulting objects could be indeed seen as generalizations of twistor diagrams rather than Feynman diagrams. The preferred extremal property strongly encourages the old and forgotten TGD inspried idea as sequences of algebraic operations with product and co-product representing 3-vertices. The sequences connect given states at the opposite boundaries of CD and have minimal length. The algebraic structure in question would be the Yangian of the super-symplectic algebra with generators identified as super-symplectic charges assignable to strings connecting partonic 2-surfaces.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction requires the introduction of generalization of twistor approach to 8-D context.

4.2.6 TGD View about Coupling Constant Evolution?

New results related to the TGD view about coupling constant evolution are discussed. The results emerge from the discussion of the recent claim of Atyiah that fine structure constant could be understood purely mathematically. The new view allows to understand the recently introduced TGD based construction of scattering amplitudes based on the analog of micro-canonical ensemble as a cognitive representation for the much more complex construction of full scattering amplitudes using real numbers rather than p-adic number fields. This construction utilizes number theoretic discretization of space-time surface inducing that of "world of classical worlds" (WCW) and makes possible adelization of quantum TGD.

The understanding of coupling constant evolution has been one of most longstanding problems of TGD and I have made several proposals during years.

Could number theoretical constraints fix the evolution? Adelization suffers from serious number theoretical problem due to the fact that the action exponentials do not in general exist p-adically for given adele. The solution of the problem turned out to be trivial. The exponentials disappear from the scattering amplitudes! Contrary to the first beliefs, adelization does not therefore seem to determine coupling constant evolution.

TGD view about cosmological constant turned out to be the solution of the problem. The formulation of the twistor lift of Kähler action led to a rather detailed view about the interpretation of cosmological constant as an approximate parameterization of the dimensionally reduced 6-D Kähler action (or energy) allowing also to understand how it can decrease so fast as a function of p-adic length scale. In particular, a dynamical mechanism for the dimensional reduction of 6-D Kähler action giving rise to the induction of the twistor structure and predicting this evolution emerges.

In standard QFT view about coupling constant evolution ultraviolet cutoff length serves as the evolution parameter. TGD is however free of infinities and there is no cutoff parameter. It turned out cosmological constant replaces this parameter and coupling constant evolution is induced by that for cosmological constant from the condition that the twistor lift of the action is not affected by small enough modifications of the moduli of the induced twistor structure. The moduli space for them corresponds to rotation group $SO(3)$. This leads to explicit evolution equations for α_K, which can be studied numerically.

The approach is also related to the view about coupling constant evolution based on the inclusions of hyper-finite factors of type II_1, and it is proposed that Galois group replaces discrete subgroup of $SU(2)$ leaving invariant the algebras of observables of the factors appearing in the inclusion.

4.2.7 The Recent View about SUSY in TGD Universe

The progress in understanding of M^8-H duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge would appear as space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with the precise understanding of quantum criticality and understand the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials. The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. M^8-H duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is indeed bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

Super-Dirac action vanishes on-mass-shell. The proposed construction relying on ZEO allows however to get scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation is however needed and makes possible to express the derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution conforms with the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

4.2.8 Could ZEO provide a new approach to the quantization of fermions?

The exact details of the quantization of fermions have remained open in TGD framework. The basic problem is the possibility of divergences coming from anti-commutators of fermions expected to involve delta functions in the continuum case. In standard framework normal ordering saves from these divergences for the "free" part of the action but higher order terms give the usual divergences of quantum field theories. In supersymmetric theories the normal ordering divergences however cancel.

In TGD the bosonic divergenges are absent due to the generalization of the notion of point-like particle to 3-surface. In fermionic sector normal ordering divergences cancel in unique number theoretic discretization based on what I call cognitive representations but in continuum case the situation is unclear.

Induction procedure plays a key role in the construction of classical TGD. The longstanding question has been whether the induction of spinor structure could be generalized to the induction of second quantization of free fermions at the level of 8-D embedding space to the level of space-time. The problem is that the anticommutators are 8-D delta functions in continuum case and could induce rather horrible divergences. It will be found that zero energy ontology (ZEO) and new view about space-time and particles allow to modify the standard quantization procedure by making modified Dirac action bi-local so that one gets rid of divergences. Also the multi-local Yangian algebras proposed on basis of physical intuition to be central in TGD emerge automatically.

4.3 PART III: TWISTORS AND TGD

4.3.1 TGD variant of Twistor Story

Twistor Grassmannian formalism has made a breakthrough in $\mathcal{N} = 4$ supersymmetric gauge theories and the Yangian symmetry suggests that much more than mere technical breakthrough is in question. Twistors seem to be tailor made for TGD but it seems that the generalization of twistor structure to that for 8-D embedding space $H = M^4 \times CP_2$ is necessary. M^4 (and S^4 as its Euclidian counterpart) and CP_2 are indeed unique in the sense that they are the only 4-D spaces allowing twistor space with Kähler structure.

The Cartesian product of twistor spaces $P_3 = SU(2,2)/SU(2,1) \times U(1)$ and F_3 defines twistor space for the embedding space H and one can ask whether this generalized twistor structure could allow to understand both quantum TGD and classical TGD defined by the extremals of Kähler action. In the following I summarize the background and develop a proposal for how to construct extremals of Kähler action in terms of the generalized twistor structure. One ends up with a scenario in which space-time surfaces are lifted to twistor spaces by adding CP_1 fiber so that the twistor spaces give an alternative representation for generalized Feynman diagrams.

There is also a very closely analogy with superstring models. Twistor spaces replace Calabi-Yau manifolds and the modification recipe for Calabi-Yau manifolds by removal of singularities can be applied to remove self-intersections of twistor spaces and mirror symmetry emerges naturally. The overall important implication is that the methods of algebraic geometry used in super-string theories should apply in TGD framework.

The physical interpretation is totally different in TGD. The landscape is replaced with twistor spaces of space-time surfaces having interpretation as generalized Feynman diagrams and twistor spaces as sub-manifolds of $P_3 \times F_3$ replace Witten's twistor strings.

The classical view about twistorialization of TGD makes possible a more detailed formulation of the previous ideas about the relationship between TGD and Witten's theory and twistor Grassmann approach. Furthermore, one ends up to a formulation of the scattering amplitudes in terms of Yangian of the super-symplectic algebra relying on the idea that scattering amplitudes are sequences consisting of algebraic operations (product and co-product) having interpretation as vertices in the Yangian extension of super-symplectic algebra. These sequences connect given initial and final states and having minimal length. One can say that Universe performs calculations.

4.3.2 From Principles to Diagrams

The recent somewhat updated view about the road from general principles to diagrams is discussed. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. Contrary to the original expectations, the twistorial approach is not mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

Second new element is the fusion of twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

4.3.3 About Twistor Lift of TGD

The twistor lift of classical TGD is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative. Cosmological constant and thus twistor lift make sense only in zero energy ontology (ZEO) involving causal diamonds (CDs) in an essential manner.

One motivation for introducing the hierarchy of Planck constants was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter twistor lift is studied in detail.

1. The first working hypothesis is that the values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. The first interesting finding is that allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.
2. The other working hypothesis is $\alpha_K(M^4) = \alpha_K(CP_2)$. The small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs. This option looks more natural.

Both options lead to a generalization of Chladni mechanism to a "dynamics of avoidance" meaning that at least asymptotically the two dynamics decouple. This leads to an interpretation with profound implications for the views about what happens in particle physics experiment and in quantum measurement, for consciousness theory and for quantum biology.

A related observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density assignable to cosmological constant and be therefore a fundamental physics length scale.

4.3.4 Some Questions Related to the Twistor Lift of TGD

In this chapter I consider questions related to both classical and quantum aspects of twistorialization.

1. The first group of questions relates to the twistor lift of classical TGD. What does the induction of the twistor structure really mean? Can the analog of Kähler form assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications? How does gravitational coupling emerge at fundamental level? Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic embedding space?

   During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift can be criticized and allows an alternative. This option led to a much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant: cosmological term would correspond to the *entire* 4-D action and the cancellation of Kähler action and cosmological term would lead to the small value of the effective cosmological constant.
2. Second group of questions relates to the construction of scattering amplitudes. The idea is to generalize the usual construction for massless states. In TGD all single particle states are massless in 8-D sense and this gives excellent hopes about the applicability of 8-D twistor approach. $M^8 - H$ duality turns out to be the key to the construction. Also the holomorphy of twistor amplitudes in helicity spinors λ_i and independence on $\tilde{\lambda}_i$ is crucial. The basic vertex

corresponds to 4-fermion vertex for which the simplest expression can be written immediately. $n > 4$-fermion scattering amplitudes can be also written immediately.

If scattering diagrams correspond to computations as number theoretic vision suggests, the diagrams should be reducible to tree diagrams by moves generalizing the old-fashioned hadronic duality. This condition reduces to the vanishing of loops which in terms of BCFW recursion formula states that the twistor diagrams correspond to closed objects in what might be called WCFW homology.

4.3.5 The Recent View about Twistorialization in TGD Framework

The recent view about the twistorialization in TGD framework is discussed.

1. A proposal made already earlier is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.
2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.
3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states.

   This idea does not make sense for incoming/outgoing particles, which light-like momenta unless they are parallel: their total momentum cannot be light-like in the general case. Rather, $P^2 = 0$ applies to the states formed inside CDs from groups of incoming and outgoing particles. BCFW deformation $p_i \rightarrow p_i + zr_i$ describes what happens for the single-particle momenta: they cease to be light-like but the total momenta for subgroups of particles in factorization channels are complex and light-like. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.
4. The key idea is number theoretical discretization in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure.

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors

In this chapter 4 topics are discussed. McKay correspondence, SUSY, and twistors are discussed from TGD point of view, and new aspects of $M^8 - H$ duality are considered.

1. McKay correspondence in TGD framework

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one.

These correspondences are discussed from number theoretic point of view suggested by TGD and based on the interpretation of discrete subgroups of $SU(2)$ as subgroups of the covering group of quaternionic automorphisms $SO(3)$ (analog of Galois group) and generalization of these groups to semi-direct products $Gal(K) \triangleleft SU(2)_K$ of Galois group for extension K of rationals with the discrete subgroup $SU(2)_K$ of $SU(2)$ with representation matrix elements in K. The identification of the inclusion hierarchy of HFFs with the hierarchy of extensions of rationals and their Galois groups is proposed.

A further mystery whether $Gal(K) \triangleleft SU(2)_K$ could give rise to quantum groups or affine algebras. In TGD framework the infinite-D group of isometries of "world of classical worlds" (WCW) is identified as an infinite-D symplectic group for which the discrete subgroups characterized by K have infinite-D representations so that hyper-finite factors are natural for their representations. Symplectic algebra SSA allows hierarchy of isomorphic sub-algebras SSA_n. The gauge conditions for SSA_n and $[SSA_n, SSA]$ would define measurement resolution giving rise to hierarchies of inclusions and ADE type Kac-Moody type algebras or quantum algebras representing symmetries modulo measurement resolution.

A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters).

2. New aspects of $M^8 - H$ duality

$M^8 - H$ duality is now a central part of TGD and leads to new findings. $M^8 - H$ duality can be formulated both at the level of space-time surfaces and light-like 8-momenta. Since the choice of M^4 in the decomposition of momentum space $M^8 = M^4 \times E^4$ is rather free, it is always possible to find a choice for which light-like 8-momentum reduces to light-like 4-momentum in M^4 - the notion of 4-D mass is relative. This leads to what might be called $SO(4) - SU(3)$ duality corresponding to the hadronic and partonic views about hadron physics. Particles, which are eigenstates of mass squared are massless in $M^4 \times CP_2$ picture and massive in M^8 picture. The massivation in this picture is a universal mechanism having nothing to do with dynamics and results in zero energy ontology automatically if the zero energy states are superpositions of states with different masses. p-Adic thermodynamics describes this massivation. Also a proposal for the realization of ADE hierarchy emerges.

4-D space-time surfaces correspond to roots of octonionic polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$. These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t = r_n, r_M \leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball. These special values of M^4 time lead to a deeper understanding of ZEO based quantum measurement theory and consciousness theory.

3. Is the identification of twistor space of M^4 really correct?

The critical questions concerning the identification of twistor space of M^4 as $M^4 \times S^2$ led to consider a more conservative identification as CP_3 with hyperbolic signature (3,-3) and replacement of H with $H = cd_{conf} \times CP_2$, where cd_{conf} is CP_2 with hyperbolic signature (1,-3). This approach reproduces the nice results of the earlier picture but means that the hierarchy of CDs in M^8 is mapped to a hierarchy of spaces cd_{conf} with sizes of CDs. This conforms with Poincare symmetry from which everything started since Poincare group acts in the moduli space of octonionic structures of M^8. Note that also the original form of $M^8 - H$ duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.

The outcome of octo-twistor approach applied at level of M^8 together with modified $M^8 - H$ duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor (super-)Grassmannian approach. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D but analogs of 6-D branes. This part of article is not a mere side track since by $M^8 - H$ duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

4.4 PART IV: CATEGORIES AND QUANTUM TGD

4.4.1 Category Theory, Quantum TGD, and TGD Inspired Theory of Consciousness

Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. Category theory might provide the desired systematic approach to fuse together the bundles of general ideas related to the construction of quantum TGD proper. Category theory might also have natural applications in the general theory of consciousness and the theory of cognitive representations.

1. The ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences could be expressed elegantly using the language of the category theory. Quantum classical correspondence might allow a mathematical formulation in terms of structure respecting functors mapping the categories associated with the three kinds of existences to each other. Basic vision is following.
   (a) Self hierarchy would have a functorial map to the hierarchy of space-time sheets and also WCW spinor fields reflect it. Thus the self referentiality of conscious experience would have a functorial formulation (it is possible to be conscious about what one *was* conscious).
   (b) The inherent logic for category defined by Heyting algebra must be modified in TGD context. Set theoretic inclusion would be replaced with the topological condensation, which can occur simultaneously to several space-time sheets.
   (c) The category of light cones with inclusion as an arrow defining time ordering appears naturally in the construction of the WCW geometry and realizes the cosmologies within cosmologies scenario.
   (d) In zero energy ontology (ZEO), which emerged many years after writing the first version of this chapter, causal diamonds (CDs) defined in terms of intersection of future and past directed light-cones form a category with arrow identified as inclusion.
   (e) The preferred extremals would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of embedding space respects associativity (co-associativity). The duality would allow to construct new preferred extremals of Kähler action.
2. Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience.

3. Categories possess inherent generalized logic based on set theoretic inclusion which in TGD framework is naturally replaced with topological condensation: the outcome is quantum variants for the notions of sieve, topos, and logic. This suggests the possibility of geometrizing the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through three-valued logic. Also the right-wrong logic of moral rules and beautiful-ugly logic of aesthetics seem to be too naive and might be replaced with a more general quantum logic.

4.4.2 Category Theory and Quantum TGD

Possible applications of category theory to quantum TGD are discussed. The so called 2-plectic structure generalizing the ordinary symplectic structure by replacing symplectic 2-form with 3-form and Hamiltonians with Hamiltonian 1-forms has a natural place in TGD since the dynamics of the light-like 3-surfaces is characterized by Chern-Simons type action. The notion of planar operad was developed for the classification of hyper-finite factors of type II_1 and its mild generalization allows to understand the combinatorics of the generalized Feynman diagrams obtained by gluing 3-D light-like surfaces representing the lines of Feynman diagrams along their 2-D ends representing the vertices.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

Years after writing this chapter a very interesting new TGD related candidate for a category emerged. The preferred extremals of Kähler action would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of embedding space respects associativity (co-associativity). The duality would allow to construct new preferred extremals of Kähler action.

4.4.3 Could categories, tensor networks, and Yangians provide the tools for handling the complexity of TGD?

TGD Universe is extremely simple locally but the presence of various hierarchies make it to look extremely complex globally. Category theory and quantum groups, in particular Yangian or its TGD generalization are most promising tools to handle this complexity. The arguments developed in the sequel suggest the following overall view.

1. Positive and negative energy parts of zero energy states can be regarded as tensor networks identifiable as categories. The new element is that one does not have only particles (objects) replaced with partonic 2-surfaces but also strings connecting them (morphisms). Morphisms and functors provide a completely new element not present in standard model. For instance, S-matrix would be a functor between categories. Various hierarchies of of TGD would in turn translate to hierarchies of categories.
2. TGD view about generalized Feynman diagrams relies on two general ideas. First, the twistor lift of TGD replaces space-time surfaces with their twistor-spaces getting their twistor structure as induced twistor structure from the product of twistor spaces of M^4 and CP_2. Secondly, topological scattering diagrams are analogous to computations and can be reduced to tree diagrams with braiding. This picture fits very nicely with the picture suggested by fusion

categories. At fermionic level the basic interaction is 2+2 scattering of fermions occurring at the vertices identifiable as partonic 2-surface and re-distributes the fermion lines between partonic 2-surfaces. This interaction is highly analogous to what happens in braiding interaction but vertices expressed in terms of twistors depend on momenta of fermions.

3. Braiding transformations take place inside the light-like orbits of partonic 2-surfaces defining boundaries of space-time regions with Minkowskian and Euclidian signature of induced metric respectively permuting two braid strands. R-matrix satisfying Yang-Baxter equation characterizes this operation algebraically.

4. Reconnections of fermionic strings connecting partonic 2-surfaces are possible and suggest interpretation in terms of 2-braiding generalizing ordinary braiding: string world sheets get knotted in 4-D space-time forming 2-knots and strings form 1-knots in 3-D space. Reconnection induces an exchange of braid strands defined by the boundaries of the string world sheet and therefore exchange of fermion lines defining boundaries of string world sheets. A generalization of quantum algebras to include also algebraic representation for reconnection is needed. Also reconnection might reduce to a braiding type operation.

Yangians look especially natural quantum algebras from TGD point of view. They are bi-algebras with co-product Δ. This makes the algebra multi-local raising hopes about the understanding of bound states. Δ-iterates of single particle system would give many-particle systems with non-trivial interactions reducing to kinematics.

One should assign Yangian to various Kac-Moody algebras (SKMAs) involved and even with super-conformal algebra (SSA), which however reduces effectively to SKMA for finite-dimensional Lie group if the proposed gauge conditions meaning vanishing of Noether charges for some sub-algebra H of SSA isomorphic to it and for its commutator $[SSA, H]$ with the entire SSA. Strong form of holography (SH) implying almost 2-dimensionality motivates these gauge conditions. Each SKMA would define a direct summand with its own parameter defining coupling constant for the interaction in question.

4.4.4 Are higher structures needed in the categorification of TGD?

The notion of higher structures promoted by John Baez looks very promising notion in the attempts to understand various structures like quantum algebras and Yangians in TGD framework. The stimulus for this article came from the nice explanations of the notion of higher structure by Urs Screiber. The basic idea is simple: replace "=" as a blackbox with an operational definition with a proof for $A = B$. This proof is called homotopy generalizing homotopy in topological sense. n-structure emerges when one realizes that also the homotopy is defined only up to homotopy in turn defined only up...

In TGD framework the notion of measurement resolution defines in a natural manner various kinds of "="s and this gives rise to resolution hierarchies. Hierarchical structures are characteristic for TGD: hierarchy of space-time sheet, hierarchy of p-adic length scales, hierarchy of Planck constants and dark matters, hierarchy of inclusions of hyperfinite factors, hierarchy of extensions of rationals defining adeles in adelic TGD and corresponding hierarchy of Galois groups represented geometrically, hierarchy of infinite primes, self hierarchy, etc...

In this article the idea of n-structure is studied in more detail. A rather radical idea is a formulation of quantum TGD using only cognitive representations consisting of points of space-time surface with embedding space coordinates in extension of rationals defining the level of adelic hierarchy. One would use only these discrete points sets and Galois groups. Everything would reduce to number theoretic discretization at space-time level perhaps reducing to that at partonic 2-surfaces with points of cognitive representation carrying fermion quantum numbers.

Even the "world of classical worlds " (WCW) would discretize: cognitive representation would define the coordinates of WCW point. One would obtain cognitive representations of scattering amplitudes using a fusion category assignable to the representations of Galois groups: something diametrically opposite to the immense complexity of the WCW but perhaps consistent with it. Also a generalization of McKay's correspondence suggests itself: only those irreps of the Lie group associated with Kac-Moody algebra that remain irreps when reduced to a subgroup defined by a Galois group of Lie type are allowed as ground states. Also the relation to number theoretic Langlands correspondence is very interesting.

4.4.5 Is Non-associative Physics and Language Possible only in Many-Sheeted Space-time?

Language is an essentially non-associative structure as the necessity to parse linguistic expressions essential also for computation using the hierarchy of brackets makes obvious. Hilbert space operators are associative so that non-associative quantum physics does not seem plausible without an extension of what one means with physics. Associativity of the classical physics at the level of *single* space-time sheet in the sense that tangent or normal spaces of space-time sheets are associative as sub-spaces of the octonionic tangent space of 8-D embedding space $M^4 \times CP_2$ is one of the key conjectures of TGD. But what about many-sheeted space-time? The sheets of the many-sheeted space-time form hierarchies labelled by p-adic primes and values of Planck constants $h_{eff} = n \times h$. Could these hierarchies provide space-time correlates for the parsing hierarchies of language and music, which in TGD framework can be seen as kind of dual for the spoken language? For instance, could the braided flux tubes inside larger braided flux tubes inside... realize the parsing hierarchies of language, in particular topological quantum computer programs? And could the great differences between organisms at very different levels of evolution but having very similar genomes be understood in terms of widely different numbers of levels in the parsing hierarchy of braided flux tubes- that is in terms of magnetic bodies as indeed proposed. If the intronic portions of DNA connected by magnetic flux tubes to the lipids of lipid layers of nuclear and cellular membranes make them topological quantum computers, the parsing hierarchy could be realized at the level of braided magnetic bodies of DNA. The mathematics needed to describe the breaking of associativity at fundamental level seems to exist. The hierarchy of braid group algebras forming an operad combined with the notions of quasi-bialgebra and quasi-Hopf algebra discovered by Drinfeld are highly suggestive concerning the realization of weak breaking of associativity.

4.5 PART V: MISCELLANEOUS TOPICS

4.5.1 Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

Contrary to the original expectations, TGD seems to allow a generalization of the space-time SUSY to its 8-D variant with masslessness in 4-D sense replaced with masslessness in 8-D sense. The algebra in question is the Clifford algebra of fermionic oscillator operators associated with given partonic 2-surface. In terms of these algebras one can in turn construct generators super-symplectic algebra as stringy Noether charges and also other super-conformal algebras and even their Yangians used to create quantum states. This also forces to generalize twistor approach to give 8-D counterparts of ordinary 4-D twistors.

The 8-D analog of super Poincare algebra emerges at the fundamental level through the anti-commutation relations of the fermionic oscillator operators. For this algebra $\mathcal{N} = \infty$ holds true. Most of the states in the representations of this algebra are massive in $4-D$ sense. The restriction to the massless sector gives the analog of ordinary SUSY with a finite value of $\mathcal{N}$ - essentially as the number of massless states of fundamental fermions to be distinguished from elementary fermions. The addition of a fermion in particular mode defines particular super-symmetry. This super-symmetry is broken due to the dynamics of the Kähler-Dirac operator, which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a restriction to a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of a fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

   The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

In superstring theory the hermiticity of super generator G_0 giving as its square scaling generator L_0 is strong argument in favor if Majorana spinors since G_0 appears as a propagator. In TGD framework the counterparts of G_0 in quark and lepton sector carry fermion number so that identification as a propagator does not make sense. The recent formulation of scattering amplitudes in terms of Yangian algebra allows to circumvent the problem. Fundamental propagators are fermion propagators for fermions massless in 8-D sense.

2. The spinor components of embedding space spinors identifiable with physical helicities and with fixed fermion number correspond to the generators of the SUSY algebra at QFT limit. This SUSY is broken due to electroweak and color interactions. Right-handed neutrinos do not have these interactions but there is a mixing with left-handed neutrinos due to the mixing of M^4 and CP_2 gamma matrices in the Kähler-Dirac gamma matrices appearing in the K-D action. Therefore also the $\mathcal{N} = 2$ sub-SUSY generated by right-handed neutrinos is broken.

In this chapter the details of the above general picture are discussed. A brief summary of the basic aspects of SUSY is included and the constraints on the formulation of the SUSY limit are discussed in detail. The formulation itself is left to the reader possessing the needed technical skills. In principle there seems to be no reason preventing the formulation in terms of super fields: the only new elements relate to the fact that baryon and lepton number are conserved now so that Majorana spinors are replaced with Weyl spinors combining to from Dirac spinors.

4.5.2 Could $\mathcal{N} = 2$ Super-conformal Theories Be Relevant For TGD?

TGD has as is symmetries super-conformal symmetry (SCS), which is a huge extension of the ordinary SCS. For instance, the infinite-dimensional symplectic group plays the role of finite-dimension Lie-group as Kac-Moody group and the conformal weights for the generators of algebra corresponds to the zeros of fermionic zeta and their number of generators is therefore infinite.

The relationship of TGD SCS to super-conformal field theories (SCFTs) known as minimal models has remained without definite answer. The most general super-conformal algebra (SCA) assignable to string world sheets by strong form of holography has $\mathcal{N}$ equal to the number of spin states of leptonic and quark type fundamental spinors but the space-time SUSY is badly broken for it. Covariant constancy of the generating spinor modes is replaced with holomorphy - kind of "half covariant constancy". Right-handed neutrino and antineutrino are excellent candidates for generating $\mathcal{N} = 2$ SCS with a minimal breaking of the corresponding space-time SUSY.

$\mathcal{N} = 2$ SCS has also some inherent problems. The critical space-time dimension is $D = 4$ but the existence of complex structure seems to require the space-time has metric signature different from Minkowskian: here TGD suggests a solution. $\mathcal{N} = 2$ SCFTs are claimed also to reduce to topological QFTs under some conditions: this need not be a problem since TGD can be characterized as almost topological QFT. What looks like a further problem is that p-adic mass calculations require half-integer valued negative conformal weight for the ground state (and vanishing weight for massless states). One can however shift the scaling generator L_0 to get rid of problem: the shift has physical interpretation in TGD framework and must be half integer valued which poses the constraint $h = K/2$, $K = 0, 1, 2..$ on the representations of SCA.

$\mathcal{N} = 2$ SCA allows a spectral flow taking Ramond representations to Neveu-Scwartz variant of algebra. The physical interpretation is as super-symmetry mapping fermionic states to bosonic states. The representations of $\mathcal{N} = 2$ SCA allowing degenerate states with positive central charge c and non-vanishing ground state conformal weight h give rise to minimal models allowing ADE classification, construction of partition functions, and even of n-point functions. This could make S-matrix of TGD exactly solvable in the fermionic sector. The ADE hierarchy suggests a direct interpretation in terms of orbifold hierarchy assignable to the hierarchy of Planck constants associated with the super-symplectic algebra: primary fields would correspond to orbifolds identified as coset spaces of ADE groups. Also an interpretation in terms of inclusions of hyper-finite factors is highly suggestive.

4.5.3 Does Riemann Zeta Code for Generic Coupling Constant Evolution?

A general model for the coupling constant evolution is proposed. The analogy of Riemann zeta and fermionic zeta $\zeta_F(s)/\zeta_F(2s)$ with complex square root of a partition function natural in Zero Energy Ontology suggests that the the poles of $\zeta_F(ks)$, $k = 1/2$, correspond to complexified critical temperatures identifiable as inverse of Kähler coupling strength itself having interpretation as inverse of critical temperature. One can actually replace the argument s of ζ_F with Möbius transformed argument $w = (as+b)/(cs+d)$ with a, b, c, d real numbers, rationals, or even integers. For α_K $w = (s+b)/2$ is proper choices and gives zeros of $\zeta(s)$ and $s = 2 - b$ as poles. The identification $\alpha_K = \alpha_{U(1)}$ leads to a prediction for α_{em}, which deviates by .7 per cent from the experimental value at low energies (atomic scale) if the experimental value of the Weinberg angle is used. The conjecture generalizes also to weak, color and gravitational interactions when general Möbius transformation leaving upper half-plane invariant is allowed. One ends up with a general model predicting successfully the entire electroweak coupling constant evolution successfully from the values of fine structure constant at atomic or electron scale and in weak scale.

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