

Overall View About TGD from Particle Physics Perspective

M. Pitkänen,

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Email: matpitka6@gmail.com.

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Postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland. ORCID: 0000-0002-8051-4364.

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Abstract

Topological Geometrostatics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the embedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces.

What GRT limit of TGD and Equivalence Principle mean in TGD framework have are problems which found a solution only quite recently (2014). GRT space-time is obtained by lumping together the sheets of many-sheeted space-time to single piece of M^4 provided by an effective metric defined by the sum of Minkowski metric and the deviations of the induced metrics of space-time sheets from Minkowski metric. Same description applies to gauge potentials of gauge theory limit. Equivalence Principle as expressed by Einstein's equations reflects Poincare invariance of TGD.

Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.
- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical constructing involving the concept of Dirac operator. As a matter fact, the construction of TGD involves four Dirac operators.
 1. The Kähler Dirac equation holds true in the interior of space-time surface: the well-definedness of em charge as quantum number of zero modes implies localization of the modes of the induced spinor field to 2-surfaces. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.
 2. Assuming measurement interaction term for four-momentum, the boundary condition for Kähler-Dirac operator gives essentially massless M^4 Dirac equation in algebraic form coupled to what looks like Higgs term but gives a space-time correlate for the stringy mass formula at stringy curves at the space-like ends of space-time surface.
 3. The ground states of the Super-Virasoro representations are constructed in terms of the modes of embedding space spinor fields which are massless in 8-D sense.
 4. The fourth Dirac operator is associated with super Virasoro generators and super Virasoro conditions defining Dirac equation in WCW. These conditions characterize

zero energy states as modes of WCW spinor fields and code for the generalization of S -matrix to a collection of what I call M -matrices defining the rows of unitary U -matrix defining unitary process.

- Twistor approach has inspired several ideas in quantum TGD during the last years. The basic finding is that M^4 resp. CP_2 is in a well-defined sense the only 4-D manifold with Minkowskian resp. Euclidian signature of metric allowing twistor space with Kähler structure. It seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness: contrary to original expectations the stringy character of the amplitudes seems necessary to guarantee finiteness.

1 Introduction

Topological Geometro-dynamics is able to make rather precise and often testable predictions. In the following I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

During these 37 years TGD has become quite an extensive theory involving also applications to quantum biology and quantum consciousness theory. Therefore it is difficult to decide in which order to proceed. Should one represent first the purely mathematical theory as done in the articles in Prespace-time Journal [L4, L5, L8, L9, L6, L3, L7, L1]? Or should one start from the TGD inspired heuristic view about space-time and particle physics and represent the vision about construction of quantum TGD briefly after that? In this and other two chapters I have chosen the latter approach since the emphasis is on the applications on particle physics.

Second problem is to decide about how much material one should cover. If the representation is too brief no-one understands and if it is too detailed no-one bothers to read. I do not know whether the outcome was a success or whether there is any way to success but in any case I have been sweating a lot in trying to decide what would be the optimum dose of details.

The third problem are the unavoidable mammoth bones and redundancy as one deals with are extensive material as TGD is. The attempts to get rid of them have turned out to be a Sisyfian task but I have done my best!

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the embedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces associated with light-like boundaries of so called causal diamonds defined as intersections of future and past directed light-cones (CDs) and with light-like 3-surfaces. Whether super-conformal symmetries imply space-time SUSY is far from a trivial question. What is suggested is a generalization of the space-time supersymmetry analogous to $\mathcal{N} = 2$ SUSY and not involving Majorana spinors since fermion numbers are conserved in TGD. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization

of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

- The understanding of the relationship between TGD and GRT and quantum and classical variants of Equivalence Principle (EP) in TGD have developed rather slowly but the recent picture is rather feasible.
 1. The recent view is that EP at quantum level reduces to Quantum Classical Correspondence (QCC) in the sense that Cartan algebra Noether charges assignable to 3-surface in case of Kähler action (inertial charges) are identical with eigenvalues of the quantum variants of Noether charges for Kähler-Dirac action (gravitational charges). The well-definedness of the latter charges is due to the conformal invariance assignable to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) at which the spinor modes are localized in generic case. This localization follows from the condition that electromagnetic charge has well defined value for the spinor modes. The localization is possibly only for the Kähler-Dirac action and key role is played by the modification of gamma matrices to Kähler-Dirac gamma matrices. The gravitational four-momentum is thus completely analogous to stringy four-momentum.
 2. At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincaré invariance suggests strongly classical EP for the GRT limit in long length scales at least. Similar procedure applies to induced gauge fields.

The classical four-momentum assignable to the light-like boundaries of string world sheets at partonic orbits can be identified as gravitational momentum naturally identifiable as inertial momentum assignable to embedding space spinor harmonics defined a ground state of super-conformal representation.
- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realize quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.
- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical construction involving the concept of Dirac operator. As a matter of fact, the construction of TGD involves several Dirac operators.
 1. The Kähler Dirac equation holds true in the interior of space-time surface and its solutions localized at string world sheets have a natural interpretation in terms of fundamental fermions forming building bricks of all particles.
 2. A very natural boundary condition at the light-like boundaries of string world sheets is that induced 1-D Dirac operator annihilates the spinor modes so that they are characterized by light-like 8-momentum crucial for 8-D tangent space twistorialization.
 3. Third Dirac operator is associated with embedding space spinor harmonics defining ground states of super-conformal representations.
 4. The fourth Dirac operator is associated with super Virasoro generators and super Virasoro conditions define Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S -matrix to a collection of what I call M -matrices defining the rows of unitary U -matrix defining unitary process.

- Twistor approach has inspired several ideas in quantum TGD during the last years and it seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness.

The discussion of this chapter is rather sketchy and the reader interesting in details can consult the books about TGD [K40, K37, K7, K19, K27, K22, K38] .

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at <http://tgdtheory.fi/tgdglossary.pdf> [L11].

2 Some Aspects Of Quantum TGD

In the following I summarize very briefly those basic notions of TGD which are especially relevant for the applications to particle physics. The representation will be practically formula free. The article series published in Prespace-time Journal [L4, L5, L8, L9, L6, L3, L7, L10] describes the mathematical theory behind TGD. The seven books about TGD [K40, K37, K7, K27, K22, K19, K12, K32] provide a detailed summary about the recent state of TGD.

2.1 New Space-Time Concept

The physical motivation for TGD was what I have christened the energy problem of General Relativity. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The way out is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S - is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [L10] , is the unique choice.

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the notion space-time in this framework is much more richer than in general relativity quite contrary to what one might expect on basis of representability as a surface in 8-D embedding space.

1. Space-time decomposes into space-time sheets (see **Fig. ??** in the appendix of this book) with finite size: this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.
2. Elementary particles are identified as topological inhomogeniuties glued to these space-time sheets (see figs. <http://tgdtheory.fi/appfigures/particletgd.jpg>, <http://tgdtheory.fi/appfigures/elparticletgd.jpg>, which are also in the appendix of this book). In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in general relativity.
3. Also the view about classical fields changes. One can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta.

Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making an ideal tool for communications [K23]. One can speak about field body or magnetic body of the system.

Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. There is evidence for the Lamb shift anomaly of muonic hydrogen [C1] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K17].

2.2 ZEO

In standard ontology of quantum physics physical states are assumed to have positive energy. In ZEO physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events. By quantum classical correspondences zero energy states must have space-time and embedding space correlates.

1. Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone (see fig. ?? in the appendix of this book). The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.
2. p -Adic length scale hypothesis [K20] motivates the hypothesis that the temporal distances between the tips of the intersecting light-cones come as octaves $T = 2^n T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by 2.5 ms for d quark [K3]. This means a direct coupling between microscopic and macroscopic scales.

ZEO conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter than the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter of fact, one must speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

For thermodynamical states this is indeed the case and this leads to the idea that quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S -matrix. M -matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S -matrix as the fundamental observable. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [K16] as the fundamental variational principle. Various M -matrices define the rows of the unitary U matrix

characterizing the unitary process part of quantum jump. From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal.

The most dramatic implications of ZEO are to the modelling of living matter since the basic unit is now a pair of space-like 3-surfaces at the opposite boundaries of CD rather than single 3-surface at either boundary. By holography the space-time surface connecting them can be taken as basic units and define space-time correlates for behavioral patterns. This modifies dramatically the views about self-organization and morphogenesis.

2.3 The Hierarchy Of Planck Constants

The motivations for the hierarchy of Planck constants come from both astrophysics [K30] and biology [K26, K11]. In astrophysics the observation of Nottale [E1] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [K30, K24].

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the embedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the embedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite (see **Fig. ??** in the appendix of this book).

A convenient technical manner to treat the situation is to replace embedding space with its n -fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively $h_{eff} = n \times h$. This splitting to multi-sheeted structure can be seen as a phase transition reducing the densities of various charges by factor $1/n$ and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to $1/h_{eff}$ and scaled down by $1/n$). The connection with fractional quantum Hall effect [D1] is suggestive [K25].

This has many profound implications, which are welcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoomed up version of quantum world at elementary particle length scales.

1. Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.
2. Photons with given frequency can in principle have arbitrarily high energies by $E = hf$ formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [J1]. Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.
3. The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.
4. The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by

the flux tubes and \hbar changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructuring of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [?] (see **Fig. ??** in the appendix of this book). The braiding of the magnetic flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [K2] .

5. p-Adic length scale hypothesis and hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like \hbar . This is not conflict with the known physics for the simple reason that we know very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.

Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [C3] . The classical Z^0 and possibly also W fields responsible for parity breaking effects must be experienced by fundamental fermions in cellular length scale. This is not possible for ordinary value of Planck constant above weak scale since the induced spinor modes are restricted on string world sheets at which W and Z^0 fields vanish: this follows from the well-definedness of em charge. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.

For the solutions of field equations which are almost vacuum extremals Z^0 field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [I2] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [K26] .

The hierarchy of Planck constants has become key part of TGD and is actually forced by the condition that strings connecting partonic 2-surfaces are correlates for the formation of bound states. The basic problem of both QFTs and string theories is the failure to describe bound states, and the generalization of quantum theory by introducing the hierarchy of Planck constant solves this problem.

2.4 P-Adic Physics And Number Theoretic Universality

p-Adic physics [K19, K35] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole analogous to adeles. One implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger structure.

2.4.1 p-Adic number fields

p-Adic number fields Q_p [A13] -one for each prime p - are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the

field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field Q_p allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers p^n of prime p have p-adic norm (magnitude) equal to p^{-n} in Q_p so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance $d(x, y) = d(x - y)$ depends on the lowest binary digit of $x - y$ only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of $x - y$. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultra-metricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality $d(x - y) \leq \text{Min}\{d(x), d(y)\}$. p-Adic topology brings in mind the decomposition of perceptive field to objects.

2.4.2 Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics -not for energy but infinitesimal scaling generator of so called super-conformal algebra [A9] acting as symmetries of quantum TGD [K37] - predicts elementary particle mass scales and also masses correctly under very general assumptions [K19]. The calculations are discussed in more detail in the second article of the series. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two: $p \simeq 2^k$, k positive integer. Those nearest to power of two correspond to Mersenne primes $M_n = 2^n - 1$. One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes $M_{G,n} = (1 + i)^n - 1$.

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-5 μm contains as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ assignable to Gaussian Mersennes $M_k = (1 + i)^k - 1$, $k = 151, 157, 163, 167$, [K26]. This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophical for p-adic numbers fields come from the question about the possible physical correlates of cognition [K21]. Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes.

Rational numbers belong to the intersection of real and p-adic continua. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

1. The intersection could consist of the rational and possibly some algebraic points in the intersection of real and p-adic partonic 2-surfaces at the ends of CD. This set is in general discrete. The interpretation could be as discrete cognitive representations.
2. The intersection could also have a more abstract meaning. For instance, the surfaces defined by rational functions with rational coefficients have a well-defined meaning in both real and p-adic context and could be interpreted as belonging to this intersection. There is strong temptation to assume that intentions are transformed to actions only in this intersection. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

Additional support for the idea comes from the observation that Shannon entropy $S = -\sum p_n \log(p_n)$ allows a p-adic generalization if the probabilities are rational numbers by replacing $\log(p_n)$ with

$-\log(|p_n|_p)$, where $|x|_p$ is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to R_p norm. Negentropic entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely (see **Fig. ??** in the appendix of this book).

Negentropy Maximization Principle [K16] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characteristics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [I1] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [?] .

3 Symmetries Of TGD

Symmetry principles play key role in the construction of WCW geometry have become and deserve a separate explicit treatment even at the risk of repetitions. Symmetries of course manifest themselves also at space-time level and space-time supersymmetry - possibly present also in TGD - is the most non-trivial example of this.

3.1 General Coordinate Invariance

General coordinate invariance is certainly of the most important guidelines and is much more powerful in TGD framework than in GRT context.

1. General coordinate transformations as a gauge symmetries so that the diffeomorphic slices of space-time surface equivalent physically. 3-D light-like 3-surfaces defined by wormhole throats define preferred slices and allows to fix the gauge partially apart from the remaining 3-D variant of general coordinate invariance and possible gauge degeneracy related to the choice of the light-like 3-surface due to the Kac-Moody invariance. This would mean that the random light-likeness represents gauge degree of freedom except at the ends of the light-like 3-surfaces.
2. GCI can be strengthened so that the pairs of space-like ends of space-like 3-surfaces at CDs are equivalent with light-like 3-surfaces connecting them. The outcome is effective 2-dimensionality because their intersections at the boundaries of CDs must carry the physically relevant information. One must however notice also the presence of string world sheets emerging from number theoretic vision and from the condition that spinor modes have well-defined cm charge. Partonic 2-surfaces (plus 4-D tangent space data) and string world sheets would carry the data about quantum states and the interpretation would be in terms of strong holography. The role of string world sheets in TGD is very much analogous to their role in AdS/CFT duality.

3.2 Generalized Conformal Symmetries

One can assign Kac-Moody type conformal symmetries to light-like 3-surfaces as isometries of H localized with respect to light-like 3-surfaces. Kac Moody algebra essentially the Lie algebra of gauge group with central extension meaning that projective representation in which representation matrices are defined only modulo a phase factor. Kac-Moody symmetry is not quite a pure gauge symmetry.

One can assign a generalization of Kac-Moody symmetries to the boundaries of CD by replacing Lie-group of Kac-Moody algebra with the group of symplectic (contact-) transformations [A16, A12, A11] of H_+ provided with a degenerate Kähler structure made possible by the effective 2-dimensionality of δM_+^4 . The light-like radial coordinate of δM_+^4 plays the role of the complex coordinate of conformal transformations or their hyper-complex analogs. The basic hypothesis is that these transformations define the isometry algebra of WCW.

p-Adic mass calculations require also second super-conformal symmetry. It is defined by Kac-Moody algebra assignable to the isometries of the embedding space or possibly those of δCD . This algebra must appear together with symplectic algebra as a direct sum. The original guess was that Kac-Moody algebra is associated with light-like 3-surfaces as a local algebra localized by hand with respect to the internal coordinates. A more elegant identification emerged in light of the wisdom gained from the solutions of the Kähler-Dirac equation. Neutrino modes and symplectic Hamiltonians generate symplectic algebra and the remaining fermion modes and Hamiltonians of symplectic isometries generate the Kac-Moody algebra and the direct sum of these algebras acts naturally on physical states.

A further physically well-motivated hypothesis inspired by holography and extended GCI is that these symmetries extend so that they apply at the entire space-time sheet and also at the level of embedding space.

1. The extension to the entire space-time surface requires the slicing of space-time surface by partonic 2- surfaces and by stringy world sheets such that each point of stringy world sheet defines a partonic 2-surface and vice versa. This slicing has deep physical motivations since it realizes geometrically standard facts about gauge invariance (partonic 2-surface defines the space of physical polarizations and stringy space-time sheet corresponds to non-physical polarizations) and its existence is a hypothesis about the properties of the preferred extremals of Kähler action.

There is a similar decomposition also at the level of CD and so called Hamilton-Jacobi coordinates for M_+^4 [K5] define this kind of slicings. This slicing can induced the slicing of the space-time sheet. The number theoretic vision gives a further justification for this hypothesis and also strengthens it by postulating the presence of the preferred time direction having interpretation in terms of real unit of octonions. In ZEO this time direction corresponds to the time-like vector connecting the tips of CD.

2. The simplest extension of the symplectic algebra at the level of embedding space is by parallel translating the light-cone boundary. This would imply duality of the formulations using light-like and space-like 3-surfaces and Equivalence Principle (EP) might correspond to this duality in turn implied by strong form of general coordinate invariance (GCI).

Conformal symmetries (see **Fig. 1**) would provide the realization of WCW as a union of symmetric spaces. Symmetric spaces are coset spaces of form G/H . The natural identification of G and H is as groups of symplectic transformations and its subgroup leaving preferred 3-surface invariant (acting as diffeomorphisms for it). Quantum fluctuating (metrically non-trivial) degrees of freedom would correspond to symplectic transformations of H_+ and fluxes of the induced Kähler form would define a local representation for zero modes: not necessarily all of them.

A highly attractive hypothesis motivated by fractality is that the algebras of conformal symmetries represent broken conformal symmetries in the sense that the sub-algebras with conformal weights coming as integer multiples of fixed integer n annihilate the physical states and corresponding Noether charges associated with Kähler and Kähler-Dirac action vanish. The hierarchies of symmetry breakings defined by the sequences $n_{i+1} = \prod_{k < i+1} m_k$ would correspond to hierarchies of Planck constants h_{eff} and hierarchies of CDs with increasing sizes characterized by the distance between the tips of CD. The transformation of generators from those of gauge symmetries to real

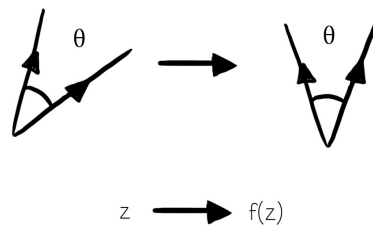


Figure 1: Conformal symmetry preserves angles in complex plane

physical symmetries would bring in new degrees of freedom increasing measurement resolution. The hierarchies would define also inclusion hierarchies of hyper-finite factors of type II_1 [K41]. The level of Kähler action n would tell the number of conformal equivalence classes connecting the 3-surfaces at the boundaries of CD.

3.3 Equivalence Principle And Super-Conformal Symmetries

Equivalence Principle (EP) is a second corner stone of General Relativity and together with GCI leads to Einstein's equations. What EP states is that inertial and gravitational masses are identical. In this form it is not well-defined even in GRT since the definition of gravitational and inertial four-momenta is highly problematic because Noether theorem is not available. Therefore the realization is in terms of local equations identifying energy momentum tensor with Einstein tensor.

Thinking EP in terms of scattering amplitudes for graviton exchange, it seems obvious that EP is true in TGD since all particles are string like objects (monopole flux tubes connecting pairs of wormhole contacts accompanied by fermionic strings). How EP is realized in TGD has been a longstanding open question [K39]. The problem has been that at the classical level EP in its GRT form can hold true only in long enough length scales and it took long time to realize that only the stringy form of this principle is required. The first question is how to identify the gravitational and inertial four-momenta. I have considered very many proposals in this regard!

One could argue that Equivalence Principle (EP) reduces to a mere tautology in TGD framework since stringy picture implies stringy scattering amplitudes for graviton exchanges. This might be the case at quantum level. There are however problems: how the exact Poincare invariance can be consistent with the non-conservation of four-momentum in GRT based cosmologies? What EP could mean at quantum level? Does EP reduce at classical level to Einstein's equations in some sense. How to take into account the many-sheetedness of TGD space-time? The following represents the latest vision about EP in TGD.

1. ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invariance of TGD is in sharp contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4 . If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this

manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

2. *Equivalence Principle at quantum level*

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

1. At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the “vibrational” parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.
2. The most recent view (2014) about understanding how EP emerges in TGD is described in [K39] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Kähler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).
3. The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the Kähler-Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

3. *Equivalence Principle at classical level*

How Einstein’s equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD space-time and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.
3. Einstein’s equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein’s equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

The latest clarification related to EP comes from the natural boundary condition that the boundaries of string world sheets at light-like orbits of the partonic 2-surfaces are light-like (if the boundary curve is not light-like, it is necessarily space-like). These orbits correspond to light-like embedding space 8-momenta classically, which leads to a generalization of 4-D twistors to 8-D ones at the level of the tangent space M^8 by introducing octonion structure and allowing to generalize twistor formalism so that it applies to particles massive in M^4 sense [K36]. If the light-like curve is light-like geodesic, the 8-momentum is conserved and its M^4 and CP_2 parts have constant length. In E^4 degrees of freedom this means $SO(4)$ symmetry, which might allow an interpretation as the symmetry of strong interactions in the description applying at hadron level. The particle states would not be eigenstates of E^4 momentum but characterized by wave functions in S^3 assignable to irreducible $SO(4)$ representations. At quark and gluon level the harmonics of CP_2 would describe color. At the level of generalized Feynman diagrams the natural identification of M^4 part of the 8-momentum would be as incoming M^4 momentum labelling the harmonics of the embedding space and this identification would provide a concrete realization of EP. In CP_2 degrees of freedom $CP_2 - E^4$ duality relating hadrons and quarks and gluons would be a more abstract realization of EP.

3.4 Extension Of Super-Conformal Symmetries

The original idea behind the extension of conformal symmetries to super-conformal symmetries was the observation that isometry currents defining infinitesimal isometries of WCW have natural super-counterparts obtained by contracting the Killing vector fields with the complexified gamma matrices of the embedding space.

This vision has generalized considerably as the construction of WCW spinor structure in terms of Kähler-Dirac action has developed. The basic philosophy behind this idea is that WCW spinor structure must relate directly to the fermionic sector of quantum physics. In particular, Kähler-Dirac gamma matrices should be expressible in terms of the fermionic oscillator operators associated with the second quantized induced spinor fields.

The explicit realization of this program leads to an identification of rich spectrum of super-conformal symmetries and generalization of the ordinary notion of space-time supersymmetry. What happens that all fermionic oscillator operator generate broken super-conformal gauge symmetries whereas in SUSYs there is only finite number of them.

One can however identify sub-algebra of super-conformal symmetries associated with right handed neutrino and this suggests $\mathcal{N} = 2$ super-symmetry respecting conservation of fermion numbers as the least broken SUSY [B3] [K29].

One must be however extremely cautious here since one can imagine several variants for space-time SUSY. The sparticles predicted by a typical supersymmetric extension of standard model have not been observed at LHC. A possible explanation is that supersymmetric matter corresponds to a non-standard value of h_{eff} and thus dark matter and does not appear in the vertices of Feynman diagrams involving ordinary matter. If this is the case, the mass scales of sparticles and particles could be same.

3.5 Does TGD Allow The Counterpart Of Space-Time Super-Symmetry?

It has been clear from the beginning that the notion of super-conformal symmetry crucial for the successes of super-string models generalizes in TGD framework. The answer to the question whether space-time SUSY makes sense in TGD framework has not been obvious at all but it seems now that the answer is affirmative. The evolution of the ideas relevant for the formulation of SUSY in TGD framework is summarized in the chapters of [K27]. The chapters devoted to the SUSY QFT limit of TGD [?], to twistor approach to TGD [K36], and to the generalization of Yangian symmetry of $\mathcal{N} = 4$ SYM manifest in the Grassmannian twistor approach [B10] to a multi-local variant of super-conformal symmetries [K36] represent a gradual development of the ideas about how super-symmetric M -matrix could be constructed in TGD framework.

Before continuing a warning to the reader is in order. In their recent form the above listed chapters do not represent the final outcome but just an evolution of ideas proceeding by trial and error.

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the Kähler-Dirac action [K42, K9]. It is possible to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation $\mathcal{N} = 2N$ SUSY algebra (an inherent cutoff on the number of fermionic modes at light-like wormhole throat) or fermionic part of super-conformal algebra with infinite number of oscillator operators results. The addition of fermion in particular mode would define particular super-symmetry. This super-symmetry is badly broken due to the dynamics of the Kähler-Dirac operator which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

ZEO combined with the analog of the twistor approach to $\mathcal{N} = 4$ SYMs and weak form of electric-magnetic duality has actually led to this kind of formulation [K36]. What is new that also virtual particles have massless fermions as their building blocks. This implies manifest finiteness of loop integrals so that the situation simplifies dramatically. What is also new element that physical particles and also string like objects correspond to bound states of massless fermions.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of a fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.
2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in $D = 10$ and $D = 11$ as the only possible candidates for TOE after it turned out that chiral anomalies cancel. It indeed turns out that TGD view about space-time SUSY is internally consistent. Even more, the separate conservation of quark and lepton number is essential for the internal consistency of this view [?].
3. The massivation of particles is the basic problem of both SUSYs and twistor approach. I have discussed several solutions to this problem [K36]. Twistor Grassmannian approach to $\mathcal{N} = 4$ SYM and the generalization of the Yangian symmetry of this theory inspires two approaches to the problem.
 - (a) In ZEO one can construct physical particles as bound states of massless particles associated with the opposite wormhole throats. If the particles have opposite 3-momenta the

resulting state is automatically massive. In fact, this forces massivation of also spin one bosons since the fermion and anti-fermion must move in opposite directions for their spins to be parallel so that the net mass is non-vanishing: note that this means that even photon, gluons, and graviton have small mass.

This mechanism makes topologically condensed fermions massive and padic thermodynamics allows to describe the massivation in terms of zero energy states and M -matrix. Bosons would receive to their mass besides the small mass coming from thermodynamics also a stringy contribution which would be the counterpart of the contribution coming from Higgs vacuum expectation value and Higgs gives rise to longitudinal polarizations. No Higgs potential is however needed. The cancellation of infrared divergences necessary for exact Yangian symmetry and the observation that even photon receives small mass suggest that scalar Higgs would disappear completely from the spectrum.

- (b) Second approach relies on the generalization of twistor approach. 4-D twistors become 8-dimensional when quaternionic sigma matrices are replaced by octonionic ones. Light-likeness in 8-D sense would allow massive particles in 4-D sense [K36]. The classical 8-momentum associated with the light-like boundary of string world sheet would realize M^8 octonionic twistoriality concretely. This approach is very elegant and allows the 4-momenta of fermions decomposing particles to be massive and there are no problems with the massivation and emergence of the third polarization. Infrared problems are automatically absent in this framework. Encouragingly, M^4 and CP_2 are indeed the unique four-manifolds allowing twistor space which is Kähler manifold. It seems that this option is the only physically plausible one.

3.5.1 Basic data bits

Let us first summarize the data bits about possible relevance of super-symmetry for TGD before the addition of the 3-D measurement interaction term to the Kähler-Dirac action [K42] .

1. Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that Dirac equation is satisfied by covariant constancy and there is no need for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [A18, A20, A15, A19].
2. In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, covariantly constant right-handed neutrino spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [K15, K15] . The negative conformal weight of the vacuum (assumption) also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.
3. The massless right-handed neutrinos would be associated with string boundaries light-like M^4 - rather than only M^8 sense. They would satisfy massless Dirac equation. What this Dirac equation is, is far from obvious and I have considered almost all possibilities that one can imagine.

The minimal option is that the gamma matrix associated with the fermion line is the light-like Kähler-Dirac gamma matrix since the K-D gamma matrix in normal direction should vanish by natural boundary conditions for the extremal of Kähler action. This gamma matrix should have a vanishing covariant divergence by field equations.

This would allow a light-like M^4 momentum with varying direction: light-likeness of M^4 momentum gives just Virasoro conditions in the same manner as for CP_2 type vacuum extremals. For general M^8 type orbits a mixing with left handed neutrino would take place but if string world sheets do not carry induced W boson fields, the mixing with charged spinor components does not occur (W gauge potential is present but can be gauge transformed away). This mixing would induce breaking of SUSY and give mass for the right-handed neutrino.

4. Space-time super-symmetry in the conventional sense of the word is impossible in TGD framework since it would require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break separate conservation of lepton and baryon numbers in TGD framework.

3.5.2 Could one generalize super-symmetry?

Could one then consider a more general space-time super-symmetry with “space-time” identified as space-time surface rather than Minkowski space?

1. The TGD variant of the super-symmetry could correspond quite concretely to the addition of right-handed neutrinos to fermion and boson states at partonic 2-surfaces. Since right-handed neutrinos do not have electro-weak interactions, the addition might not appreciably affect the mass formula although it could affect the p-adic prime defining the mass scale.
2. The problem is to understand what this addition of the right-handed neutrino means. To begin with, notice that in TGD Universe fermions reside at light-like 3-surfaces at which the signature of induced metric changes. Bosons correspond to pairs of light-like wormhole throats with wormhole contact having Euclidian signature of the induced metric.

The long standing head ache has been that for bosons with parallel light-like four-momenta with same sign of energy the spins of fermion and anti-fermion are opposite so that one would obtain only scalar bosons! The problem disappears when 4-D light-likeness is replaced with 8-D light-likeness. The massless Dirac equation using induced gamma matrices at the light-like boundary of string world sheet indeed allows momenta which are light-like in 8-D sense and massive in M^4 sense so that a mixing of M^4 chiralities occurs. This allows to have both spin one bosonic states.

3. The super-symmetry as an addition of a fermion carrying right handed neutrino quantum numbers to the wormhole throat opposite to that carrying many-fermion state does not make sense since the resulting state cannot be distinguished from gauge boson or Higgs type particle. The light-like 3-surfaces can however carry fermion numbers up to the number of modes of the induced spinor field, which is expected to be infinite inside string like objects having wormhole throats at ends and finite when one has space time sheets containing the throats [K42]. In very general sense one could say that each mode defines a very large broken N -super-symmetry with the value of N depending on state and light-like 3-surface. The breaking of this super-symmetry would come from electro-weak - , color - , and gravitational interactions. Right-handed neutrino would by its electro-weak and color inertness define a minimally broken super-symmetry.
4. What this addition of the right handed neutrinos or more general fermion modes could precisely mean? One cannot assign fermionic oscillator operators to right handed neutrinos which are covariantly constant in both M^4 and CP_2 degrees of freedom since the modes with vanishing energy (frequency) cannot correspond to fermionic oscillator operator creating a physical state since one would have $a = a^\dagger$. The intuitive view is that all the spinor modes move in an exactly collinear manner - somewhat like quarks inside hadron do approximately. This would suggest right-handed neutrinos have a non-vanishing but massless four-momentum so that there is an unavoidable breaking of SUSY.

3.5.3 TGD counterpart of space-time super-symmetry

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

1. One can in principle construct many-fermion states containing both fermions and anti-fermions at given light-like 3-surface. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the Kähler-Dirac gamma matrices from the ordinary M^4 gamma matrices. In

particular, the fact that $\hat{\Gamma}^\alpha$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.

2. For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the TGD counterpart of super-symmetry.
3. The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\bar{\nu}_R, e_R\bar{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1, 1, 0, 2)$ and spins $(1/2, 1/2, 0, 1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\bar{\nu}_R, e_L\bar{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1, 1, 0, 2)$ and spins $(1/2, 1/2, 1, 0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2, 1, 0, -1, -2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

1. For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.
2. The quantum field theoretic description should be based on QFT limit of TGD formulated in terms of bosonic emergence . This formulation should allow to calculate the propagators of the super-partners in terms of fermionic loops.
3. This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true. The states inside super-multiplets have identical electro-weak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.
4. It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The fact that spins $J = 0, 1, 2, 3/2, 2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8 -fold degeneracy.

To sum up, this approach does not suggest that particles and sparticles should have different p-adic mass scales. A possible way out of the problem is that the p-adic mass scales are same but sparticles have different h_{eff} and dark relative to particles so that they are not observable in particle physics experiments. The breaking of super-conformal symmetry indeed occurs and could mean a transformation of super-conformal gauge degrees of freedom to dynamical ones and increase of $h_{eff}/h = n$ characterizing the breaking of the conformal symmetry.

3.6 What Could Be The Generalization Of Yangian Symmetry Of $\mathcal{N} = 4$ SUSY In TGD Framework?

There has been impressive steps in the understanding of $\mathcal{N} = 4$ maximally supersymmetric YM theory possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in $AdS_5 \times S^5$ background. Second stringy representation was discovered by Witten and is based on 6-D Calabi-Yau manifold defined by twistors. The unifying proposal is that so called Yangian symmetry is behind the mathematical miracles involved.

The notion of Yangian symmetry would have a generalization in TGD framework obtained by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of CP_2 . The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and CP_2 allow a description in terms of twistors. These observations inspire the proposal that a generalization of Witten's twistor string theory relying on the identification of twistor string world sheets with certain holomorphic surfaces assigned with Feynman diagrams could allow a formulation of quantum TGD in terms of 3-dimensional holomorphic surfaces of $CP_3 \times CP_3$ mapped to 6-surfaces dual $CP_3 \times CP_3$, which are sphere bundles so that they are projected in a natural manner to 4-D space-time surfaces. Very general physical and mathematical arguments lead to a highly unique proposal for the holomorphic differential equations defining the complex 3-surfaces conjectured to correspond to the preferred extremals of Kähler action.

3.6.1 Background

I am outsider as far as concrete calculations in $\mathcal{N} = 4$ SUSY are considered and the following discussion of the background probably makes this obvious. My hope is that the reader had patience to not care about this and try to see the big pattern.

The developments began from the observation of Parke and Taylor [B11] that n-gluon tree amplitudes with less than two negative helicities vanish and those with two negative helicities have unexpectedly simple form when expressed in terms of spinor variables used to represent light-like momentum. In fact, in the formalism based on Grassmannian integrals the reduced tree amplitude for two negative helicities is just "1" and defines Yangian invariant. The article *Perturbative Gauge Theory As a String Theory In Twistor Space* [B9] by Witten led to so called Britto-Cachazo-Feng-Witten (BCFW) recursion relations for tree level amplitudes [B5, B6, B5] allowing to construct tree amplitudes using the analogs of Feynman rules in which vertices correspond to maximally helicity violating tree amplitudes (2 negative helicity gluons) and propagator is massless Feynman propagator for boson. The progress inspired the idea that the theory might be completely integrable meaning the existence of infinite-dimensional un-usual symmetry. This symmetry would be so called Yangian symmetry [K36] assigned to the super counterpart of the conformal group of 4-D Minkowski space.

Drumond, Henn, and Plefka represent in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory* [B8] an argument suggesting that the Yangian invariance of the scattering amplitudes is an intrinsic property of planar $\mathcal{N} = 4$ super Yang Mills at least at tree level.

The latest step in the progress was taken by Arkani-Hamed, Bourjaily, Cachazo, Carot-Huot, and Trnka and represented in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory* [B10]. At the same day there was also the article of Rutger Boels entitled *On BCFW shifts of integrands and integrals* [B12] in the archive. Arkani-Hamed *et al* argue that a full Yangian symmetry of the theory allows to generalize the BCFW recursion relation for tree amplitudes to all loop orders at planar limit (planar means that Feynman diagram allows embedding to plane without intersecting lines). On mass shell scattering amplitudes are in question.

3.6.2 Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K36]. Besides

ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product in terms of particle reactions. Particle annihilation is analogous to annihilation of two particles so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4 - or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and concrete manner in the article *Yangian Symmetry in $D=4$ superconformal Yang-Mills theory* [B7]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h . Serre's relations characterize the difference and involve the deformation parameter h . Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$ -local in the sense that they involve $n + 1$ -forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

3.6.3 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, I have nothing to say. I am just perplexed. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A4] and Virasoro algebras [A9] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.
2. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super

Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.
2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of partonic 2-surface. A coset construction is applied to these two Virasoro algebras so that the differences of the corresponding Super-Virasoro generators and Kac-Moody generators annihilate physical states. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.
3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.
4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.6.4 Is there any hope about description in terms of Grassmannians?

At technical level the successes of the twistor approach rely on the observation that the amplitudes can be expressed in terms of very simple integrals over sub-manifolds of the space consisting of k -dimensional planes of n -dimensional space defined by delta function appearing in the integrand. These integrals define super-conformal Yangian invariants appearing in twistorial amplitudes and the belief is that by a proper choice of the surfaces of the twistor space one can construct all invariants. One can construct also the counterparts of loop corrections by starting from tree diagrams and annihilating pair of particles by connecting the lines and quantum entangling the states at the ends in the manner dictated by the integration over loop momentum. These operations can be defined as operations for Grassmannian integrals in general changing the values of n and k . This description looks extremely powerful and elegant and nosta importantly involves only the external momenta.

The obvious question is whether one could use similar invariants in TGD framework to construct the momentum dependence of amplitudes.

1. The first thing to notice is that the super algebras in question act on infinite-dimensional representations and basically in the world of classical worlds assigned to the partonic 2-surfaces correlated by the fact that they are associated with the same space-time surface. This does not promise anything very practical. On the other hand, one can hope that everything related to other than M^4 degrees of freedom could be treated like color degrees of freedom in $\mathcal{N} = 4$ SYM and would boil down to indices labeling the quantum states. The Yangian conditions coming from isometry quantum numbers, color quantum numbers, and electroweak quantum numbers are of course expected to be highly non-trivial and could fix the coefficients of various singlets resulting in the tensor product of incoming and outgoing states.
2. The fact that incoming particles can be also massive seems to exclude the use of the twistor space. The following observation however raises hopes. The Dirac propagator for wormhole throat is massless propagator but for what I call pseudo momentum. It is still unclear how this momentum relates to the actual four-momentum. Could it be actually equal to it? The recent view about pseudo-momentum does not support this view but it is better to keep mind open. In any case this finding suggests that twistorial approach could work in in more or less standard form. What would be needed is a representation for massive incoming particles as bound states of massless partons. In particular, the massive states of super-conformal representations should allow this kind of description.

Could zero energy ontology allow to achieve this dream?

1. As far as divergence cancellation is considered, zero energy ontology suggests a totally new approach producing the basic nice aspects of QFT approach, in particular unitarity and coupling constant evolution. The big idea related to zero energy ontology is that all virtual particle particles correspond to wormhole throats, which are pairs of on mass shell particles. If their momentum directions are different, one obtains time-like continuum of virtual momenta and if the signs of energy are opposite one obtains also space-like virtual momenta. The on mass shell property for virtual partons (massive in general) implies extremely strong constraints on loops and one expect that only very few loops remain and that they are finite since loop integration reduces to integration over much lower-dimensional space than in the QFT approach. There are also excellent hopes about Cutkoski rules.
2. Could zero energy ontology make also possible to construct massive incoming particles from massless ones? Could one construct the representations of the super conformal algebras using only massless states so that at the fundamental level incoming particles would be massless and one could apply twistor formalism and build the momentum dependence of amplitudes using Grassmannian integrals.

One could indeed construct on mass shell massive states from massless states with momenta along the same line but with three-momenta at opposite directions. Mass squared is given by $M^2 = 4E^2$ in the coordinate frame, where the momenta are opposite and of same magnitude. One could also argue that partonic 2-surfaces carrying quantum numbers of fermions and their superpartners serve as the analogs of point like massless particles and that topologically condensed fermions and gauge bosons plus their superpartners correspond to pairs of wormhole throats. Stringy objects would correspond to pairs of wormhole throats at the same space-time sheet in accordance with the fact that space-time sheet allows a slicing by string worlds sheets with ends at different wormhole throats and defining time like braiding.

The weak form of electric magnetic duality indeed supports this picture. To understand how, one must explain a little bit what the weak form of electric magnetic duality means.

1. Elementary particles correspond to light-like orbits of partonic 2-surfaces identified as 3-D surfaces at which the signature of the induced metric of space-time surface changes from Euclidian to Minkowskian and 4-D metric is therefore degenerate. The analogy with black

hole horizon is obvious but only partial. Weak form of electric-magnetic duality states that the Kähler electric field at the wormhole throat and also at space-like 3-surfaces defining the ends of the space-time surface at the upper and lower light-like boundaries of the causal diamond is proportional to Kähler magnetic field so that Kähler electric flux is proportional to Kähler magnetic flux. This implies classical quantization of Kähler electric charge and fixes the value of the proportionality constant.

2. There are also much more profound implications. The vision about TGD as almost topological QFT suggests that Kähler function defining the Kähler geometry of the “world of classical worlds” (WCW) and identified as Kähler action for its preferred extremal reduces to the 3-D Chern-Simons action evaluated at wormhole throats and possible boundary components. Chern-Simons action would be subject to constraints. Wormhole throats and space-like 3-surfaces would represent extremals of Chern-Simons action restricted by the constraint force stating electric-magnetic duality (and realized in terms of Lagrange multipliers as usual).

If one assumes that Kähler current and other conserved currents are proportional to current defining Beltrami flow whose flow lines by definition define coordinate curves of a globally defined coordinate, the Coulombic term of Kähler action vanishes and it reduces to Chern-Simons action if the weak form of electric-magnetic duality holds true. One obtains almost topological QFT. The absolutely essential attribute “almost” comes from the fact that Chern-Simons action is subject to constraints. As a consequence, one obtains non-vanishing four-momenta and WCW geometry is non-trivial in M^4 degrees of freedom. Otherwise one would have only topological QFT not terribly interesting physically.

Consider now the question how one could understand stringy objects as bound states of massless particles.

1. The observed elementary particles are not Kähler monopoles and there much exist a mechanism neutralizing the monopole charge. The only possibility seems to be that there is opposite Kähler magnetic charge at second wormhole throat. The assumption is that in the case of color neutral particles this throat is at a distance of order intermediate gauge boson Compton length. This throat would carry weak isospin neutralizing that of the fermion and only electromagnetic charge would be visible at longer length scales. One could speak of electro-weak confinement. Also color confinement could be realized in analogous manner by requiring the cancellation of monopole charge for many-parton states only. What comes out are string like objects defined by Kähler magnetic fluxes and having magnetic monopoles at ends. Also more general objects with three strings branching from the vertex appear in the case of baryons. The natural guess is that the partons at the ends of strings and more general objects are massless for incoming particles but that the 3-momenta are in opposite directions so that stringy mass spectrum and representations of relevant super-conformal algebras are obtained. This description brings in mind the description of hadrons in terms of partons moving in parallel apart from transversal momentum about which only momentum squared is taken as observable.
2. Quite generally, one expects for the preferred extremals of Kähler action the slicing of space-time surface with string world sheets with stringy curves connecting wormhole throats. The ends of the stringy curves can be identified as light-like braid strands. Note that the strings themselves define a space-like braiding and the two braidings are in some sense dual. This has a concrete application in TGD inspired quantum biology, where time-like braiding defines topological quantum computer programs and the space-like braidings induced by it its storage into memory. Stringlike objects defining representations of super-conformal algebras must correspond to states involving at least two wormhole throats. Magnetic flux tubes connecting the ends of magnetically charged throats provide a particular realization of stringy on mass shell states. This would give rise to massless propagation at the parton level. The stringy quantization condition for mass squared would read as $4E^2 = n$ in suitable units for the representations of super-conformal algebra associated with the isometries. For pairs of throats of the same wormhole contact stringy spectrum does not seem plausible since the wormhole contact is in the direction of CP_2 . One can however expect generation of small mass as deviation of vacuum conformal weight from half integer in the case of gauge bosons.

If this picture is correct, one might be able to determine the momentum dependence of the scattering amplitudes by replacing free fermions with pairs of monopoles at the ends of string and topologically condensed fermions gauge bosons with pairs of this kind of objects with wormhole throat replaced by a pair of wormhole throats. This would mean suitable number of doublings of the Grassmannian integrations with additional constraints on the incoming momenta posed by the mass shell conditions for massive states.

3.6.5 Could zero energy ontology make possible full Yangian symmetry?

The partons in the loops are on mass shell particles have a discrete mass spectrum but both signs of energy are possible for opposite wormhole throats. This implies that in the rules for constructing loop amplitudes from tree amplitudes, propagator entanglement is restricted to that corresponding to pairs of partonic on mass shell states with both signs of energy. As emphasized in [B10], it is the Grassmannian integrands and leading order singularities of $\mathcal{N} = 4$ SYM, which possess the full Yangian symmetry. The full integral over the loop momenta breaks the Yangian symmetry and brings in IR singularities. Zero energy ontologist finds it natural to ask whether QFT approach shows its inadequacy both via the UV divergences and via the loss of full Yangian symmetry. The restriction of virtual partons to discrete mass shells with positive or negative sign of energy imposes extremely powerful restrictions on loop integrals and resembles the restriction to leading order singularities. Could this restriction guarantee full Yangian symmetry and remove also IR singularities?

3.6.6 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n , it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$ -local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

4 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B2] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K10]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.
3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.
4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.
5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current. Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

4.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the embedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

4.1.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of δM_{\pm}^4 at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.
2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.
3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.
4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \quad (4.1)$$

A more general form of this duality is suggested by the considerations of [K13] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}\sqrt{g_4} . \quad (4.2)$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} , \quad (4.3)$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

4.1.2 Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar} \oint B dS = n .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L2] , [L2] read as

$$\begin{aligned} \gamma &= \frac{e F_{em}}{\hbar} = 3J - \sin^2(\theta_W) R_{03} , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} . \end{aligned} \quad (4.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar} F_{em} + \sin^2(\theta_W) \frac{g_Z}{6\hbar} F_Z . \quad (4.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar} Q_{em} + \frac{g_Z^2 p}{6} Q_{Z,V} &= K \oint J = Kn , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} , \quad p = \sin^2(\theta_W) . \end{aligned} \quad (4.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + \sin^2(\theta_W) Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em} Q_{em} + p \frac{\alpha_Z}{2} Q_{Z,V} &= \frac{3}{4\pi} \times rnK , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} , \quad \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} . \end{aligned} \quad (4.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z . This however requires weak isospin invariance.

4.1.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should be invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is fine structure constant in electron length scale and \hbar_0 is the standard value of Planck constant.
2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD and CP_2 . The point is that in this case a given value of Planck constant corresponds to a finite number of pages of the “Big Book”. The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r . This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K25] supports this interpretation.
3. The identification of J as a counterpart of eB/\hbar means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to \hbar . This implies that for large values of \hbar Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.
4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z . \quad (4.8)$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{\hbar \text{bar}} . \quad (4.9)$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kähler form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

4.1.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kähler field and classical Z^0 field

$$\begin{aligned}\gamma &= 3J - \sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ .\end{aligned}\tag{4.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L2]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.
3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K26]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength must be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.
2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and

CP_2 are allowed as simplest possible solutions of field equations [K39]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.
4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

4.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

4.2.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \bar{\nu}_R$ or $X_{1/2} = \bar{\nu}_L \nu_R$. $\nu_L \bar{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.
2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

4.2.2 Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4.2.3 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1 + i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D5] .

4.2.4 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [?] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities X_{\pm} with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127} . It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.
2. The addition of the particles X^{\pm} replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.
3. How should one describe the bound state formed by the fermion and X^{\pm} ? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy.

In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K16]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K17].

4.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.
2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \rightarrow n \times h$ would effectively describe this. Boundary conditions would however give $1/n$ factor so that \hbar would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute “almost” would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like (“massless extremals” for which weak self-duality condition does not make sense [K5]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.
2. The original naïve conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates

Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K \epsilon^{n\alpha\beta\gamma} J_{\beta \text{ gamma}}) \sqrt{g_4} d^3x . \quad (4.11)$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = \text{constant}$ sphere - call it J^1 . The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K (J_{\gamma\delta} + \epsilon J_{\gamma\delta}^1)$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.
4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha . \quad (4.12)$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_\beta^K \partial_\gamma j_{\delta}^K = 0 . \quad (4.13)$$

j_K is a four-dimensional counterpart of Beltrami field [B4] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K5]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = *(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K . By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ . These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence

a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 . \quad (4.14)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J \phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2 . It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.
7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ . This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.
8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about

the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

5 Quantum TGD Very Briefly

5.1 Two Approaches To Quantum TGD

There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry [A5] for the “world of classical worlds” (WCW) identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein’s geometrization of physics program is in question. The second vision is the identification of physics as a generalized number theory involving p-adic number fields and the fusion of real numbers and p-adic numbers to a larger structure, classical number fields, and the notion of infinite prime.

With a better resolution one can distinguish also other visions crucial for quantum TGD. Indeed, the notion of finite measurement resolution realized in terms of hyper-finite factors, TGD as almost topological quantum field theory, twistor approach, ZEO, and weak form of electric-magnetic duality play a decisive role in the actual construction and interpretation of the theory. One can however argue that these visions are not so fundamental for the formulation of the theory than the first two.

5.1.1 Physics as infinite-dimensional geometry

It is good to start with an attempt to give overall view about what the dream about physics as infinite-dimensional geometry is. The basic vision is generalization of the Einstein’s program for the geometrization of classical physics so that entire quantum physics would be geometrized. Finite-dimensional geometry is certainly not enough for this purposed but physics as infinite-dimensional geometry of what might be called world of classical worlds (WCW) -or more neutrally WCW of some higher-dimensional imbeddign space- might make sense. The requirement that the Hermitian conjugation of quantum theories has a geometric realization forces Kähler geometry for WCW. WCW defines the fixed arena of quantum physics and physical states are identified as spinor fields in WCW. These spinor fields are classical and no second quantization is needed at this level. The justification comes from the observation that infinite-dimensional Clifford algebra [A1] generated by gamma matrices allows a natural identification as fermionic oscillator algebra.

The basic challenges are following.

1. Identify WCW.
2. Provide WCW with Kähler metric and spinor structure
3. Define what spinors and spinor fields in WCW are.

There is huge variety of finite-dimensional geometries and one might think that in infinite-dimensional case one might be drowned with the multitude of possibilities. The situation is however exactly opposite. The loop spaces associated with groups have a unique Kähler geometry due to the simple condition that Riemann connection exists mathematically [A17]. This condition requires that the metric possesses maximal symmetries. Thus raises the vision that infinite-dimensional Kähler geometric existence is unique once one poses the additional condition that the resulting geometry satisfies some basic constraints forced by physical considerations.

The observation about the uniqueness of loop geometries leads also to a concrete vision about what this geometry could be. Perhaps WCW could be regarded as a union of symmetric spaces [A10] for which every point is equivalent with any other. This would simplify the construction of the geometry immensely and would mean a generalization of cosmological principle to infinite-D context [K13, K28], [L5].

This still requires an answer to the question why $H = M^4 \times CP_2$ is so unique. Something in the structure of this space must distinguish it in a unique manner from any other candidate.

1. The uniqueness of M^4 factor can be understood from the miraculous conformal symmetries of the light-cone boundary but in the case of CP_2 there is no obvious mathematical argument of this kind although physically CP_2 is unique [L10].
2. The observation that $M^4 \times CP_2$ has dimension 8, the space-time surfaces have dimension 4, and partonic 2-surfaces, which are the fundamental objects by holography have dimension 2, suggests that classical number fields [A7, A2, A8] are involved and one can indeed end up to the choice $M^4 \times CP_2$ from physics as generalized number theory vision by simple arguments [K35], [L6]. In particular, the choices M^8 -a subspace of complexified octonions (for octonions see [A7]), which I have used to call hyper-octonions- and $M^4 \times CP_2$ can be regarded as physically equivalent: this “number theoretical compactification” is analogous to spontaneous compactification in M-theory. No dynamical compactification takes place so that $M^8 - H$ duality is a more appropriate term. Octonionic spinor structure required to be equivalent with the ordinary one makes also possible to generalize the twistors from 4-D to 8-D context and replaced 4-D light-likeness with 8-D one.
3. A further powerful argument in favor of H is that M^4 and CP_2 are the only twistor spaces with Kähler structure. The twistor lift of space-time surfaces to their twistor spaces with twistor structure induced from that of $M^4 \times CP_2$ indeed provides a new approach to TGD allowing to utilize powerful tools of algebraic geometry [K36].

5.1.2 Physics as generalized number theory

Physics as a generalized number theory (for an overview about number theory see [A6]) program consists of three separate threads: various p-adic physics and their fusion together with real number based physics to a larger structure [K34] , [L9], the attempt to understand basic physics in terms of classical number fields [K35], [L6] (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes [K33] , [L3], whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. In this article a summary of the philosophical ideas behind this dream and a summary of the technical challenges and proposed means to meet them are discussed.

The construction of p-adic physics and real physics poses formidable looking technical challenges: p-adic physics should make sense both at the level of the embedding space, the “world of classical worlds” (WCW), and space-time and these physics should allow a fusion to a larger coherent whole. This forces to generalize the notion of number by fusing reals and p-adics along rationals and common algebraic numbers. The basic problem that one encounters is definition of the definite integrals and harmonic analysis [A3] in the p-adic context [K20]. It turns out that the representability of WCW as a union of symmetric spaces [A10] provides a universal group theoretic solution not only to the construction of the Kähler geometry of WCW but also to this problem. The p-adic counterpart of a symmetric space is obtained from its discrete invariant by replacing discrete points with p-adic variants of the continuous symmetric space. Fourier analysis [A3] reduces integration to summation. If one wants to define also integrals at space-time level, one must pose additional strong constraints which effectively reduce the partonic 2-surfaces and perhaps even space-time surfaces to finite geometries and allow assign to a given partonic 2-surface a unique power of a unique p-adic prime characterizing the measurement resolution in angle variables. These integrals might make sense in the intersection of real and p-adic worlds defined by algebraic surfaces.

The dimensions of partonic 2-surface, space-time surface, and embedding space suggest that classical number fields might be highly relevant for quantum TGD. The recent view about the connection is based on hyper-octonionic representation of the embedding space gamma matrices, and the notions of associative and co-associative space-time regions defined as regions for which the Kähler-Dirac gamma matrices span quaternionic or co-quaternionic plane at each point of the region. A further condition is that the tangent space at each point of space-time surface contains a preferred hyper-complex (and thus commutative) plane identifiable as the plane of non-physical polarizations so that gauge invariance has a purely number theoretic interpretation. WCW can be regarded as the space of sub-algebras of the local octonionic Clifford algebra [A1] of the embedding space defined by space-time surfaces with the property that the local sub-Clifford algebra spanned

by Clifford algebra valued functions restricted at them is associative or co-associative in a given region.

The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy. The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave functions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of embedding space and space-time surface are subject to a number theoretic evolution.

One fascinating aspect of infinite primes is that besides the simplest infinite primes analogous to Fock states of a supersymmetric arithmetic QFT constructed from single particle states labelled by primes, also infinite primes having interpretation as bound states emerge. They correspond to polynomials characterized by degree n . Since the formation of bound states in TGD framework corresponds to a hierarchy of conformal symmetry breakings labelled by integer $n = h_{eff}/h$, the natural question is whether these two integers correspond to each other.

5.1.3 Questions

The experience has shown repeatedly that a correct question and identification of some weakness of existing vision is what can only lead to a genuine progress. In the following I discuss the basic questions, which have stimulated progress in the challenge of constructing WCW geometry.

1. What is WCW?

Concerning the identification of WCW I have made several guesses and the progress has been basically due to the gradual realization of various physical constraints and the fact that standard physics ontology is not enough in TGD framework.

1. The first guess was that WCW corresponds to all possible space-like 3-surfaces in $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 denotes complex projective space of two complex dimensions having also representation as coset space $SU(3)/U(2)$ (see the separate article summarizing the basic facts about CP_2 and how it codes for standard model symmetries [L2], [L7, L2]). What led to this particular choice H was the observation that the geometry of H codes for standard model quantum numbers and that the generalization of particle from point like particle to 3-surface allows to understand also remaining quantum numbers having no obvious explanation in standard model (family replication phenomenon). What is important to notice is that Poincare symmetries act as exact symmetries of M^4 rather than space-time surface itself: this realizes the basic vision about Poincare invariant theory of gravitation. This lifting of symmetries to the level of embedding space and the new dynamical degrees of freedom brought by the sub-manifold geometry of space-time surface are absolutely essential for entire quantum TGD and distinguish it from general relativity and string models. There is however a problem: it is not obvious how to get cosmology.
2. The second guess was that WCW consists of space-like 3-surfaces in $H_+ = M_+^4 \times CP_2$, where M_+^4 future light-cone having interpretation as Big Bang cosmology at the limit of vanishing mass density with light-cone property time identified as the cosmic time. One obtains cosmology but loses exact Poincare invariance in cosmological scales since translations lead out of future light-cone. This as such has no practical significance but due to the metric 2-dimensionality of light-cone boundary δM_+^4 the conformal symmetries of string model assignable to finite-dimensional Lie group generalize to conformal symmetries assignable to an infinite-dimensional symplectic group of $S^2 \times CP_2$ and also localized with respect to the

coordinates of 3-surface. These symmetries are simply too beautiful to be important only at the moment of Big Bang and must be present also in elementary particle length scales. Note that these symmetries are present only for 4-D Minkowski space so that a partial resolution of the old conundrum about why space-time dimension is just four emerges.

3. The third guess was that the light-like 3-surfaces inside CD are more attractive than space-like 3-surfaces. The reason is that the infinite-D conformal symmetries characterize also light-like 3-surfaces because they are metrically 2-dimensional. This leads to a generalization of Kac-Moody symmetries [A4] of super string models with finite-dimensional Lie group replaced with the group of isometries of H . The natural identification of light-like 3-surfaces is as 3-D surfaces defining the regions at which the signature of the induced metric changes from Minkowskian $(1, -1, -1, -1)$ to Euclidian $(-1 -1 -1 -1)$ - I will refer these surfaces as throats or wormhole throats in the sequel. Light-like 3-surfaces are analogous to blackhole horizons and are static because strong gravity makes them light-like. Therefore also the dimension 4 for the space-time surface is unique.

This identification leads also to a rather unexpected physical interpretation. Single light-like wormhole throat carries elementary particle quantum numbers. Fermions and their superpartners are obtained by glueing Euclidian regions (deformations of so called CP_2 type vacuum extremals of Kähler action) to the background with Minkowskian signature. Bosons are identified as wormhole throats with two throats carrying fermion *resp.* anti-fermionic quantum numbers. These can be identified as deformations of CP_2 vacuum extremals between two parallel Minkowskian space-time sheets. One can say that bosons and their superpartners emerge. This has dramatic implications for quantum TGD [K8] and QFT limit of TGD .

The question is whether one obtains also a generalization of Feynman diagrams. The answer is affirmative. Light-like 3-surfaces or corresponding Euclidian regions of space-time are analogous to the lines of Feynman diagram and vertices are replaced by 2-D surface at which these surfaces glued together. One can speak about Feynman diagrams with lines thickened to light-like 3-surfaces and vertices to 2-surfaces. The generalized Feynman diagrams are singular as 3-manifolds but the vertices are non-singular as 2-manifolds. Same applies to the corresponding space-time surfaces and space-like 3-surfaces. Therefore one can say that WCW consists of generalized Feynman diagrams- something rather different from the original identification as space-like 3-surfaces and one can wonder whether these identification could be equivalent.

4. The fourth guess was a generalization of the WCW combining the nice aspects of the identifications $H = M^4 \times CP_2$ (exact Poincare invariance) and $H = M^4_+ \times CP_2$ (Big Bang cosmology). The idea was to generalize WCW to a union of basic building bricks -causal diamonds (CDs) - which themselves are analogous to Big Bang-Big Crunch cosmologies breaking Poincare invariance, which is however regained by the allowance of union of Poincare transforms of the causal diamonds.

The starting point is General Coordinate Invariance (GCI). It does not matter, which 3-D slice of the space-time surface one choose to represent physical data as long as slices are related by a diffeomorphism of the space-time surface. This condition implies holography in the sense that 3-D slices define holograms about 4-D reality.

The question is whether one could generalize GCI in the sense that the descriptions using space-like and light-like 3-surfaces would be equivalent physically. This requires that finite-sized space-like 3-surfaces are somehow equivalent with light-like 3-surfaces. This suggests that the light-like 3-surfaces must have ends. Same must be true for the space-time surfaces and must define preferred space-like 3-surfaces just like wormhole throats do. This makes sense only if the 2-D intersections of these two kinds of 3-surfaces -call them partonic 2-surfaces- and their 4-D tangent spaces carry the information about quantum physics. A strengthening of holography principle would be the outcome. The challenge is to understand, where the intersections defining the partonic 2-surfaces are located.

ZEO (ZEO) allows to meet this challenge.

- (a) Assume that WCW is union of sub-WCWs identified as the space of light-like 3-surfaces assignable to $CD \times CP_2$ with given CD defined as an intersection of future and past directed light-cones of M^4 . The tips of CDs have localization in M^4 and one can perform for CD both translations and Lorentz boost for CDs. Space-time surfaces inside CD define the basic building brick of WCW. Also unions of CDs allowed and the CDs belonging to the union can intersect. One can of course consider the possibility of intersections and analogy with the set theoretic realization of topology.
- (b) ZEO property means that the light-like boundaries of these objects carry positive and negative energy states, whose quantum numbers are opposite. Everything can be created from vacuum and can be regarded as quantum fluctuations in the standard vocabulary of quantum field theories.
- (c) Space-time surfaces inside CDs begin from the lower boundary and end to the upper boundary and in ZEO it is natural to identify space-like 3-surfaces as pairs of space-like 3-surfaces at these boundaries. Light-like 3-surfaces connect these boundaries.
- (d) The generalization of GCI states that the descriptions based on space-like 3-surfaces must be equivalent with that based on light-like 3-surfaces. Therefore only the 2-D intersections of light-like and space-like 3-surfaces - partonic 2-surfaces- and their 4-D tangent spaces (4-surface is there!) matter. Effective 2-dimensionality means a strengthened form of holography but does not imply exact 2-dimensionality, which would reduce the theory to a mere string model like theory. Once these data are given, the 4-D space-time surface is fixed and is analogous to a generalization of Bohr orbit to infinite-D context. This is the first guess. The situation is actually more delicate due to the non-determinism of Kähler action motivating the interaction of the hierarchy of CDs within CDs.

In this framework one obtains cosmology: CDs represent a fractal hierarchy of big bang-big crunch cosmologies. One obtains also Poincare invariance. One can also interpret the non-conservation of gravitational energy in cosmology which is an empirical fact but in conflict with exact Poincare invariance as it is realized in positive energy ontology [K39, K31]. The reason is that energy and four-momentum in ZEO correspond to those assignable to the positive energy part of the zero energy state of a particular CD. The density of energy as cosmologist defines it is the statistical average for given CD: this includes the contributions of sub-CDs. This average density is expected to depend on the size scale of CD density is should therefore change as quantum dispersion in the moduli space of CDs takes place and leads to large time scale for any fixed sub-CD.

Even more, one obtains actually quantum cosmology! There is large variety of CDs since they have position in M^4 and Lorentz transformations change their shape. The first question is whether the M^4 positions of both tips of CD can be free so that one could assign to both tips of CD momentum eigenstates with opposite signs of four-momentum. The proposal, which might look somewhat strange, is that this not the case and that the proper time distance between the tips is quantized as integer multiples of a fundamental time scale $T = R/c$ defined by CP_2 size R .

A stronger - maybe un-necessarily strong - condition would be that the quantization is in octaves. This would explain p-adic length scale hypothesis, which is behind most quantitative predictions of TGD. That the time scales assignable to the CD of elementary particles correspond to biologically important time scales [K11] forces to take this hypothesis very seriously.

The interpretation for T could be as a cosmic time. Even more general quantization is proposed to take place. The relative position of the second tip with respect to the first defines a point of the proper time constant hyperboloid of the future light cone. The hypothesis is that one must replace this hyperboloid with a lattice like structure. This implies very powerful cosmological predictions finding experimental support from the quantization of redshifts for instance [K31]. For quite recent further empirical support see [E2].

One should not take this argument without a grain of salt. Can one really realize ZEO in this framework? The geometric picture is that translations correspond to translations of CDs.

Translations should be done independently for the upper and lower tip of CD if one wants to speak about zero energy states but this is not possible if the proper time distance is quantized. If the relative M_{\pm}^4 coordinate is discrete, this pessimistic conclusion is strengthened further.

The manner to get rid of problem is to assume that translations are represented by quantum operators acting on states at the light-like boundaries. This is just what standard quantum theory assumes. An alternative- purely geometric- way out of difficulty is the Kac-Moody symmetry associated with light-like 3-surfaces meaning that local M^4 translations depending on the point of partonic 2-surface are gauge symmetries. For a given translation leading out of CD this gauge symmetry allows to make a compensating transformation which allows to satisfy the constraint.

This picture is roughly the recent view about *WCW*. What deserves to be emphasized is that a very concrete connection with basic structures of quantum field theory emerges already at the level of basic objects of the theory and GCI implies a strong form of holography and almost stringy picture.

2. Some Why's

In the following I try to summarize the basic motivations behind quantum TGD in form of various Why's.

1. Why WCW?

Einstein's program has been extremely successful at the level of classical physics. Fusion of general relativity and quantum theory has however failed. The generalization of Einstein's geometrization program of physics from classical physics to quantum physics gives excellent hopes about the success in this project. Infinite-dimensional geometries are highly unique and this gives hopes about fixing the physics completely from the uniqueness of the infinite-dimensional Kähler geometric existence.

2. Why spinor structure in WCW?

Gamma matrices defining the Clifford algebra [A1] of WCW are expressible in terms of fermionic oscillator operators. This is obviously something new as compared to the view about gamma matrices as bosonic objects. There is however no deep reason denying this kind of identification. As a consequence, a geometrization of fermionic oscillator operator algebra and fermionic statistics follows as also geometrization of super-conformal symmetries [A9, A4] since gamma matrices define super-generators of the algebra of WCW isometries extended to a super-algebra.

3. Why Kähler geometry?

Geometrization of the bosonic oscillator operators in terms of WCW vector fields and fermionic oscillator operators in terms of gamma matrices spanning Clifford algebra. Gamma matrices span hyper-finite factor of type II_1 and the extremely beautiful properties of these von Neuman algebras [A21] (one of the three von Neumann algebras that von Neumann suggests as possible mathematical frameworks behind quantum theory) lead to a direct connection with the basic structures of modern physics (quantum groups, non-commutative geometries, .. [A14]).

A further reason why is the finiteness of the theory.

- (a) In standard QFTs there are two kinds of infinities. Action is a local functional of fields in 4-D sense and one performs path integral over **all** 4-surfaces to construct S-matrix. Mathematically path integration is a poorly defined procedure and one obtains diverging Gaussian determinants and divergences due to the local interaction vertices. Regularization provides the manner to get rid of the infinities but makes the theory very ugly.
- (b) Kähler function defining the Kähler geometry is expected to be non-local functional of the space-like 3-surfaces at the ends of space-time surface reducing by strong form of holography to a functional of partonic 2-surfaces and their 4-D tangent space data

(Kähler action for the Euclidian regions of the preferred extremal and having as interpretation in terms of generalized Feynman diagram).

Path integral is replaced with a functional integral, which is mathematically well-defined procedure and one performs functional integral only over the unions of 3-surfaces at opposite boundaries of CD and having vanishing super-conformal charges for a sub-algebras of conformal algebras with conformal weights coming as multiples of integer $h = h_{eff}/h$. This realizes the strong form of holography. The exponent of Kähler function - Kähler action for the Euclidian space-time regions - defines a unique vacuum functional whereas Minkowskian contribution to Kähler action gives the analog of ordinary imaginary exponent of action.

The local divergences of local quantum field theories are expected to be absent since there are no local interaction vertices. Also the divergences associated with the Gaussian determinant and metric determinant cancel since these two determinants cancel each other in the integration over WCW. As a matter of fact, symmetric space property suggests a much more elegant manner to perform the functional integral by reducing it to harmonic analysis in infinite-dimensional symmetric space [K42].

- (c) One can imagine also the possibility of divergences in fermionic degrees of freedom but the generalization of the twistor approach to 8-D context [K36] suggests that the generalized Feynman diagrams in ZEO are manifestly finite: in particular IR divergences plaguing ordinary twistor approach should be absent by 8-D masslessness. The only fermionic interaction vertex is 2- vertex associated with the discontinuity of K-D operator assignable to string world sheet boundary at partonic 2-surfaces serving as geometric vertices. At fermionic level scattering amplitudes describe braiding and OZI rule is satisfied so that the analog of topological QFT is obtained. The topological vertices describing the joining of incoming light-like orbits of partonic 2-surface at the vertices imply the non-triviality of the scattering amplitudes.

4. Why infinite-dimensional symmetries?

WCW must be a union of symmetric spaces in order that the Riemann connection exists (this generalizes the finding of Freed for loop groups [A17]). Since the points of symmetric spaces are metrically equivalent, the geometrization becomes tractable although the dimension is infinite. A union of symmetric spaces is required because 3-surfaces with a size of galaxy and electron cannot be metrically equivalent. Zero modes distinguish these surfaces and can be regarded as purely classical degrees of freedom whereas the degrees of freedom contributing to the WCW line element are quantum fluctuating degrees of freedom.

One immediate implication of the symmetric space property is constant curvature space property meaning that the Ricci tensor is proportional to metric tensor. Infinite-dimensionality means that Ricci scalar either vanishes or is infinite. This implies vanishing of Ricci tensor and vacuum Einstein equations for WCW.

5. Why ZEO and why causal diamonds?

The consistency between Poincaré invariance and GRT requires ZEO. In positive energy ontology only one of the infinite number of classical solutions is realized and partially fixed by the values of conserved quantum numbers so that the theory becomes obsolete. Even in quantum theory conservation laws mean that only those solutions of field equations with the quantum numbers of the initial state of the Universe are interesting and one faces the problem of understanding what the initial state of the universe was. In ZEO these problems disappear. Everything is creatable from vacuum: if the physical state is mathematically realizable it is in principle reachable by a sequence of quantum jumps. There are no physically non-reachable entities in the theory. ZEO leads also to a fusion of thermodynamics with quantum theory. Zero energy states are defined as entangled states of positive and negative energy states and entanglement coefficients define what I call M -matrix identified as “complex square root” of density matrix expressible as a product of diagonal real and positive density matrix and unitary S -matrix [K8].

There are several good reasons why for causal diamonds. ZEO requires CDs, the generalized form of GCI and strong form of holography (light-like and space-like 3-surfaces are physically

equivalent representations) require CDs, and also the view about light-like 3-surfaces as generalized Feynman diagrams requires CDs. Also the classical non-determinism of Kähler action can be understood using the hierarchy CDs and the addition of CDs inside CDs to obtain a fractal hierarchy of them provides an elegant manner to understand radiative corrections and coupling constant evolution in TGD framework.

A strong physical argument in favor of CDs is the finding that the quantized proper time distance between the tips of CD fixed to be an octave of a fundamental time scale defined by CP_2 happens to define fundamental biological time scale for electron, u quark and d quark [K11]: there would be a deep connection between elementary particle physics and living matter leading to testable predictions.

5.2 Overall View About Kähler Action And Kähler Dirac Action

In the following the most recent view about Kähler action and the Kähler-Dirac action (Kähler-Dirac action) is explained in more detail. The proposal is one of the many that I have considered.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action. The action could contain also Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term could be chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD). For Euclidian wormhole contacts Chern-Simons term need not reduce to a mere boundary terms since the gauge potential is not globally defined. One can also consider the possibility that only Minkowskian regions involve the Chern-Simons boundary term. One can also argue that Chern-Simons term is actually an un-necessary complication not needed in the recent interpretation of TGD.

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

The vanishing of conformal Noether charges for sub-algebras of various conformal algebras are also posed. They could be also realized as Lagrange multiplied terms at the ends of 3-surface.

2. By supersymmetry requirement the Kähler-Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with embedding space gamma matrices to obtain K-D gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. As explained, it is assumed that localization to 2-D string world sheets occurs. At the light-like boundaries the limit of K-D equation gives K-D equation at the fermionic lines expressing 8-D light-likeness or 4-D light-likeness in effective metric.

5.2.1 Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.
3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface. This however leads to an unphysical outcome.

5.2.2 Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naïve guess has been that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This is however a mere guess and need not be correct. The outcome is actually that the limit of K-D equation at string world sheets defines the Dirac equation at the boundaries of string world sheets.

One should try to make first clear what one really wants.

1. What one wants are generalized Feynman diagrams demanding massless Dirac propagators in 8-D sense at the light-like boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that 8-D generalization of the twistor Grassmannian approach works. The localization of spinors at string world sheets is crucial for achieving this.

In ordinary QFT fermionic propagator results from the kinetic term in Dirac action. Could the situation be same also now at the boundary of string world sheet associated with parton orbit? One can consider the Dirac action

$$L_{ind} = \int \bar{\Psi} \Gamma_{ind}^t \partial_t \Psi \sqrt{g_1} dt$$

defined by the induced gamma matrix Γ_{ind}^t and induced 1-metric. This action need to be associated only to the Minkowskian side of the space-surface. By supersymmetry Dirac action must be accompanied by a bosonic action $\int \sqrt{g_1} dt$. It forces the boundary line to be a geodesic line. Dirac equation gives

$$\Gamma_{ind}^t D_t \Psi = ip^k (M^8) \gamma_k \Psi = 0 .$$

The square of the Dirac operator gives $(\Gamma_{ind}^t)^2 = 0$ for geodesic lines (the components of the second fundamental form vanish) so that one obtains 8-D light-likeness.

Boundary line would behave like point-like elementary particle for which conserved 8-momentum is conserved and light-like: just as twistor diagrammatics suggests. 8-momentum must be real since otherwise the particle orbit would belong to the complexification of H . These conditions can be regarded as boundary conditions on the string world sheet and spinor modes. There would be no additional contribution to the Kähler action.

2. The special points are the ends of the fermion lines at incoming and outgoing partonic 2-surfaces and at these points M^4 mass squared is assigned to the embedding space spinor harmonic associated with the incoming fermion. CP_2 mass squared corresponds to the eigenvalue of CP_2 spinor d'Alembertian for the spinor harmonic.

At the end of the fermion line $p(M^4)^k$ corresponds to the incoming fermionic four-momentum. The direction of $p(E^4)^k$ is not fixed and one has $SO(4)$ harmonic at the mass shell $p(E^4)^2 = m^2$, m the mass of the incoming particle. At embedding space level color partial waves correspond to $SO(4)$ partial waves ($SO(4)$ could be seen as the symmetry group of low energy hadron physics giving rise to vectorial and axial isospin).

5.2.3 Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n \Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

5.2.4 Associativity (co-associativity) and quantum criticality

Quantum criticality is one of the basic notions of TGD. It was originally introduced to fix the value(s) of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current: this current vanishes for Cartan algebra of isometries. A clearer formulation of criticality is as a condition that the various conformal charges vanish for 3-surfaces at the ends of space-time surface for conformal weights coming as multiples of integer n . The natural expectation is that the numbers of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations is finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The conjecture is that quantum critical space-time surfaces are associative (co-associative) in the sense that the tangent vectors span a associative (co-associative) subspace of complexified octonions at each point of the space-time surface is consistent with what is known about preferred extremals. The notion of octonionic tangent space can be expressed by introducing octonionic structure realized in terms of vielbein in manner completely analogous to that for the realization of gamma matrices.

One can also introduce octonionic representations of gamma matrices but this is not absolutely necessarily. The condition that both the Kähler-Dirac gamma matrices and spinors are quaternionic at each point of the space-time surface leads to a precise ansatz for the general solution of the Kähler-Dirac equation making sense also in the real context. The octonionic version of the Kähler-Dirac equation is very simple since $SO(7,1)$ as vielbein group is replaced with G_2 acting as automorphisms of octonions so that only the neutral Abelian part of the classical electro-weak gauge fields survives the map.

This condition is analogous to what happens for the spinor modes when they are restricted at string worlds sheets carrying vanishing induced W fields (and also Z^0 fields above weak length scale) to guarantee well-definedness of em charge and it might be that this strange looking condition makes sense. The possibility to define G_2 structure would thus be due to the well-definedness of em charge and in the generic case possible only for string world sheets and possibly also partonic 2-surfaces.

Octonionic gamma matrices provide also a non-associative representation for the 8-D version of Pauli sigma matrices and encourage the identification of 8-D tangent space twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Quaternionicity condition implies that octo-twistors reduce to something closely related to ordinary twistors.

The sigma matrices are however an obvious problem since their commutators are proportional to M^4 sigma matrices. This raises the question whether the equivalence with ordinary Kähler-Dirac equation should be assumed. This assumption very strongly suggests a localization string

world sheets implied also by the condition that electromagnetic charge is well-defined for the spinor modes. The weakest manner to satisfy the equivalence would be for Dirac equation restricted to the light-like boundaries of string world sheets and giving just 8-D light-likeness condition but with random direction of light-like momentum.

5.2.5 The analog AdS/CFT duality

Although quantum criticality in principle predicts the possible values of Kähler coupling strength coming as a series of critical temperatures $\alpha_K = g_K^2/4\pi\hbar_{eff}$, $\hbar_{eff}/h = n$ characterizing quantum criticalities, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

Since WCW Kähler metric can be defined as anti-commutators of WCW gamma matrices identified as super-conformal super-charges for the K-D action, one would have the analog of AdS/CFT duality between bosonic definition of Kähler metric in terms of Kähler function defined by Euclidian contribution to Kähler action and fermionic definition in terms of anti-commutator of conformal supercharges.

This encourages to ask whether Dirac determinant - if it can be defined - could be identified as exponent of Kähler function or Kähler action. This might be of course un-necessary and highly unpractical outcome: it seems Kähler function is easy to obtain as Kähler action and Kähler metric as anti-commutators of super-charges. This is discussed in [K1].

5.3 Various Dirac Operators And Their Interpretation

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the Kähler-Dirac gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats. This applies in particular to condensed matter physics.

5.3.1 Four Dirac equations

To begin with, Dirac equation appears in four forms in TGD.

1. The Dirac equation in the world of classical worlds codes (WCW) for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the counterpart of string like objects (throats correspond to the ends of the string. WCW Dirac operator generalizes the Dirac operator of 8-D embedding space by bringing in vibrational degrees of freedom. This Dirac equation should give as its solutions zero energy states and corresponding M-matrices generalizing S-matrix.

The unitary U-matrix realizing discrete time evolution in the moduli space of CDs can be constructed as an operator in the space of zero energy states relating M-matrices [K18]. The natural application of U-matrix appears in consciousness theory as a coder of what Penrose calls U-process. The ground states to which super-conformal algebras act correspond to embedding space spinor modes in accordance with the idea that point like limit gives QFT in embedding space.

2. The analog of massless Dirac equation at the level of 8-D embedding space and satisfied by fermionic ground states of super-conformal representations.
3. Kähler Dirac equation is satisfied in the interior of space-time. In this equation the gamma matrices are replaced with Kähler-Dirac gamma matrices defined by the contractions of canonical momentum currents $T_k^\alpha = \partial L/\partial_\alpha h^k$ with embedding space gamma matrices Γ_k . This replacement is required by internal consistency and by super-conformal symmetries. The well-definedness of em charge implies that the modes of induced spinor field are localized at 2-D surfaces so that a connection with string theory type approach emerges.
4. At the light-like boundaries of string world sheets K-D equation gives rise to an analog of 4-D massless Dirac equation also one has light-like 8-momentum corresponding to the light-like

tangent vector of the fermion carrying line. This equation is equivalent with its octonionic counterpart.

Kähler-Dirac equation defines Dirac equation at space-time level. Consider first K-D equation in the interior of space-time surface.

1. The condition that electromagnetic charge operator defined in terms of em charge expressed in terms of Clifford algebra is well defined for spinor modes (completely analogous to spin defined in terms of sigma matrices) leads to the proposal that induced spinor fields are necessarily localized at 2-dimensional string worlds sheets [K42]. Only the covariantly constant right handed neutrino and its modes assignable to massless extremals (at least) generating supersymmetry (super-conformal symmetries) would form an exception since electroweak couplings would vanish. Note that the Kähler-Dirac gamma matrices possess CP_2 and this must vanish in order to have de-localization.
2. This picture implies stringy realization of super Kac-Moody symmetry elementary particles can be identified as string like objects albeit in different sense than in string models. At light-like 3-surfaces defining the orbits of partonic 2-surfaces spinor fields carrying electroweak quantum numbers would be located at braid strands as also the notion of finite measurement resolution requires.
3. Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed matter physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.
4. The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.
5. Sound as a concept is usually assigned with a rather high level of description. Stringy world sheets could however dramatically raise the status of sound in this respect. The oscillations of string world sheets connecting wormhole throats describe non-local 2-particle interactions. Holography suggests that this interaction just “gravitational” dual for electroweak and color interactions. Could these oscillations inducing the oscillation of the distance between wormhole throats be interpreted at the limit of weak “gravitational” coupling as analogs of sound waves, and could sound velocity correspond to maximal signal velocity assignable to the effective metric?
6. The latest progress in the understanding of quantum TGD imply that the area of string world sheet in the effective metric defined by the K-D gamma matrices indeed plays a fundamental role in quantum TGD (of course, WCW Kähler metric also involves this effective metric). By conformal invariance this metric could be equivalent with the induced metric. The string tension would be dynamical and the conjecture is that one can express Kähler action as total effective area of string world sheets. The hierarchy of Planck constants is essential in making possible to understand the description of not only gravitational but all bound states in terms of strings connecting partonic 2-surfaces. This description is analogous to AdS/CFT correspondence. That the string tension is defined by the Kähler action rather than assumed to be determined by Newton’s constants allows to avoid divergences.

The status of the Chern-Simons counterpart of K-D action has remained unclear. K-D action reduces to Chern-Simons boundary terms in Minkowskian space-time regions at least. I have considered Chern-Simons boundary term as an additional term in Kähler action and considered also Chern-Simons-Dirac operator. The localization of spinors to string world sheets however suggests that its introduction produces more problems than solves them. One reason is that C-S-D action involves only CP_2 gamma matrices so that one cannot realize 8-D masslessness for the spinor localized at fermion line defining the boundary of string world sheet.

5.3.2 Does energy metric provide the gravitational dual for condensed matter systems?

The Kähler-Dirac gamma matrices define an effective metric via their anti-commutators quadratic in components of energy momentum tensor (canonical momentum densities). This effective metric vanishes for vacuum extremals. Note that the use of the Kähler-Dirac gamma matrices guarantees among other things internal consistency and super-conformal symmetries of the theory.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. The energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anti-commutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1, -1, -1, -1)$. The 3-vector v_i/v_0 has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\bar{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the “energy metric”. One can associate with it analogs of curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton’s constant appearing as constant of proportionality. Note however that the besides ordinary metric and “energy” metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography could provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Could this give a general dual gravitational description of dissipative effects in terms of the “energy” metric and induced gauge fields? Does one obtain the analogs of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete determinism for Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity, which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [D4]. The lower bound for η/s is satisfied in good approximation by what should have been

QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [K17]).

An encouraging sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propagate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteeing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a de-coherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

5.3.3 Preferred extremals as perfect fluids

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional confirm the discovery [C2]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D3] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D2] .

1. The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3} \partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3} \partial_k v^k g_{ij}) . \quad (5.1)$$

The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta (\partial_i v_j + \partial_j v_i - \frac{2}{3} \partial_k v^k g_{ij}) . \quad (5.2)$$

From $dF^i = T^{ij} S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

2. The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} . \quad (5.3)$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p)u^\alpha u^\beta + pg^{\alpha\beta} - \sigma_{visc}^{\alpha\beta} . \quad (5.4)$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric $((1, -1, -1, -1))$.

3. If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p)g^{tt}\delta_t^\alpha \delta_t^\beta + pg^{\alpha\beta} - \sigma_{visc}^{\alpha\beta} . \quad (5.5)$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense. The existence of a global flow parameter means that one has

$$v_i = \Psi \partial_i \Phi . \quad (5.6)$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} . \quad (5.7)$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

1. Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the “topological” half of Maxwell’s equations (Faraday’s induction law and the statement that no non-topological magnetic are possible) is satisfied.
2. Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one’s tastes) define what I identify as generalized Feynman diagrams.

Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x . What causes the failure of the exact perfect fluid property?

1. Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At embedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).
2. The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of superconductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is fed into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

3. The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of h_{eff} can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large h_{eff} is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

1. The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before

continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by v_{\parallel} *resp.* v_{\perp} the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

2. The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs}mv_{\parallel}v_{\perp}S . \quad (5.8)$$

This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\parallel}}{d} \times S . \quad (5.9)$$

From this one obtains

$$\eta = n_{abs}mv_{\perp}d . \quad (5.10)$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = mv_{\perp}d . \quad (5.11)$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x , Angular momentum quantization suggests strongly itself as origin of the quantization.

3. Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities v_{\perp} . Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.

Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon} \quad (5.12)$$

Quantization condition would give

$$\epsilon = 4\pi \quad . \quad (5.13)$$

One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d .

4. One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

5.3.4 Is the effective metric one- or two-dimensional?

The following argument suggests that the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices is effectively one- or two-dimensional. Effective one-dimensionality would conform with the observation that the solutions of the modified Dirac equations can be localized to one-dimensional world lines in accordance with the vision that finite measurement resolution implies discretization reducing partonic many-particle states to quantum superpositions of braids. The localization to 1-D curves occurs always at the 3-D orbits of the partonic 2-surfaces. Note that the localization of induced spinor fields to string world sheets with 2-D CP_2 projection and carrying vanishing classical W fields would require only 2-D property.

The localization requires that the embedding space 1-forms associated with the K-D gamma matrices define lower-dimensional linearly independent set with elements proportional to gradients of embedding space coordinates defining coordinates for the lower-dimensional manifold. Therefore Frobenius conditions would be satisfied.

The argument is based on the following assumptions.

1. The Kähler-Dirac gamma matrices for Kähler action are contractions of the canonical momentum densities T_k^α with the gamma matrices of H .
2. The strongest assumption is that the isometry currents

$$J^{A\alpha} = T_k^\alpha j^{Ak} \quad (5.14)$$

for the preferred extremals of Kähler action are of form

$$J^{A\alpha} = \Psi^A (\nabla \Phi)^\alpha \quad (5.15)$$

with a common function Φ guaranteeing that the flow lines of the currents integrate to coordinate lines of single global coordinate variables (Beltrami property). Index raising is carried out by using the ordinary induced metric.

3. A weaker assumption is that one has two functions Φ_1 and Φ_2 assignable to the isometry currents of M^4 and CP_2 respectively.:

$$\begin{aligned} J_1^{A\alpha} &= \Psi_1^A (\nabla \Phi_1)^\alpha \quad , \\ J_2^{A\alpha} &= \Psi_2^A (\nabla \Phi_2)^\alpha \quad . \end{aligned} \quad (5.16)$$

The two functions Φ_1 and Φ_2 could define dual light-like curves spanning string world sheet. In this case one would have effective 2-dimensionality and decomposition to string world sheets [K14]. Isometry invariance does not allow more than two independent scalar functions Φ_i .

Consider now the argument.

1. One can multiply both sides of this equation with j^{Ak} and sum over the index A labeling isometry currents for translations of M^4 and $SU(3)$ currents for CP_2 . The tensor quantity $\sum_A j^{Ak} j^{Al}$ is invariant under isometries and must therefore satisfy

$$\sum_A \eta_{AB} j^{Ak} j^{Al} = h^{kl} , \quad (5.17)$$

where η_{AB} denotes the flat tangent space metric of H . In M^4 degrees of freedom this statement becomes obvious by using linear Minkowski coordinates. In the case of CP_2 one can first consider the simpler case $S^2 = CP_1 = SU(2)/U(1)$. The coset space property implies in standard complex coordinate transforming linearly under $U(1)$ that only the isometry currents belonging to the complement of $U(1)$ in the sum contribute at the origin and the identity holds true at the origin and by the symmetric space property everywhere. Identity can be verified also directly in standard spherical coordinates. The argument generalizes to the case of $CP_2 = SU(3)/U(2)$ in an obvious manner.

2. In the most general case one obtains

$$\begin{aligned} T_1^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla \Phi_1)^\alpha \equiv f_1^k (\nabla \Phi_1)^\alpha , \\ T_2^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla \Phi_2)^\alpha \equiv f_2^k (\nabla \Phi_2)^\alpha . \end{aligned} \quad (5.18)$$

3. The effective metric given by the anti-commutator of the modified gamma matrices is in turn is given by

$$G^{\alpha\beta} = m_{kl} f_1^k f_1^l (\nabla \Phi_1)^\alpha (\nabla \Phi_1)^\beta + s_{kl} f_2^k f_2^l (\nabla \Phi_2)^\alpha (\nabla \Phi_2)^\beta . \quad (5.19)$$

The covariant form of the effective metric is effectively 1-dimensional for $\Phi_1 = \Phi_2$ in the sense that the only non-vanishing component of the covariant metric $G_{\alpha\beta}$ is diagonal component along the coordinate line defined by $\Phi \equiv \Phi_1 = \Phi_2$. Also the contravariant metric is effectively 1-dimensional since the index raising does not affect the rank of the tensor but depends on the other space-time coordinates. This would correspond to an effective reduction to a dynamics of point-like particles for given selection of braid points. For $\Phi_1 \neq \Phi_2$ the metric is effectively 2-dimensional and would correspond to stringy dynamics.

One can also develop an objection to effective 1- or 2-dimensionality. The proposal for what preferred extremals of Kähler action as deformations of the known extremals of Kähler action could be leads to a beautiful ansatz relying on generalization of conformal invariance and minimal surface equations of string model [K5]. The field equations of TGD reduce to those of classical string model generalized to 4-D context.

If the proposed picture is correct, field equations reduce to purely algebraically conditions stating that the Maxwellian energy momentum tensor for the Kähler action has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1, 1) and second fundamental form H^k a tensor of type (2, 0) and (0, 2) so that $Tr(TH^k) = 0$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric

Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

There is however an important consistency condition involved. The Maxwell energy momentum tensor for Kähler action must have vanishing covariant divergence. This is satisfied if it is linear combination of Einstein tensor and metric. This gives Einstein's equations with cosmological term in the general case. By the algebraic character of field equations also minimal surface equations are satisfied and Einstein's General Relativity would be exact part of TGD.

In the case of Kähler-Dirac equation the result means that modified gamma matrices are contractions of linear combination of Einstein tensor and metric tensor with the induced gamma matrices so that the TGD counterpart of ordinary Dirac equation would be modified by the addition of a term proportional to Einstein tensor. The condition of effective 1- or 2-dimensionality seems to pose too strong conditions on this combination.

6 Summary Of Generalized Feynman Diagrammatics

This section gives a summary about the recent view about generalized Feynman diagrammatics, which can be seen as a hybrid of Feynman diagrammatics and stringy diagrammatics. The analogs of Feynman diagrams are realized at the level of space-time topology and geometry and the lines of these diagrams are Euclidian space-time regions identifiable as wormhole contacts. For fundamental fermions one has the usual 1-D propagator lines.

Physical particles can be seen as bound state of massless fundamental fermions and involve two wormhole contacts forming parts of closed Kähler magnetic flux tubes carrying monopole flux. The orbits of wormhole throats are connected by fermionic string world sheets whose boundaries correspond to massless fermion lines defining strands of braids. String world sheets in turn can form 2-braids.

It is a little bit matter of taste whether one refers to these diagrams generalized Feynman diagrams, generalized stringy diagrams, generalized Wilson loops or generalized twistor diagrams. All these labels are partly misleading.

In the sequel the basic action principles - Kähler action and Kähler-Dirac action are discussed first, and then a proposal for the diagrams describing M -matrix elements is discussed.

6.1 The Basic Action Principle

In the following the most recent view about Kähler action and the Kähler-Dirac action (Kähler-Dirac action) is explained in more detail. The proposal is one of the many that I have considered.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action. The action could contain also Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term could be chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD). For Euclidian wormhole contacts Chern-Simons term need not reduce to a mere boundary terms since the gauge potential is not globally defined. One can also consider the possibility that only Minkowskian regions involve the Chern-Simons boundary term. One can also argue that Chern-Simons term is actually an un-necessary complication not needed in the recent interpretation of TGD.

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

The vanishing of conformal Noether charges for sub-algebras of various conformal algebras are also posed. They could be also realized as Lagrange multiplied terms at the ends of 3-surface.

2. By supersymmetry requirement the Kähler-Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with embedding space gamma matrices to obtain K-D gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. As explained, it is assumed that localization to 2-D string world sheets occurs. At the light-like boundaries the limit of K-D equation gives K-D equation at the fermionic lines expressing 8-D light-likeness or 4-D light-likeness in effective metric.

6.1.1 Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.
2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.
3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface. This however leads to an unphysical outcome.

6.1.2 Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naïve guess has been that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This is however a mere guess and need not be correct. The outcome is actually that the limit of K-D equation at string world sheets defines the Dirac equation at the boundaries of string world sheets.

One should try to make first clear what one really wants.

1. What one wants are generalized Feynman diagrams demanding massless Dirac propagators in 8-D sense at the light-like boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that 8-D generalization of the twistor Grassmannian approach works. The localization of spinors at string world sheets is crucial for achieving this.

In ordinary QFT fermionic propagator results from the kinetic term in Dirac action. Could the situation be same also now at the boundary of string world sheet associated with parton orbit? One can consider the Dirac action

$$L_{ind} = \int \bar{\Psi} \Gamma_{ind}^t \partial_t \Psi \sqrt{g_1} dt$$

defined by the induced gamma matrix Γ_{ind}^t and induced 1-metric. This action need to be associated only to the Minkowskian side of the space-surface. By supersymmetry Dirac action must be accompanied by a bosonic action $\int \sqrt{g_1} dt$. It forces the boundary line to be a geodesic line. Dirac equation gives

$$\Gamma_{ind}^t D_t \Psi = ip^k (M^8) \gamma_k \Psi = 0 .$$

The square of the Dirac operator gives $(\Gamma_{ind}^t)^2 = 0$ for geodesic lines (the components of the second fundamental form vanish) so that one obtains 8-D light-likeness.

Boundary line would behave like point-like elementary particle for which conserved 8-momentum is conserved and light-like: just as twistor diagrammatics suggests. 8-momentum must be real since otherwise the particle orbit would belong to the complexification of H . These conditions can be regarded as boundary conditions on the string world sheet and spinor modes. There would be no additional contribution to the Kähler action.

2. The special points are the ends of the fermion lines at incoming and outgoing partonic 2-surfaces and at these points M^4 mass squared is assigned to the embedding space spinor harmonic associated with the incoming fermion. CP_2 mass squared corresponds to the eigenvalue of CP_2 spinor d'Alembertian for the spinor harmonic.

At the end of the fermion line $p(M^4)^k$ corresponds to the incoming fermionic four-momentum. The direction of $p(E^4)^k$ is not fixed and one has $SO(4)$ harmonic at the mass shell $p(E^4)^2 = m^2$, m the mass of the incoming particle. At embedding space level color partial waves correspond to $SO(4)$ partial waves ($SO(4)$ could be seen as the symmetry group of low energy hadron physics giving rise to vectorial and axial isospin).

6.1.3 Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n \Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

6.2 A Proposal For M -Matrix

The proposed general picture reduces the core of U -matrix to the construction of S -matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to imagine how the construction of M -matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k \gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-like geodesics of embedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.
3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.
4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K36].

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in CP_2 length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface the vertices would be represented by partonic 2-surfaces. In [K36] the interpretation of scattering amplitudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are “free”. At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K4] is a remnant of an “idea that came too early”. The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond to partial wave in the space

S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures ??, ?? (<http://tgdtheory.fi/appfigures/elparticletgd.jpg> <http://tgdtheory.fi/appfigures/tgdgrpahs.jpg>) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K36] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

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