The recent view about TGD and applications to condensed matter

M. Pitkänen,

February 2, 2024
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland. ORCID: 0000-0002-8051-4364.

Contents

Abstract

Condensed matter physics is under rapid evolution, and one might even speak of revolution. New exotic states of matter are discovered and their theoretical understanding in the existing theoretical framework is highly challenging. The findings challenge the existing reductionistic framework and it is quite possible that new physics is required. This motivates the question whether the new physics provided by TGD could provide some understanding.

The purpose of this article is to give a rough overall view about Topological Geometrodynamics (TGD) and to consider possible applications of TGD to condensed matter physics. The preparation of this article led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

2. The understanding of the details of the M^8-H duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. M^8-H duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H. Mass shell (its positive and negative energy parts) is mapped to a light-like boundary of CD with size $T=h_{eff}/m$, m the mass associated with momentum. This understanding of is crucial for the understanding of condensed matter physics since the 4-surfaces in M^8 are analogous to Fermi balls of condensed matter physics.

3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states, and is bound to have profound implications for condensed matter physics.

4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition, BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

After a summary of TGD as it is now, the basic notions of condensed matter physics are discussed from the TGD point view, some concrete problems of condensed matter are considered, and some tests are proposed.

1 Introduction

The purpose of this article is to give a rough overall view about Topological Geometrodynamics (TGD) and to consider its possible applications in condensed matter physics at the general level. It must be emphasized that TGD is only a vision, not a theory able to provide precise rules for calculating scattering amplitudes. A collective theoretical and experimental effort would be needed to achieve this. The proposal for a model of superconductivity [L54] provides a representative example about what TGD could possibly give for condensed matter physics.

It is perhaps good to explain what TGD is not and what it is or hoped to be. The article [L45] gives an overview of various aspects of TGD and is warmly recommended.

1. "Geometro-" refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with sub-manifold geometry. The basic motivation is the loss of classical conservation laws in

General Relativity Theory (GRT)(see **Fig. 1**). Also the interpretation as a generalization of string models by replacing string with 3-D surface is natural.

Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [L2]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely so that TGD is unique [L22, L27](see **Fig. 6**). The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "world of classical worlds" (WCW) - becomes the basic object endowed with Kähler geometry (see **Fig. 7**). General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. Given 3-surface fixes the space-time surface almost completely as analog of Bohr orbit (preferred extremal).This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces.

2. Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$.

   The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 3**).

   Practically any GCI action has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correletes for massless particles. World as a set or particles is replaced with a network having particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

   "Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 10**).

3. Adelic physics fusing real and various p-adic physics are part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical" orders. Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in all scales.

   The concrete realization of the number theoretic vision is based on $M^8 - H$ duality (see **Fig. 8**). The physics in the complexification of M^8 is algebraic - field equations as partial differential equations are replaced with algebraic equations associating to a polynomial with rational coefficients a X^4 mapped to H by $M^8 - H$ duality. The dark matter hierarchy corresponds to a hierarchy of algebraic extensions of rationals inducing that for adeles and has interpretation as an evolutionary hierarchy (see **Fig. 9**).

   $M^8 - H$ duality provides two complementary visions about physics (see **Fig. 2**), and can be seen as a generalization of the q-p duality of wave mechanics, which fails to generalize to quantum field theories (QFTs).

4. In Zero energy ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 12** and **Fig. 13**).

   Quantum jumps occur between these and the basic problem of standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 14**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions.

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary of CD and states at it remain unchanged (Zeno effect).

This work led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

2. The understanding of the details of the $M^8 - H$ duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. $M^8 - H$ duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H.

3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

The improved vision allows us to develop the TGD interpretation for various condensed matter notions.

1. TGD is analogous to hydrodynamics in the sense that field equations at the level of H reduce to conservation laws for isometry charges. The preferred extremal property meaning that space-time surfaces are simultaneous extremals of volume action and Kähler action allows interpretation in terms of induced gauge fields. The generalized Beltrami property implies the existence of an integrable flow serving as a correlate for quantum coherence. Conserved Beltrami flows currents correspond to gradient flows. At the QFT limit this simplicity would be lost.

2. The fields H, M, B and D, P, E needed in the applications of Maxwell's theory could emerge at the fundamental level in the TGD framework and reflect the deviation between Maxwellian and the TGD based view about gauge fields due to CP_2 topology.

3. The understanding of macroscopic quantum phases improves. The role of the magnetic body carrying dark matter is central. The understanding of the role of WCW degrees of freedom improves considerably in the case of Bose-Einstein condensates of bosonic particles such as polaritons. M^8 picture allows us to understand the notion of skyrmion. The formation of Cooper pairs and analogous states with higher energy would correspond to a formation of Galois singlets liberating energy used to increase h_{eff}. What is new is that energy feed makes possible supra-phases and their analogs above the critical temperature.

4. Fermi surface emerges as a fundamental notion at the level of M^8 but has a counterpart also at the level of H. Galois groups would be crucial for understanding braids, anyons and fractional Quantum Hall effect. Space-time surface could be seen as a curved quasicrystal associated with the lattice of M^8 defined by algebraic integers in an extension of rationals. Also the TGD analogs of condensed matter Majorana fermions emerge.

In section 1 this picture is discussed in more detail. In section 2 some concepts of condensed matter physics are discussed from the TGD view. In section 3 some concrete questions about condensed matter are discussed Hydrodynamical turbulence represents one of the unsolved problems of physics and therefore as an excellent test bench for the TGD based vision and is discussed in the 4^{th} section. The last section lists some tests for the TGD based vision. The approach is rather general: this is the only possible option since I am not a condensed matter specialist.

2 Physics as geometry

The following provides a sketchy representation of TGD based on the vision about physics as geometry which is complementary to the vision of physics as number theory. $M^8 - H$ duality relates these two visions. A longer representation can be found in [L45].

2.1 Space-time as 4-surface in $H = M^4 \times CP_2$

1. The energy problem of GRT means that since space-time is curved, one cannot define Poincare charges as Noether charges (see **Fig. 1**). If space-time X^4 is a surface in $H = M^4 \times CP_2$, the situation changes. Poincare symmetries are lifted to the level of $M^4 \subset H$.

2. Generalization of the notion of particle is in question: point-like particle $\rightarrow$ 3-surface so that TGD can be seen also as a generalization of string model. String $\rightarrow$ 3-surface. String world sheet $\rightarrow X^4$. The notions of the particle and space are unified.

3. Einstein's geometrization program is extended to standard model interactions. CP_2 codes for standard model symmetries and gauge fields. Isometries $\leftrightarrow$ color SU(3). Holonomies of spinor connection $\leftrightarrow$ electroweak U(2) [L2]. Genus-generation correspondence provides a topological explanation of the family replication phenomenon of fermions [K10]: 3 fermion families are predicted.

4. Induction of spinors structure as projection of components of spinor connection from CP_2 to X^4 is central for the geometrization. The projections of Killing vectors of color isometries yield color gauge potentials. Parallel translation at X^4 using spinor connection of H. Also spinor structure is induced and means projection of gamma matrices.

5. Dynamics for X^4 is determined by an action S consisting of Kähler action plus volume term (cosmological constant) following from the twistor lift of TGD [K39, L27].

6. The dynamics for fermions at space-time level is determined by modified Dirac action determined by S being super-symmetrically related to it. Gamma matrices are replaced with modified gamma matrices determined by the S as contractions of canonical momentum currents with gamma matrices. Preferred extremal property follows as a condition of hermiticity for the modified Dirac operator.

   Second quantized H-spinors, whose modes satisfy free massless Dirac equation in H restricted to X^4: this induces second quantization to X^4 and one avoids the usual problems of quantization in a curved background. This picture is consistent with the modified Dirac equation satisfied by the induced spinors in X^4.

   Only quarks are needed if leptons are 3-quark composites in CP_2 scale: this is possible only if one accepts the TGD view about color symmetries. This also provides a new view about matter antimatter asymmetry [L36, L53]. CP violation is forced by the M^4 part of Kähler form forced by the twistor lift.

2.1.1 Basic extremals of classical action

Practically any GCI action allows the same basic extremals (for basic questions related to classical TGD see **Fig. 3**).

1. CP_2 type extremals having light-like geodesic as M^4 projection and Euclidian signature of the induced metric serve as building bricks of elementary particles. If the volume term is absent as it might be at infinite volume limit, the geodesics become light-like curves [L71]. Wormhole contacts connecting two Minkowskian space-time sheets can be regarded as a piece of a deformed CP_2 type extremal. Monopole flux through contact stabilizes the wormhole contact.

2. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow superposition of modes with single direction of ligth-like momentum. Ideal laser beam is a convenient analogy here.

3. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and their thickenings to flux tubes are also a central notion.

2.1.2 QFT limit of TGD

The induced gauge fields and gravitational field are expressible in terms of only 4 H- coordinates. Locally the theory is too simple to be physical.

1. Many-sheeted space-time means that X^4 is topologically extremely complex. CP_2 coordinates are many-valued functions of M^4 coordinates or vice versa or both. In contrast to this, the space-time of EYM theory is topologically extremely simple.

2. Einsteinian space-times have 4-D projection to M^4. Small test particle experiences the sum of the classical gauge potentials associated with various space-time sheets. At QFT limit the sheets are replaced with a single region of M^4 made slightly curved and gauge potentials are defined as the sums of gauge potentials from different space-time sheets having common M^4 projection. Topological complexity and local simplicity are replaced with topological simplicity and local complexity. (see **Fig. 3**).

2.2 World of classical worlds (WCW)

The notion of WCW emerges as one gives up the idea about quantizing by path integral.

2.2.1 The failure of path integral forces WCW geometry

The extreme non-linearity implies that the path integral for surfaces space-time surfaces fails. A possible solution is generalize Einstein's geometrization program to the level of the entire quantum theory.

1. "World of classical worlds" (WCW) can be identified as the space of 3-surfaces endowe with a metric and spinor structure (see **Fig. 7**). Hermitian conjugation must have a geometrization. This requires Kähler structure requiring also complex structure. WCW has Kähler form and metric.

2. WCW spinors are Fock states created by fermionic oscillator operators assignable to spinor modes of H basically [L43]. WCW gamma matrices as linear combinations of fermionic (quark) oscillator operators defining analog of vielbein.

   WCW has also spinor connection and curvature in WCW. correspond The quantum states of world correspond formally to *classical* spinor fields in WCW. Gamma matrices of WCW expressinble in terms of fermionic oscillator operators are also purely classical objects.

2.2.2 Implications of General Coordinate Invariance

General Coordinate Invariance (GCI) in 4-D sense forces to assign to 3-surface X^3 a 4-surface $X^4(X^3)$, which is as unique as possible. This gives rise to Bohr orbitology and quantum classical correspondence (QCC), and holography. Also zero energy ontology (ZEO) emerges.

Quantum states quantum superpositions of space-time surfaces as analogs of Bohr orbits. QCC means that the classical theory is an exact part of quantum theory (QCC).

A solution to the basic paradox of quantum measurement theory emerges [L35]: superposition of deterministic time evolutions is replaced with a new one in state function reduction (SFR): SFR does not force any failure of determinism for individual time evolutions.

2.2.3 WCW Kähler geometry from classical action

WCW geometry is determined by a classical action defining Kähler function $K(X^3)$ for a preferred extremal $X^4(X^3)$ defining the preferred extremal/Bohr orbit [K19] (see **Fig. 7**).

1. QCC suggests that the definition of Kähler function assigns a more or less unique 4-surface $X^4(X^3)$ to 3-surface X^3. Finite non-uniqueness is however possible [L71].

2. $X^4(X^3)$ is identified as a *preferred* extremal of some general coordinate invariant (GCI) action forcing the Bohr orbit property/holography/ZEO. This means a huge reduction of degrees of freedom.

   Remark:: Already the notion of induced gauge field and metric eliminates fields as primary dynamical variables and GCI leaves locally only 4 H-coordinates as dynamical variables.

3. Twistor lift [L22, L27] of TGD geometrizes the twistor Grassmann approach to QFTs. The 6-D extremal X^6 of 6-D Kähler action as a 6- surface in the product $T(M^4) \times T(CP_2)$ of twistor spaces of M^4 and CP_2 represents the twistor space of X^4.

   The condition that X^6 reduces to an S^2 bundle with X^4 as base space, forces a dimensional reduction of 6-D Kähler action to 4-D Kähler action + volume term, whose value for the preferred extremal defines the Kähler function for $X^4(X^3)$.

4. The volume term corresponds to a p-adic length scale dependent cosmological constant Λ approach zero at long p-adic length scale so that a solution of the cosmological constant problem emerges. Preferred extremal/Bohr orbit property means a simultaneous extremal property for *both* Kähler action and volume term. This forces X^4 to have a generalized complex structure (Hamilton-Jacobi structure) so that field equations trivialize and there is no dependence on coupling parameters. Universality of dynamics follows and the TGD Universe is quantum critical. In particular, Kähler coupling strength is analogous to a critical temperature and is quantized [L62].

5. Soap film analogy is extremely useful [L71]: the analogs of soap film frames are singular surfaces of dimension $D < 4$. At the frame the space-time surface fails to be a simultaneous extremal of both actions separately and Kähler and volume actions couple to each other. The corresponding contributions to conserved isometry currents diverge but sum up to a finite contribution. The frames define the geometric analogs for the vertices of Feynman diagrams.

2.2.4 WCW geometry is unique

WCW geometry is fixed by the existence of Riemann connection and requires maximal symmetries.

1. Dan Freed [A3] found that loop space for a given Lie group allows a unique Kähler geometry: maximal isometries needed in order to have a Riemann connection. Same expected to be true now [K11, K34].

2. Twistor lift of TGD [L22, L27] means that one can replace X^4 with its twistor space $X^6(X^4)$ in the product $T(M^4) \times T(CP_2)$ of the 6-D twistor spaces $T(M^4)$ and $T(CP_2)$. $X^6(X^4)$ is 6-surface with the structure of S^2 bundle.

Dimensionally reduced 6-D Kähler action gives sum of 4-D Kähler action and volume term. Twistor space must however have a Kähler structure and only the twistor spaces of M^4,E^4, and CP_2 have Kähler structure [A6]. TGD is unique both physically and mathematically!

2.2.5 Isometries of WCW

What can one say about the isometries of WCW? Certainly, they should generalize conformal symmetries of string models.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [K11].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

2.2.6 A possible problem related to the twistor lift

The twistor lift strongly suggests that the Kähler form of M^4 exists. The Kähler gauge potential would be the sum of M^4 and CP_2 contributions. The definition of M^4 Kähler structure is however not straightforward [L39, L40]. The naive guess would be that J represents an imaginary unit as the square root of -1 represented by the metric tensor. This would give the condition $J^2 = -g$ for the tensor square but this leads to problems.

To understand the situation, notice that the analogs of symplectic/Kähler structures in $M^4 \subset H$ have a moduli space, whose points correspond to what I have called Hamilton-Jacobi structures defined by integrable distributions of orthogonal decompositions $M^4 = M^2(x) \times E^2(x)$: $M^2(x)$ is analogous to string world sheet and Y^2 to partonic 2-surface. This means the presence of slicing by string world sheets $X^2(x)$, where x labels a point of Y^2. $X^2(x)$ is orthogonal to Y^2 at x. One can interchange the roles X^2 ad Y^2 in the slicing.

The induced Kähler form has an analogous decomposition. The decomposition is completely analogous to the decomposition of polarizations to non-physical time-like ones and physical space-like ones. This decomposition allows a natural modification of the definition of the symplectic structure so that the problem caused by $J^2 = -g$ conditions is avoided.

Consider first the problem. The $E^2(x)$ part of M^4 Kähler metric produces no problems since the signature of the metric is Euclidean. For $M^2(x)$ part, the Minkowskian signature produces problems. If one assumes that the $M^2(x)$ part of the Kähler form is non-vanishing, it should be imaginary in order to satisfy $J^2(M^2(x)) = -g(M^2(x))$. This implies that Kähler gauge potential is imaginary and this spoils the hermiticity of the modified Dirac equation [K43]. Also the electric contribution to the Kähler energy is negative.

The solution of the problem turned out to be ridiculously simple and I should have noticed it a long time ago.

1. $M^2(x)$ has a hypercomplex structure, which means that the imaginary unit e satisfies $e^2 = 1$ rather than $e^2 = -1$. Hamilton-Jacobi structure allows one to decompose J locally

into two parts $J = J(M^2(x)) + J(E^2(x))$ such that $J^2 = g(M^2(x)) - g(E^2(x))$. This gives $J^4 = g(M^4)$. The Kähler energy of the canonically embedded M^4 is non-vanishing and positive whereas Kähler action vanishes by self-duality. Situation is identical to that in Maxwell's electrodynamics.

2. Kähler action for the canonically embedded M^4 vanishes and it is possible to define also Lagrangian 2-surfaces as surfaces for which the induced Kähler form vanishes. These are of special interest since they would guarantee small CP violation: string world sheets could be examples of these surfaces. Note that since the magnetic part of J induces violation of CP, the violation is vanishing for CP_2 type extremals and cosmic strings and also small for flux tubes.

If the notion of symplectic/canonical transformation generated by Hamiltonian preserving J generalizes, one could generate an infinite number of slicings.

Consider first ordinary symplectic transformations.

1. For the ordinary symplectic transformations, the closedness of the symplectic for J is essential ($dJ = 0$ corresponds to topological half of Maxwell's equations).

2. Second essential element is that symplectic transformation is generated as a flow for some Hamiltonian H: $j_H = i_{dH}J$ or more explicitly: $j^l_H = J^{kl}\partial_l H$. It is essential that one has $i_{j_H}J = -dH$: having a vanishing exterior derivative. In other words, $J_{kl}j^l_H = -\partial_k H$ is a gradient vector field and has therefore a vanishing curl. Together with $dJ = 0$, this guarantees the vanishing of the Lie derivative of J: $d_{j_H}J = d(i_{j_H}J) + i_{j_H}dJ = ddH + dJ(j_H) = 0$ so that J is preserved.

Could one talk about symplectic transformations in M^4?

1. The analogs of symplectic/canonical transformations should map the Hamilton-Jacobi structure to a new one and leave $J(M^2(x))$ and $J(E^2(x))$ invariant. The induced metrics of X^2 and Y^2 need not be preserved since only the diagonal metric $g^k_l(X^2/Y^2)$ appears in the conditions $J^2 = g(X^2) - g(Y^2)$.

2. The symplectic transformation generated by the Hamiltonian H would be a flow defined by the vector field $j_H = i_{dH}J$ and one would have $i_{j_H}J = -d_1H + d_2H$, where d_1 and d_2 are gradients operators in X^2 and Y^2. Usually one would have $J_{kl}j^l = dH$ satisfying $d^2H = 0$.

   The condition $ddH = 0$ satisfied by the ordinary symplectic transformations is replaced with the condition $d(-d_1H + d_2H) = 0$. This can be written as $-d_1^2H + d_2^2H + [d_2, d_1]H = 0$, and is satisfied. Therefore this part is not a problem.

3. Also the orthogonality of $M^2(x)$ and $E^2(x)$ must be preserved. This is a highly non-trivial condition since the metrics are induced and the symplectic transformations change the slicing and the metrics. An arbitrary Hamiltonian flow f, which depends on the coordinates of Y^2 only, maps Y^2 to itself but takes the tangent space $E^2(x)$ to $E^2(f(x))$. Unless the slicing satisfies special conditions, $E^2(f(x))$ is not orthogonal to $M^2(x)$.

4. The orthogonality is expressed as orthogonality of the projectors $P(X^2)$ and $P(Y^2)$: $P(X^2)P(Y^2) = 0$. This condition must be respected by the Hamiltonian flow. The product involves 4 components giving 4 conditions which turn out to be partial differential equations for Hamiltonian. The naive expectation is that there are very few solutions. The Lie-derivative of the product must therefore vanish:

$$L_{j_H}[P(X^2)P(Y^2)] = L_{j_H}(P(X^2))P(Y^2) + P(X^2))L_{j_H}(P(Y^2)) = 0 \ . \quad (2.1)$$

The projector $P_{mn}(X^2)$ can be expressed as

$$P^{mn} = g^{\alpha\beta}\partial_\alpha m^k \partial_\beta m^l \quad . \tag{2.2}$$

Here $g_{\alpha\beta} = m_{kl}\partial_\alpha m^k \partial_\beta m^l$ is the induced metric of X^2 or Y^2. m_{kl} is Minkowski metric and one can use linear Minkowski coordinates so that m_{kl} is constant.

The Lie derivative of $P^{mn}(X^2) \equiv P$ can be written as

$$L_j P^{mn} = L_j(g^{\alpha\beta})\partial_\alpha m^k \partial_\beta m^l + g^{\alpha\beta})(\partial_r j^k \partial_\alpha m^r \partial_\beta m^l + \partial_r j^l k \partial_\alpha m^r \partial_\beta m^k \quad . \tag{2.3}$$

The Lie derivative of the induced metric is

$$\begin{aligned} L_j g^{\alpha\beta} &= g^{\alpha\mu} g^{\beta\nu} L_J g_{\mu\nu} \quad , \\ L_j g_{\alpha\beta} &= m_{kl}(\partial_\alpha j^k \partial_\beta m^l + \partial_\alpha m^k \partial_\beta j^l \quad . \end{aligned} \tag{2.4}$$

Although the existence of symplectic transformations in the general case seems implausible, one can construct special slicings for which symplectic transformations are possible.

1. One can start from a trivial slicing defined by $M^2 \times E^2$ decomposition and perform slicings of M^2 and E^2. The orthogonality is trivially true for all slicings of this kind since $Y^2(y)$ is orthogonal to X^2 not only at y but at every point x. Symplectic transformations of M^2 and Y^2 produce new slicings of this kind. Even symplectic flowqs defined by general Hamiltonians respect the orthogonality.

2. Second example is provided by the slicing of the light-one boundary by light-like 2-surfaces Y^2_v labelled by the value of light-like radial coordinate v with metrics differing by r^2 factor. The surfaces X^2 would be planes $X^2(y)$ orthogonal to Y^2 at y with light-like coordinates u and v. The orthogonality would be preserved by symplectic transformations.

   The open question is whether these slicings are the only possible slicings allowing symplectic transformations. Although the construction of these slicings looks trivial, they are not trivial physically.

2.3 Should unitarity be replaced with the Kähler-like geometry of the fermionic state space?

Physical states correspond to WCW spinor fields and in ZEO. WCW spinors at a given point of WCW correspond to pairs of Fock states assignable to the 3-surfaces at the opposite boundaries of CD defining space-time surface. These pairs of many-fermion states in fermionic degrees of freedom define the TGD counterpart of the S-matrix.

Unitary is a natural notion in non-relativistic wave-mechanics but already in quantum field theory it becomes problematic. In the twistor approach to the scattering amplitudes of massless gauge theories both unitarity and locality are problematic. Whether TGD can give rise to a unitary S-matrix has been a continual head-ache. This leads to a heretic question.

Is unitarity possible at all in TGD framework and should it be replaced with some deeper principle? I have considered these questions several times and in [L49] a rather radical solution was proposed. The implications of this proposal for the construction of scattering amplitudes are discussed in [L51].

Assigning an S-matrix to a unitary time evolution works in non-relativistic theory but fails already in the generic QFT and correlation functions replace S-matrix.

1. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space could replace the unitary S-matrix.

2. An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of the Teichmüller matrix.

   Teichmüller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong form of holography (SH), the most natural candidate would be Cartesian product of Teichmüller spaces of partonic 2 surfaces with punctures and string world sheets.

3. Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

4. In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmüller condition guaranteeing non-negative probabilities.

5. Equivalence Principle generalizes to the level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

6. There is also an objection. The transition probabilities would be given by $P(A,B) = g^{A,\overline{B}}g_{\overline{B},A}$ and the analogs for unitarity conditions would be satisfied by $g^{A,\overline{B}}g_{\overline{B},C} = \delta^A_C$. The problem is that $P(A,B)$ is not real without further conditions. Can one imagine any physical interpretation for the imaginary part of $Im(P(A,B))$?

In this framework, the twistorial scattering amplitudes as zero energy states define the covariant Kähler metric $g_{A\overline{B}}$, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric. The problem with the unitarity would disappear.

This view is developed in detail in [L51] and one ends up with a very concrete and surprisingly simple number theoretic view about scattering amplitudes.

2.4 About Dirac equation in TGD framework

2.4.1 Three Dirac equations

In TGD spinors appear at 3 levels:

1. At the level of embedding space $H = M^4 \times CP_2$ the spinor field embedding space $M^4 \times CP_2$spinor fields (quark field) is a superposition of the harmonics of the Dirac operator. In the complexified M^8 having interpretation as complexified octonions, spinors are octonionic

spinors. In accordance with the fact that M^8 is analogous to momentum space, the Dirac equation is purely algebraic and its solutions correspond to discrete points analogous to occupied points of Fermi ball.

2. The spinors at the level of 4-surfaces $X^4 \subset H$ are restrictions of the second quantized embedding space spinor field in X^4 so that the problematic second quantization in curved background is avoided. At the level of M^8 the restriction selects the points of M^8 belonging to 4-surface and carrying quark. The simplest manner to realize Fermi statistics is to assume that there is at most a single quark at a given point.

3. The third realization is at the level of the "world of classical worlds" (WCW) assigned to H consisting of 4-surfaces as preferred extremals of the action. Gamma matrices of WCW are expressible as superpositions of quark oscillator operators so that anti-commutation relations are geometrized. The conditions stating super-symplectic symmetry are a generalization of super-Kac-Moody symmetry and of super-conformal symmetry and give rise to the WCW counterpart of the Dirac equation [K34] [L45].

4. What the realization of WCW at the level of M^8 is, has remained unclear. The notion of WCW geometry does not generalize to his level and should be replaced with an essentially number theoretic notion.

   Adelic physics as a fusion of real and p-adic physics suggests a possible realization. Given extension of rationals induces extensions of various p-adic number fields. These can be glued to a book-like structure having as pages real numbers and the extensions of p-adic number fields.

   The pages would intersect along points with coordinates in the extension of rationals. These points form a cognitive representation. The additional condition that the active points are occupied by quarks guarantees that this makes sense also for octonions, quaternions and 4-surface in M^8. The p-adic sector could consist of discrete and finite cognitive representations continued to the p-adic surface and define the counterpart of WCW at the level of M^8?

2.4.2 The relationship between Dirac operator of H and modified Dirac operator

At the level of $X^4 \subset H$, the proposal is that modified Dirac action for the induced spinor fields defines the dynamics somehow. Modified Dirac equation or operator should be also consistent with the second quantization of induced spinor fields performed at the level of H and inducing the second quantization at the level of X^4.

1. The modified gamma matrices Γ^α are defined by the contractions of H gamma matrices Γ_k and canonical momentum currents $T^{k\alpha}$ associated with the action defining space-time surface. The modified Dirac operator $D = \Gamma^\alpha D_\alpha$, where D_α is X^4 projection of the vector defined by the covariant derivative operators of H ($D_\alpha = \partial_\alpha h^k D_k$). Hermiticity requires $D_\alpha \Gamma^\alpha = 0$ implying that classical field equations are satisfied.

2. Can one assume that the modified Dirac equation is satisfied? Or is it enough to assume that this is not the case so that the modified Dirac operator defines the propagator as its inverse as the QFT picture would suggest?

   In fact, the propagators in H allow to compute N-point functions involving quarks and at the level of H the theory is free and the restriction to the space-time surface brings in the interactions. Therefore the notion of space-time propagator is not absolutely necessary. One can however ask whether some weaker condition could be satisfied and provide new insights.

   One can also ask whether the solutions of the modified Dirac equation correspond to external particles, which correspond to space-time surfaces for which the solution of the modified Dirac equation is consistent with the solution of the Dirac equation in H. Are these kinds of space-time surfaces possible?

3. The intuitive picture is that the solutions of the modified Dirac equation correspond to the external particles of a scattering diagram having an interpretation on mass shell states and are possible only for a very special kind of preferred extremals. Intuitively they should

correspond to singular surfaces in M^8 and their mapping to H would involve blow-up due to the non-uniqueness of the normal space along lower than 4-D surface. String like objects and CP_2 type extremals would be basic entities of this kind. Could the modified Dirac equation or its weakened form hold true for these surfaces.

The strong form of equivalence of modified Dirac equation and ordinary Dirac equation would mean the equivalence of the actions of two Dirac operators acting on the second quantized induced spinor field.

1. The modified Dirac operator is given by $\Gamma_k T^{\alpha k}\partial_\alpha h^k D_k$ and its action should be same as H Dirac operator $\Gamma^k D_k$. This would require

$$\Gamma_k T^{\alpha k}\partial_\alpha h^k D_k\Psi \quad = \quad \Gamma^k D_k \Psi \ . \tag{2.5}$$

Not surprisingly, it turns out that this condition is too strong.

2. One can express Γ_k using an overcomplete basis defined by the Killing vector fields j_A^k for H isometries. In the case of M^4 it is enough to use translations by using the identity $\sum_A j_A^k j_A^l = h^{kl}$. This allows to define gamma matrices $\Gamma_A = \Gamma_k j_A^k$ and to write the equation in the form

$$\Gamma_A T^{A\alpha}\partial_\alpha h^k D_k\Psi \quad = \quad \Gamma_A j_A^k D_k\Psi \ . \tag{2.6}$$

Here $T^{A\alpha}$ is the conserved isometry current associated with the Killing vector j_A^k. Is it possible to satisfy the condition

$$T^{A\alpha}\partial_\alpha h^k = j_A^k \tag{2.7}$$

or its suitably weakened form?

The strong form of the condition cannot be satisfied. The left hand side of the equation is determined by the gradients of H coordinates and parallel to X^4 whereas the right hand side also involves the component normal to X^4. Therefore the condition cannot be satisfied in the general case.

3. By projecting the condition to the tangent space, one obtains a weaker condition stating that the tangential parts of two Dirac operators are proportional to each other with a position dependent proportionality factor $\Lambda(x)$:

$$\begin{aligned} T^{A\alpha} &= \Lambda(x) j_A^\alpha \\ j_A^\alpha &= j_A^k \partial^\alpha h_k = j_A^k h_{kl} g^{\alpha\beta}\partial_\beta h^l \ . \end{aligned} \tag{2.8}$$

The conserved isometry current is proportional to the projection of the Killing vector to the tangent space of X^4. $\Lambda(x)$ is proportionality constant depending on the point of X^4. Isometry current is analogous to a Hamiltonian vector field being parallel to the Killing vector field.

4. If the action were a mere cosmological volume term, the isometry currents would be proportional to j^α so that the conditions would be automatically satisfied. The contribution to $\Lambda(x)$ is proportional to the p-adic length scale dependent cosmological constant.

Kähler action receives contributions from both M^4 and CP_2. Both add to $T^{A\alpha}$ a term of form $T^{\alpha\beta} j_{A\beta}$ coming from the variation of the Kähler action with respect to $g_{\alpha\beta}$. $T^{\alpha\beta}$ is the energy momentum tensor with a form similar to that for Maxwell action.

Besides this, M^4 *resp.* CP_2 contribute a term proportional to $J^{\alpha\beta}J_{kl}\partial_\beta h^k j_A^k$ coming from the variation of the Kähler action with respect to $J_{\alpha\beta}$ contributing only to M^4 *resp.* CP_2 isometries. These contributions make the conditions non-trivial. The Kähler contribution to $\Lambda(x)$ need not be constant. Note that the Kähler contributions to the energy momentum tensor vanish if X^4 is (minimal) surface of form $X^2 \times Y^2 \subset M^4 \times CP_2$ so that both X^2 and Y^2 are Lagrangian.

5. The vanishing of the divergence of $T^{A\alpha}$ using the Killing property $D_l j_{Ak} + D_k j_{Al} =$ of j_{Ak} gives

$$j^{A\alpha}\partial_\alpha \Lambda = 0 \ . \tag{2.9}$$

   Λ is constant along the flow lines of $j^{A\alpha}$ and is therefore analogous to a Hamiltonian. The constant contribution from the cosmological term to Λ does not contribute to this condition.

6. An attractive hypothesis, consistent with the hydrodynamic interpretation, is that the proposed condition is true for all preferred extremals. The conserved isometry current along the X^4 projection of the flow line is proportional to the projection of Killing vector: this conservation law is analogous to the conservation of energy density $\rho v^2/2 + p$ along the flow line). One can say that isometries as flows in the embedding space are projected to flows along the space-time surface. One could speak of projected or lifted representation.

7. The projection to the normal space does not vanish in the general case. One could however ask whether a weaker condition stating that the second fundamental form $H^k_{\alpha\beta} = D_\alpha h^k$, which is normal to X^4, defines the notion of the normal space in terms of data provided by space-time surface. If X^4 is a geodesic submanifold of H, in particular a product of geodesic submanifolds of M^4 and CP_2, one has $H^k_{\alpha\beta} = 0$.

2.4.3 Gravitational and inertial representations of isometries

The lift/projection of the isometry flows to X^4 strongly suggests a new kind of representation of isometries as analog of the braid representation considered earlier.

1. Projected/lifted representation would clarify the role of the classical conserved charges and currents and generalize hydrodynamical conservation laws along the flow lines of isometries. In particular, quark lines would naturally correspond to time-like flow lines of time translations. In the case of CP_2 type extremals, quark momenta for the lifted representations would be light-like.

2. The conservation conditions along the flow lines are very strong, and one can wonder if they might provide a new formulation of the preferred extremal property. It is quite possible that the conditions apply only to a sub-algebra. Quantum classical correspondence (QCC) suggests Cartan algebra for which the quantum charges can have well-defined eigen values simultaneously. In accordance with QCC, the choice of the quantization axes would affect the space-time surfaces considered and could be interpreted as a higher level quantum measurement.

3. Projected/lifted representation provides a new insight also to the Equivalence Principle (EP) stating that gravitational and inertial masses are identical. At the level of scattering amplitudes involving isometry charges defined at the level of H, the isometries affect the entire space-time surface, and one could see EP as an almost trivial statement. QCC however forces us to consider EP more seriously.

   I have proposed that QCC could be seen as the identification of the eigenvalues of Cartan algebra isometry charges for quantum states with the classical charges associated with the preferred extremals. EP would follow from QCC: gravitational charges would correspond to the representation of the flows defined by isometries as their projections/lifts to X^4 whereas inertial charges would correspond to the representation at the level of H with isometries affecting the entire space-time surfaces.

4. The lifted/projected/gravitational representation of isometries, which seems possible in 4-D situation, is analogous to braid group representation making sense only in 2-D situation. Indeed, for the many-sheeted space-time surfaces assignable to $h_{eff} > h_0$, it can happen that rotation by 2π leads to a new space-time sheet and that the $SO(2)$ subgroup of the rotation group associated with the Cartan algebra is lifted to n-fold covering. Same can can happen in the case of color rotations. This leads to a fractionation of quantum numbers usually assigned with quantum group representations suggested to correspond to $h_{eff} > h$ [K27].

   Also for the quantum groups, Cartan algebra plays a special role. In the case of the Poincare group, the 2-D nature of braid group representations would correspond to the selection $M^2 \times SO(2)$ as a Cartan subgroup implying effective 2-dimensionality in the case rotation group. Gravitational representations could therefore correspond to quantum group representations.

5. The gravitational representation provides also a new insight on $M^8 - H$ duality. The source of worries has been whether Uncertainty Principle (UP) is realized if a given 4-surface in M^8 is mapped to a single space-time surface in M^8. It seems that UP can be realized both in terms of inertial and gravitational representations.

   (a) In the case of the "inertial" representation of H-isometries at the level of H, one must regard $X^4 \subset H$ representing images of particle-like 4-surface in M^8 analog of Bohr orbit (holography) and map it to an analog of plane wave define as superposition of its translates and by the total momentum associated with the either boundary of CD associated with the particle. The same applies to the transforms to other Cartan algebra generators.

   In a cognitive representation based on extension of rationals, the shifts for Cartan algebra would be discrete: the values of the plane wave would be roots of unity belonging to the extension and satisfy periodic boundary conditions at the boundary of larger CD.

   Periodic boundary conditions pose rather strong conditions on the time evolution by scaling between two SSFRs. The scaling must respect the boundary conditions. If the momenta assignable to the plane waves of massive particles are conserved and h_{eff} is conserved, the scaling must multiply CD size by integers. The iterations of integer scalings, in particular $n = 2$ scalings (period doubling), are in a preferred position.

   (b) If one replaces the inertial representation of isometries with the gravitational representation, the quantum states can be realized at the level of a single space-time surface. One would have two representations: gravitational and inertial -subjective and objective, one might say.

   (c) Gravitational representations make also sense for the super-symplectic group acting at the boundary of light-cone as well as for the Kac-Moody type algebra associated with the isometries of H realized the light-like orbits of partonic 2-surfaces.

2.5 Different ways to understand the "complete integrability" of TGD

There are several ways to see how TGD could be a completely integrable theory.

2.5.1 Preferred extremal property

Preferred extremal property requires Bohr orbit property and holography and is an extremely powerful condition.

1. Twistor lift of TGD implies that X^4 in H is simultaneous extremal of volume action and Kähler action. Minimal surface property is counterpart for massless field equations and extremality for Kähler action gives interpretation for massless field as Kähler form as part of induced electromagnetic field.

   The simultaneous preferred extremal property strongly suggests that 2-D complex structure generalizes for 4-D space-time surfaces and so called Hamilton-Jacobi structure [L32] meaning a decomposition of M^4 to orthogonal slicings by string world sheets and orthogonal partonic 2-surfaces would realize this structure.

2. Generalized Beltrami property [L54] implies that 3-D Lorentz force and dissipation for Kähler form vanish. The Kähler form is analogous to the classical Maxwell field. Energy momentum tensor has vanishing divergence, which makes it plausible that QFT limit is analogous to Einstein-Maxwell theory.

   The condition also implies that the Kähler current defines an integrable flow so that there is global coordinate varying along flow lines. This is a natural classical correlate for quantum coherence. Quantum coherence would be always present but broken only by the finite size of the region of the space-time considered.

   Beltrami property plus current conservation implies gradient flow and an interesting question is whether conserved currents define gradient flows: non-trivial space-time topology would allow this at the fundamental level. Beltrami condition is a very natural classical condition in the models of supraphases.

3. The condition that the isometry currents for the Cartan algebra of isometries are proportional to the projections of the corresponding Killing vectors is a strong condition and could also be at least an important aspect of the preferred extremal property.

2.5.2 Supersymplectic symmetry

The third approach is based on the super-symplectic symmetry of WCW. Isometry property would suggest that an infinite number of super-symplectic Noether charges are defined at the boundaries of CD by the action of the theory. They need not be conserved since supersymplectic symmetries cannot be symmetries of the action: if they were, the WCW metric would be trivial.

The gauge conditions for Virasoro algebra and Kac-Moody algebras suggest a generalization. Super-symplectic algebra (SSA) involves only non-negative conformal weights n suggesting extension to a Yangian algebra (this is essential!). Consider the hierarchy of subalgebras SSA_m for which the conformal weights are m-tiples of those of entire algebra. These subalgebras are isomorphic with the entire algebra and form a fractal hierarchy.

Assume that the sub-algebra SSA_m and commutator $[SSA_m, SSA]$ have vanishing classical Noether charges for $m > m_{max}$. These conditions could fix the preferred extremal. One can also assume that the fermionic realizations of these algebras annihilate physical states. The remaining symmetries would be dynamical symmetries.

The generators are Hamiltonians of $\delta M^4_+ \times CP_2$. The symplectic group contains Hamiltonians of the isometries as a normal sub-algebra. Also the Hamiltonians of and one could assume that only the isometry generators correspond to non-trivial classical and quantal Noether charges. Could the actions of SSA and Kac-Moody algebras of isometries be identical if a similar construction applies to Kac-Moody half-algebras associated with the light-like partonic orbits. Super-symplectic symmetry would reduce to a hierarchy of gauge symmetries.

3 Physics as number theory

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L20, L21], and classical number fields [K38].

3.1 p-Adic physics

The motivation for p-adicization came from p-adic mass calculations [K22, K10].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Boltzmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1+i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [K25] is interpreted as a correlate for cognition. Motivation comes from the observation that piecewise constant functions depending on a finite number of pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

Unlike the Higgs mechanism, p-adic thermodynamics provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry which guarantees the existence of p-adic Boltzmann weights.

The number theoretic picture leads to a deeper understanding of a long standing objection against p-adic thermodynamics [K22] as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra.

If one requires super-Virasoro symmetry and identifies mass squared with a scaling generator L_0, one can argue that only massless states are possible since L_0 must annihilate these states! All states of the theory would be massless, not only those of fundamental particles as in conformally invariant theories to which twistor approach applies! This looks extremely beautiful mathematically but seems to be in conflict with reality already at single particle level!

The resolution of the objection is that *thermodynamics* is indeed in question.

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and describes approximately the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors. The excited states of the entire system indeed are massless [?]

2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment and the entire mass squared would remain vanishing. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

3. In the p-adic mass calculations [K22, K10], the effective environment for quarks and leptons would in a good approximation consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons). The many-quark state many-quark state associated with the wormhole throat (single quark state for quarks and 3-quark-state for leptons [L53].

4. In M^8 picture [L39, L40], tachyonicity is unavoidable since the real part of the mass squared as a root of a polynomial P can be negative. Also tachyonic real but algebraic mass squared values are possible. At the H level, tachyonicity corresponds to the Euclidean signature of the induced metric for a wormhole contact.

   Tachyonicity is also necessary: otherwise one does not obtain massless states. The super-symplectic states of quarks would entangle with the tachyonic states of the wormhole contacts by Galois confinement.

5. The massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-symplectic representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Galois confinement is satisfied. The tachyonic mass squared is assigned with wormhole contacts with the Euclidean signature of the induced metric, whose throats in turn carry the fermions so that the wormhole contact would form the nearby environment.

The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state remains massless and massive single particle states receive a compensating negative mass squared from the wormhole contact. Thermal mass squared corresponds to a single particle mass squared and does not take into account the contribution of wormhole contacts except for the ground state.

6. There is a further delicate number theoretic element involved [L61, L71]. The choice of $M^4 \subset M^8$ for the system is not unique. Since M^4 momentum is an M^4 projection of a massless M^8 momentum, it is massless by a suitable choice of $M^4 \subset M^8$. This choice must be made for the environment so that both the state of the environment and the single particle ground state are massless. For the excited states, the choice of M^4 must remain the same, which forces the massivation of the single particle excitations and p-adic massivation.

These arguments strongly suggest that pure states, in particular the state of the entire Universe, are massless. Mass would reflect the statistical description of entanglement using the density matrix. The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural. This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K26] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and M^8-H duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L72, L73].

3.2 Adelic physics

Adelic physics fuses real and various p-adic physics to a single structure [L21].

1. One can combine real numbers and p-adic number fields to a product: number fields would be like pages of a book intersecting along rationals acting as the back of the book.

2. Each extension of rational induces extensions of p-adic number fields and extension of the basic adele. Points in the extension of rationals are now common to the pages. The infinite hierarchy of adeles defined by the extensions forms an infinite library.

3. This leads to an evolutionary hierarchy (see **Fig. 9**) . The order n of the Galois group as a dimension of extension of rationals is identified as a measure of complexity and of evolutionary level, "IQ". Evolutionary hierarchy is predicted.

4. Also a hierarchy of effective Planck constants interpreted in terms of phases of ordinary matter is predicted. X^4 decomposes to n fundamental regions related by Galois symmetry. Action is n times the action for the fundamental region. Planck constant h is effectively replaced with $h_{eff} = nh$. Quantum coherence scales are typically proportional to h_{eff}. Quantum coherence in arbitrarily long scales is implied. Dark matter at the magnetic body of the system would serve as controller of ordinary matter in the TGD inspired quantum biology [L92].

   $h_{eff} = nh_0$ is a more general hypothesis. Reasons to believe that h/h_0 could be the ratio R^2/L_p^2 for CP_2 length scale R deduced from p-adic mass calculations and Planck length L_P [L62]. The CP_2 radius R could actually correspond to L_P and the value of R deduced from the p-adic mass calculations would correspond to a dark CP_2 radius $\sqrt{h/h_0}l_P$.

3.3 Adelic physics and quantum measurement theory

Adelic physics [L21] forces us to reconsider the notion of entanglement and what happens in state function reductions (SFRs). Let us leave the question whether the SFR can correspond to SSFR or BSFR or both open for a moment.

1. The natural assumption is that entanglement is a number-theoretically universal concept and therefore makes sense in both real and various p-adic senses. This is guaranteed if the entanglement coefficients are in an extension E of rationals associated with the polynomial Q defining the space-time surface in M^8 and having rational coefficients.

   In the general case, the diagonalized density matrix ρ produced in a state function reduction (SFR) has eigenvalues in an extension E_1 of E. E_1 is defined by the characteristic polynomial P of ρ.

2. Is the selection of one of the eigenstates in SFR possible if E_1 is non-trivial? If not, then one would have a number-theoretic entanglement protection.

3. On the other hand, if the SFR can occur, does it require a phase transition replacing E with its extension by E_1 required by the diagonalization?

Let us consider the option in which E is replaced by an extension coding for the measured entanglement matrix so that something also happens to the space-time surface.

1. Suppose that the observer and measured system correspond to 4-surfaces defined by the polynomials O and S somehow composed to define the composite system and reflecting the asymmetric relationship between O and S. The simplest option is $Q = O \circ S$ but one can also consider as representations of the measurement action deformations of the polynomial $O \times P$ making it irreducible. Composition conforms with the properties of tensor product since the dimension of extension of rationals for the composite is a product of dimensions for factors.

2. The loss of correlations would suggest that a classical correlate for the outcome is a union of uncorrelated surfaces defined by O and S or equivalently by the reducible polynomial defined by the $O \times S$ [L57]. Information would be lost and the dimension for the resulting extension is the sum of dimensions for the composites. O however gains information and quantum classical correspondence (QCC) suggests that the polynomial O is replaced with a new one to realize this.

3. QCC suggests the replacement of the polynomial O the polynomial $P \circ O$, where P is the characteristic polynomial associated with the diagonalization of the density matrix ρ. The final state would be a union of surfaces represented by $P \circ O$ and S: the information about the measured observable would correspond to the increase of complexity of the space-time surface associated with the observer. Information would be transferred from entangled Galois degrees of freedom including also fermionic ones to the geometric degrees of freedom $P \circ O$. The information about the outcome of the measurement would in turn be coded by the Galois groups and fermionic state.

4. This would give a direct quantum classical correspondence between entanglement matrices and polynomials defining space-time surfaces in M^8. The space-time surface of O would store the measurement history as kinds of Akashic records. If the density matrix corresponds to a polynomial P which is a composite of polynomials, the measurement can add several new layers to the Galois hierarchy and gradually increase its height.

   The sequence of SFRs could correspond to a sequence of extensions of extensions of..... This would lead to the space-time analog of chaos as the outcome of iteration if the density matrices associated with entanglement coefficients correspond to a hierarchy of powers P^k [L41, L56].

Does this information transfer take place for both BSFRs and SSFRs? Concerning BSFRs the situation is not quite clear. For SSFRs it would occur naturally and there would be a connection with SSFRs to which I have associated cognitive measurement cascades [?]

1. Consider an extension, which is a sequence of extensions $E_1 \rightarrow ..E_k \rightarrow E_{k+1}.. \rightarrow E_n$ defined by the composite polynomial $P_n \circ .... \circ P_1$. The lowest level corresponds to a simple Galois group having no non-trivial normal subgroups.

2. The state in the group algebra of Galois group $G = G_n$ having G_{n-1} as a normal subgroup can be expressed as an entangled state associated with the factor groups G_n/G_{n-1} and subgroup G_{n-1} and the first cognitive measurement in the cascade would reduce this entanglement. After that the process could but need not to continue down to G_1. Cognitive measurements considerably generalize the usual view about the pair formed by the observer and measured system and it is not clear whether $O - S$ pair can be always represented in this manner as assumed above: also small deformations of the polynomial $O \times S$ can be considered.

   These considerations inspire the proposal the space-time surface assigned to the outcome of cognitive measurement G_k, G_{k-1} corresponds to polynomial the $Q_{k,k-1} \circ P_n$, where $Q_{k,k-1}$ is the characteristic polynomial of the entanglement matrix in question.

3.4 Entanglement paradox and new view about particle identity

A brain teaser that the theoretician sooner or later is bound to encounter, relates to the fermionic and bosonic statistics. This problem was also mentioned in the article of Keimer and Moore [D2] discussing quantum materials `https://cutt.ly/bWdTRj0`. The unavoidable conclusion is that both the fermions and bosons of the entire Universe are maximally entangled. Only the reduction of entanglement between bosonic and fermionic states of freedom would be possible in SFRs. In the QFT framework, gauge boson fields are primary fields and the problem in principle disappears if entanglement is between states formed by elementary bosons and fermions.

In the TGD Universe, all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) so that if Fock space the Fock states of fermions and bosons express everything worth expressing, SFRs would not be possible at all!

Remark: In the TGD Universe all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) localized at the points of space-time surface defining a number theoretic discretization that I call cognitive representation. Besides this there are also degrees of freedom associated with the geometry of 3-surfaces representing particles. These degrees of freedom represent new physics. The quantization of quarks takes place at the level of H so that anticommutations hold true over the entire H.

Obviously, something is entangled and this entanglement is reduced. What these entangled degrees of freedom actually are if Fock space cannot provide them?

1. Mathematically entanglement makes sense also in a purely classical sense. Consider functions $\Psi_i(x)$ and $\Psi_j(y)$) and form the superposition $\Psi(x) = \sum_{ij} c_{ij}\Psi_i(x)\Psi_j x)$. This function is completely analogous to an entangled state.

2. Number theoretical physics implies that the Galois group becomes the symmetry group of physics and quantum states are representations of the Galois group [L47, L52]. For an extension of extension of ...., the Galois group has decomposition by normal subgroups to a hierarchy of coset groups.

   The representation of a Galois group can be decomposed to a tensor product of representations of these coset groups. The states in irreps of the Galois group are entangled and the SFR cascade produces a product of the states as a product of representations of the coset groups. Galois entanglement allows us to express the asymmetric relation between observer and observed very naturally. This cognitive SSFR cascade - as I have called it - could correspond to what happens in at least cognitive SFRs.

   If so, then SFR would in TGD have nothing to do with fermions and bosons (consisting of quarks too) since the maximal fermionic entanglement remains. For instance, when one for instance talks about long range entanglement the entanglement that matters would correspond to entanglement between degrees of freedom, which do not allow Fock space description.

In the TGD framework, the replacement of particles with 3-surfaces brings in an infinite number of non-Fock degrees of freedom. Could it make sense to speak about the reduction of entanglement in WCW degrees of freedom? There is no second quantization at WCW level so that one cannot talk about Fock spaces WCW level but purely classical entanglement is possible as observed.

1. In WCW unions of disjoint 3-surfaces correspond to classical many-particle states. One can form single particle wave functions for 3-surfaces with a single component, products of these single particle wave functions, and also analogs of entangled states as their superposition realized as building bricks of WCW spinor fields.

   If one requires that these wave functions are completely symmetric under the exchange of 3-surfaces, maximal entanglement in this sense would be realized also now and SFR would not be possible. But can one require the symmetry? Under what conditions one can regard two 3-surfaces as identical? For point-like particles one has always identical particles but in TGD the situation changes.

2. Here theoretical physics and category theory meet since the question when two mathematical objects can be said to be identical is the basic question of category theory. The mathematical answer is they are isomorphic in some sense. The physical answer is that the two systems are identical if they cannot be distinguished in the measurement resolution used.

4 $M^8 - H$ duality

There are several observations motivating $M^8 - H$ duality (see **Fig. 8**).

1. There are four classical number fields: reals, complex numbers, quaternions, and octonions with dimensions $1, 2, 4, 8$. The dimension of the embedding space is $D(H) = 8$, the dimension of octonions. Spacetime surface has dimension $D(X^4) = 4$ of quaternions. String world sheet and partonic 2-surface have dimension $D(X^2) = 2$ of: complex numbers. The dimension $D(string) = 1$ of string is that of reals.

2. Isometry group of octonions is a subgroup of automorphism group G_2 of octonions containing $SU(3)$ as a subgroup. $CP_2 = SU(3)/U(2)$ parametrizes quaternionic 4-surfaces containing a fixed complex plane.

Could M^8 and $H = M^4 \times CP_2$ provide alternative dual descriptions of physics (see **Fig. 8**)?

1. Actually a complexification $M^8_c \equiv E^8_c$ by adding an imaginary unit i commuting with octonion units is needed in order to obtain sub-spaces with real number theoretic norm squared. M^8_c fails to be a field since $1/o$ does not exist if the complex valued octonionic norm squared $\sum o_i^2$ vanishes.

2. The four-surfaces $X^4 \subset M^8$ are identified as "real" parts of 8-D complexified 4-surfaces X^4_c by requiring that $M^4 \subset M^8$ coordinates are either imaginary or real so that the number theoretic metric defined by octonionic norm is real. Note that the imaginary unit defining the complexification commutes with octonionic imaginary units and number theoretical norm squared is given by $\sum_i z_i^2$ which in the general case is complex.

3. The space H would provide a geometric description, classical physics based on Riemann metric, differential geometric structures and partial differential equations deduced from an action principle. M^8_c would provide a number theoretic description: no partial differential equations, no Riemannian metric, no connections...

   M^8_c has only the number theoretic norm squared and bilinear form, which are real only if M^8_c coordinates are real or imaginary. This would define "physicality". One open question is whether all signatures for the number theoretic metric of X^4 should be allowed? Similar problem is encountered in the twistor Grassmannian approach.

4. The basic objection is that the number of algebraic surfaces is very small and they are extremely simple as compared to extremals of action principle. Second problem is that there are no coupling constants at the level of M^8 defined by action.

Preferred extremal property realizes quantum criticality with universal dynamics with no dependence on coupling constants. This conforms with the disappearance of the coupling constants from the field equations for preferred extremals in H except at singularities, with the Bohr orbitology, holography and ZEO. $X^4 \subset H$ is analogous to a soap film spanned by frame representing singularities and implying a failure of complete universality.

5. In M^8, the dynamics determined by an action principle is replaced with the condition that the *normal* space of X^4 in M^8 is associative/quaternionic. The distribution of normal spaces is always integrable to a 4-surface.

   One cannot exclude the possibility that the normal space is complex 2-space, this would give a 6-D surface [L39, L40]. Also this kind of surfaces are obtained and even 7-D with a real normal space. They are interpreted as analogs of branes and are in central role in TGD inspired biology.

   Could the twistor space of the space-time surface at the level of H have this kind of 6-surface as M^8 counterpart? Could $M^8 - H$ duality relate these spaces in 16-D M_c^8 to the twistor spaces of the space-time surface as 6-surfaces in 12-D $T(M^4) \times T(CP_2)$?

6. Symmetries in M^8 number theoretic: octonionic automorphism group G_2 which is complexified and contains $SO(1,3)$. G_2 contains $SU(3)$ as M^8 counterpart of color $SU(3)$ in H. Contains also $SO(3)$ as automorphisms of quaternionic subspaces. Could this group appear as an (approximate) dynamical gauge group?

   $M^8 = M^4 \times E^4$ as $SO(4)$ as a subgroup. It is not an automorphism group of octonions but leaves the octonion norm squared invariant. Could it be analogous to the holonomy group $U(2)$ of CP_2, which is not an isometry group and indeed is a spontaneously broken symmetry.

   A connection with hadron physics is highly suggestive. $SO(4) = SU(2)_L \times SU(2)_R$ acts as the symmetry group of skyrmions identified as maps from a ball of M^4 to the sphere $S^3 \subset E^4$. Could hadron physics $\leftrightarrow$ quark physics duality correspond to $M^8 - H$ duality. The radius of S^3 is proton mass: this would suggest that M^8 has an interpretation as an analog of momentum space.

7. What is the interpretation of M^8? Massless Dirac equation in M^8 for the octonionic spinors must be algebraic. This would be analogous to the momentum space Dirac equation. Solutions would be discrete points having interpretation as quark momenta! Quarks pick up discrete points of $X^4 \subset M^8$.

   States turn out to be massive in the M^4 sense: this solves the basic problem of 4-D twistor approach (it works for massless states only). Fermi ball is replaced with a region of a mass shell (hyperbolic space H^3).

   M^8 duality would generalize the momentum-position duality of the wave mechanics. QFT does not generalize this duality since momenta and position are not anymore operators.

4.1 Associative dynamics in M_c^8

How to realize the associative dynamics in M_c^8 [L39, L40]?

1. Number theoretical vision requires hierarchy of extensions of rationals and polynomials with rational coefficients would realize them. Rational coefficients make possible the interpretation as a polynomial with p-adic argument and therefore number theoretical universality.

   One cannot exclude the possibility that also real argument is allowed and that number theoretic universality and adelization applies only for the space-time surfaces defined by polynomials with rational coefficients.

2. Algebraic physics suggests that X^4 is in some sense a root of a M_c^8 valued polynomial. One can continue polynomials P with rational coefficients to M_c^8 by replacing the real argument with a complexified octonion.

3. The algebraic conditions should imply that the normal space of X^4 is quaternionic/associative. One can decompose octonions to sums $q_1 + I_4 q_2$, or "real" and "imaginary" parts q_i, which are quaternions and I_4 is octonion unit orthogonal to quaternions. The condition is that the "real" part of the octionic polynomial vanishes. Complexified 4-D surface whose projection to a real section (M^8 coordinates imaginary or real so that complexified octonion norm squared is real) is 4-D.

4. $M^8 - H$ duality requires an additional condition. The normal space contains also a complex plane M^2 which is commutative. This guarantees that normal spaces correspond to a point of CP_2. This is necessary in order to define $M^8 - H$ duality mapping X^4 from M^8 to H. M^2 can be replaced with an integrable distribution of M^2s if the assignment of the CP_2 point to tangent space can be made unique. This is the case if the spaces $M^2(x)$ are obtained from $M^2(y)$ by a unique G_2 automorphism $g(x, y)$.

4.1.1 Associativity condition at the level of M^8

Associativity condition for polynomials allows to characterize space-time surfaces in terms of polynomials with rational coefficients and possibly also analytic functions with rational Taylor coefficients at M^8 level. $M^8 - H$ duality would map $X^4 \subset M^8$ to $X^4 \subset H$. In M_c^8 the space-time surfaces could be also seen as graphs of local (complex) G_2 gauge transformations.

Remark: Even non-rational coefficients can be considered. In this case polynomials with rational coefficients would define a unique discretion of WCW and allow p-adicization and adelization.

In the generic case the set of points in the extension of rationals defining cognitive representation is discrete and finite. The surprise was that the "roots"can be solved explicitly and that the discrete cognitive representation is dense so that momentum quantization due to the finite volume of CD must be assumed to obtain finite cognitive representation inside CD. Cognitive representation could be defined by the points which correspond to the 8-momenta solving octonionic Dirac equation. This is excellent news concerning practical applications.

The outcome of a detailed examination of the "roots" of the octonionic polynomial having real part $X = Re_Q(P)$ and imaginary part $Y = Im_Q(P)$ in quaternionic sense, yielded a series of positive and negative surprises and demonstrated the failure of the naive arguments based on dimension counting.

1. Although no interesting associative space-time surfaces are possible, every distribution of normal associative planes (co-associativity) is integrable. Note that the distribution of normal spaces must have an integrable distribution of commutative planes in order to guarantee the existence of $M^8 - H$ duality. Generic arguments fail in the presence of symmetries.

2. Another positive surprise was that Minkowski signature is the only possible option. Equivalently, the image of M^4 as real co-associative subspace of O_c (complex valued octonion norm squared is real valued for them) by an element of local $G_{2,c}$ or its subgroup $SU(3, c)$ gives a real co-associative space-time surface.

3. The conjecture based on naive dimensional counting, which was not correct, was that the polynomials P determine these 4-D surfaces as roots of $Re_Q(P)$. The normal spaces of these surfaces possess a fixed 2-D commuting sub-manifold or possibly their distribution allowing the mapping to H by $M^8 - H$ duality as a whole.

   If this conjecture were correct, strong form of holography (SH) would not be needed and would be replaced with extremely powerful number theoretic holography determining space-time surface from its roots and selection of real subspace of O_c characterizing the state of motion of a particle.

4. One of the cold showers during the evolution of the ideas about $M^8 - H$ duality was that the naive expectation that one obtains complex 4-D surfaces as solutions is wrong. The equations for $Re_Q(P) = 0$ ($ImQ_P = 0$) reduce to roots of ordinary real polynomials defined by the odd (even() parts of P and have interpretation as complex values of 8-D mass squared. These surfaces have complex dimension 7. 4 complex dimensions should be eliminated in order to have a complex 4-D surface, whose real parts would give a real 4-surface X^4. The explanation

for the unexpected result comes from the symmetries of the octonionic polynomial implying that generic arguments fail.

4.1.2 How does one obtain 4-D space-time surfaces?

Contrary to the naive expections, the solutions of the vanishing conditions for the $Re_Q(P)$ $(Im_Q(P))$ (real (imaginary) part in quaternionic sense) are 7-D complex mass shells $r^2 = r_{n,1}$ as roots of $P_1(r) = 0$ or $r^2 = r_{n,2}$ of $P_2(r) = 0$ rather than 4-D complex surfaces (for a detailed discussion see [K7]) A solution of both conditions requires that P_1 and P_2 have a common root but the solution remains a 7-D complex mass shell! This was one of the many cold showers during the development of the ideas about $M^8 - H$ duality! It seems that the adopted interpretation is somehow badly wrong. Here zero energy ontology (ZEO) and holography come to the rescue.

1. Could the roots of P_1 or P_2 define only complex mass shells of the 4-D complex momentum space identifiable as M_c^4? ZEO inspires the question whether a proper interpretation of mass shells could be as pre-images of boundaries of cd:s (intersections of future and past directed light-cones) as pairs of mass shells with opposite energies. If this is the case, the challenge would be to understand how X_c^4 is determined if P does not determine it.

   Here holography, considered already earlier, suggests itself: the complex 3-D mass shells belonging to X_c^4 would only define the 3-D boundary conditions for holography and the real mass shells would be mapped to the boundaries of cds. This holography can be restricted to X_R^4. Bohr orbit property at the level of H suggests that the polynomial P defines the 4-surface more or less uniquely.

2. Let us take the holographic interpretation as a starting point. In order to obtain an X_c^4 mass shell from a complex 7-D light-cone, 4 complex degrees of freedom must be eliminated. $M^8 - H$ duality requires that X_c^4 allows M_c^4 coordinates.

   Note that if one has $X_c^4 = M_c^4$, the solution is trivial since the normal space is the same for all points and the H image under $M^8 - H$ duality has constant $CP_2 = SU(3)/U(2)$ coordinates. X_c^4 should have interpretation as a non-trivial deformation of M_c^4 in M^8.

3. By $M^8 - H$ duality, the normal spaces should be labelled by $CP_2 = SU(3)/U(2)$ coordinates. $M^8 - H$ duality suggests that the image $g(p)$ of a momentum $p \in M_c^4$ is determined essentially by a point $s(p)$ of the coset space $SU(3)/U(2)$. This is achieved if M_c^4 is deformed by a local $SU(3)$ transformation $p \rightarrow g(p)$ in such a way that each image point is invariant under $U(2)$ and the mass value remains the same: $g(p)^2 = p^2$ so that the point represents a root of P_1 or P_2.

   Remark: I have earlier considered the possibility of G_2 and even $G_{2,c}$ local gauge transformation. It however seems that that local $SU(3)$ transformation is the only possibility since G_2 and $G_{2,c}$ would not respect $M^8 - H$ duality. One can also argue that only real $SU(3)$ maps the real and imaginary parts of the normal space in the same manner: this is indeed an essential element of $M^8 - H$ duality.

4. This option defines automatically $M^8 - H$ duality and also defines causal diamonds as images of mass shells $m^2 = r_n$. The real mass shells in H correspond to the real parts of r_n. The local $SU(3)$ transformation g would have interpretation as an analog of a color gauge field. Since the H image depends on g, it does not correspond physically to a local gauge transformation but is more akin to an element of Kac-Moody algebra or Yangian algebra which is in well-defined half-algebra of Kac-Moody with non-negative conformal weights.

The following summarizes the still somewhat puzzling situation as it is now.

1. The most elegant interpretation achieved hitherto is that the polynomial P defines only the mass shells so that mass quantization would reduce to number theory. Amusingly, I started to think about particle physics with a short lived idea that the d'Alembert equation for a

scalar field could somehow give the mass spectrum of elementary particles so that the issue comes full circle!

2. Holography assigns to the complex mass shells complex 4-surfaces for which M^8-H duality is well-defined even if these surfaces would fail to be 4-D co-associative. These surfaces are expected to be highly non-unique unless holography makes them unique. The Bohr orbit property of their images in H indeed suggests this apart from a finite non-determinism [L71]. Bohr orbit property could therefore mean extremely powerful number theoretical duality for which the roots of the polynomial determine the space-time surface almost uniquely. $SU(3)$ as color symmetry emerges at the level of M^8. By M^8-H duality, the mass shells are mapped to the boundaries of CDs in H.

3. Do we really know that X_r^4 co-associative and has distribution of 2-D commuting subspaces of normal space making possible M^8-H duality? The intuitive expectation is that the answer is affirmative [A2]. In any case, M^8-H duality is well-defined even without this condition.

4. The special solutions to $P=0$, discovered already earlier, are restricted to the boundary of CD_8 and correspond to the values of energy (rather than mass or mass squared) coming as roots of the real polynomial P. These mass values are mapped by inversion to "very special moments in the life of self" (a misleading term) at the level of H as special values of light-cone proper time rather than linear Minkowski time as in the earlier interpretation [L31]. The new picture is Lorenz invariant.

4.1.3 Octonionic Dirac equation requires co-associativity

The octonionic Dirac equation allows a second perspective on associativity [L40].

1. Everything is algebraic at the level of M^8 and therefore also the octonionic Dirac equation should be algebraic. The octonionic Dirac equation is an analog of the momentum space variant of ordinary Dirac equation and also this forces the interpretation of M^8 as momentum space.

2. Fermions are massless in the 8-D sense and massive in 4-D sense. This suggests that octonionic Dirac equation reduces to a mass shell condition for massive particle with $q \cdot q = m^2 = r_n$, where $q \cdot q$ is octonionic norm squared for quaternion q defined by the expression of momentum p as $p = I_4 q$, where I_4 is octonion unit orthogonal to q. r_n represents mass shell as a root of P.

3. For the co-associative option, the co-associative octonion p representing the momentum is given in terms of quaternion q as $p = I_4 q$. One obtains $p \cdot p = q\overline{q} = m^2 = r_n$ at the mass shell defined as a root of P. Note that for M^4 subspace the space-like components of p p are proportional to i and the time-like component is real. All signatures of the number theoretic metric are possible.

4. For associative option, one would obtain $qq = m^2$, which cannot be satisfied: q reduces to a complex number $zx + Iy$ and one has analog of equation $z^2 = z^2 - y^2 + 2Ixy = m_n^2$, which cannot be true. Hence co-associativity is forced by the octonionic Dirac equation.

This picture combined with zero energy ontology leads also to a view about quantum TGD at the level of M^8. Local $SU(3)$ element g has properties suggesting a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P. The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

4.1.4 Hamilton-Jacobi structure and Kähler structure of $M^4 \subset H$ and their counterparts in $M^4 \subset M^8$

The Kähler structure of $M^4 \subset H$, forced by the twistor lift of TGD, has deep physical implications and seems to be necessary. It implies that for Dirac equation in H, modes are eigenstates of only the longitudinal momentum and in the 2 transversal degrees of freedom one has essentially harmonic oscillator states [L67, L61], that is Gaussians determined by the 2 longitudinal momentum components. For real longitudinal momentum the exponents of Gaussians are purely imaginary or purely real.

The longitudinal momentum space $M^2 \subset M^4$ and its orthogonal complement E^2 is in a preferred role in gauge theories, string models, and TGD. The localization of this decomposition leads to the notion of Hamilton-Jacobi (HJ) structure of M^4 and the natural question is how this relates to Kähler structures of M^4. At the level of H spinors fields only the Kähler structure corresponding to constant decomposition $M^2 \oplus E^2$ seems to make sense and this raises the question how the H-J structure and Kähler structure relate. TGD suggests the existence of two geometric structure in M^4: HJ structure and Kähler structure. It has remained unclear whether HJ structure and Kähler structure with covariantly constant self-dual Kähler form are equivalent notions or whether there several H-J structures accompaning the Kähler structure.

In the following I argue that H-J structures correspond to different choices of symplectic coordinates for M^4 and that the properties of $X^4 \subset H$ determined bt M^-H duality make it natural to to choose particular symplectic coordinates for M^4.

Consider first what H-J structure and Kähler structure could mean in H.

1. The H-J structure of $M^4 \subset H$ would correspond to an integrable distribution of 2-D Minkowskian sub-spaces of M^4 defining a distribution of string world sheets $X^2(x)$ and orthogonal distribution of partonic 2-surfaces $Y^2(x)$. Could this decomposition correspond to self-dual covariantly Kähler form in M^4?

   What do we mean with covariant constancy now? Does it mean a separate covariant constancy for the choices of $M^2(x)$ and $Y^2(x)$ or only of their sum, which in Minkowski coordinates could correspond to a constant electric and magnetic fields orthogonal to each other?

2. The non-constant choice of $M^2(x)$ ($E^2(x)$) cannot be covariantly constant. One can write $J(M^4) = J(M^2(x)) \oplus J(E^2(x)$ corresponding to decomposition to electric and magnetic parts. Constancy of $J(M^2(x)$ would require that the gradient of $J(M^2(x)$ is compensated by the gradient of an antisymmetric tensor with square equal to the projector to $M^2(x)$. Same condition holds true for $J(E^2(x))$. The gradient of the antisymmetric tensor would be parallel to itself implying that the tensor is constant.

3. H-J structure can only correspond to a transformation acting on J but leaving $J_{kl}dm^kdm^l$ invariant. One should find analogs of local gauge transformations leaving J invariant. In the case of CP_2, these correspond to symplectic transformations and now one has a generalization of the notion. The M^4 analog of the symplectic group would parameterize various decompositions of $J(M^4)$.

   Physically the symplectic transformations define local choices of 2-D space $E^2(x)$ of transversal polarization directions and longitudinal momentum space M^2 emerging in the construction of extremals of Kähler action.

4. For the simplest Kähler form for $M^4 \subset H$, this decomposition in Minkowski coordinates would be constant: orthogonal constant electric and magnetic fields. This Kähler form extends to its number theoretical analog in M^8. The local $SU(3)$ element g would deform M^4 to $g(M^4)$ and define an element of local CP_2 defining $M^8 - H$ duality. g should correspond to a symplectic transformation of M^4.

Consider next the number theoretic counterparts of H-J- and Kähler structures of $M^4 \subset H$ in $M^4 \subset M^8$.

1. In M^4 coordinates H-J structure would correspond to a constant $M^2 \times E^2$ decomposition. In M^4 coordinates Kähler structure would correspond to constant E and B orthogonal to

each other. Symplectic transformations give various representations of this structure as H-J structures.

2. The number theoretic analog of H-J structure makes sense also for $X^4 \subset M^8$ as obtained from the distribution of quaternionic normal spaces containing 2-D commutative sub-space at each point by multiplying then by local unit $I_4(x)$ orthogonal to the quaternionic units $\{1, I_1 = I_2 = I_3\}$ with respect to octonionic inner product. There is a hierarchy of CDs and the choices of these structures would be naturally parameterized by G_2.

   This would give rise to a number theoretically defined slicing of $X_c^4 \subset M_c^8$ by complexified string world sheets X_c^2 and partonic 2-surfaces Y_c^2 orthogonal with respect to the octonionic inner product for complexified octonions.

3. In $M^8 - H$ duality defined by $g(p) \subset SU(3)$ assigns a point of CP_2 to a given point of M^4. $g(p)$ maps the number theoretic H-J to H-J in $M^4 \subset M^8$. The space-time surface itself - that is $g(p)$ - defines these symplectic coordinates and the local $SU(3)$ element g would naturally define this symplectic transformation.

4. For $X^4 \subset M^8$ g reduces to a constant color rotation satisfying the condition that the image point is $U(2)$ invariant. Unit element is the most natural option. This would mean that g is constant at the mass and energy shells corresponding to the roots of P and the mass shell is a mass shell of M^4 rather than some deformed mass shell associated with images under $g(p)$.

   This alone does not yet guarantee that the 4-D tangent space corresponds to M^4. The additional physically very natural condition on g is that the 4-D momentum space at these mass shells is the same. $M^8 - H$ duality maps these mass shells to the boundaries of these cd:s in M^4 (CD= $cd \times CP_2$). This conforms with the identification of zero energy states as pairs of 3-D states at the boundaries of CD.

This generalizes the original intuitive but wrong interpretation of the roots r_n of P as "very special moments in the life of self" [L31].

1. Since the roots correspond to mass squared values, they are mapped to the boundaries of cd with size $L = \hbar_{eff}/m$ by $M^8 - H$ duality in M^4 degrees of freedom. During the sequence of SSFRs the passive boundary of CD remains does not shift only changes in size, and states at it remain unaffected. Active boundary is shifted due to scaling of cd.

   The hyperplane at which upper and lower half-cones of CD meet, is shifted to the direction of geometric future. This defines a geometric correlate for the flow of experienced time.

2. A natural proposal is that the moments for SSFRs have as geometric correlates the roots of P defined as intersections of geodesic lines with the direction of 4-momentum p from the tip of CD to its opposite boundary (here one can also consider the possibility that the geodesic lines start from the center of cd). Also energy shells as roots $E = r_n$ of P are predicted. They decompose to a set of mass shells $m_{n.,k}$ with the same $E = r_n$: similar interpretation applies to them.

3. What makes these moments very special is that the mass and energy shells correspond to surfaces in M^4 defining the Lorentz quantum numbers. SSFRs correspond to quantum measurements in this basis and are not possible without this condition. At $X^4 \subset M^8$ the mass squared would remain constant but the local momentum frame would vary. This is analogous to the conservation of momentum squared in general relativistic kinematics of point particle involving however the loss of momentum conservation.

4. These conditions, together with the assumption that g is a rational function with real coefficients, strongly suggest what I have referred to as preferred extremal property, Bohr orbitology, strong form of holography, and number theoretical holography.

In principle, by a suitable choice of M^4 one can make the momentum of the system light-like: the light-like 8-momentum would be parallel to M^4. I have asked whether this could be behind the fact that elementary particles are in a good approximation massless and whether the small mass of elementary particles is due to the presence of states with different mass squares in the zero state allowed by Lorentz invariance.

The recent understanding of the nature of right-handed neutrinos based on M^4 Kähler structure [L61] makes this mechanism un-necessary but poses the question about the mechanism choosing some particular M^4. The conditions that $g(p)$ leaves mass shells and their 4-D tangent spaces invariant provides this kind of mechanism. Holography would be forced by the condition that the 4-D tangent space is same for all mass shels representing inverse images for very special moments of time.

4.2 Uncertainty Principle and $M^8 - H$ duality

The detailed realization of $M^8 - H$ duality involves still uncertainties. The quaternionic normal spaces containing fixed 2-space M^2 (or an integrable distribution of M^2) are parametrized by points of CP_2. One can map the normal space to a point of CP_2.

The tough problem has been the precise correspondence between M^4 points in $M^4 \times E^4$ and $M^4 \times CP_2$ and the identification of the sizes of causal diamonds (CDs) in M^8 and H. The identification is naturally linear if M^8 is analog of space-time but if M^8 is interpreted as momentum space, the situation changes. The option discussed in [L39, L40] maps mass hyperboloids to light-cone proper time =constant hyperboloids and it has turned out that this correspondence does not correspond to the classical picture suggesting that a given momentum in M^8 corresponds in H to a geodesic line emanating from the tip of CD.

4.2.1 $M^8 - H$ duality in M^4 degrees of freedom

The following proposal for $M^8 - H$ duality in M^4 degrees of freedom relies on the intuition provided by UP and to the idea that a particle with momentum p^k corresponds to a geodesic line with this direction emanating from the tip of CD.

1. The first constraint comes from the requirement that the identification of the point $p^k \in X^4 \subset M^8$ should classically correspond to a geodesic line $m^k = p^k\tau/m$ ($p^2 = m^2$) in M^8 which in Big Bang analogy should go through the tip of the CD in H. This geodesic line intersects the opposite boundary of CD at a unique point.

   Therefore the mass hyperboloid H^3 is mapped to the 3-D opposite boundary of $cd \subset M^4 \subset H$. This does not fix the size nor position of the CD ($= cd \times CP_2$) in H. If CD does not depend on m, the opposite light-cone boundary of CD would be covered an infinite number of times.

2. The condition that the map is 1-to-1 requires that the size of the CD in H is determined by the mass hyperboloid M^8. Uncertainty Principle (UP) suggests that one should choose the distance T between the tips of the CD associated with m to be $T = \hbar_{eff}/m$.

   The image point m^k of p^k at the boundary of $CD(m, h_{eff})$ is given as the intersection of the geodesic line $m^k = p^k\tau$ from the origin of $CD(m, h_{eff})$ with the opposite boundary of $CD(m, h_{eff})$:

   $$m^k = \hbar_{eff} X \frac{p^k}{m^2} \quad , X = \frac{1}{1+p_3/p_0} \ . \tag{4.1}$$

   Here p_3 is the length of 3-momentum.

   The map is non-linear. At the non-relativistic limit ($X \to 1$), one obtains a linear map for a given mass and also a consistency with the naive view about UP. m^k is on the proper time constant mass shell so the analog of the Fermi ball in $H^3 \subset M^8$ is mapped to the light-like boundary of $cd \subset M^4 \subset H$.

3. What about massless particles? The duality map is well defined for an arbitrary size of CD. If one defines the size of the CD as the Compton length $\hbar_{eff}/m$ of the massless particle, the size of the CD is infinite. How to identify the CD? UP suggests a CD with temporal distance $T = 2\hbar_{eff}/p_0$ between its tips so that the geometric definition gives $p^k = \hbar_{eff} p^k/p_0^2$ as the point at the 2-sphere defining the corner of CD. p-Adic thermodynamics [K22]) strongly suggests

that also massless particles generate very small p-adic mass, which is however proportional to $1/p$ rather than $1/\sqrt{p}$. The map is well defined also for massless states as a limit and takes massless momenta to the 3-ball at which upper and lower half-cones meet.

4. What about the position of the CD associated with the mass hyperboloid? It should be possible to map all momenta to geodesic lines going through the 3-ball dividing the largest CD involved with T determined by the smallest mass involved to two half-cones. This is because this 3-ball defines the geometric "Now" in TGD inspired theory of consciousness. Therefore all CDs in H should have a common center and have the same geometric "Now".

   $M^8 - H$ duality maps the slicing of momentum space with positive/negative energy to a Russian doll-like slicing of $t \geq 0$ by the boundaries of half-cones, where t has origin at the bottom of the double-cone. The height of the $CD(m, h_{eff})$ is given by the Compton length $L(m, h_{eff}) = \hbar_{eff}/m$ of quark. Each value of h_{eff} corresponds its own scaled map and for $h_{gr} = GMm/v_0$, the size of $CD(m, h_{eff}) = GM/v_0$ does not depend on m and is macroscopic for macroscopic systems such as Sun.

5. The points of cognitive representation at quark level must have momenta with components, which are algebraic integers for the extension of rationals considered. A natural momentum unit is $m_{Pl} = \hbar_0/R$, h_0 is the minimal value of $h_{eff} = h_0$ and R is CP_2 radius. Only "active" points of $X^4 \subset M^8$ containing quark are included in the cognitive representation. Active points give rise to active CD:s $CD(m, h_{eff})$ with size $L(m, h_{eff})$.

   It is possible to assign $CD(m, h_{eff})$ also to the composites of quarks with given mass. Galois confinement suggest a general mechanism for their formation: bound states as Galois singlets must have a rational total momentum. This gives a hierarchy of bound states of bound states of ..... realized as a hierarchy of CDs containing several CDs.

6. This picture fits nicely with the general properties of the space-time surfaces as associative "roots" of the octonionic continuation of a real polynomial. A second nice feature is that the notion of CD at the level H is forced by this correspondence. "Why CDs?" at the level of H has indeed been a longstanding puzzle. A further nice feature is that the size of the largest CD would be determined by the smallest momentum involved.

7. Positive and negative energy parts of zero energy states would correspond to opposite boundaries of CDs and at the level of M^8 they would correspond to mass hyperboloids with opposite energies.

8. What could be the meaning of the occupied points of M^8 containing fermion (quark)? Could the image of the mass hyperboloid containing occupied points correspond to sub-CD at the level of H containing corresponding points at its light-like boundary? If so, $M^8 - H$ correspondence would also fix the hierarchy of CDs at the level of H.

It is enough to realize the analogs of plane waves only for the actualized momenta corresponding to quarks of the zero energy state. One can assign to CD as total momentum and passive *resp.* active half-cones give total momenta $P_{tot,P}$ *resp.* $P_{tot,A}$, which at the limit of infinite size for CD should have the same magnitude and opposite sign in ZEO.

The above description of $M^8 - H$ duality maps quarks at points of $X^4 \subset M^8$ to states of induced spinor field localized at the 3-D boundaries of CD but necessarily delocalized into the interior of the space-time surface $X^4 \subset H$. This is analogous to a dispersion of a wave packet. One would obtain a wave picture in the interior.

4.2.2 Does Uncertainty Principle require delocalization in H or in X^4?

One can argue that Uncertainty Principle (UP) requires more than the naive condition $T = \hbar_{eff}/m$ on the size of sub-CD. I have already mentioned two approaches to the problem: they could be called inertial and gravitational representations.

1. The inertial representations assigns to the particle as a space-time surface (holography) an analog of plane wave as a superposition of space-time surfaces: this is natural at the level of WCW. This requires delocalization space-time surfaces and CD in H.

2. The gravitational representation relies on the analog of the braid representation of isometries in terms of the projections of their flows to the space-time surface. This does not require delocalization in H since it occurs in X^4.

Consider first the inertial representation. The intuitive idea that a single point in M^8 corresponds to a discretized plane wave in H in a spatial resolution defined by the total mass at the passive boundary of CD. UP requires that this plane wave should be realized at the level of H and also WCW as a superposition of shifted space-time surfaces defined by the above correspondence.

1. The basic observation leading to TGD is that in the TGD framework a particle as a point is replaced with a particle as a 3-surface, which by holography corresponds to 4-surface.

   Momentum eigenstate corresponds to a plane wave. Now planewave could correspond to a delocalized state of 3-surface - and by holography that of 4-surface - associated with a particle.

   A generalized plane wave would be a quantum superposition of shifted space-time surfaces inside a larger CD with a phase factor determined by the 4-momentum. $M^8 - H$ duality would map the point of M^8 containing an object with momentum p to a generalized plane wave in H. Periodic boundary conditions are natural and would force the quantization of momenta as multiples of momentum defined by the larger CD. Number theoretic vision requires that the superposition is discrete such that the values of the phase factor are roots of unity belonging to the extension of rationals associated with the space-time sheet. If momentum is conserved, the time evolutions for massive particles are scalings of CD between SSFRs are integer scalings. Also iterated integer scalings, say by 2 are possible.

2. This would also provide WCW description. Recent physics relies on the assumption about single background space-time: WCW is effectively replaced with M^4 since 3-surface is replaced with point and CP_2 is forgotten so that one must introduce gauge fields and metric as primary field variables.

As already discussed, the gravitational representation would rely on the lift/projection of the flows defined by the isometry generators to the space-time surface and could be regarded as a "subjective" representation of the symmetries. The gravitational representation would generalize braid group and quantum group representations.

The condition that the "projection" of the Dirac operator in H is equal to the modified Dirac operator, implies a hydrodynamic picture. In particular, the projections of isometry generators are conserved along the lifted flow lines of isometries and are proportional to the projections of Killing vectors. QCC suggests that only Cartan algebra isometries allow this lift so that each choice of quantization axis would also select a space-time surface and would be a higher level quantum measurement.

4.2.3 Exact ZEO emerges only at the limit of CD with infinite size

At the limit when the volume of CD becomes infinite, the sum of the momenta associated with opposite boundaries of CD should automatically vanish and one would obtain ideal zero energy states. The original assumption that ideal zero energy states are possible for finite size of CD, is not strictly true. The situation is the same for quantization in a finite volume.

1. Denote the sum of the total momenta with positive energy associated with passive boundaries of all CDs by $P_{tot,P} =\equiv P_{tot}$. For finite size of CD, $P_{tot,P}$ need not be the same as the total momentum $P_{tot,A}$ associated with the active boundary which can change during the sequence of SSFRs. Denote the difference $P_{tot,P} - P_{tot,A}$ by ΔP.

   This momentum is P_{tot} is large for large CDs, and naturally defines the spatial resolution. Denote by $M^k = nXh_{eff}P^k_{tot}/ \cdot P^2_{tot}$, $X = 1/(1 + P_3/P_0)$, the shift defined by P_{tot}. The analogs of plane waves for the sub-CDs should be discretized with this spatial resolution and at the limit of large total mass the discretization improves.

2. The image of X^4 in H for a given mass hyperboloid H^3 should define a geometric analog of a plane wave in WCW for the total momentum $P^k = \sum_i p^k_i$, $p^2_i = m^2$ of H^3, associated with

the CD(M) in M^8. It is also possible to include the momenta with different masses since they have images also at the boundaries of all CDs in the Russian doll hierarchy. For $\hbar_{gr}$ there is a common CD for all particle masses with size Λ_{gr}.

The WCW plane wave would not be a superposition of points but of shifted space-time surfaces. The argument of the plane wave would correspond to the shift of the $X^4 \subset CD(M) \subset H$.

Maximal spatial resolution is achieved if one shifts the X^4 and corresponding CD(m) in H inside the large CD by nM^k, $M^k = nh_{eff}XP^k_{tot}/ \cdot P^2_{tot}$ and forms the WCW spinor field as a superposition of shifted space-time surfaces $X^4(m)$ with $U_n = exp(i\Delta P \cdot nM)$ appearing as plane wave phase factor.

3. At the limit when the size of the largest CD becomes infinite (the mass M defining Λ_{gr} becomes very large), the sum $\sum_n U_n$ obtained as integral over the identical shifted copies of the space-time surfaces is non-vanishing only for $\Delta P = 0$ and one obtains an momentum conserving ideal zero energy state.

These states would be analogs of single particle states as plane waves, with particle replaced with many-quark state inside $CD(m)$. The generalization is obvious: perform the analog of second quantization by forming N-particle states in which one has N $CD(m)$ plane waves.

4.2.4 The revised view about $M^8 - H$ duality and the "very special moments in the life of self"

The polynomial equations allow at M^8 level also highly unique brane-like solutions having the topology of 6-sphere S^6 and intersecting M^4 along $p^0 = E = constant$ hyperplane. These quantizd values of energy E correspond to the roots of the polynomial defining the solution and are algebraic numbers and algebraic integers for monic polynomials of form $P(x) = x^n + p_{n-1}x^{n-1} + ...$

The TGD inspired theory of consciousness motivated the interpretation of these hyperplanes as "very special moments in the life of self": this interpretation [L31] emerged before the realization that M^8 corresponds to momentum space. The images of these planes under $M^8 - H$ duality should however allow this interpretation also in the new picture. Is this possible?

To answer the question one must understand what the image of S^6 under $M^8 - H$ duality is.

1. The image must belong to $M^4 \times CP_2$. The 2-D normal space of the point of S^6 is a complex commutative plane of octonions. Since 4-D normal planes of space-time surface containing complex plane correspond to points of CP_2, the natural proposal is that the image now corresponds to point of CP_1 identified as homologically trivial geodesic sub-manifold S^2_G of CP_2 carrying Kähler magnetic charge.

2. The first thing to notice about the H-image of the 3-D $E = constant$ surface $X^3(E) \subset M^4$ is that it is indeed 3-D rather than 4-D. In M^4 the map has the form $m^k = X\hbar_{eff}/m^2$, $X = 1/(1 + p_3/p_0)$ already discussed.

   The value of $m^2 = E^2 - p_3^2$ decreases as p_3^2 increases so that the values of light-cone proper time $a = t^2 - r^2$ for the image are larger than $a_{min} = \hbar_{eff}/m$. "Fermi-spheres" $S^2_F(p_3)$ are mapped to 2-spheres $S^2(r) \subset M^4 \subset H$ with an increasing radius $r(t) = \sqrt{t^2 - a^2_{min}}$. 2-sphere is born at $t = a_{min}$ and starts to increase in size and the expansion velocity approaches light velocity asymptotically. This expanding sphere would be magnetically charged.

   The sequence a_n of "very special moments in the life of self" in the life of self would mean the birth of this kind of expanding sphere and a_n would correspond to the roots of the polynomial considered identified as quantized energies. The dispersion relation $E = constant$ means that energy does not depend on the momentum: plasmons provide the condensed matter analogy.

3. There are interesting questions to be answered. Do the surfaces $X^3(E)$ intersect the 4-D space-time surface $X^4 \subset H$? At the level of M^8 the intersections of 4-D and 6-D surfaces are 2-D. The proposal is that these 2-surfaces M^8 are mapped to partonic vertices identified as 2-surfaces $X^2 \subset X^4 \subset H$ at which 4-D surfaces representing particles meet. This should happen also for the new identification of $M^8 - H$ duality.

However, in the generic case the intersections of 3-surfaces and 4-surfaces in H are empty. The recent situation is however not a generic one since the S^6 solutions are non-generic (one would expect only 4-D solutions) and 4-D and 6-D solutions are determined by the same polynomial. Therefore the points to which the 2-spheres contract for $t = a_{min}$ should be mapped to partonic 2-surfaces in H. Single point should correspond to the geodesic sphere S^2_G.

Does this conform with the view that 4-D CP_2 type extremals in H correspond to "blow-ups" of 1-D line singularities of $X^4 \subset M^8$ for which the quaternionic tangent spaces at singularity are not unique and define 3-D surface as points of CP_2. Now the 2-D normal spaces of S^2_F would span $S^2_G \subset CP_2$ and at the limit of S^2_F contracting to a point, one would have a 2-D singularity having an interpretation as a partonic vertex.

4. Cosmic strings $X^4 = X^2 \times S^2_G \subset M^4 \times CP_2$ carrying monopole charge are basic solutions of field equations. Could these cosmic strings relate to the images of $X^3(E)$? For instance, could $X^3(E_1)$ and $X^3(E_2)$ correspond to the ends of a cosmic string thickening to a monopole flux tube? Thickening would correspond to the growth of M^4 projection $S^2(r(t))$ of the flux tube having $r(t) = \sqrt{t^2 - a^2_{min}}$. The interpretation would be as a pair of magnetic poles connected by a monopole flux tube. Cosmic strings would be highly dynamical entities if this is the case.

4.2.5 An objection against $M^8 - H$ duality

Objections are the best manner to proceed. $M^8 - H$ duality maps the point M^8 at mass shell m to points of CD corresponding to the Compton length $\hbar_{eff}/m$ obtained as intersection of line with momentum p starting at the center point of CD and intersecting either boundary of CD. Each quaternionic normal space contains a commuting subspace (in octonionic sense) such that the distribution of the latter spaces is integrable. These normal spaces are parameterized by CP_2. This implies a complete localization in CP_2 so that the restriction of the induced quark field does not have well-defined color quantum numbers.

How to circumvent this objection? The proposed identification of string-like and particle-like space-time surfaces suggests a solution to the problem. Consider first CP_2 type extremals.

1. Consider first CP_2 type extremals as analogs of particles proposed to correspond to line singularities of algebraic 4-surfaces in M^8 with the property that the normal co-quaternionic space is not unique and the normal spaces at given point of the line are parametrized by a 3-D surface of CP_2 at each point of the light-like curve. Algebraic geometers speak of blow-up singularity. This kind of singularity is analogous to the tip of a cone.

   For polynomials the M^4 projection is a light-like geodesic. Also the octonionic continuations of analytic functions of real argument with rational Taylor coefficients can define space-time surfaces and in this case more general light-like curves are expected to be possible. This gives rise to a 4-D surface of H, which has the same Euclidean metric and Kähler form as CP_2 and only the induced gamma matrices are different.

2. The induced spinor field as restriction of the second quantized spinor field of H decomposes into modes, which are modes of H d'Alembertian. The modes have well-defined color quantum numbers so that one can speak of color quarks. This would mean that one can speak about colored quarks only inside CP_2 type extremals and possibly also inside string-like objects. This would trivialize the mysteries of quark and color confinement.

   Gluons would correspond to pairs of quark and antiquark associated with distinct wormhole throats or even - contacts. The mass squared for a given mode is well-defined but at the level of H only the right-handed neutrino is massless. Other states have mass of order CP_2 mass.

3. One can argue that the average momenta associated with these kinds of states have M^4 projection parallel to the light-like geodesic so that the momentum is light-like. There are several justifications for the claim.

   (a) The gravitational representation of isometries already discussed as lift/projection of the corresponding flows in H to X^4 restricts the action of M^4 isometries to a light-like geodesic and implies that the states are massless in this sense.

(b) The claim conforms with an earlier intriguing observation that the restriction of a massive quark propagator to a pair of space-time points with light-like M^4 distance is essentially a massless propagator irrespective of the value of the mass.

(c) With a suitable choice of $M^4 \subset M^8$ the ground state mass can be chosen to vanish. The reason is that the 8-D momentum is light-like and if M^4 contains the momentum, then also the M^4 mass vanishes. This choice can be made only for a single mode in the superposition. p-Adic thermodynamics would describe the contribution of higher modes in the quantum superposition of states to the mass squared having interpretation as thermal mass squared.

(d) One can look at the situation also at the space-time level. If one has a light-like curve or a curve consisting of segments, which are light-like geodesic lines, the situation changes. Since the average velocity for this kind of zigzag (zitterbewegung) curve is below light velocity, the intuitive expectation is that this represents the TGD analog of the Higgs mechanism having interpretation as massivation.

    This finding was the original motivation for p-adic thermodynamics. The conditions stating the light-likeness of the projection are nothing but Virasoro conditions. p-Adic thermodynamics involves also the inclusion of supersymplectic symmetries.

    $H(M^4)$ is orthogonal to the space-time surface and has an interpretation as a local acceleration of the space-time surface as an extended particle. The CP_2 part of H was the original proposal for the Higgs field considered in my thesis. Indeed, $H(CP_2)$ behaves like a complex doublet in complex coordinates. The physical interpretation is that the minimal surface property forces zitterbewegung with acceleration $H(M^4) = H(CP_2)$, which in turn means that light-like curve looks in the average sense like time-like geodesic for a massive particle.

    The problem is that the proposed Higgs field vanishes in the interiors of space-time surfaces. However, the general field equations do not imply minimal surface property and also for preferred extremals it fails at singularities analogous to frames of soap films. At these point one can have non-vanishing $H(CP_2)$. 8-D light-likeness suggests that at these points $H(H)$ is light-like.

What happens to string like-objects corresponding to 2-D singularities such that the normal spaces at a given point correspond to a 2-D surface of CP_2, which in the most general situation can be either complex 2-surface of CP_2 or a minimal Lagrangian 2-manifold? One cannot exclude 1-D singularities associated with surfaces $X^3 \times X^1 \subset M^4 \times CP_2$ for which CP_2 projection is 1-D, presumably a geodesic circle.

(a) The simplest string-like objects come in 2 variants corresponding to CP_2 projection, which is a geodesic sphere, which can be homologically non-trivial or non-trivial. M^4 projection is in the simplest situation 2-D plane M^2.

    These two options correspond to the reduction of $SU(3)$ to $U(2)$ or $SO(3)$. The interpretation in terms of spontaneous symmetry breaking is highly suggestive. The representations of $SU(3)$ decompose to those of $U(2)$ or $SO(3)$. Color confinement could weaken to that for $U(2)$ or $SO(3)$ so that the total color quantum numbers I_3 and Y would still vanish but color multiplets would allow these kinds of states.

(b) The simplest symmetry breaking to $U(1)$ could correspond to extremals of form $M^3 \times S^1$ and only $U(1)$ confinement would hold true. In the case of M^4 it does not make sense to speak of color quantum numbers.

4.3 Generalizations related to $M^8 - H$ duality

It has become clear that $M^8 - H$ duality generalizes and there is a connection with the twistorialization at the level of H.

4.3.1 M^8-H duality at the level of WCW and p-adic prime as the maximal ramified prime of polynomial

The vacuum functional as an exponent of the Kähler function determines the physics at WCW level. $M^8 - H$ duality suggests that it should have a counterpart at the level of M^8 and appear as a weight function in the summation. Adelic physics requires that weight function is a power of p-adic prime and ramified primes of the extension are the natural candidates in this respect.

1. The discriminant D of the algebraic extension defined by a polynomial P with rational coefficients (`https://en.wikipedia.org/wiki/Discriminant`) is expressible as a square for the product of the non-vanishing differences $r_i - r_j$ of the roots of P. For a polynomial P with rational coefficients, D is a rational number as one can see for polynomial $P = ax^2 + bx + c$ from its expression $D = b^2 - 4ac$. For monic polynomials of form $x^n + a_{n-1}x^{m-1} + ...$ with integer coefficients, D is an integer. In both cases, one can talk about ramified primes as prime divisors of D.

   If the p-adic prime $p$is identified as a ramified prime, D is a good candidate for the weight function since it would be indeed proportional to a power of p and have p-adic norm proportional to negative power of p. Hence the p-adic interpretation of the sum over scattering amplitudes for polynomials P is possible if p corresponds to a ramified prime for the polynomials allowed in the amplitude.

   p-Adic thermodynamics [K22] suggest that p-adic valued scattering amplitudes are mapped to real numbers by applying to the Lorentz invariants appearing in the amplitude the canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adics to reals in a continuous manner

2. For monic polynomials, the roots are powers of a generating root, which means that D is proportional to a power of the generating root, which should give rise to some power of p. When the degree of the monic polynomial increases, the overall power of p increases so that the contributions of higher polynomials approach zero very rapidly in the p-adic topology. For the p-adic prime $p = M_{127} = 2^{127} - 1 \sim 10^{38}$ characterizing electrons, the convergence is extremely rapid.

   Polynomials of lowest degree should give the dominating contribution and the scattering amplitudes should be characterized by the degree of the lowest order polynomial appearing in it. For polynomials with a low degree n the number of particles in the scattering amplitude could be very small since the number n of roots is small. The sum $x_i + p_i$ cannot belong to the same mass shell for timelike p_i so that the minimal number of roots r_n increases with the number of external particles.

3. $M^8 - H$ duality requires that the sum over polynomials corresponds to a WCW integration at H-side. Therefore the exponent of Kähler function at its maximum associated to a given polynomial should be apart from a constant numerical factor equal to the discriminant D in canonical identification.

   The condition that the exponent of Kähler function as a sum of the Kähler action and the volume term for the preferred extremal $X^4 \subset H$ equals to power of D apart from a proportionality factor, should fix the discrete number theoretical and p-adic coupling constant evolutions of Kähler coupling strength and length scale dependent cosmological constant proportional to inverse of a p-adic length scale squared. For Kähler action alone, the evolution is logarithmic in prime p since the function reduces to the logarithm of D.

$M^8 - H$ duality suggests that the exponent $exp(-K)$ of Kähler function has an M^8 counterpart with a purely number theoretic interpretation. The discriminant D of the polynomial P is the natural guess. For monic polynomials D is integer having ramified primes as factors.

There are two options for the correspondence between $exp(-K)$ at its maximum and D assuming that P is monic polynomial.

1. In the real topology, one would naturally have $exp(-K) = 1/D$. For monic polynomials with high degree, D becomes large so that $exp(-K)$ is large.

2. In a p-adic topology defined by p-adic prime p identified as a ramified prime of D, one would have naturally $exp(-K) = I(D)$, where one has $I(x) = \sum x_n p^n = \sum x_n p^{-n}$.

   If p is the largest ramified prime associated with D, this option gives the same result as the real option, which suggests a unique identification of the p-adic prime p for a given polynomial P. P would correspond to a unique p-adic length scale L_p and a given L_p would correspond to all polynomials P for which the largest ramified prime is p.

   This might provide some understanding concerning the p-adic length scale hypothesis stating that p-adic primes tend to be near powers of integer. In particular, understanding about why Mersenne primes are favored might emerge. For instance, Mersennes could correspond to primes for which the number of polynomials having them as the largest ramified prime is especially large. The quantization condition $exp(-K) = D(p)$ could define which p-adic primes are the fittest ones.

The condition that $exp(-K)$ at its maximum equals to D via canonical identification gives a powerful number theoretic quantization condition.

4.3.2 Space-time surfaces as images of associative surfaces in M^8

$M^8 - H$ duality would provide an explicit construction of space-time surfaces as algebraic surfaces with an associative normal space [L39, L40]. M^8 picture codes space-time surface by a real polynomial with rational coefficients. One cannot exclude coefficients in an extension of rationals and also analytic functions with rational or algebraic coefficients can be considered as well as polynomials of infinite degree obtained by repeated iteration giving rise algebraic numbers as extension and continuum or roots as limits of roots.

$M^8 - H$ duality maps these solutions to H and one can consider several forms of this map. The weak form of the duality relies on holography mapping only 3-D or even 2-D data to H and the strongest form maps entire space-time surfaces to H. The twistor lift of TGD allows to identify the space-time surfaces in H as base spaces of 6-D surfaces representing the twistor space of space-time surface as an S^2 bundle in the product of twistor spaces of M^4 and CP_2. These twistor spaces must have Kähler structure and only the twistor spaces of M^4 and CP_2 have it [A6] so that TGD is unique also mathematically.

An interesting question relates to the possibility that also 6-D commutative space-time surfaces could be allowed. The normal space of the space-time surface would be a commutative subspace of M^8_c and therefore 2-D. Commutative space-time would be a 6-D surface X^6 in M^8.

This raises the following question: Could the inverse image of the 6-D twistor-space of 4-D space-time surface X^4 so that X^6 would be M^8 analog of twistor lift? This requires that $X^6 \subset M^8_c$ has the structure of an S^2 bundle and there exists a bundle projection $X^6 \rightarrow X^4$.

The normal space of an associative space-time surface actually contains this kind of commutative normal space! Its existence guarantees that the normal space of X^4 corresponds to a point of CP_2. Could one obtain the M^8_c analog of the twistor space and the bundle bundle projection $X^6 \rightarrow X^4$ just by dropping the condition of associativity. Space-time surface would be a 4-surface obtained by adding the associativity condition.

One can go even further and consider 7-D surfaces of M^8 with real and therefore well-ordered normal space. This would suggest dimensional hierarchy: $7 \rightarrow 6 \rightarrow 4$.

This leads to a possible interpretation of twistor lift of TGD at the level of M^8 and also about generalization of $M^8 - H$ correspondence to the level of twistor lift. Also the generalization of twistor space to a 7-D space is suggestive. The following arguments representa vision about "how it must be" that emerged during the writing of this article and there are a lot of details to be checked.

4.3.3 Commutative 6-surfaces and twistorial generalization of $M^8 - H$ correspondence

One can generalize the notion of complex 4-surface $X^6_c \subset M^8_c$ to that of complex 6-surface $X^6_c \subset M^8$ with a complexified commutative normal space. The 6-surface would correspond to a surface obtained by a local $SU(3)$ element invariant under $U(1) \times U(1) \subset SU(2)$. In complete analogy with 4-D case, these 6-surfaces would contain 5-D mass shells determined by the roots of P. The space $F = SU(3)/U(1) \times U(1)$ of points is nothing but the twistor space of CP_2!

The deformed M^6 defining $X^6 \subset M^8$ regarded as surface in M^8 suggests an interpretation as an analog of 6-D twistor space of M^4. Maybe one could identify the M^6 as the projective space $C^4/C_\times$ obtained from C^4 by dividing with complex scalings? This would give the twistor space $CP_3 = SU(4)/U(3)$ of M^4. This is not obvious since one has (complexified) octonions rather than C^4 or its hypercomplex analog. This would be analogous to using several (4) coordinate charts glued together as in the case of sphere CP_1.

The map $M^6 \to F$ obtained in this manner would define mapping of the twistor spaces of M^4 and CP_2 to each other. The twistor lift of TGD indeed defines this kind of map. The twistor lift involves the additional assumption that the S^2 fibers of these twistor spaces correspond to each other isometrically. This could correspond to a choice of Hamilton-Jacobi structure defining a local decomposition of $M^6 = M^2 \oplus E^4$ such that M^2 defines the analog of the Riemann sphere for M^6.

It might be also possible to identify the octonionic analog of the projective space $CP_3 = C^4/C_\times$. Could the octonionic M^8 momenta be scaled down by dividing with the momentum projection in the commutative normal space so that one obtains an analog of projective space? Could one use these as coordinates for M^6? The scaled 8-momenta would correspond to the points of the octonionic analog of CP_3. The scaled down 8-D mass squared would have a constant value.

A possible problem is that one must divide either from left or right and results are different in the general case. Could one require that the physical states are invariant under the automorphisms generated $o \to gog^{-1}$, where g is an element of the commutative subalgebra in question?

4.3.4 Physical interpretation of the counterparts of twistors at the level of M^8_c

What about the physical interpretation at the level of M^8_c. The twistor space allows a geometrization of spin so that momentum and spin would combine to a purely geometric entity with 6 components. The active points would correspond to fermions (quarks) with a given momentum and spin.

1. The first thing to notice is that in the twistor Grassmannian approach twistor space provides an elegant description of spin. Partial waves in the fiber S^2 of twistor space representation of spin as a partial wave. All spin values allow a unified treatment.

   The problem is that this requires massless particles. In the TGD framework 4-D masslessness is replaced with its 8-D variant so that this difficulty is circumvented. This kind of description in terms of partial waves is expected to have a counterpart at the level of the twistor space $T(M(^4) \times T(CP_2)$. At level of M^8 the description is expected to be in terms of discrete points of M^8_c.

2. Consider first the real part of $X^6_c \subset M^8_c$. At the level of M^8 the points of X^4 correspond to points. The same must be true also at the level of X^6. Single point in the fiber space S^2 would be selected. The interpretation could be in terms of the selection of the spin quantization axis.

   Spin quantization axis corresponds to 2 diametrically opposite points of S^2. Could the choice of the point also fix the spin direction? There would be two spin directions and in the general case of a massive particle they must correspond to the values $S_z = \pm 1/2$ of fermion spin. For massless particles in the 4-D sense two helicities are possible and higher spins cannot be excluded. The allowance of only spin 1/2 particles conforms with the idea that all elementary particles are constructed from quarks and antiquarks. Fermionic statistics would mean that for fixed momentum one or both of the diametrically opposite points of S^2 defining the same and therefore unique spin quantization axis can be populated by quarks having opposite spins.

3. For the 6-D tangent space of X^6_c or rather, its real projection, an analogous argument applies. The tangent space would be parametrized by a point of $T(CP_2)$ and mapped to this point. The selection of a point in the fiber S^2 of $T(CP_2)$ would correspond to the choice of the quantization axis of electroweak spin and diametrically opposite points would correspond to opposite values of electroweak spin 1/2 and unique quantization axis allows only single point or pair of diametrically opposite points to be populated.

   Spin 1/2 property would hold true for both ordinary and electroweak spins and this conforms with the properties of $M^4 \times CP_2$ spinors.

4. The points of $X_c^6 \subset M_c^8$ would represent geometrically the modes of H-spinor fields with fixed momentum. What about the orbital degrees of freedom associated with CP_2?

   M^4 momenta represent orbital degrees of M^4 spinors so that E^4 parts of E^8 momenta should represent the CP_2 momenta. The eigenvalue of CP_2 Laplacian defining mass squared eigenvalue in H should correspond to the mass squared value in E^4 and to the square of the radius of sphere $S^3 \subset E^4$.

   This would be a concrete realization for the $SO(4) = SU(2)_L \times SU(2)_R \leftrightarrow SU(3)$ duality between hadronic and quark descriptions of strong interaction physics. Proton as skyrmion would correspond to a map S^3 with radius identified as proton mass. The skyrmion picture would generalize to the level of quarks and also to the level of bound states of quarks allowed by the number theoretical hierarchy with Galois confinement. This also includes bosons as Galois confined many quark states.

5. The bound states with higher spin formed by Galois confinement should have the same quantization axis in order that one can say that the spin in the direction of the quantization axis is well-defined. This freezes the S^2 degrees of freedom for the quarks of the composite.

What does the map of the twistor space $T(M^4)$ to $T(CP_2)$ mean physically? Does spin correspond to color isospin or electroweak spin? Color $U(2)$ corresponds to electroweak $U(2)$ as the holonomy group of CP_2 as symmetric space so that the latter option is possible.

Quarks are doublets with respect to spin and electroweak spin but color triplet contains also isospin singlet. This is not a problem since color is not a spin-like quantum number in TGD but corresponds to color partial waves. This leaves spin-ew spin correspondence realized for quarks. Does the map between spin and electroweak degrees of freedom allow all pairings of spin and electroweak isospin doublets? The map between the spheres S^2 is determined only modulo relative rotation so that this might be the case for spin and color isospin. For composites of quarks obtained as Galois singlets, the relation between spin and ew spin could be more complex.

4.3.5 7-surfaces with real normal space and generalization of the notion of twistor space

The next step is to ask whether it makes sense to consider 7-surfaces with a real normal space allowing well-ordering? This would give a hierarchy of surfaces of M^8 with dimensions 7, 6, and 4. The 7-D space would have bundle projection to 6-D space having bundle projection to 4-D space.

One can also consider the complex 7-D surfaces with a complexified normal space for which the real projection is well-ordered so that the hierarchy of number fields would be realized. These surfaces would be realized by local elements of $SU(3)$ invariant under $U(1) \subset SU(3)$ and would define maps to $SU(3)/U(1)$ defining a generalization of twistor space. Now 6-D complex mass shells would take the role of 3-D complex mass shells and would correspond to the roots of P-

For the 7-D surface also the 7:th component of H- momentum should have some physical interpretation. Fermi statistics at the level of M^8 could be expressed purely geometrically: a single point of X^7 can contain only a single fermion (quark).

What could be the physical interpretation of 7-D surfaces of M^8 with real normal space in the octonionic sense and of their H images?

1. The first guess is that the images in H correspond to 7-D surfaces as generalizations of 6-D twistor space in the product of similar 7-D generalization of twistor spaces of M^4 and CP_2. One would have a bundle projection to the twistor space and to the 4-D space-time.

2. $SU(3)/U(1) \times U(1)$ is the twistor space of CP_2. $SU(3)/SU(2) \times U(1)$ is the twistor space of M^4? Could 7-D $SU(3)/U(1)$ *resp.* $SU(4)/SU(3)$ correspond to a generalization of the twistor spaces of M^4 *resp.* CP_2? What could be the interpretation of the fiber added to the twistor spaces of M^4, CP_2 and X^4? S^3 isomorphic to $SU(2)$ and having $SO(4)$ as isometries is the obvious candidate.

3. The analog of $M^8 - H$ duality in Minkowskian sector in this case could be to use coordinates for M^7 obtained by dividing M^8 coordinates by the real part of the octonion. Is it possible to

identify $RP_7 = M^8/R_\times$ with $SU(4)/SU(3)$ or at least relate these spaces in a natural manner. It should be easy to answer these questions with some knowhow in practical topology.

A possible source of problems or of understanding is the presence of a commuting imaginary unit implying that complexification is involved in Minkowskian degrees of freedom whereas in CP_2 degrees of freedom it has no effect. RP_7 is complexified to CP_7 and the octonionic analog of CP_3 is replaced with its complexification.

What could be the physical interpretation of the extended 7-D twistor space?

1. Twistorialization takes care of spin and electroweak spin and correlates them for quarks. The remaining standard model quantum numbers are Kähler and Kähler magnetic charges for M^4 and CP_2. Could the additional dimension allow a geometrization of these quantum numbers in terms of partial waves in the 3-D fiber? The example with the twistorialization suggests that the M^4 and CP_2 Kähler charges are identical apart from the sign.

2. The first thing to notice is that it is not possible to speak about the choice of quantization axis for $U(1)$ charge. It is however possible to generalize the momentum space picture also to the 7-D branes X^7 of M^8 with real normal space and select only discrete points of cognitive representation carrying quarks. The coordinate of 7-D generalized momentum in the 1-D fiber would correspond to some charge interpreted as a $U(1)$ momentum in the fiber of 7-D generalization of the twistor space.

3. One can start from the level of the 7-D surface with a real normal space. For both M^4 and CP_2, a plausible guess for the identification of 3-D fiber space is as 3-sphere S^3 having Hopf fibration $S^3 \to S^2$ with $U(1)$ as a fiber.

   At H side one would have a wave $exp(iQ\phi/2\pi)$ in $U(1)$ with charge Q and at M^8 side a point of X^7 representing Q as 7:th component of 7-D momentum.

   Note that for X^6 as a counterpart of twistor space the 5:th and 6:th components of the generalized momentum would represent spin quantization axis and sign of quark spin as a point of S^2. Even the length of angular momentum might allow this kind representation.

4. Since both M^4 and CP_2 allow induced Kähler field, a possible identification of Q would be as a Kähler magnetic charge. These charges are not conserved but in ZEO the non-conservation allows a description in terms of different values of the magnetic charge at opposite halfs of the light-cone of M^8 or CD.

   Instanton number representing a change of magnetic charge would not be a charge in strict sense and drops from consideration.

One expects that the action in the 7-D situation is analogous to Chern-Simons action associated with 8-D Kahler action, perhaps identifiable as a complexified 4-D Kähler action.

1. At M^4 side, the 7-D bundle would be $SU(4)/SU(3) \to SU(4)/SU(3) \times U(1)$. At CP_2 side the bundle would be $SU(3)/U(1) \to SU(3)/U(1) \times U(1)$.

2. For the induced bundle as 7-D surface in the $SU(4)/SU(3) \times SU(3)/U(1)$, the two $U(1)$:s are identified. This would correspond to an identification $\phi(M^4) = \phi(CP_2)$ but also a more general correspondence $\phi(M^4) = (n/m)\phi(CP_2)$ can be considered. m/n can be seen as a fractional $U(1)$ winding number or as a pair of winding numbers characterizing a closed curve on torus.

3. At M^8 level, one would have Kähler magnetic charges $Q_K(M^4)$, $Q_K(CP_2)$ represented associated with $U(1)$ waves at twistor space level and as points of X^7 at M^8 level involving quark. The same wave would represent both M^4 and CP_2 waves that would correlate the values of Kähler magnetic charges by $Q_{K,m}(M^4)/Q_{K,m}(CP_2) = m/n$ if both are non-vanishing. The value of the ratio m/n affects the dynamics of the 4-surfaces in M^8 and via twistor lift the space-time surfaces in H.

4.3.6 How could the Grassmannians of standard twistor approach emerge number theoretically?

One can identify the TGD counterparts for various Grassmann manifolds appearing in the standard twistor approach.

Consider first, the various Grassmannians involved with the standard twistor approach (`https://cutt.ly/XE3vDKj`) can be regarded as flag-manifolds of 4-complex dimensional space T.

1. Projective space is FP_{n-1} the Grasmannian $F_1(F^n)$ formed by the k-D planes of V^n where F corresponds to the field of real, complex or quaternionic numbers, are the simplest spaces of this kind. The F-dimension is $d_F = n-1$. In the complex case, this space can be identified as $U(n)/U(n-1) \times U(1) = CP_{n-1}$.

2. More general flag manifolds carry at each point a flag, which carries a flag which carries ... so that one has a hierarchy of flag dimensions $d_0 = 0 < d_1 < d_2 ... d_k = n$. Defining integers $n_i = d_i - d_{i-1}$, this space can in the complex case be expressed as $U(n)/U(n_1) \times .....U(n_k)$. The real dimension of this space is $d_R = n^2 - \sum_i n_i^2$.

3. For $n = 4$ and $F = C$, one has the following important Grassmannians.

   (a) The twistor space CP_3 is projective is of complex planes in $T = C^4$ and given by $CP_3 = U(4)/U(3) \times U(1)$ and has real dimension $d_R = 6$.

   (b) $M = F_2$ as the space of complex 2-flags corresponds to $U(4)/U(2) \times U(2)$ and has $d_R = 16 - 8 = 8$. This space is identified as a complexified Minkowski space with $D_C = 4$.

   (c) The space $F_{1,2}$ consisting of 2-D complex flags carrying 1-D complex flags has representation $U(4)/U(2) \times U(1) \times U(1)$ and has dimension $D_R = 10$.

   $F_{1,2}$ has natural projection ν to the twistor space CP_3 resulting from the symmetry breaking $U(3) \to U(2) \times U(1)$ when one assigns to 2-flag a 1-flag defining a preferred direction. $F_{1,2}$ also has a natural projection μ to the complexified and compactified Minkowski space $M = F_2$ resulting in the similar manner and is assignable to the symmetry breaking $U(2) \times U(2) \to U(1) \times U(1)$ caused by the selection of 1-flag.

   These projections give rise to two correspondences known as Penrose transform. The correspondence $\mu \circ \nu_{-1}$ assigns to a point of twistor space CP_3 a point of complexified Minkowski space. The correspondence $\nu \circ \mu_{-1}$ assigns to the point of complexified Minkowski space a point of twistor space CP_3. These maps are obviously not unique without further conditions.

This picture generalizes to TGD and actually generalizes so that also the real Minkowski space is obtained naturally. Also the complexified Minkowski space has a natural interpretation in terms of extensions of rationals forcing complex algebraic integers as momenta. Galois confinement would guarantee that physical states as bound states have real momenta.

1. The basic space is $Q_c = Q^2$ identifiable as a complexified Minkowski space. The idea is that number theoretically preferred flags correspond to fields R, C, Q with real dimensions 1,2,4. One can interpret Q_c as Q^2 and Q as C^2 corresponding to the decomposition of quaternion to 2 complex numbers. C in turn decomposes to $R \times R$.

2. The interpretation $C^2 = C^4$ gives the above described standard spaces. Note that the complexified and compactified Minkowski space is not same as $Q_c = Q^2$ and it seems that in TGD framework Q_c is more natural and the quark momenta in M_c^4 indeed are complex numbers as algebraic integers of the extension.

Number theoretic hierarchy $R \to C \to Q$ brings in some new elements.

1. It is natural to define also the quaternionic projective space $Q_c/Q = Q^2/Q$ `https://cutt.ly/LE3vMOG`, which corresponds to real Minkowski space. By non-commutativity this space has two variants corresponding to left and right division by quaternionic scales factor. A natural condition is that the physical states are invariant under automorphisms $q \to hqh^{-1}$

and depend only on the class of the group element. For the rotation group this space is characterized by the direction of the rotation axis and by the rotation angle around it and is therefore 2-D.

This space is projective space QP_1, quaternionic analog of Riemann sphere CP_1 and also the quaternionic analog of twistor space CP_3 as projective space. Therefore the analog of real Minkowski space emerges naturally in this framework. More generally, quaternionic projective spaces Q^n have dimension $d = 4n$ and are representable as coset spaces of symplectic groups defining the analogs of unitary/orthogonal groups for quaternions as $Sp(n+1)/Sp(n) \times Sp(1)$ as one can guess on basis of complex and real cases. M^4_R would therefore correspond to $Sp(2)/Sp(1) \times SP(1)$.

QP_1 is homeomorphic to 4-sphere S^4 appearing in the construction of instanton solutions in E^4 effectively compactified to S^4 by the boundary conditions at infinity. For Minkowski signature it would be replaced by 4-D hyperboloid $H^4 = SO(1,4)/SO(3)$ known also as anti-de Sitter space AdS(4,1) (`https://cutt.ly/RRuXIBS`). An interesting question is whether the self-dual Kähler forms in E^4 could give rise to M^4 Kähler structure and could correspond to this kind of self-dual instantons and therefore what I have called H-J structures.

2. The complex flags can also contain real flags. For the counterparts of twistor spaces this means the replacement of $U(1)$ with a trivial group in the decompositions.

   The twistor space CP_3 would be replaced $U(4)/U(3)$ and has real dimension $d_R = 7$. It has a natural projection to CP_3. The space $F_{1,2}$ is replaced with representation $U(4)/U(2)$ and has dimension $D_R = 12$.

To sum up, the Grassmannians associated with M^4 as 6-D twistor space and its 7-D extension correspond to a complexification by a commutative imaginary unit i - that is "vertical direction". The Grassmannians associated with CP_2 correspond to "horizontal ", octonionic directions and to associative, commutative and well-ordered normal spaces of the space-time surface and its 6-D and 7-D extensions. Geometrization of the basic quantum states/numbers - not only momentum - representing them as points of these spaces is in question.

4.3.7 How could the quark content of the physical state determine the geometry of the space-time surface?

In the standard quantum field theory, fermionic currents serve as sources of the gauge fields. This correlation must have a counterpart in the TGD framework. Somehow the selection of the active points of the cognitive representation containing quarks must determine the 4-surface of M^8 determined by a polynomial P with rational coefficients. $M8 - H$ duality would in turn determine the space-time surface.

This requirement gives a motivation for the earlier assumption that the roots of P defining 6-D surfaces fix P. Two kinds of surfaces appear.

1. The special $E = E_n$ roots of P having interpretation as energy have 3-D hyperplanes as M^4 intersections that I have misleadingly called "special moments in the life of self".

   The proposal [L39, L40] was that quarks are associated with the 2-D intersections of 4-D space-time surfaces with these planes. At the level of H, these 2-D intersections were assigned to partonic 2-surfaces serving as vertices of topological Feynman diagrams represented as space-time surfaces. Knowledge of the values of energy E_n defining 3-D complex planes at which the quarks of the quantum state are located in momentum space fixes the minimal polynomial P and therefore also space-time surface.

2. Besides energy hyper-planes there are also complex mass hyperboloids. The general 4-D solution of co-associativity conditions is 4-D (in real sense) intersection of two complex mass shells with mass squared $m^2_{c,odd}$ *resp.* $m^2_{c,even}$ with complex mass squared equal to a root of the odd *resp.* even part of the polynomial P defining the 4-surface [L39]. The real projection of the 4-D intersection is 2-D and might have interpretation as counterpart of a partonic 2-surface.

This complex surface has complex dimension 4 and 4-D real projection in the sense that the number theoretic quadratic form is real. The 6-surface defined by the root reduces to a 3-D real mass shell if the imaginary part of m_c^2 can vanishes: this is possible for real roots only. The 4-D intersection of these complex mass shells provide natural seats for the quark momenta as algebraic integers, which in general are complex. This data can fix the roots of the imaginary part of P as complex mass squared values.

3. Interestingly, also 6-D surfaces having these 4-surfaces as sub-manifolds emerge. A good guess is that these are just the surfaces with commutative normal space and serve as M^8 counterparts of twistor space.

4.3.8 How to understand leptons as bound states of 3 quarks?

A benchmark test for the view about the twistorial aspects of M^8 is the challenge of describing leptons as bound states of 3 quarks assignable to single wormhole contact, single throat, or even single point. The assumption that wormhole contacts correspond to blow-ups of line singularities in M^8 containing quarks favors the strongest option.

1. At the level of H , quarks with different colors (color partial waves in CP_2) could have exactly the same M^4 location inside a single wormhole throat but different CP_2 locations to realize statics. Color can be realized as H partial waves and this would require that the oscillator operators act at the level of M^8 allowing to put several oscillators at a single M^4 point at the level of H.

2. At the level of M^8 the Fermi statistics would state that only a single quark corresponds to a given point. If one works at the level of 4-surface so that only momentum is taken into account, this is not possible. Could the 3 quarks be at different points in the 7-D extension of the twistor space bringing in quark spin and Kähler magnetic charge?

   The total spin of lepton is 1/2 so that two spins are opposite. Kähler magnetic charges of quarks are proposed to be proportional to color hypercharge (2,-1,-1) for quarks to realize Fermi statistics topologically. The points (p,1/2,-1),(p,1/2,-1) and (p,-1/2,2) and the states obtained by permuting Kähler charges would allow arealization of lepton as a 3 quark state with identical momenta.

4.4 Hierarchies of extensions for rationals and of inclusions of hyperfinite factors

TGD suggests 3 different views of finite measurement resolution.

1. At the space-time level, finite measurement resolution is realized in terms of cognitive representations at the level of M^8 actualized in terms of fermionic momenta with momentum components identifiable as algebraic integers. Galois group has natural action on the momentum components.

2. The inclusion $N \subset M$ of group algebras of Galois groups is proposed to realize finite measurement resolution for which the number theoretic counterpart is Galois singlet property of N with respect to the Galois group of M relative to N identifiable as the coset group of Galois groups of M and N. If the origin serves as a root of all polynomials considered, the composite $P \circ Q$ inherits the roots of Q.

   The idea generalizes to infinite-D Galois groups [L56, L52]. The HFF in question would be infinite-D group algebra of infinite Galois group for a polynomial R obtained as a composite $R = P_{infty} \circ Q$ of an infinite iterate P_{infty} of polynomial P and of some polynomial Q of finite degree (inverse limit construction). The roots of R at the limit correspond to the attractor basin associated with P_∞, which is bounded by the Julia set so that a connection with fractals emerges.

3. The inclusions $N \subset M$ of hyperfinite factors of type II_1 (HFFs) [K42, K17] is a natural candidate for the representation of finite measurement resolution. N would represent the degrees

of freedom below measurement resolution mathematically very similar to gauge degrees of freedom except that gauge algebra would be replaced with the super-symplectic algebra and analogs of Kac Moody algebra with non-negative conformal weights and gauge conditions would apply to sub-algebra with conformal weights larger than the weight h_{max} defining the measurement resolution.

For HFFs, the index $[M : N]$ of the inclusion defines the quantum dimension $d(N \subset M) \leq 1$ as a quantum trace of the projector $P(M \rightarrow N)$ (the identify operator of M has quantum trace equal to one). $d(N \subset M)$ is defined in terms of quantum phase q and serves as a dimension for the analog of factor space M/N representing the system with N regarded as degrees of freedom below the measurement resolution and integrated out in "quantum algebra" M/N. Quantum groups and quantum spaces are closely related notions [K42, K17].

Galois confinement would suggest that $N \subset M$ corresponds to the algebra creating Galois singlets with respect to the Galois group of N relative to M whereas M includes also operators which are not this kind of singlets. In the above example $R = P \circ Q$, the Galois group of P would be represented trivially and the Galois group of Q or its subgroup would act non-trivially. In the case of hadrons, color degrees of freedom perhaps assignable to the Galois group Z^3 in the case of quarks would correspond to the degrees of freedom below the measurement resolution.

The universality of the quantum dimension and its expressibility in terms of quantum phase suggests that the integer m in $q = rxp(i2\pi/m)$ is closely related to the dimension for the extension of rationals $n = h_{eff}/h_0$ and depends therefore only very weakly on the details of the extension. The simplest guess is $m = n$. This conforms with the concrete interpretation of charge fractionation as being due to the many-valuedness of the graphs of space-time surfaces as maps from $M^4 \rightarrow CP_2$ or vice versa.

4.5 Galois confinement

The notion of Galois confinement emerged in TGD inspired biology [L92, L44, L52, L58]. Galois group for the extension of rationals determined by the polynomial defining the space-time surface $X^4 \subset M^8$ acts as a number theoretical symmetry group and therefore also as a physical symmetry group.

1. The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L44, L58, L57]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units.

2. The idea generalizes also to nuclear physics and suggests an interpretation for the findings claimed by Eric Reiter [L68] in terms of dark N-gamma rays analogous to BECs and forming Galois singlets. They would be emitted by N-nuclei - also Galois singlets - quantum coherently [L68]. Note that the findings of Reiter are not taken seriously because he makes certain unrealistic claims concerning quantum theory.

4.5.1 Galois confinement as a number theoretically universal manner to form bound states?

It seems that Galois confinement might define a notion much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Why not also hadrons could be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L34, L42, L15].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states. The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action inside the space-time surface is not well-defined. At the level of M^8 octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H. In contrast to this, weak gauge transformations acting as holonomies act in the tangent space of H.

   One can argue that the symmetries of H and even of WCW should/could have a reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (momenta) of M^8_c with coordinates, which are algebraic integers for the extension.

   Momenta as points of M^8_c would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of (in general complex) momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are algebraic, possibly complex, integers.

   Also other quantum numbers, in particular color, would correspond to representations of the Galois group. In the case of angular moment Galois confinement would allow algebraic half-integer valued angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential factors which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

4.5.2 Explicit conditions for Galois confinement

It is interesting to look more explicitly at the conditions for the Galois confinement.

Single quark states have momenta, which are algebraic integers generated by so called integral basis (`https://cutt.ly/SRuZySX`) spanning algebraic integers as a lattice and analogous to unit vectors of momentum lattice but for single component of momentum as a vector in extension. There is also a theorem stating that one can form the basis of extension as powers of a single root. It is also known that irreducible monic polynomials have algebraic integers as roots.

1. In its minimal form Galois confinement states that only momenta, which are rational integers are allowed by Galois confinement. Note that for irreducible polynomials with rational coefficients one does not obtain any rational roots. Monic polynomials with integer coefficients can allow integer roots. If one assumes that single particle states can have arbitrary algebraic integer as momentum, one obtain also rational integers for momentum values. These states are not at mass - or energy shell associated with the single particle momenta.

2. A stronger condition would be that also the inner products of the momenta involved are real so that one has $Re(p_i) \cdot Im(p_j) = 0$. For $i = j$ this gives a condition is possible only for the real roots for the real polynomials defining the space-time surface.

To see that real roots are necessary, some facts about the realization of the co-associativity condition [L39] are necessary.

1. The expectation is that that the vanishing condition for the real part (in quaternionic sense) of the octonionic polynomial gives a co-associative surface. By the Lorentz symmetry one actually obtains as a solution a 6-D complex mass shell $m_c^2 \equiv m_{Re}^2 - m_{Im}^2 + 2iRe(p) \cdot Im(p) = r_1$, where the real and imaginary masses are defined are $m_{Re}^2 = Re(p)^2$ and $m_{Im}^2 = Im(p)^2$ and r_1 is some root for the odd part of the polynomial P assumed to determining the 4-surface.

2. This surface can be co-associative but would be also co-commutive. Maximally co-associative surface requires quaternionic normal space. The first proposal is that the space-time surface is the intersection of the surface defined by the polynomial and its conjugate with respect to i. This gives 4-D surface as the intersection of the two 6-D surfaces.

   Second proposal is that the 6-surface having a structure of S^2 bundle defines as its base space quaternionic 4-surface. This space would correspond to a gauge choices selecting point of S^2 at very point of M^4. To a given polynomial one could assign entire family of 4-surfaces mapped to different space-time surfaces in H. A possible interpretation of gauge group would be as quaternionic automorphisms acting on the 2-sphere.

   These proposals are equivalent if the base base is the intersection of the 6-D bundle spaces. One could say that the fibers are conjugates of each other. This might be relevant for ZEO.

Concerning Galois confinement, the basic result is that for complex roots r_1 the conditions $Re(p_i) \cdot Im(p_i) = 0$ cannot be satisfied unless one requires that r_1 is real. Therefore the stronger option makes sense for real roots only.

1. Galois confinement allows the momenta p_i forming the bound state to be in an extension of rationals defined by the polynomial defining the space-time surface. Galois confinement condition states that the total momentum is rational integer when a suitable unit defined by the size of CD is used (periodic boundary conditions).

2. Another natural condition is the vanishing of the inner products between the real part $Re(p)$ and imaginary part $Im(p)$ of p. This guarantees that the number theoretical norm squared for the momentum is real. For time-like p, this means that $Im(p)$ belongs to the 3-D orthogonal complement E^3 of $Re(p)$. For light-like p, $Im(p)$ belongs to 2-D orthogonal complement E^2.

3. Suppose one has several number theoretic momenta p_i such that $\sum p_i = p$ is rational integer and $p_i \propto p$ holds true. Also in this case, the number theoretic inner products must be real. The orthogonality conditions read as

$$Re(p_i) \cdot Im(p_j) = 0 \quad . \tag{4.2}$$

   For a given pair (i, j), one has several conditions corresponding to algebraically independent imaginary momentum components and it is quite possible that very few solutions exist besides $Im(p_i) = 0$. If $Re(p_i)$ is not a rational integer, the number of conditions still increases.

4. The proposal for Galois confinement is that the real parts of p_i are parallel or even identical: $Re(p_i) \propto Re(\sum p_i) = p$, which is a rational integer. In this case the conditions reduce to $Re(p) \cdot Im(p_i) = 0$ and their number is much smaller.

5. For a given momentum component, the basis $p_{i,k}$ has the dimension n of extension. The basis contains m complex elements e_k and their conjugates $\overline{e}_k$ plus $n - 2m - 1$ real but algebraically trivial elements r_k besides the real unit 1. The sums $E_k = e_k + \overline{e}_k$ are algebraic integers and give m real basis elements. Note that $F_k = e_k - \overline{e}_k$ are purely imaginary algebraic integers.

   r_k and E_i give $n-m-1$ algebraically non-trivial real momenta. The momentum components $p_{i,k}$ formed as linear combinations of r_k, E_i, and 1 are real. This gives $n - m$-dimensional real subspace and momenta formed in this way satisfy the reality conditions for the inner products.

6. One can also construct complex momenta such that $Im(p_i)$ is a linear combination $Im(p_i) = \sum n_{i,k}F_k$. If $Re(p_i)$ are parallel and rational integers and $p_i \propto p$ holds true, the reality conditions reduce to

$$p \cdot Im(p_i) = \sum_k p^i n_{i,k} F_k = 0 \ . \tag{4.3}$$

One can construct a maximal set of complex momenta P_K characterized by matrices n_{ik}^K satisfying these conditions. Also linear combinations of P_K satisfy the reality conditions and one obtains a lattice of momenta.

This looks like nice construction but it seems that mere Galois confinement is more realistic.

4.6 $M^8 - H$ duality at the level of WCW

WCW emerges in the geometric view of quantum TGD. $M^8 - H$ duality should lso work for WCW. What is the number theoretic counterpart of WCW? What is the geometric counterpart of the discretization characteristic to the number theoretic approach?

In the number theoretic vision in which WCW is discretized by replacing space-time surfaces with their number theoretical discretizations determined by the points of $X^4 \subset M^8$ having the octonionic coordinates of M^8 in an extension of rationals and therefore making sense in all p-adic number fields? How could an effective discretization of the real WCW at the geometric H level, making computations easy in contrast to all expectations, take place?

1. The key observation is that any functional or path integral with integrand defined as exponent of action, can be *formally* calculated as an analog of Gaussian integral over the extrema of the action exponential $exp(S)$. The configuration space of fields would be effectively discretized. Unfortunately, this holds true only for the so called integrable quantum field theories and there are very few of them and they have huge symmetries. But could this happen for WCW integration thanks to the maximal symmetries of the WCW metric?

2. For the Kähler function K, its maxima (or maybe extrema) would define a natural effective discretization of the sector of WCW corresponding to a given polynomial P defining an extension of rationals.

   The discretization of the sector defined by P should be equivalent with the number theoretical discretization induced by the number theoretical discretization of space-time surfaces. Various p-adic physics and corresponding discretizations should emerge naturally from the real physics in WCW.

3. The physical interpretation is clear. The TGD Universe is analogous to the spin glass phase [?] The discretized WCW corresponds to the energy landscape of spin glass having an ultrametric topology. Ultrametric topology of WCW means that discretized WCW decomposes to p-adic sectors labelled by polynomials P. The ramified primes of P label various p-adic topologies associated with P.

4.7 Some questions and ideas related to $M^8 - H$ duality

In the following some questions and ideas, which do not quite fit under the titles of the previous sections, are considered.

4.7.1 A connection with Langlands program

Langlands correspondence [A7, K20, A5, A4], which I have tried to understand several times [K20] [L1, L4, L14] relates in an interesting manner to $M^8 - H$ duality and Galois confinement.

1. Global Langlands correspondence (GLC) states that there is connection between representations of continuous groups and Galois groups of extensions of rationals.

2. Local LC states (LLC) states this in the case of p-adics.

There is a nice interpretation for the two LCs in terms of sensory experience and cognition in TGD inspired theory of consciousness.

1. In adelic physics real numbers and p-adic number fields define the adele. Sensory experience corresponds to reals and cognition to p-adics. Cognitive representations are in their discrete intersection and for extensions of rationals belonging to the intersection.

   (a) Sensory world, "real" world corresponds to representation of continuous groups/Galois groups of rationals. GLC.

   (b) "p-Adic" worlds correspond to cognition and representations of p-adic variants of continuous groups and Galois groups over p-adics. Local LLC.

   (c) One could perhaps talk also about Adelic LC: ALC in the TGD framework. Adelic representations would combine real and p-adic representations for all primes and give as complete information about reality as possible.

   TGD provides a geometrization for the identification of Galois groups as discrete subgroups of Lie groups, not only of the isometry (automorphism) groups of H (M^8) but perhaps also as discrete sub-groups of more general Lie groups to which the action of super-symplectic representations could reduce. A naive guess is that these groups correspond to the ADE groups appearing in the McKay correspondence [L16, L37, L38].

   The representation of real continuous groups assignable to the real numbers as a piece of adele [L20, L21] would be related to the representations of Galois groups GLC. Also p-adic representations of groups are needed to describe cognition and these p-adic group representations and representations of p-adic Galois groups would be related by LLC.

4.7.2 Could the notion of emergence of space-time have some analog in the TGD Universe?

The idea about the emergence of space-time from entanglement is as such not relevant for TGD. One can however ask whath the emergence of *observed* space-time could mean in TGD. Space-time surface as a continuum exists in TGD but they are not directly observable due to a finite measurement resolution. One can ask what a body with an outer boundary means physically. The space-time regions defined by solid bodies have boundaries. What makes the boundaries of the bodies "hard"?

1. In momentum space Fermi statistics does not allow fermions to get through the boundary of Fermi ball. This is a good guideline.

2. Second feature of a spatial object such as an atom is that it is a bound state quantum mechanically. If it has parts they stay together. In QFT theory the notion of a bound state is however poorly understood.

3. Quantum coherence is a further property considered in the article. Spatial objects correspond to quantum coherent structures. Quantum coherence reduces to entanglement. Quantum coherence length and time determine the size of a quantum object. Somehow one must have stable entanglement in long scales.

Let us see what these guidelines could give in the framework of M^8-H duality which generalizes the wave particle duality of wave mechanics.

1. In adelic physics space-times can be seen as either surfaces in M^8 or $H = M^4 \times CP_2$. $X^4 \subset M^8$ is analogous to momentum space cognitive representations consist of points of $X^4 \subset M^8$, whose points are algebraic integers in the extension of rationals defined by the polynomial defining the space-time surface and are algebraic integers as roots of monic polynomials of form $x^n + ...$ This defines a unique discretization of the space-time surface. The discretization guarantees number theoretical universality: the cognitive representation makes sense also p-adically and space-time has also p-adic variants.

   Cognitive representations give rise to "cognitive emergence" of the space-time in cognitive sense and since cognitive representations are intersection of reality and p-adicities they must closely related to the "sensory emergence"".

2. $X^4 \subset M^8$ is mapped to H by $M^8 - H$ duality determined by the condition that it momentum is mapped to a geodesic with a direction of momentum and starting from either tip of CD: the image point is its intersection with the opposite light-like boundary of CD and selects a point of space-time surface. The size of CD is $T = h_{eff}/m$ for quark with mass m to satisfy Uncertainty Principle. The map generalizes to bound states of quarks (whatever they are).

Consider the problem of "sensory emergence" in this framework.

1. What makes a point of a cognitive representation "hard"? Quarks are associated with points (not necessarily all) of a cognitive representation: one can say that the point is activated when there is a quark at it. Fermi ball corresponds to a discrete set of activated points at the level of momentum space. These points define activated points also in $X^4 \subset H$ by $M^8 - H$ duality. One could perhaps say that these activated points in M^8 and their H-image containing fermions define the spatial objects as something "hard" and having a boundary. Another fermion knows that there is a space-time point there because it cannot get to this point. The presence of a fermion (quark) would make a space-time point "hard".

2. What about the role of entanglement? The size and duration of the space-time surface (inside a causal diamond CD) defines quantum coherence length and time. Fermionic statistics makes fundamental fermions - to be distinguished from elementary fermions - maximally entangled. One cannot reduce fermionic entanglement in SFR and quantum measurements would be impossible. The entanglement in the WCW degrees of freedom comes to the rescue. This entanglement can be reduced in SFRs since the particles as surfaces are identical under very special - naturally number theoretical - conditions.

   Negentropy Maximization Principle and hierarchy of $h_{eff} = n \times h_0$ phases favor the generation of stable entanglement in the TGD Universe. Also, if the coefficients of the entanglement matrix belong to extension of rationals, entanglement probabilities in general belong to its extension and the density matrix is not diagonalizable without going to a larger extension. This might require "big" SFR increasing the extension: only after this state function reduction to an eigenstate could occur. This leads to a concrete proposal for how the information about the diagonal form of the density matrix expressed by its characteristic polynomial is coded into the geometry of the space-time surface [L52].

3. Bound state formation is third essential element. Momenta are points of the space-time surface $X^4 \subset M^8$ with components which are algebraic integers. Physical momenta are however ordinary integers for a particle in a finite volume defined by causal diamond (CD). This means that one can allow only composites of quarks with rational integer valued momenta which correspond to Galois singlets.

   Galois confinement would be the universal mechanism behind formation of all bound states and also give rise to stable entanglement. One would obtain a hierarchy of bound states corresponding to a hierarchy of polynomials and corresponding Galois groups and extensions of rationals. By $M^8 - H$ duality, bound states of quarks and higher structures formed from them in M^8 would give rise to spatial objects.

5 Neutrinos and TGD

Neutrinos are problematic from the point of view of the standard model. It has become clear that neutrinos experience an analog of CKM mixing for quarks but there are anomalous findings related to the mixing. MiniBoone collaboration published 2018 findings [?] (see `https://arxiv.org/abs/1805.12028`) related to the mixing between muon and electron neutrinos for incoming muon beam.

The transformation of electron neutrino to electron via charged current reaction was used as a signature for the electron neutrinos and the findings forced the conclusion that the number of electrons produced is too high to be consistent with the neutrino CKM matrix deduced from other experiments. The sterile neutrino was one of the many proposed explanations (see `https://cutt.ly/DRKPZYz`).

The recent experiment of Micro-Boone collaboration however shows no evidence for sterile neutrinos (`https://cutt.ly/QRKDsUA` and `https://cutt.ly/oRKS77W`). The only remaining anomaly is associated with the channel producing an electron but no hadrons in the final state. If this finding is taken seriously, it is difficult to avoid the conclusion that some new physics, which is not caught by the standard model, is involved. Could the transformation of neutrino to an electron occur in some unknown way?

As it often happens, this rather specific question led to a thorough reconsideration of the TGD view about particles and their massivation: what is really understood and what is really certain? The basic idea of the TGD based solution described at the end of the article, would not have required these considerations so that an inpatient reader can directly skip to the last section.

5.1 Two problems related to neutrinos

The following considerations were motivated by two problems related to neutrinos.

5.1.1 What is the role of right handed neutrinos in TGD?

The new view led to the conclusion that the right-handed neutrino predicted by TGD and analogous to the inert neutrino solves some long-standing problems of TGD.

1. TGD in its recent form predicts an entire tower of color excitations as modes of second quantized $H = M^4 \times CP_2$ spinor field identified as a quark field. The mass scale determined by CP_2 length scale and these give rise to bound states of 3 antiquarks having quantum numbers of leptons if TGD view about color symmetry is accepted [L53]. In particular, covariantly constant right-handed neutrino ν_R in some respects analogous to a sterile neutrino is predicted.

   It is intuitively clear that ν_R must have a very special physical role. The naive proposal that ν_R and ν_R could generate the analog $N = 2$ SUSY [L36] has not led to a breakthrough. Spartners would have been created by adding zero momentum right-handed neutrinos and antineutrinos to the state: the problem is that the norm of these states vanishes if the only CP_2 Kähler form is present as in the formulation of TGD before the discovery of the twistor lift of TGD.

2. The twistor lift of TGD [L22] predicts that also M^4 has Kähler structure. This implies a breaking of Lorentz symmetry within causal diamond CD to $M^2 \subset M^4$ emerging also in the dual M^8 picture based on number theoretical view about physics [L39, L40, L67] as a prerequisite of $M^8 - H$ duality.

   M^4 mass squared m^2 is replaced with M^2 mass squared as in the quark model of hadrons, in string models, and also in p-adic mass calculations [K22]. The M^2 mass squared spectrum for $H = M^4 \times CP_2$ spinor modes is very much like in conformal field theories and the two integers (n_1, n_2) characterizing analogs of cyclotron states are analogous to conformal weights.

   The key point is that the massless ν_R transforms to a tachyon. This is due to the presence of spin term $J^{kl}(M^4)\Sigma_{kl}$ in $D^2(H)$ vanishing for left-handed leptons. On the other hand, p-adic mass calculations [K22] require a tachyon- like ground state: otherwise massless states are impossible. The origin of tachyonicity has remained a mystery. The tachyobiuc right-handed neutrinos could provide the long sought-for mechanism allowing to reduce the conformal weight of a given many-quark state to obtain a massless state.

3. The hard problem is that neutrinos are massive but only the left-handed neutrinos are observed. The problem is that the left-handed neutrinos mix with the right-handed ones if H Dirac operator $D(H)$ determines the time evolution operator. This should be seen in neutrino mixing experiments.

   The proposed solution of the problem is based on the TGD view about time evolution in zero energy ontology (ZEO). It has become clear that the time evolution between "small" state function reductions (SSFRs) corresponds to a scaling rather than time translation, and is induced by Virasoro generator L_0 - essentially mass squared operator - rather than by Hamiltonian.

   This suggests that for the spinor modes of H, the mass squared operator, that is the square $D^2(H)$ of Dirac operator $D(H)$ - or rather, its longitudinal M^2 part - should determine the time evolution operator rather than $D(H)$. Different M^4 chiralities would *not* mix.

4. This alone does not explain why only left-handed neutrinos are observed since different M^4 chiralities for leptons can appear as superpositions if left and right M^4 chiralities have the same value of $m^2(M^2)$. However, the $J^{kl}(M^4)\Sigma_{kl}$ term in $D^2(H)$ implies L-R splitting of mass squared eigenvalues. Degeneracy is possible if different values of n_1+n_2 can compensate for this splitting.

   Empirical facts require that R-L mixing is possible for charged leptons but not for neutrino states. Right-handed neutrinos would not mix with left-handed ones and would couple only to M^4 Kähler form but not to electroweak interactions. This could explain why they are not detected but also suggests that their detection might be possible.

5.1.2 Mini-Boone-Micro-Boone conflict and the TGD view about dark matter

This picture looks nice but does not explain the conflict between Mini-Boone and Micro-Boone experiments. Because Micro-Boone observes the anomaly for single electron final states only, it seems that neutrinos must scatter from some new form of matter.

TGD indeed predicts $h_{eff} > h$ phases of ordinary particles behaving like dark matter. The anomalous production of electrons by charged currents could be understood by the presence of dark protons or nuclei in the detector and having large enough h_{eff}. This could scale up weak interaction Compton length by h_{eff}/h above nuclear or even atomic length scale so that weak bosons would be effectively massless particles and the scattering cross section could be of the same order of magnitude as electroweak scattering cross section.

5.2 Some background about TGD

Some background about TGD is necessary in order to tackle the problems related to neutrinos.

5.2.1 Spinor fields in TGD

Spinor fields appear in TGD at three levels. At the level of embedding space $H = M^4 \times CP_2$, at the level of space-time surface $X^4 \subset H$, and at the level of "world of classical worlds" (WCW).

1. Spinor fields in H

Consider first spinor fields and their quantization at the level of H, which actually induces the spinor structure at the level of X^4 and WCW.

1. In the TGD Universe space-times are 4-surfaces X^4 in 8-D $H = M^4 \times CP_2$. The only fundamental fermions are quarks and the TGD view about color allows us to identify leptons as composites of 3 antiquarks in the scale of CP_2: this is not possible in QCD [L36, L53]. In what follows a key assumption is that leptons behave effectively like H spinor field having a chirality opposite to that for quarks and have the same electroweak quantum numbers apart from em charge. Therefore the Dirac equation in H applies to them.

2. The quantization of spinors is carried out at the level of H and quantized quark fields in X^4 are induced, that is restricted, to X^4 so that one avoids all problems related to second quantization in curved background. One of them is the difficulty in defining what positive and negative energy solutions to the Dirac equation do really mean.

3. If the Kähler form of $J(M^4)$ of M^4 vanishes (the more general case will be discussed later on), the square $D^2(H)$ of the H Dirac operator $D(H) = D(M^4) + D(CP_2)$ allows solutions satisfying $D^2(H)\Psi = 0$ that is massless modes in 8-D sense. The solutions of $D(H)\Psi = 0$ are of form $D(M^4)\Psi_1 \otimes \Psi_2 + \Psi_1 \otimes D(CP_2)\Psi_2$. Ψ_1 is a plane wave and Ψ_2 is an eigenstate of $D^2(CP_2)$ with a quantized mass squared eigenvalue m^2. Note that chiralities are mixed in accordance with the massivation in H.

   Covariantly constant right-handed neutrino is the only massless solution of $D(H)\Psi = 0$ in the M^4 sense. Since it does not have electroweak couplings it satisfies $D(CP_2)\nu_R = 0$ and is covariantly constant in CP_2. One can say that masslessness in 4-D sense is replaced with masslessness in 8-D sense and this is crucial also for why the twistor lift of TGD applies also to massive particles.

   One can say that $D(CP_2)$ is the analog of $D(M^4) = \gamma^k p^k$ in M^4 degrees of freedom. However, it cannot be algebraized. One could also say that it acts as an analog of the Higgs field which is not a H scalar but a CP_2 vector.

2. Spinor fields in X^4

Consider next the spinor fields at the level of X^4.

1. One can define modified Dirac operator [L67] at the level of X^4 in terms of the modified Gamma matrices determined as contractions of H gamma matrices Γ^k and the canonical momentum currents T^α_k determined by the action, which for twistor lift involves volume term (length scale dependent cosmological constant) and Kähler action analogous to Maxwell action. Preferred extremals are actually minimal surfaces which are also extremals of the Kähler action in the interior of X^4 [L71].

2. Modified Dirac equation cannot be satisfied generally as an operator equation. It could be however satisfied at the boundaries of causal diamond (CD) (one might say for external free quarks there) or possibly even in the interior of X^4 for the physical states but not generally. In any case the oscillator operator algebra for quarks in H would be used to construct quantum states.

   The intuitive guess is that the inverse of D canappear as a propagator. Its construction looks however a horrible problem. Fortunately, the problem disappears since $D(H)$ naturally defines a propagator between points restricted to the space-time surface.

   What is remarkable is that quite generally, the propagation between points with light-like distance is essentially like massless propagation. Particle-like entities are light-like orbits of partonic 2-surfaces so that the geometric character of particles forces massive modes effectively masslessness. A more precise formulation is discussed in [L67].

The induction procedure generalizes to the level of the isometry algebra (IA) and even super-symplectic algebra (SSA) [K34] [L45, L67].

1. One can construct the representations of IA and SSA in H for the Dirac action associated with $D(H)$ and construct the Noether currents of super symplectic algebra and project the currents to the space-time surface. A natural condition would be that these currents are equal to the corresponding currents assigned to the modified Dirac action for the physical states defined at the boundaries of CD.

2. An analogous condition for classical currents was proposed in [L67] and stated that the conserved classical current for given isometry with Killing vector j^k_A is proportional to its projection to the space-time surface.

$$
\begin{aligned}
T^{A\alpha}_B &= \Lambda(x) j^{\alpha}_A \ , \\
j^{\alpha}_A &= j^k_A \partial^{\alpha} h_k \equiv j^k_A h_{kl} g^{\alpha\beta}\partial_{\beta} h^l \quad , \\
\partial_{\alpha}\Lambda j^{A\alpha} &= 0 \ .
\end{aligned} \tag{5.1}
$$

This condition could be true for the entire space-time surface or at the ends of X^4 at the boundaries of CD. The conserved bosonic current in H corresponds to j^k_A satisfying $D_k j^k_A = 0$. The conservation condition requires that Λ is constant along the flow lines of j^k_A.

Quantum classical correspondence suggests that the condition can be true only for Cartan algebra. For the volume part of the action the condition is identically true and $\Lambda(x)$ corresponds to length scale dependent cosmological constant in this case. For Kähler action, the condition is non-trivial.

3. In the fermionic case, the condition would state that the conserved second quantized quark current at the level of H projected to the space-time surface is equal to the conserved fermionic current for the Dirac action in X^4. In the general case, this could hold true for the Cartan algebra and in the case of H isometries at the entire space-time surface. For the symplectic currents it could hold true at the 3-D ends of the space-time surface at boundaries of CD. The condition reads as

$$
T^{A\alpha}_F = \overline{\Psi}\Gamma_k \partial_{\alpha} h^k \delta_A \Psi = k(x)\overline{\Psi}\Gamma_k T^{k\alpha}_B \delta_A \Psi \ . \tag{5.2}
$$

If the bosonic condition for $T^{k\alpha}_B$ holds true, this condition and the conservation condition are trivially satisfied for $k(x) = \Lambda(x)$ as also the conservation condition. The condition also generalizes to super-currents obtained by replacing $\overline{\Psi}$ or Ψ by a mode of H spinor field in the expression of the fermionic current.

3. WCW spinors

The third realization is at the level of the "world of classical worlds" (WCW) assigned to H consisting of 4-surfaces as preferred extremals of the action. Gamma matrices of WCW are expressible as superpositions of quark oscillator operators so that anti-commutation relations are geometrized. WCW spinors are Fock states of quarks. The conditions stating super-symplectic symmetry are a generalization of super-Kac-Moody symmetry and of super-conformal symmetry and give rise to the WCW counterpart of the Dirac operator [K34] [L45, L67] as a non-hermitian super-Virasoro generator G which however carries fermion number.

Bosonic conditions and the fermionic condition implied by them have been already discussed and would dramatically simplify the construction of the quantums states as super-symplectic representations.

WCW gamma matrices would be simply SSA super charges for the induced spinor fields obtained by integrating the 3-D SSA super currents over 3-surfaces X^3 defining the ends of X^4 at the boundaries of CD. That they are projections of 8-D conserved currents in H would make life simple.

One could construct also WCW Kähler metric and in principle all related geometric entities in terms of SSA.

1. The matrix element of the WCW Kähler metric would be obtained as anticommutators

$$
g_{\overline{A},B} = \frac{1}{2}\{Q^{\dagger}_A, Q_B\} \tag{5.3}
$$

of the super symplectic charges. Super charge Q_A is obtained as a 3-D integral of super current J_A carrying quark number over the 3-surface X^3:

$$Q_A = \int_{X^3} d^3x J_A \quad . \tag{5.4}$$

The anticommutators of the fermionic oscillator operators for H spinors give Kronecker deltas for both momenta and color quantum numbers.

2. The localization at 3-surface implies that $g_{\overline{A}B}$ is given by an integral of form

$$\int_{X^3 \times X^3} d^3x_1 d^3x_2 \sum_{p,n} \overline{T}_{A_1}(p,n,x_1) T_{A_2}(p,n,x_2) \quad . \tag{5.5}$$

The plane waves in the product give a factor $exp[ip[\cdot(m(x_1) - m(x_2))]$ giving rise to interference. CP_2 spinor harmonics give a product of $\overline{\Psi}_n(s(x_1)\Psi_n(s(x_2)$. The products of factors at different points give rise to interference effects and could save from infinities.

The replacement of point-like particles with 3-surfaces is essential since the 7-D equal-time anti-commutation relations for quark oscillator operators give a 7-D delta function in H. Indeed, for a point-like particle instead of a 3-surface, one would obtain a sum over terms $\overline{\Psi}_n(s(x_1)\Psi_n(s(x_1)$ multiplied by the volume of the corresponding mass shell.

3. More generally, the double 3-D integral over a particle like n-surface should compensate for the 7-D delta function divergence so that for $2n > 7$ divergences would be absent. For 3-D objects one has $2n = 6$, so that one cannot exclude logarithmic divergences typically present also in gauge theories. Does this mean that the divergence cancellation cannot rely on mere non-locality.

4. Could the preferred extremal property be crucial? As a matter of fact, the condition guaranteeing that SSA currents for the action are equal to the projections of SSA currents for H spinors (at least at boundary CD) has been already assumed.

   Number theoretic holography fixes the space-time region in terms of roots of a polynomial with rational coefficients and is an extremely powerful condition also on 3-surfaces at the boundary of CD.

   Also the geometry of $\delta CD = \delta cd \times CP_2$ might be relevant as also the precise definition of the integral. One has a 6-D integral over $\delta cd \times \delta cd$. It seems that this is the correct intuition.

The following argument indeed shows that the geometry of CD (and thus ZEO) is highly relevant.

1. For $m_1 - m_2 = 0$, the CP_2 anticommutator gives a 4-D delta function in CP_2 as a singularity for $s(m_1) = s(m_2)$. For $m_1 = m_2$, one also has a 3-D delta function corresponding to equal time anticommutation relations. This would give 7-D delta function and the integral would diverge and be ill-defined. This is the source of troubles and raises the question whether one should one define the integral as a limit in which the ill-defined 7-D delta function contribution is avoided.

2. Denote by D the diagonal set $Diag(\delta cd \times \delta cd)$ of points $m_1 = m2$ of $\delta cd \times \delta cd$. Assign to D a thin 3-D layer $D \times L$ with L having a thickness l and define the integral over the volume $cd \times cd \setminus D \times L$ and take the limit $l \to 0$. This removes the problematic 7-D delta function singularity and leaves only the 1-D light-ray singularity at δcd [L49, L48] under consideration so that the anticommutator is well-defined and finite.

3. Irrespective of mass, fermion anticommutator has 1-D delta function type singurity as a 1-D delta function $\delta(a)$, $a^2 = (m_1 - m_2)^2$. Now both m_1 and m_2 are points at δcd, and the delta function defines light-like geodesic rays from origin connecting m_1 and m_2. This delta function eliminates 1 integration variable from 6 integration variables in the integration measure $dV = d^3m_1 d^3m_2$ associated with $\delta cd \times \delta cd$.

d^3m is determined by the determinant of the induced metric and if the CP_2 coordinates are not constant, the determinant is manifestly non-trivial even if one uses radial light-like coordinate r and angle coordinates Ω of $R_+ \times S^2$ as coordinates. This leaves a 5-D integration volume $X^5 \subset \delta cd \times \delta cd$. Note that for canonically embedded M^4 as a minimal surface extremal the integration measure is trivial so that the 3-surfaces do not belong to WCW.

4. The geometry of δcd would be highly relevant. If one had E^3 as time= constant slice instead of M^4, the same definition of the integral would give a vanishing result since light-like radial rays as singularities would be lost. This picture supports the importance of light-cone boundary as a basic notion but strictly speaking does not force CD.

One could worry for the somewhat ad hoc elimination of 7-D delta function singularity and perhaps take it as a signal telling that something important is still missing. There indeed exists a variant of gamma matrices with which I ended up from the cancellation of fermionic divergences in ZEO. This option is inspired by the multi-locality of the Yangian variants of the super symplectic algebra and isometry algebra for H.

1. The fermionic creation and annihilation operators appearing as building bricks of super symplectic (SSA) charges defining the gamma matrices would be at the opposite boundaries of CD and 3-D states at the opposite boundaries would relate like bras and kets. Annihilation operators would act like creation operators at the opposite boundary of CD.

   The conserved isometry currents in H would be replaced by bilocals with Ψ and $\overline{\Psi}$ and opposite boundaries of CD and remain conserved currents thanks to the (covariant) constancy of M^4 gamma matrices. Note that although SSA currents are not conserved, the Noether charges at the boundaries of CD are well-defined.

2. Can one apply this recipe to the WCW gamma matrices as bi-local entities having 3-surfaces at opposite boundaries as arguments? For supersymmetry generators associated with H isometries, the conservation laws hold and one can calculate the anticommutators. They are non-vanishing and the dominating contributions come from pairs of points with light-like separations. One can use the same CP_2 and S^2 coordinates at both light-like boundaries and only the radial light-like coordinates are different. The 3-D delta function singularity does not appear at all. This would justify the notion of CD rather than only light-cone boundary.

3. The commutators of SSA charges associated with 3-surfaces at different boundaries of CDs or even at boundaries of different CDs generate a poly-local algebra, which could have an interpretation as the Yangian algebra of SSA acting as isometries for WCW.

5.2.2 Twistor lift predicts M^4 Kähler force

The twistor lift of TGD suggests also a modification of the neutral weak forces.

1. The twistor lift of TGD requires that there is a covariantly constant self-dual Kähler form also in M^4. This would contribute to the electromagnetic and Z^0 fields an additional coupling analogous to that of electroweak hypercharge to U(1) gauge potential.

2. M^4 Kähler form contributes to the Kähler action an additional term. The M^4 contribution is fixed by the condition that the M^4 metric is the square of the Kähler form. Also H-spinors couple to M^4 Kähler gauge potential defining a self-dual Abelian field: essentially constant electric and magnetic fields, which are orthogonal and have the same strength, is in question.

   The scale of the M^4 metric defines the normalization of $J(M^4)$. Here one however encounters a problem since M^4 does not have any inherent scale in its geometry. The size scale L causal diamond ($CD = cd \times CP_2$), where cd is the intersection of light-cones with opposite direction, serves as a natural scale allowing to identify dimensionless coordinates for M^4 in such a way that the range of variation for the dimensionless coordinates does not depend on the size of CD.

   In these coordinates the self-dual Kähler form scales $E = B = k/L^2$, k a constant near unity. At the limit of long length scales $E = B$ would approach zero. The identification of L as a length scale determined by the cosmological constant is attractive. The breaking of Lorentz symmetry to that of M^4 for the Dirac operator $D(H)$ would be small in long length scales. In very short length scales associated with quarks, the breaking would be large.

Remark: One cannot completely exclude the alternative option $E = B = k/R^2$, where R is CP_2 scale for which the breaking of Lorentz invariance would be large in all scales.

The presence of M^4 Kähler structure has non-trivial implications also at the level of particle physics.

1. In particular, M^4 Kähler gauge potential $A(M^4)$ couples also to neutrinos unlike $A(CP_2)$, where the net coupling vanishes. The effects are expected to be small in the TGD view about space-time sheets at particle level.

2. The prediction is that all particles have an additional M^4 contribution in their Z^0 and em force and also right-handed neutrinos couple to M^4 Kähler gauge potential.

   Remark: The Kähler gauge potential A does not correspond to a genuine gauge invariance and each choice defines a different physics. The proposal is that the so-called Hamilton-Jacobi structures could correspond to different choices of A.

3. At the level of H the square $D^2(H)$ of the modified Dirac operator would allow spinors to be eigen states of energy and single momentum component. Self duality and covariant constancy imply that $D^2(H)$ contains a term proportional to charge matrix $J^{kl}(M^4)\Sigma_{kl} \propto (\sigma_{03} + \Sigma_{12})$, which vanishes for the second M^4 chirality.

4. 2 components of the 3-momentum would correspond to harmonic oscillator states so that the states would be confined to a finite transversal volume to a harmonic oscillator state characterized by transversal momenta of order magnetic length $\sqrt{B_K}$.

   Suppose that for the transversal degrees of freedom in E^2 with signature (-1,-1), Kähler gauge potential can be chosen to be $A_x = B_K y$. For an eigenstate of p_x, one obtains for the square of the E^2 part of the square D^2 of the Dirac operator,

   $$D^2(E^2) = -(\partial_x - B_K y)^2 - \partial_y^2 = p_x^2 + \partial_y^2 - B_K y^2 - 2ip_x B_K y \ .$$

   The sign of the harmonic oscillator term is correct and the complex shift does not produce problems if the notion of hermiticity is generalized so that PT replaces complex conjugation. Eigenvalues of $p_y^2 + ..$ are essentially the eigenvalues of energy in harmonic oscillator potential and proportional to $2nB_K$ with $n = 1$ assignable to the ground state.

5. In the longitudinal degrees of freedom M^2, the signature of the metric is (1,-1). If A is given by $A_t = B_K z$, the M^2 part of the square of the Dirac operator for an energy eigenstate reduces to $D^2(M^2) = (iE - iB_K z)^2 - \partial_z^2 = -E^2 - \partial_z^2 - B_K^2 z^2 - 2EB_K z$. One obtains a harmonic oscillator potential with a wrong sign and has suffered a complex shift by $z \to z + iE/B_K$. Harmonic oscillator Gaussian would be replaced with an imaginary exponential - this is of course familiar from free quantum field theories based on path integral defined by Gaussian. The size scale of CD would bring to the theory an arbitrarily long p-adic length scale as a fundamental level scale but expressible in terms of CP_2 radius.

Some physics inspired comments are in order.

1. This picture brings strongly in mind the parton model of hadrons. If cosmological constant Λ characterizes the size scale L, it must correspond to the scale which is essentially geometric mean of Planck length and the p-adic length scaled defined defining the length scale dependent cosmological constant Λ (of order Hubble scale). In the TGD framework, cosmological constant is length scale dependent, and the value of Λ assignable to cosmology would correspond to length L of order 10^{-4} meters assignable to a large neuron.

2. The spectrum of the M^2 mass squared operator is integer valued using B as a unit. The mass squared spectrum is similar to the spectrum in string models. This picture also conforms with the idea that the transversal Kac-Moody modes in $M^2 \times E^2$ are dynamical. Also transversality of polarizations in gauge theories conforms with this picture. Also the properties of "massless extermals" support this picture.

3. What comes to mind is that the values of integers n_i characterizing harmonic oscillator states are analogous to fermionic conformal weights. One has conformal weight for both the light-like radial coordinate of super symplectic representations and for the Kac-Moody type representations associated with light-like orbits of partons: the light-likeness of the partonic 2-surfaces and of light-cone boundary make them metrically 2-D and implies a generalization of conformal invariance.

   This conforms with the notion of induction. The fermion super symplectic charges should be constructible in terms of the quark oscillator operators for the second quantized quark fields of H.

5.2.3 How can massless particles exist at all and how do they become massive?

One must understand why there are light particles at all and what makes them massive.

1. The mass scale for CP_2 is about 10^{-4} Planck masses and the only massless particle is a right-handed neutrino of only $J(CP_2)$ is present. Also the color quantum numbers depend on the em charge. Therefore physical elementary particles cannot correspond to the quarks as such. The situation remains essentially the same if $J(M^4)$ is present.

   The proposal has been that H spinor modes define ground states for super-symplectic representations and operators carrying conformal weight contribute to mass squared additively create the physical states. The lowest states have vanishing mass squared. The introduction of $J(M^4)$ suggests that the quark oscillator operators labelled by two integers could actually be interpreted as conformal weights and that M^2 momentum would take the role of M^4 momentum. The number of ground states of super-symplectic representations could be much smaller.

2. p-Adic thermodynamics however mixes these states with states of higher conformal weight and this gives rise to the mass of the light particles. One must assume that there is a negative tachyonic contribution to the ground state conformal weight since only the right-handed neutrino is massless in 4-D sense. The origin of this negative conformal weight has remained a mystery.

3. M^8-H duality provides a possible insight to the mystery of the tachyonic conformal weight. The map of 4-surfaces in M^8_c (complexified octonions) by M^8-H duality involves selection of M^4 as a 4-D linear subspace in M^8. This choice is not unique. Momenta and color quantum numbers in H correspond to 8-momenta in M^8 such that 8-D mass squared vanishes at both sides and M^4 momenta are identical. For a suitable choice of $M^4 \subset M^8$, the 8-momentum is parallel to M^4 and the state is massless!

   Could the introduction of negative tachyonic conformal weight provide an alternative description of this choice? This choice can be made only for a single, naturally dominant contribution of the state, and the remaining contributions to mass squared coming from higher conformal weights give rise to massivation described by p-adic thermodynamics.

4. Here the twistor lift comes to rescue. Twistor lift of TGD requires that also M^4 has Kähler structure defined by a self-dual Kähler form $J_{kl}(M^4)$ (constant E and B with $vertB| = |E|$ orthogonal to each other). Depending on the selected correlation between M^4 and CP_2 chiralities guaranteeing that quarks correspond to a fixed H chirality, $D^2(H)$ contains for either left- or right-handed M^4 modes a nonvanishing spin term $J^{kl}(M^4)\Sigma_{kl}$. The reason is that for left-/right-handed mode the eigenvalues of Σ_{03} and Σ_{12} have the same/opposite sign or vice versa.

   This would give a mass splitting between left-and right-handed modes and also spin splitting for left- or right-handed modes. The spin-splitting could give rise to a negative contribution to the mass squared in the case of right-handed neutrinos. Could the tachyonic state of the right-handed neutrino give rise to the mysterious tachyonic ground states required by p-adic mass calculations? Could a suitable number of tachyonic right-handed neutrinos allow to nullify arbitrarily high conformal weight of ground state?

5.2.4 How to describe the unitary time evolution of quantum states in the TGD Universe?

The first question is how to describe the time evolution of quantum states in general. The time evolution at the single particle level is involved with the mixing of neutrinos.

Remark: One must remember that physical particles are multiquark composites: even leptons are local composites of 3 antiquarks). Therefore the description in terms of H-spinors applied in the sequel can be criticized.

1. In the TGD framework the standard 4-D approach based on the Hamiltonian picture can be only an approximate description since it neglects masslessness in the 8-D sense and is not relativistically invariant.

2. The empirical fact is that neutrinos are massive but always left- handed. The trivial explanation could be that right-handed neutrinos have only gravitational interaction so that their detection is not possible. The mixing of left-handed neutrinos with right-handed ones should however be visible in neutrino mixing experiments.

   In the TGD framework Dirac equation in H forces the mixing of quark chiralities for the modes of H-spinors. The covariantly constant right-handed neutrino is an exception. Induction as a mere restriction to the space-time surface respects this property! This implies that left-handed neutrino modes mix with right-handed ones and this could make itself visible in the neutrino beam experiments like Mini-Boone and Micro-Boone.

   The problem can be avoided if it is possible to have massive neutrinos with well-defined M^4 chirality and a time evolution which does not mix the chiralities. Could this kind of time evolution allow a realization?

3. Certainly, if the Dirac operator in H, or equivalently, the modified Dirac operator in X^4 defines the phenomenological Hamiltonian operator, the chirality mixing seems unavoidable. There is however no deep reason why $D(H)$ or $D(X^4)$ should define the propagation.

4. To get some guidance, one can also consider the level of "world of classical worlds" (WCW). The gamma matrices of WCW are constructed in terms of anticommuting oscillator operators of H-spinors and at tat that level the analog of the Dirac operator is a generator G of super-conformal algebra whereas the scaling generator L_0 is essentially $GG^\dagger$. However, G carries a quark number and therefore it does not make sense to talk about a propagator defined by G or an analog of Hamiltonian.

   The only reasonable unitary time evolution operator at WCW level is defined by the exponent of L_0, which is essentially mass squared operator obtained as "square" of WCW Dirac operator and has at the level of H counterpart of mass squared operator $D^2(H)$.

   In fact, in superstring models, the time evolution operator for the string world sheet is defined by L_0 so that this idea is not new. Also p-adic thermodynamics is defined by the exponent of L_0, at this time real, and its existence in the p-adic sense is responsible for the predictive power of p-adic thermodynamics.

   Here one must be more precise. Entire L_0 cannot be in question if it annihilates the physical states. In p-adic mass calculations L_0 is identified as the vibrational part $L_{0,vib}$ and for physical states in the string model satisfy $L_0\Psi = (p^2 - kL_{0,vib})\Psi = 0$. One could say that one has thermodynamics for states with different values of mass squared but satisfying the Virasoro condition. p^2 could also correspond to the longitudinal M^2 momentum and transversal momentum would be absorbed to $L_{0,vib}$. Both p-adic mass calculations and M^4 Kähler form favor this option and this picture conforms also with the stringy picture with M^2 effectively replacing the string world sheet.

Also the TGD based quantum measurement theory [L35] [K44] leads to the conclusion that the unitary time evolutions between "small" state function reductions (SSFRs) correspond to the exponential of L_0. Unitary time evolution as a time translation is replaced with a scaling which is a Lorenz invariant notion and better suited for relativistic purposes.

5. L_0 does not mix chiralities! If the initial state of a neutrino is left-handed, it remains left-handed. But how can the initial state of a neutrino be left-handed if spinor modes at the level of H are mixtures of left and right-handed modes as $D(H)\Psi = 0$ demands?

   Massless Dirac equation cannot be satisfied at the level of X^4 and at the level of WCW it does not make sense. Could one consider the radical possibility of giving it up altogether so that at the level of H one would require only that $D^2(H)\Psi = 0$ is satisfied and $D^2(H)$ would define counterpart of fermionic L_0 and time evolution.

   If so, the number of modes is doubled except for the right-handed neutrino. This implies mirror neutrinos. Could left and right-handed charged leptons and quarks be interpreted in terms of the mirror modes? Mirror neutrino hypothesis does not however have empirical support at available energies. One explanation is that the right-handed neutrino modes are very massive or somehow special.

6. If $J(M^4)$ is present, the masses of the left-handed mode and corresponding right-handed mode differ by the $S = J^{kl}(M^4)\Sigma_{kl}$ whose eigenvalues define the vacuum conformal weight $\pm h_{vac}$. Assume that S is non-vanishing for the right-handed mode. The number of right-handed modes with tachyonic mass squared would be the number of CP_2 modes with mass squared smaller than h_{vac}. Covariantly constant neutrino would certainly define this kind of state.

   If the mass is identified as the longitudinal M^2 mass, it might be possible to select the values of the conformal weights n_1 and n_2 for the modes in such a way that the masses are identical for the left- and right-handed modes and they can superpose. This should happen for charged modes. If this is not possible for neutrinos, the mixing of chiralities could not occur. This does not work.

   The masses of modes related by multiplication with Dirac operator have always identical mass squared values as follows from the commutativity of D and D^2. However, the covariantly constant right-handed neutrino does not have a left-handed companion. Both mixed states as modes of D and unmixed states satisfy $D^2\Psi = 0$. Why would neutrinos always have a definite handedness? Does the absence of standard model interactions for ν_R imply that the state preparation and reduction involving weak interactions creates only purely left-handed neutrinos?

   In the TGD Universe, even covariantly constant right-handed neutrino mode couples to M^4 Kähler form. Could this make it possible to project from mostly left-handed neutrino the non-covariantly constant right-handed part? Could their large mass make their creation impossible?

5.3 Problems related to neutrinos

In what follows, the problem of missing right-handed neutrinos and the problem created by apparently contradictory findings of Mini-Boone and Micro-Boone about neutrino mixing are discussed. Also the topological model for neutrino and D-quark CKM mixing is briefly considered.

5.3.1 Why only left-handed neutrinos are observed?

A basic theoretical motivation for the sterile neutrinos is the difficulty posed by the fact that the neutrinos behave like massive particles. This is not consistent with their left-handedness, which is an experimental fact.

As a matter of fact, the sterile neutrinos would be analogous to the covariantly constant right-handed neutrinos in TGD if only $J(cP_2)$ would be present.

Remark: As already stated, in the sequel it is assumed that leptons as bound states of 3 antiquarks can be described using spinors of H with chirality opposite to that for quarks. They have colored modes and the action of super-symplectic algebra is assumed to neutralize the color and also give rise to a massless state getting its small mass by p-adic thermodynamics.

How could one understand the fact that only left-handed neutrinos are observed although neutrinos are massive? One can consider two approaches leading to the same conclusion.

Is it possible to have time evolution respecting M^4 chirality and neutrinos with fixed chirality possible despite their mass?

1. All spinor modes in CP_2 are of the form Φ_L or $D(CP_2)\Phi_L$ and therefore generated from left-handed spinors Φ_L.

   If one assumes $D(H)\Psi = 0$, the spinor modes of H are of the form $D(M^4)\Psi_R \otimes \Phi_L + \Psi_R \otimes D(CP_2)\Phi_L$. The modes of form $D(M^4)\Psi_L \otimes \Phi_R + \Psi_L \otimes D(CP_2)\Phi_R$ are therefore of the form $D(M^4)\Psi_L \otimes D\Phi_L + \Psi_L \otimes D^2(CP_2)\Phi_L$. The mixing of chiralities is unavoidable.

2. However, if one assumes only the condition $D^2(H)\Psi = 0$, one can obtain both left- and right-handed modes without mixing of M^4 chiralities and M^4 Kähler structure could make the lowest mass second right-handed neutrino (covariantly constant in CP_2) tachyonic. The time evolution generated by the exponent of L_0 would respect M^4 chirality.

   This does not prevent superpositions of right- and left-handed fermions if their masses are the same. If only charged leptons can satisfy this condition, one can understand why right-handed neutrinos are not observed.

An alternative approach would rely on quantum measurement theory but leads to the same conclusion.

1. Suppose that neutrinos can appear as superpositions of both right- and left-handed components. To detect a right-handed neutrino, one must have a measurement interaction, which entangles both length and right-handed components of the neutrino with the states of the measuring system. Measurement would project out the right-handed neutrino. If only the $J(CP_2)$ form is present, the right-handed neutrino has only gravitational interactions, and this kind of measurement interaction does not seem to be realizable.

2. Putting it more explicitly, the reduction probability should be determined by a matrix element of a neutral (charged) weak current between a massive neutrino (charged lepton) spinor with a massless right-handed neutrino spinor. This matrix element should have the form $\overline{\Psi}_R O \Psi_L$, where O transforms like a Dirac operator. If it is proportional to $D(H)$, the matrix element vanishes by the properties of the massless right-handed neutrino.

3. There is however a loophole: the transformation of left- to right-handed neutrinos analogous to the transformation to sterile neutrino in the neutrino beam experiments could demonstrate the existence of ν_R just like it was thought to demonstrate the existence of the inert neutrino in Mini-Boone experiment. Time evolution should thus respect M^4 chirality.

If $J(M^4)$ is present, one might understand why right- and left-handed neutrinos have different masses.

1. Also the right-handed neutrino interacts with Kähler gaug potential $A(M^4)$ and one can consider an entanglement distinguishing between right- and left-handed components and the measurement would project out the right-handed component. How could this proposal fail?

   Could it be that right- and left-handed neutrinos cannot have modes with the same mass so that these superpositions are not possible as mass eigen states? Why charged modes could have the same mass squared but not the neutral ones?

2. The modes with right-handed CP_2 chirality are constructed from the left-handed ones by applying the CP_2 Dirac operator to them and they have the same CP_2 contribution to mass squared. However, for the right-handed modes the $J^{kl}(M^4)\Sigma_{kl}$ term splits the masses. Could it be that for right- and left-handed charged leptons the same value of mass is possible.

   The presence of $J(M^4)$ breaks the Poincare symmetry to that for M^2 which corresponds to a Lagrangian manifold. This suggests that the physical mass is actually M^2 mass and the

QCD picture is consistent with this. Also the p-adic mass calculations strongly support this view. The E^2 degrees of freedom would be analogous to Kac-Moody vibrational degrees of freedom of string. This would allow right- and left-handed modes to have different values of "cyclotron" quantum numbers n_1 and n_2 analogous to conformal weights. This could allow identical masses for left- and right-handed modes. For a Lagrangian manifold M^2, one would have $n_1 = n_2 = 0$, which could correspond to ground states of super-symplectic representation.

3. Why identical masses would be impossible for right- and left-handed neutrinos? Something distinguishing between right- and left-handed neutrinos should explain this. Could the reason be that Z^0 couples to left-handed neutrinos only? Could the fact that charged leptons and neutrinos correspond to different representations of color group explain why only charged states can have right and left chiralities with the same mass?

   Perhaps it is of interest to notice that the presence of $J^{kl}(M^4)\Sigma_{kl}$ for right-handed modes makes possible the existence of a mode for which mass can vanish for a suitable selection of B.

5.3.2 Mini-Boone and Micro-Boone anomalies and TGD

After these preliminaries we are ready to tackle the anomalies associated with the neutrino mixing experiments. The incoming beam consists of muonic neutrinos mixing with electron neutrinos. The neutrinos are detected as they transform to electrons by an exchange of W boson with nuclei of the target and the photon shower generated by the electron serves as the experimental signature.

The basic findings are as follows.

1. Mini-Boone collaboration reported 2018 [?] an anomalously large number of electrons generated in the charged weak interaction assumed to occur between neutrino and a nucleus in the detector. "Anomalous" meant that the fit of the analog of the CKM matrix of neutrinos could not explain the finding. Various explanations including also inert neutrinos were proposed. Muonic inert neutrino would transform to inert neutrino and then to electron neutrino increasing the electro neutrino excess in the beam.

2. The recently published findings of Micro-Boone experiment [?] studied several channels denoted by $1eNpM\pi$ where $N = 0, 1$ is the number of protons and $M = 0, 1$ is the number of pions. Also the channel 1eX, where "X" denotes all possible final states was studied.

   It turned out that the rate for the production of electrons is below or consistent with the predictions for channels 1e1p, 1eNp0π and 1eX. Only one channel was an exception and corresponds to 1e0p0π.

   If one takes the finding seriously, it seems that a neutrino might be able to transform to an electron by exchanging the W boson with a nucleus or hadron, which does not belong to the target.

In TGD, the only imaginable candidate for this interaction could be charged current interaction with a dark nucleus or with a nucleon with $h_{eff} > h$. This could explain the absence of ordinary hadrons in the final state for 1e events.

1. Dark particles are identified as $h_{eff} > h$ phases of the ordinary matter because they are relatively dark with respect to phases with a different value of h_{eff}. Dark protons and ions play a key role in the TGD inspired quantum biology [L92] and even in the chemistry of valence bonds [L17]. Dark nuclei play a key role in the model for "cold fusion" [L10, L42] and also in the description of nuclear reactions with nuclear tunnelling interpreted as a formation of dark intermediate state [L34].

2. I have proposed that dark protons are also involved with the lifetime anomaly of the neutron [L23] [L23]. The explanation relies on the transformation of some protons produced in the decay of neutrons to dark protons so that the measured life time would appear to be longer than real lifetime. In this case, roughly 1 percent of protons from the decay of n had to transform to dark protons.

3. If dark protons have a high enough value of h_{eff} and weak bosons interacting with them have also the same value of h_{eff}, their Compton length is scaled up and dark W bosons behave effectively like massless particles below this length scale. The minimum scale seems to be nuclear or atomic scale. This would dramatically enhance the dark rate for $\nu p \rightarrow e + n$ so that it would have the same order of magnitude as the rates for electromagnetic interactions. Even a small fraction of dark nucleons or nuclei could explain the effect.

5.3.3 CKM mixing as topological mixing and unitary time evolution as a scaling

The scaling generator L_0 describes basically the unitary time evolution between SSFRs [L35] [K44] involving also the deterministic time evolutions of space-time surfaces as analogs of Bohr orbits appearing in the superposition defining the zero energy state. How can one understand the neutrino mixing and more generally quark and lepton mixing in this picture?

1. In the TGD framework, quarks are associated with partonic 2-surfaces as boundaries of wormhole contacts, which connect two Minkowskian space-time sheets and have an Euclidean signature of induced metric and light-like projection to M^4 [K10, K22].

2. For some space-time surfaces in their superposition defining a zero energy state, the topology of the partonic 2-surfaces can change in these time evolutions. The mixing of boundary topologies would explain the mixing of quarks and leptons. The CKM matrix would describe the difference of the mixings for U and D type quarks and for charged and neutral leptons. The topology of a partonic 2-surface is characterized by the genus g as the number of handles attached to a sphere to obtain the topology.

   The 3 lowest genera with $g \leq 2$ have the special property that they always allow Z_2 as a conformal symmetry [K10, K22]. The proposal is that handles behave like particles and thanks to Z_2 symmetry $g = 2$ the handles form a bound state. For $g > 2$ one expects a quasi-continuous spectrum of mass eigenvalues. These states could correspond to so-called unparticles introduced by Howard Georgi (`https://cutt.ly/sRZKSFm`).

3. The time evolution operator defined by L_0 induces mixing of the partonic topologies and in a reasonable idealization one can say that L_0 has matrix elements between different genera. The dependence of the time evolution operator on mass squared differences is natural in this framework. In standard description it follows from the approximation of relativistic energies as $p_0 \simeq p + m^2/2p$. Also the model of hadronic CKM relies on mass squared as a basic notion and involves therefore L_0 rather than Hamiltonian.

5.4 Could inert neutrinos be dark neutrinos in the TGD sense?

I learned about a new-to-me anomaly related to nuclear physics and possible neutrino physics (`https://cutt.ly/mKb9265`). The so-called Gallium anomaly [?] is actually well-known but had escaped my attention. Baksan Experiment on Sterile Transitions (BEST) studies the nuclear reaction $\nu_e +^{71} Ga \rightarrow e^+ +^{71} Ga$ in which an electronic neutrino is produced in the beta decay of ^{51}Cr. The reaction rate has been found to be about 20-24 per cent lower than predicted. The articles [?, ?] by Barinov et al published in Phys Rev Letters and Phys Rev C of the experiment, can be found from arXiv (`https://arxiv.org/abs/2109.11482`, `https://arxiv.org/abs/2201.07364`).

Gallium anomaly is reported to be consistent with the sterile neutrino explanation stating that part of the electron neutrinos from the beta decay of ^{51}Cr transform to their sterile counterparts so that the reaction rate is reduced. A thorough discussion of the standard nuclear physics predictions for the reaction rate of the reaction can be found in the article "The gallium anomaly revisited" by Kostensalo et al [?](`https://cutt.ly/5Kb95nz`.

The already discussed MicroBoone experiment [?] however seems to exclude intert/sterile neutrinos.

1. What was reported is the following. Liquid Argon scintillator was used as a target. Several channels denoted by $1eNpM\pi$ where $N = 0, 1$ is the number of protons and $M = 0, 1$ is the number of pions, were studied. Also the channel 1eX, where "X" denotes all possible final

states was studied. It turned out that the rate for the production of electrons is *below or consistent* with the predictions for channels 1e1p, 1eNp0π and 1eX.

Only one channel was an exception and corresponds to 1e0p0π. This anomalous scattering without hadrons in the final state was interpreted in terms of the scattering of ν_e on dark weakly interacting matter. Also the neutrino must be dark and the values of h_{eff} must be identical for this dark matter and dark neutrino if they interact.

2. The strange scattering in the 0-proton channel would take place from weakly interacting matter, which is dark in the sense that it has non-standard value of effective Planck constant $h_{eff} = nh_0$: this proposal has a number theoretic origin in the TGD framework. Darkness implies that the particles with the same value of h_{eff} appear in the vertices of scattering diagrams. Dark and ordinary particles can however transform to each other in 2-vertex and this corresponds to mixing. The identification of what this weakly interaction dark matter might be, was not considered.

The anomaly associated with the neutron life-time is another anomaly, which the dark proton hypothesis explains [L23]. The two methods used to determine the lifetime of neutrons give different results. The first method measures the number of protons emerging to the beam in neutron decays. Second method measures the number of neutrons. The TGD explanation of the anomaly is that a fraction of neutrons decay to a dark proton, which remains unobserved in the first method. Second method detects the reduction of the intensity of the neutron beam and is insensitive to what happens to the proton so that the measurements give slightly different results.

These findings inspire the question whether the inert neutrinos are dark neutrinos in the TGD sense and therefore have $h_{eff} > h$? The mixing of the incoming neutrinos with their dark variants would take place in the ^{71}Ga experiment. Dark neutrinos would not interact with ^{71}Ga target since neutrons inside the ^{71}Ga nuclei are expected to be ordinary so that the $\nu_e + n \rightarrow p + e^-$ scattering rate would be lower as observed.

The identification of sterile neutrinos as dark neutrinos can be consistent with the Micro-Boone anomaly if one can identify the weakly interacting dark matter.

1. Dark neutrons should not be present in the liquid Argon. Could the weakly interacting dark matter be meson-like states consisting of dark d or $\overline{u}$ quarks? Since the scattering from them cannot contribute to the nuclear weak interaction, these flux tubes must be outside Argon nuclei. By the large value of h_{eff}, they would connect Argon nuclei.

2. The TGD inspired model of nuclei describes them as nuclear strings consisting of nucleons connected by meson-like strings with quark and antiquark at its ends. The model of "cold fusion" [L10, L15, L42] inspired the proposal that dark nuclei consisting of dark protons connected by dark meson-like strings are formed in a water environment and give rise to what might be called dark nuclei.

   The nuclear binding energy of the dark nuclei is scaled down by the ratio of the length scale defined by the distance between dark protons to nuclear length scale. The decay of dark nuclei to ordinary nuclei liberates more nuclear energy than ordinary nuclear reactions. Also strings of nuclei connected by dark meson-like flux tubes can be imagined. One can also consider flux tube bonded clusters of nuclei.

3. The TGD based model for living matter involves in an essential way the formation of dark proton sequences at flux tubes when water is irradiated in presence of gel, by say infrared light.

   Could these dark flux tube bonds between nuclei relate to hydrogen bonds and hydrogen bonded clusters of water molecules? Could the "$Y \cdots H$" of the hydrogen bond $Y \cdots H - X$ actually correspond to a dark meson-like flux tube bond between nuclei of Y and H? Could the attractive nuclear interaction between Y and X generated in this way increase the density of the liquid phase and explain the strange finding that the density of water above freezing point is higher than the density of the solid state?

   Interestingly, according to the Wikipedia article (`https://cutt.ly/5KWribE`), Ga has some strange thermodynamic properties. The density of Ga above freezing point is higher than that of solid state. This property is shared also by water, silicon, germanium, bismuth, and plutonium. Ga has a strong tendency to supercool down to temperatures below 90 K.

4. This suggests that liquid phases could in some situations form structures connected by dark meson-like flux tubes. If $h_{eff} > h$ phases are generated as long range quantum fluctuations at quantum criticality and if quantum criticality is behind the thermodynamic criticality, this could happen near or above criticality for solid-liquid phase transition and even solid-gas phase transition.

   If this kind of flux tubes connecting Argon nuclei (Argon does not have anomalous thermodynamics) are present in a liquid Argon detector, they explain the observed anomalous contribution to neutrino-Argon scattering.

   Also in Gallium this could be the case as suggested by the higher density above freezing point. Could one detect the anomalous scattering of neutrinos from the proposed flux tube bonds connecting Ga atoms and study the anomalous scattering as a function of the temperature?

6 Zero energy ontology (ZEO)

ZEO [K44] forms the cornerstone of the TGD inspired quantum theory extending to a theory of consciousness. ZEO has so far reaching consequences that it would have deserved a separate section. Since it involves in an essential manner the notion of CD, it is natural to include it to the section discussing $M^8 - H$ duality.

6.1 The basic view about ZEO and causal diamonds

The following list those ideas and concepts behind ZEO that seem to be rather stable.

1. GCI for the geometry of WCW implies holography, Bohr orbitology and ZEO [L35] [K44].

2. X^3 is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. Finite failure of determinism is however possible [L71]. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD) defined as intersection of future and past directed light cones ($\times CP_2$) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. 12 Fig. 13**).

4. The opposite light-like boundaries of CD correspond for fermions dual vacuums (bra and ket) annihilated by fermion annihilation *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD. This seems to be the correct view in H. At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing would be by mass shells (hyperboloids) at the level of M^8 and by CDs with same center point at the level of H.

5. Zeno effect can be understood if the states at either cone of CD do not change in "small" state function reductions (SSFRs). SSFRs are analogs of weak measurements. One could call this half-cone call as a passive half-cone. I have earlier used a somewhat misleading term passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate - geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change. The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like deterministic time evolution leading to the final state [L29] as observed by Minev *et al* [L29].

7. h_{eff} hierarchy [K12, K13, K14, K15] implied by the number theoretic vision [L39, L40] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

8. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. Provides also a mechanism of self-organization [L33]: dissipation with reversed arrow of time looks like self-organization [L92].

6.2 Open questions related to ZEO

There are many unclear details related to the time evolution in the sequence of SSRs. Before discussing these unclear details let us make the following assumptions.

1. The size of CDs increases at least in a statistical sense in the sequence of CD and the second boundary remains stationary apart from scaling (note that one can also consider the possibility that the entire CD is scaled and temporal shift occurs in both directions).

2. Mental mentals (say after images) are in kind of Karma's cycle: they are born and die roughly periodically.

3. I do not experience directly mental images with the opposite arrow of time.

4. I can have memories only about states of consciousness with the same arrow of time that I have. This explains why I do not have memories about periods of sleep if sleep is interpreted as a time reversed state of some subself of me responsible for self-ness.

One can use three empirical inputs in an attempt to fix the model.

1. After images appear and disappear roughly periodically. Also I fall asleep and wake up with a standard arrow of time roughly periodically.

   (a) The first interpretation is that as a sequence of wake up-sleep periods I am a time crystal-like structure consisting of nearly copies of the mental image, such that each mental image - including me as mental images of higher level self - continues Karma's cycle in my geometric past. How "me" is transferred to a new almost copy of my biological body? Does my MB just redirect its attention?

   (b) The second interpretation is that me and my mental images somehow drift towards my geometric future, while performing the Karma's cycle so that my mental images follow me in my time travel. This would require that the sub-CDs of mental images drift towards the geometric future.

   Also sleep could be a "small" death at some layer of the personal hierarchy of MBs. I do not however wake-up in BSFR at the moment of geometric time defined by the moment of falling asleep but later. So it seems that my CD must drift to the geometric future with the same speed that those of other living beings in the biosphere.

2. There is however an objection. In cosmology the observation of stars older than the Universe would have a nice solution if the stars evolve forth and back in time in our distant geometric past rather than drifting towards the future so that they could age by continuing their Karma's cycle with a constant center of mass value of time. Can these three observations be consistent?

6.2.1 Could the scaling dynamics CD induce the temporal shifting of sub-CDs as 4-D perceptive fields?

Suppose that the sub-CDs within a bigger CD "follow the flow". How the dynamics of the bigger CD could induce this flow?

1. The scalings of bigger CD in unitary evolutions between SSFRs induce the scaling of sub-CS. This would not be shifting but scaling and the distance between given CD and larger CDs would gradually scale up.

   This would remove the objection. The astrophysical objects in distant geometric past would move towards the geometric future but with much smaller velocity as the objects with cosmic scale so that the temporal distance to future observers would increase. These objects would be aging in their personal Karma's cycle, and the paradox would disappear.

2. The flow would be defined by the scalings of a larger CD containing our CDs and those of others at my level. Each CD would define a shared time for its sub-CDs. If the CDs form a hierarchy structure with a common center, this is indeed true of the time evolutions as scalings of CDs. There would be scalings induced by scalings at higher levels and "personal" scalings.

3. It however seems that the common center is too strong an assumption and shifted positions for the sub-CDs and associated hierarchy inside a given CD are indeed possible for the proposed realization of $M^8 - H$ duality and actually required by Uncertainty Principle.

A further open question is what happens to the size of CD in the BSFR. Does it remain the same so that the size of the CD would increase indefinitely? Or is the size reduced in the sense that there would be scaling, reducing the size of the CD in which the passive boundary of the CD would be shifted towards the active one. After every BSFR, the self would experience a "childhood".

6.2.2 Are we sure about what really occurs in BSFR?

It has been assumed hitherto that a time reversal occurs in BSFR. The assumption that SSFRs correspond to a sequence of time evolutions identified as scalings, forces to challenge this assumption. Could BSFR involve a time reflection T natural for time translations or inversion $I : T \rightarrow 1/T$ natural for the scalings or their combination TI?

I would change the scalings increasing the size of CD to scalings reducing it. Could any of these options: time reversal T, inversion I, or their combination TI take place in BSFRs whereas arrow would remain as such in SSFRs? T (TI) would mean that the active boundary of CD is frozen and CD starts to increase/decrease in size.

There is considerable evidence for T in BSFRs identified as counterparts of ordinary SFRs but could it be accompanied by I?

1. Mere I in BSFR would mean that CD starts to decrease but the arrow of time is not changed and passive boundary remains passive boundary. What comes to mind is blackhole collapse.

   I have asked whether the decrease in size could take place in BSFR and make it possible for the self to get rid of negative subjective memories from the last moments of life, start from scratch and live a "childhood". Could this somewhat ad hoc looking reduction of size actually take place by a sequence of SSFRs? This brings into mind the big bang and big crunch. Could this period be followed by a BSFR involving inversion giving rise to increase of the size of CD as in the picture considered hitherto?

2. If BSFR involves TI, the CD would shift towards a fixed time direction like a worm, and one would have a fixed arrow of time from the point of view of the outsider although the arrow of time would change for sub-CD. This modified option does not seem to be in conflict with the recent picture, in particular with the findings made in the experiments of Minev *et al* [L29] [L29].

   This kind of shifting must be assumed in the TGD inspired theory of consciousness. For instance, after images as a sequence of time reversed lives of sub-self, do not remain in the geometric past but follow the self in travel through time and appear periodically (when their arrow of time is the same as of self). The same applies to sleep: it could be a period with a reversed arrow of time but the self would shift towards the geometric future during this period: this could be interpreted as a shift of attention towards the geometric future. Also this option makes it possible for the self to have a "childhood"".

3. However, the idea about a single arrow of time does not look attractive. Perhaps the following observation is of relevance. If the arrow of time for sub-CD correlates with that of sub-CD, the change of the arrow of time for CD, would induce its change for sub-CDs and now the sub-CDs would increase in the opposite direction of time rather than decrease.

To sum up, TI or T can be considered as competing options for what happens in BSFR. T should however be able to explain why sub-selves (sub- CDs) drift to the direction of the future. If the time evolutions between SSFRs correspond to scalings rather than time translations, and if the scalings occur also for sub-CDs this can be understood. The dynamics of spin glasses strongly suggests that SSFRs correspond to scalings [L64].

6.3 What happens in quantum measurement?

According to the proposed TGD view about particle identity, the systems for which mutual entanglement can be reduced in SFR must be non-identical in the category theoretical sense.

When SFR corresponds to quantum measurement, it involves the asymmetric observer-system $O-S$ relationship. One cannot exclude SFRs without this asymmetry. Some kind of hierarchy is suggestive.

The extensions of rationals realize this kind of $O-S$ hierarchy naturally. The notion of finite measurement resolution strongly suggests discretization, which favors number theoretical realization. The hierarchies of effective Planck constants and p-adic length scale hierarchies reflect this hierarchy. What about the topological situation: can one order topologies to a hierarchy by their complexity and could this correspond to $O-S$ relationship?

The intuitive picture about many-sheeted space-time is as a hierarchical structure consisting of sheets condensed at larger sheets by wormhole contacts, whose throats carry fermion number. Intuitively, the larger sheet serves as an observer. p-Adic primes assignable to the space-time sheet could arrange them hierarchically and one could have entanglement between wavefunctions for the Minkowskian regions of the space-time sheets and the surface with a larger value for p would be in the role of O

6.3.1 Number theoretic view about measurement interaction

Quantum measurement involves also a measurement interaction. There must be an interaction between two different levels O and S of the hierarchy.

One can look at the measurement interaction from a number theoretic point of view.

1. For cognitive measurements the step forming the composite $O \circ S$ of polynomials would represent the measurement interaction. Before measurement interaction systems would be represented by O and S and measurement interaction would form $O \circ S$ and after the measurement the situation would be as proposed.

   Could one think that in BSFR the pair of uncorrelated surface defined by $O \times S$ with degree $n_O + n_S$ (analog for the additivity of classical degrees of freedom) is replaced with $O \circ S$ with degree $n_O \times n_S$ (analog for multiplicativity of degrees of freedom in tensor product) in BSFR? This would mean that the formation of $O \circ S$ is like a formation of an intermediate state in particle reaction or in chemical reaction.

   Could the subsequent SSFR cascade define a cascade of cognitive measurements [L47]. I have proposed that this occurs in all particle reactions. For instance, nuclear reactions involving tunneling would involve formation of dark nuclei with $h_{eff} > h$ in BSFR and a sequence of SSFRs in opposite time direction performing cognitive quantum measurement cascade [L34] and also the TGD based model for "cold fusion" relies on this picture [L15, L42]. After the SSFR cascade, a second BSFR would occur and bring back the original arrow of time and lead to the final state of the nuclear reaction.

From the point of view of cognition, BSFR would correspond to the heureka moment and the sequences of SSFRs to the cognitive analysis decomposing the space-time surface defined by $O \circ S$ to pieces.

2. One can also consider small perturbations of the polynomials $O \circ S$ as a measurement interaction. For instance, quantum superpositions of space-time surfaces determined by polynomials depending on rational valued parameters are possible. The Galois groups for two polynomials with parameters which are near to each other are the same but for some critical values of the parameters the polynomials separate into products. This would reduce the Galois group effectively to a product of Galois groups. Quantum measurement could be seen as a localization in the parameter space [L52].

6.3.2 Topological point of view about measurement interaction

The measurement interaction can be also considered from the topological point of view.

1. Wormhole contacts are Euclidean regions of $X^4 \subset H$ couples two parallel space-time regions with Minkowskian signature and could give rise to measurement interaction. Wormhole contact carries a monopole flux and there must be a second monopole contact to make flux loop possible. This structure has an interpretation as an elementary particle, for instance a boson. The measurement interaction could correspond to the formation of this structure and splitting by reconnection to flux loops associated with the space-time sheets after the interaction has ceased.

   Remark: Wormhole contacts for $X^4 \subset H$ correspond in $M^8 - H$ duality images of singularities of $X^4 \subset M^8$. The quaternionic normal space at a given point is not unique but has all possible directions, which correspond to all points of CP_2. This is like the monopole singularity of an electric or magnetic field. At the level of CP_2 wormhole contact is the "blow-up" of this singularity.

2. Flux tube pairs connecting two systems serve also as a good candidate for the measurement interaction. U-shaped monopole flux tubes are like tentacles and their reconnection creates a flux tube pair connecting two systems. SFR would correspond geometrically to the splitting of the flux tube pair by inverse re-connection.

6.3.3 Geometric view about SSFR

The considerations of [L51] strongly suggest the following picture about SSFRs.

In the measurement interaction a quantum superposition of functional composites of polynomials P_i defining the space-time surfaces of external states as Galois singlets is formed. A priori all orders for the composites in the superposition are allowed but if one requires that the same SSFR cascade can occur for all of them simultaneously, only single ordering and its cyclic permutations can be allowed.

The SSFR cascade can of course begin with a reduction selection single permutation and its cyclic permutations: localization in S_n/Z_n would take place.

Incoming states at passive boundary of CD correspond to prepared states and outgoing states at active boundary to state function reduced states. The external states could correspond to products of polynomials as number theoretical correlates for the absence of correlations in unentangled states.

Number theoretic existence for the scattering amplitudes [**?**] require that the p-adic primes characterizing the external states correspond to maximal ramified primes of the corresponding polynomials and therefore also to unique p-adic length scales L_p. In the interaction regions this ramified prime is the largest p-adic (that is ramified) prime for particles participating in the reaction. This correlation between polynomial and p-adic length scale allows a rather concrete geometric vision about what happens in the cascade.

SSFR cascade begins with a reduction of the state to a superposition of single composite with its cyclic variants for positive and negative energy parts separately: this kind of cyclic superpositions appear also in the twistor Grassmann picture [L51] and in string models. In the recent situation this makes possible a well-defined state preparation and SFR cascades at the two sides of CD. In ZEO, the cascade could take place for positive energy states only during SSFR.

A number theoretic SFR cascade would take place and decompose the Galois state group of the composite having decomposition to normal sub-groups to a product of states for the relative Galois groups for the composite.

A given step of the cascade would be a measurement of a density matrix ρ producing information coded by its reduction probabilities as its eigenvalues in turn coded by the characteristic polynomial P_M of the density matrix.

The simplest guess is that the final state polynomial is simply the product $\prod P_{i_-}$ of the polynomials P_{i_-} for the passive boundary of CD and product $\prod P_{i_+}$ for the active boundary.

6.3.4 Question of quantum information theorist

Quantum information theorists could however ask what happens to the information yielded by a given step of the measurement cascade.

1. Could the information about the measured ρ coded by P_M as its algebraic roots be stored to the final state coded by the final state polynomials $P_{i,+}$?

   Could the outcome at the active boundary of CD for which the SSFR cascade is actually not the 4-surface determined by the polynomials $P_{i,+}$ but $P_{M_{i_+}} \circ P_i$, or more generally a quantum superposition of $P_{M_{i_+}} \circ P_i$, and $P_i \circ P_{M_{i_+}}$.

   The "unitary time evolution" preceding the next SSFR would correspond to a functional composite of these polynomials so that the space-time surface would evolve during the SSFR sequence. The basic process would be a formation of functional composite followed by SSFR cascade storing the information about the measured density matrices to the space-time surface.

2. There are strong constraints on this proposal. $P_{M_{i_+}}$ should have rational coefficients in the extension of rationals defined by the composite polynomial, or even polynomial P_i. Monic polynomial property would pose even stronger conditions on entanglement coefficients and the representations of the entire Galois group.

   There is also the notion of Galois confinement for physical states. What constraints does this give?

   These conditions pose very strong conditions on the allowed entanglement matrix and could make the proposal unrealistic.

6.4 About TGD based description of entanglement

The general classification of possible quantum entanglements is an interesting challenge and there are many approaches (`https://cutt.ly/iREIg1u`). One interesting approach relies on the irreducible representations of the unitary group $U(n)$ acting as the isometry group of n-D Hilbert space (`https://cutt.ly/ZREIEAT`). The assumption about irreducibility is however not essential for what follows.

1. A system with n-D state space H_n identified as a sub-system of a larger system with N-D state space H_N can entangle with its $M = N - n$-D complement H_M. Suppose $n \leq M$. Entanglement implies that the n-D state space or its sub-space is embedded isometrically into a subspace of the M-D state space. For a non -trivial subspace one can replace H_n with this subspace H_m in what follows. The diagonal form of the density matrix describes this correspondence explicitly. If the subspace is 1-D one has an unentangled situation.

2. $U(n)$ and its subgroups act as automorphism groups of H_n This inspires the idea that the irreducible representations of $U(n)$ define physically very special entanglements $H_n \subset H_M$. The isometric inclusions $H_n \subset H_M$ are parametrized by a flag-manifold $F_{n,M} = U(M)/U(n) \times$

$U(M-n)$. If one allows second quantization in the sense that the wave functions in the space of entanglements make sense, this flag manifold represents additional degrees of freedom for entanglements $H_n \subset H_M$. If the entanglement does not have maximal dimension, the product of flag manifolds $F_{n,M}$ and $F_{m,n}$ characterizes the space of entanglements.

3. Flag manifold has a geometric interpretation as the space of n-D spaces C^n (flags) embedded in C^M. Interestingly,twistor spaces and more general spaces of twistor Grassmannian approach are flag manifolds and twistor spaces are also related to Minkowski space.

4. I have not been personally enthusiastic about the notion of emergence of 3-space or space-time from entanglement but one can wonder whether flag manifolds related naturally to entanglement could lead to the emergence of Minkowski space. Or perhaps better, whether the notion of entanglement and Minkowski space could be natural aspects of a more general description.

5. One can also have flags inside flags inside leading to more complex flag manifolds $F(n_1, n_2, .., n_k = M) = U(M)/U(m_1) \times ... \times U(m_k)$, $m_k = n_k - n_{k-1}$ assuming $n_0 = 0$. In consciousness theories, the challenge is to understand the quantum correlates of attention. Entanglement is the most obvious candidate in this respect. Attention seems to be something with a directed arrow. This is difficult to understand in terms of the ordinary entanglement. Flag hierarchy would suggest a hierarchical structure of entanglement in which the system entangles with a higher-D system, which entangles with a higher-D system. In this picture the state function reduction would be replaced by a cascade starting from the top.

6. The analog of flags inside flags is what happens in what I call number theoretic measurement cascades for wavefunctions [L47] in the Galois groups which are associated with extension of extensions of..... The already mentioned cognitive measurement cascade corresponds to a hierarchy of normal subgroups of Galois group and one can perhaps say that discret Galois group replaces the unitary group. Each normal subgroup in the hierarchy is the Galois group of the extension of the extension below it. This automatically realizes the hierarchical entanglement as an attentional hierarchy. The cognitive measurement cascade can actually start at any level of the hierarchy of extensions of extensions and if it starts from the top all factors are reduced to a pure state.

   If the polynomials defining the 4-surfaces in M^8 satisfy $P(0) = 0$, the the composite polynomial $P_n \circ P_{n-1} ... \circ P_1$ has the roots of $P_1, ..., P_{n-1}$ as its roots. In this case the inclusion of state spaces are unique so that flag manifolds are not needed.

6.5 Negentropy Maximization Principle

Negentropy Maximization Principle (NMP) [L60] is the basic variational principle of TGD based quantum measurement theory giving rise to a theory of consciousness.

1. The adelic entanglement entropy is the sum of the real entanglement entropy and p-adic entropies. The adelic negentropy is its negative.

   The real part of adelic entropy is non-negative but p-adic negentropies can be positive. The sum of p-adic negentropies can be larger than the real entropy for non-trivial extensions of rationals. NMP is expected to take care that this is indeed the case. Second law for the real entropy would still hold true and guarantee NMP.

2. NMP states that SFRs cannot reduce the *overall* entanglement entropy although this can happen to subsystems. In SFRs this local reduction of negentropy would happen. Entanglement is not destroyed in SFRs in general and new entanglement negentropy can be generated.

3. Although real entanglement entropy tends to increase, the positive p-adic negentropies assignable to the cognition would do the same so that net negentropy would increase. This would not mean only entanglement protection, but entanglement generation and cognitive evolution. This picture is consistent with the paradoxical proposal of Jeremy England [I3] [L9] that biological evolution involves an increase of entropy.

4. It should be noticed that the increase of real entanglement entropy as such does not imply the second law. The reduction of real entropy transforms it to ensemble entropy since the outcome of the measurement is random. This entropy is entropy of fermions at space-time sheets. The fermionic entanglement would be reduced but transformed to Galois entanglement.

7 Some notions of condensed matter physics from the TGD point of view

Before continuing I must emphasize that I am not a condensed matter physicist and have no practical experience about experimental physics. Therefore I cannot propose any experimental protocols. I dare to hope that the new vision about space-time and quantum theory could inspire people who are doing real condensed matter physics.

7.1 The notion of Brillouin zone from the TGD viewpoint

In condensed matter physics the notions of lattice, reciprocal lattice, unit cell and Brillouin zone at its counterpart in reciprocal lattices are central notions.

The reciprocal lattice in momentum space is the dual of the lattice in 3-space. This follows automatically from the periodicity of properties of wave functions in the lattice : they force wave vectors to be in the reciprocal lattice. The diffraction amplitude has peaks at the photon momenta in the reciprocal lattice.

M^8-H duality can be seen as the counterpart of position-momentum duality. Therefore it is interesting to look at these notions from the point of view of M^8_H duality. Recall that 4-surfaces in $H=M^4\times CP_2$ is identified as space-time whereas the 4-surface in $M^8=M^4\times E^4$ is analogous to momentum space with slicing induced by the mass shells (hyperboloids) of M^4. In H the corresponding slicing is by CDs inside CDs with size given by the Compton length associated with mass m.

1. At the level of H, periodic minimal surfaces would nicely produce lattice-like structures and the momenta associated with the peaks of Fourier transforms would belong to the reciprocal lattice. I have considered the construction of also more general structures in [L71].

2. At the level of M^8, the allowed momenta as points of $X^4\subset M^8$ belong to cognitive representations: the momentum components are algebraic integers in the extension defined by the polynomial defined the 4-surface in M^8. This guarantees the theoretical universality of the adelic physics [L20, L21]) so that the points make sense also as points of the p-adic variants of space-time surface defining geometric correlates of cognition.

   Lattice-like structures are naturally associated with the lattice of algebraic integers and one obtains a hierarchy of lattices. The lattices can be seen as products of ordinary lattices in E^3 and lattices in the extension of rationals having dimension n: this feature is completely new.

7.1.1 Construction of bound states

Number theoretic vision suggests a universal way to construct bound states as Galois confined states. This would mean that many quark states in M^8 consisting of points of cognitive representation carrying quark are Galois singlets. In the case of momentum degrees of freedom this would mean that the total momentum is (rational) integer.

The physical motivation for Galois confinement is that periodic boundary conditions require integer value 4-momenta which are rational integers using a suitable momentum unit determined by the size scale of CD (Compton length $\hbar_{eff}/m$ for some particle would be in question for $\hbar_{eff}=\hbar_{gr}=GMm/v_0$ the gravitational Compton length $\Lambda_{gr}=GM/v_0=r_s(M)/2v_0$ would not depend at all on mass of the particle.

1. The condition that the total 4-momentum is integer-valued poses a strong condition on the bound states.

2. Second condition is that the inner products of the momenta (algebraic integers which can have an imaginary part) defining number theoretical metric are real valued. This poses strong quantization conditions, and one obtains also lattice structures in the lattice defined by the unit vectors of extension and by 3-space. These lattice structures are sublattices of lattice E^3, whose points are n-D number theoretical lattices defined by the unit vectors of the extension of rationals.

3. The fundamental entities are quarks and the construction gives a hierarchy of increasingly complex bound states of them. One obtains also atoms and their lattices. Quasi-crystals are obtained as cut and project construction and it is feasible that number theoretical lattices makes them possible also now.

4. The lattices in M^8 involving particles with the same mass are actually lattices in 3-D hyperbolic space and called tessellations. In good approximation they are lattices in E^3 since H^3 can be approximated by E^3 below length scale given by h_{eff}/m which is Λ_{gr} for $\hbar_{gr}$ (.9 cm for Earth and of the order of radius of Earth for Sun).

   The structure of tessellations is extremely rich and perhaps the simplest tessellations known as icosa-tetrahedral tessellations involve all basic Platonic solids and are proposed to give rise to universal realization of genetic code having chemical realization only as a special case and having besides DNA also higher dimensional realizations [L58].

   M^8 picture allows also universal 6-D brane-like solutions with a topology of 6-sphere, whose projection to CD is its intersection with 3-D hyperplane E^3 of constant energy. This plane would allow many quarks states with an ordinary lattice structure. There both hyperbolic tessellations and Euclidian lattices would be allowed.

5. Even the lattice formed by atoms would be a bound state of this kind. The reciprocal lattice in M^8 has an interpretation in terms of cognitive representation in M^8 mapped to H by M^8-H duality defined by particle momenta, which are basically bound states of quarks (also leptons).

7.1.2 M^8-H duality and the relation between lattices and reciprocal lattices

M^8 and H descriptions are related by M^8-H duality as an analog for momentum-position duality. Uncertainty Principle (UP) must be respected but what does this really require is not quite clear. The map of $X^4 \subset M^8$ to $X^4 \subset H$ is certainly involved. This would be the M^8-H duality for space-time surfaces. This description is not enough: M^8-H duality is required also at the level of "world of classical worlds" (WCW).

1. M^8-H duality at the level of 4-surfaces

Consider first the M^8-H duality at space-time level.

1. Uncertainty Principle (UP) is the basic constraint on M^8-H duality and fixes the form of M^8-H duality at the space-time level.

   One takes the momentum projection p in M^4 - an algebraic integer for cognitive representations and quarks are at these points, not all - and maps it to a point of $M^4 \subset M^4 \times CP_2$ that is to a point of $X^4 \subset H$. One assigns to p a geodesic line in the direction of momentum beginning at the common center of all CDs. In this way the slicing by mass shells of $M^4 \subset M^8$ is mapped to a slicing by CDs inside CDs (Russian doll-like structure).

2. p is mapped to the intersection of this geodesic line with the boundary of CD. One obtains the analog of the pattern produced by diffraction from the lattice. In particular, the intersections of the geodesics with the $t=T$ plane above the center point of CD form a reciprocal lattice, whose projection to the 2-D surface of a large 2-sphere corresponds to the standard diffraction pattern. One would be happy if one would obtain a lattice, rather than its reciprocal.

   As if there were a lattice around the center of the ball producing the diffraction pattern as a projection of the reciprocal lattice to the heavenly sphere. Intuition would suggest that this must be the case but one must be very cautious.

3. The momenta of quarks (or atoms) are therefore mapped to the light-cone boundaries of CD and basically define boundary values for the induced quark fields for quarks composing both proton, nuclei, and even electrons. These fields would be localized at these points at the boundary of CD and disperse in the interior. Induced spinor fields are second quantized H-spinor fields restricted to space-time surface and obeying modified Dirac equation for induced geometry and determined by variational principle.

   One can assign to the points at the boundary of CD corresponding to the image of the reciprocal lattice localized states of atoms of the lattice (many-quark states). At quark level this corresponds to a superposition of spinor harmonics of H localized to the point of the boundary (this corresponds to so-called light-cone quantization). This would dictate the time evolution of the induced spinor field inside the space-time surface and it would reflect the data coding for the reciprocal lattice.

4. Does this mean the emergence of lattice (as desired) or of reciprocal lattice in the interior? Since the lattice points by definition would correspond to peaks of plane waves generated by the reciprocal lattice at the boundary of CD would expect that the peak positions define the lattice.

One can also wonder whether one could one define M^8-H duality so that it would take momentum lattice in M^8 to its dual in H? The notion of dual lattice makes sense for the lattice defined by the extension. If one defines the cognitive representation in M^8 by selecting a tessellation at the mass shell of M^8 (this might follow the conditions for bound states), one could map the momenta of tessellations to their duals and would obtain the desired result in H. It is however not clear whether the map of tessellation to its dual (if it exists) can be completed to a continuous map of H^3 to itself.

2. M^8-H duality at the level of WCW

It seems that the proposed description need not be enough to realize UP at the level of H and the "world of classical worlds" (WCW) . The objection is that localized states in M^8 correspond to delocalized states at the level of H.

The above description maps quarks at points of $X^4 \subset M^8$ to states of induced spinor field localized at the 3-D boundaries of CD but necessarily delocalized into the interior of the space-time surface $X^4 \subset H$. This is analogous to a dispersion of a wave packet. One would obtain a wave picture in the interior and the lattice should emerge.

1. The basic observation leading to TGD is that in the TGD framework a particle as a point is replaced with a particle as a 3-surface, which by holography corresponds to 4-surface.

   Momentum eigenstate corresponds to a plane wave. Now planewave could correspond to a delocalized state of 3-surface associated with a particle in M^4 and by holography that of 4-surface.

2. A generalized plane wave would be a quantum superposition of shifted space-time surfaces with a phase factor determined by 4-momentum. This suggests that M^8-H duality should map the point of M^8 containing an object with momentum p to a generalized plane wave and this is assumed.

   This would also define WCW description. Recent physics relies on the assumption about single background space-time: WCW is effectively replaced with M^4 since 3-surface is replaced with point and CP_2 is forgotten so that one must introduce gauge fields and metric as primary field variables.

3. For cognitive representations, momenta are given by algebraic integers. Lattice plane waves can be idealized as waves in a discrete lattice. This would suggest that the plane wave is replaced by a discretized plane wave corresponding to the points of H at which the plane wave has the same value. One can say that one counts only the wave crests and thus only the information about wavelength and frequency.

4. For reciprocal momenta, one obtains a wave function in H for the shifted images of the 3-surface/4-surface labelled by a vector of the reciprocal lattice in H and this wave function can be regarded as a wave function with the periodicities of lattice.

The WCW picture is necessary if one wants to take into account WCW degrees of freedom. In the approximate description of phenomena involving only elementary particles constructible from quarks, WCW is not absolutely necessary.

7.1.3 Galois confinement and lattice like structures

It is interesting to look more explicitly at the conditions for the Galois confinement.

Single quark states have momenta, which are algebraic integers generated by so called integral basis (`https://cutt.ly/SRuZySX`) analogous to unit vectors of momentum lattice but for single component of momentum as vector in extension. There is a theorem stating that one can form the basis as powers of a single root. It is also known that irreducible monic polynomials have algebraic integers as roots.

1. In its minimal form Galois confinement states that only momenta, which are rational integers, are allowed by Galois confinement. Note that for irreducible polynomials with rational coefficients one does not obtain any rational roots. If one assumes that single particle states can have an arbitrary algebraic integer as a momentum, one also obtains rational integers for momentum values. These states are not at mass - or energy shell associated with the single particle momenta.

2. A stronger condition would be that also the inner products of the momenta involved are real so that one has $Re(p_i) \cdot Im(p_j) = 0$. For $i = j$ this gives a condition possible only for the real roots for the real polynomials defining the space-time surface.

To see that real roots are some facts about the realization of the co-associativity condition [L39] are necessary.

1. The expectation is that the vanishing condition for the real part (in a quaternionic sense) of the octonionic polynomial gives a co-associative surface. By the Lorentz symmetry one actually obtains as a solution a 6-D complex mass shell $m_c^2 \equiv m_{Re}^2 - m_{Im}^2 + 2iRe(p) \cdot Im(p) = r_1$, where the real and imaginary masses are defined are $m_{Re}^2 = Re(p)^2$ and $m_{Im}^2 = Im(p)^2$ and r_1 is some root for the odd part of the polynomial P assumed to determining the 4-surface.

2. This surface can be co-associative but would also be co-commutative. Maximally co-associative surface requires quaternionic normal space and the proposal is that the 6-surface having a structure of S^2 bundle defines as its base space quaternionic 4-surface. This space would correspond to a gauge choice selecting a point of S^2 at every point of M^4. To a given polynomial one could assign an entire family of 4-surfaces mapped to different space-time surfaces in H. A possible interpretation of gauge group would be as quaternionic automorphisms acting on the 2-sphere.

Concerning Galois confinement, the basic result is that for complex roots r_1 the conditions $Re(p_i) \cdot Im(p_i) = 0$ cannot be satisfied unless one requires that r_1 is real. Therefore the stronger option makes sense for real roots only.

Despite this one can also consider the strong option for real roots. There are two cases to consider. The first case corresponds to complex 4-surfaces for which complex mass squared is equal to a root of the odd part of the polynomial determining the space-time surface. The real part of these surfaces in the sense that the imaginary part of mass squared vanishes is 4-D.

These conditions lead to a spectrum of 4-momenta and masses with each mass involving a subset of momenta. One can form Galois singlets also from states with different masses.

1. One can assign to each algebraic integer n_A a Galois invariant defined as the determinant $det(N(n_A))$ of the matrix $N(n_A)$ of the linear transformation defined by a multiplication of the units of algebraic integers by n_A. The algebraic integers n_A with the same value of $det(N(n_A))$ can belong to the orbit of Galois group. Physical intuition suggests that the values of mass squared (energy) are the same for these integers in the case of H^3.

2. One expects that the group $SL(2, Z_A)$, where Z_A denotes algebraic integers associated with the polynomial defining the space-time surface produces new solutions from a given solution. This would be a discrete version of Lorentz invariance. Tessellations of H^3 are highly suggestive as bound states.

3. Since Galois group is finite, the only possibility is that Galois groups corresponds to a subgroup of rotations permuting algebraic integers with the same time-component of 4-momentum. Therefore the discrete subgroups of $SO(3)$ associated with the inclusions of hyper-finite factors of type II_1 would emerge.

The situation for the surfaces $E = E_n$, E_n the root of the polynomial P defined the 4-surface situation is different.

1. Single particle states correspond to a discrete set of in general complex mass values extending from E_n to 0. The number of momenta with given m is finite and one obtains a slicing of the space of 3-momenta by spheres $S^2(m)$ with constant mass having the allowed points of $S^2(m)$ at the orbits of Galois group. Also now single particle states are impossible but one obtains many-particle states and also lattice like structures are expected. A given mass m can correspond to several energies $E_n(m)$ giving this value of mass.

2. Also now it is possible to construct Galois singlets as many-particle states and these have rational integer valued momenta. In condensed matter, one has energy bands such that the energy inside the band depends on the momentum k. Could one think that the values of energy form bands decomposing to discrete energy levels?

Two further remarks are in order.

1. Besides the simplest realization also a higher level realization is possible: Galois singlets are not realized in the space of momenta but in the space of wavefunctions of momenta. States of an electron in an atom serve as an analogy. Origin is invariant under the rotation group and electron at origin would be the classical analog of a rotationally invariant state. In quantum theory, this state is replaced with an s-wave invariant under rotations although its argument is not.

   In the recent situation, one would have a wave function in the space of algebraic integers representing momenta, which are not Galois invariants but if one has Galois singlet, the average momentum as Galois invariant is ordinary integer. Also single-quark states could be Galois invariant in this sense.

2. The proposal inspired by TGD inspired quantum biology is that the polynomials defining 4-surface in M^8 vanish at origin: $P(0) = 0$. One can form increasingly complex 4-surfaces in M^8 by forming composite polynomials $P_n \circ P_{n-1} \circ ... \circ P_1$ and these polynomials have roots of P_1....and P_{n-1} as their roots. These roots are like conserved genes: also the momentum spectra of Galois singlets are analogous to conserved genes. This construction applies to Galois singlets in both classical and quantal sense.

   At the highest level one can construct states as singlets under the entire Galois group. One can use non-singlets of previous level as building bricks of these singlets.

7.1.4 About the analogs of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

1. Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L81] define a unique discretization of 4-surface in M^4 and, by M^8-H duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L81, L77] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L58]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3)\setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

   The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

   Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and M^8-H duality in terms of associativity [L39, L40]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

2. De Sitter manifolds as tachyonic analogs of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ. (`https://cutt.ly/XVsdLwY`).

3. Do complexified hyperbolic manifolds as analogs of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

   Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T = 1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H^3_c with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogs of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogs of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H^3_c would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H^3_c is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$ contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H^3_c could be such in a complexified sense.

7.2 Topological condensed matter physics and TGD

Topological considerations have become an essential part of condensed matter physics. In condensed matter physics the topology of patterns of order parameters and of Fermi surface play a key role. In the TGD framework the topology of space-time surface in X^4 and the dual 4-surface in M^8 having an interpretation as an analog of momentum space are non-trivial and the question how this could reflect itself in condensed matter physics.

7.2.1 Topology of the energy bands in solids

The notions of 2-D face states, edge states, and corner states seem to be behind many topological states. It is interesting to see what they could correspond to in the TGD framework.

One can imagine two alternative guesses.

1. At H level 4-surfaces as analogous to 4-D complexified momentum space are algebraic surfaces, that is 4-D "roots" of polynomials.nThese algebraic surfaces have singularities at the level of H mapped to singularities at the level of H. They can have corners, edges, and

intersection points, 2-D singular surfaces. At the level of H they correspond to strings, string world sheets, and light-like orbits of partonic 2-surfaces: in this case the line singularity is blown up to a 3-D singularity.

2. These singularities need not however correspond as such to the above listed singularities since the active points of cognitive representation defined by mometa which are algebraic integers do not correspond as such to the physical states. Rather, physical states are Galois confined bound states of quarks for a given extension of rationals and it is the energy and momentum spectrum of these states which is relevant.

The second guess is based on the idea that the energy bands correspond to substructures formed by discrete 4-momenta of Galois confined states.

1. Cognitive representation consists of momenta for which momentum components are algebraic integers. Some of these points are occupied by quarks, they are "active" (this brings in mind Bohm's notion of active information).

   Physical states must have total momentum which is rational integer using the unit defined by the largest CD involved defining IR cutoff. Smallest CD defines the UV cutoff. This means Galois confinement in momentum degrees of freedom. Same happens also in spinorial degrees of freedom.

2. Bloch waves are of the form $exp(ikx)u(x)$ where u is a periodic function with the periods of lattices and k is continuous pseudo-momentum. k can be restricted to the first Brillouin zone defined as the counterpart of a lattice cell in momentum space. For Bloch states the translational symmetry is broken down to a discrete subgroup of the translation group acting as symmetries of lattices and therefore of u.

   For Bloch waves, the wave vectors and also energies would be quantized by periodic boundary conditions which would mean in the TGD framework that the momenta are integer valued using a suitable unit. The phase factors $epx(iknL)$ would be roots of unity and therefore number theoretically universal. This requires that $kL = m$ is a rational integer.

3. Mass shells as hyperboloids $H^3(m)$ are of special interest as are also the 3-D M^4 projections of 6-D universal brane-like entities. The latter are 3-surfaces $E = E_n$ where E_n is the root of the polynomial defining the 4-surface in M^8. Hyperboloid allows tessellations and the Euclidean 3-space E_3 defined $E = E_n$ surfaces inside light-cone allows lattices expected to emerge naturally from Galois confinement.

4. This picture suggests that each $E = E_n$ shell gives rise to real energy shells with rational integer valued energy and momentum components as sums of the multiples of algebraic integers for quarks. The allowed momenta for given total energy would correspond to states assignable to a given total energy analogous to a given $E = constant$ 2-surface of an energy band. The singular topologies could correspond to intersections or touchings of these bands.

One cannot exclude the possibility that the states with quarks with momenta at the singular pieces of 4-surfaces (touching along 0,1, or 2-D surface) could correspond to these singularities. For instance, the touching of two energy bands could correspond to this kind of singularity.

The article of Carpentier [D4] gives a nice introduction to the topology of bands in solids and it is interesting to see the situation from the TGD point of view. Topological insulators, semimetals, so called Majorana fermions, etc. involve singular situations in which energy bands touch each other and the question is what this means at the level of M^8.

Can one have a situation in which different energy bands touch each other at a single point or possibly along 1-D or 2-D (discrete) surfaces? The discussion is very similar for mass shells $H^3(m)$ and energy bands $E^3(E_n)$ so that only the case of E^3 is discussed.

1. Consider first energy bands E_n. For a given mass m, one obtains a set of energies E_n corresponding to the roots of P. When two roots co-incide, entire energy bands coincide. This would be however the situation for single quark states which are not possible by Galois confinement for irreducible polynomials with rational coefficients.

2. Two Galois confined states belonging to different energy bands E_n have energies, which are sums of the integer combinations of rational parts of energies E_n of single particle states. These sums are identical for some states associated with E_n and E_n.

   One can imagine that these bound states energies are the same for two different values of E_n so that bands formed by bound states can touch. Even higher-dimensional intersections can be considered. Similar situation might occur for the Galois confined states associated with different mass hyperboloids.

3. In condensed matter situation momenta are defined only modulo the addition of lattice momentum, which is multiple of $\hbar_{eff}/a = N\hbar_{eff}/L$ where a and L are UV and IR length scale cutoffs defined by the smallest and largest CDs. This condition would loosen the conditions for touching.

7.2.2 Topological insulators in the TGD framework

There is a nice summary by Suichi Murakami about topological insulators [D27] helpful for a newcomer to the field.

Let us summarize the basic physical properties of spin waves.

1. Topological insulator is an insulator in the bulk and therefore has a gap between valence and conduction bands. TIs have conducting surface states, which can be edge states for 2-D TIs and surface states for 3-D TIs (Dirac cone in momentum space). The edge/surface states correspond to edges/surfaces in x-space. Fig. 1 of [D27] `https://cutt.ly/yRGDV1U`) provides an illustration of edge and surface states. As Fig.3 associated with a simple model for surface states illustrates, edge and surface states have a finite penetration depth to the bulk.

   For 2-D TIs, valence and conduction bands touch in 1-D k-space (see Fig. 2 of `https://cutt.ly/yRGDV1U`), which also illustrates the Dirac cone). The states with degenerate energies correspond to pairs of electrons with opposite spins and momenta related by the condition $k_1 = -k_2$ modulo lattice momentum. The electrons at opposite edges/surfaces move in opposite directions and have opposite spins. The net charge current vanishes but there is net spin current.

2. Spin orbit coupling is present. Orbital momentum is mathematically like magnetic field B effectively replaced with angular momentum L. The analog of torque for B is replaced with torque $s \times L$. This gives rise to counter propagating opposite spins and spin currents.

3. For TIs, T is not violated but PT and P are violated. The presence of magnetic fields breaking T thus destroys the edge/surface conductivity. The states are helical and have no definite parity since P changes the helicity. Superposition of states with opposite momenta and spins occurs so that spin current is formed. By the absence of magnetic field back scattering destroying the conductivity is not possible since this would require change of both spin direction and momentum direction.

   Spin orbit $(L \cdot S)$ interaction is required for the formation of spin currents. L comes from the rotational motion of electrons along the surface or edge; it tends to turn L and S in the same direction so that spin waves emerge.

4. Z_2 topological quantum number Z is conserved and reflects time reflection invariance. Z can be understood from the graph of energy at the conduction band, which has suffered splitting due to the spin orbit interaction so that energy is reduced in the conduction band. The graph of energy has two topologically non-equivalent forms. The graph either connects valence and conduction bands or not. In the latter case one has an ordinary insulator (I). In the first case one has TI.

   For I, the graph has 2 or 0 intersections with the graph for the lower energy of the spin-slit state. TI has only one intersection. Perturbations invariant under time reversal do not affect the situation. More general formulation for the Z_2 invariance is in terms of the odd/even character of intersections.

Could TGD add something interesting to the notion of TI?

1. Mathematically the spin-orbit interaction is analogous to that between magnetic moment and magnetic field except that it couples orbital motion and spin and forces the correlation between spin direction and momentum and therefore the formation of a spin wave. Magnetic field does not cause this although it would parallelize spins with itself.

2. In Quantum hydrodynamics (QHD) according to TGD [L66], the circular orbital motion could be accompanied by a Kähler magnetic field in the direction of angular motion possibly assignable to a monopole flux tube.

   Could this make sense now? There would be 2 magnetic fields of opposite direction associated with the two directions of rotation of electrons. They should reside at different space-time sheets. At QFT limit the net B would vanish but make itself visible as a spin current. The effect could be therefore seen as evidence in favour of many-sheeted space-time.

3. One can also consider variants of this picture. Kähler magnetic field at flux tubes would be an essential element. This can come from both M^4 and CP_2 and one can ask whether only M^4 contribution is present. Velocity of current flow would be proportional to Kaehler gauge potential which would be of opposite sign a 2 space-time sheets. This would not break T at the QFT limit.

   Note that neutrinos would experience this contribution and this provides an experimental test: could the strange behavior of solar solar neutrinos and also in laboratory be understood in terms of M^4 Kähler field in Sun or in laboratory?

7.2.3 Discrete symmetries at the level of M^8

Discrete symmetries T, PT, and CP and their violations are closely involved with the phenomena of topological condensed matter physics. The challenge is to understand T, PT, and CP violations at the level of M^8.

The definition of discrete symmetries in $H = M^4 \times CP_2$ was discussed already in my thesis [K1] [L3] about TGD. In particular, geometrically C corresponds to a complex conjugation in CP_2. At the level of M^8, these discrete symmetries should allow a realization as symmetries of the polynomials defining the space-time surface.

P changes the direction of 3-momenta. The counterpart of the Fermi surface should therefore become reflection asymmetric in the violation of P. The reflections are with respect to the middle point of the CD. T changes the sign of energy and half cones of CD in H and mass shells with opposite sign in M^8 are permuted. Also the time reversed classical time evolutions are different if T is violated. One can ask whether the violation of P implies a compensating violation of T (by CPT)?

Both M^4 and CP_2 contributions to Kähler magnetic field could induce T violation and M^4 contribution could do this in long scales. If T violation takes place at the fundamental level, topological instanton term which is divergence of axial current appearing in Kähler action could induce it. The analogs of instantons induce a violation of the conservation of monopole charge. This is possible only if the M^4 projection of the space-time surface is 4-dimensional. Analogous statement applies in the case of CP_2 and CP_2 type extremals have indeed 4-D CP_2 projection.

C involves a complex conjugation and changes the signs of charges. What does this mean in M^8? The normal spaces of 4-surface in M^8 containing a preferred complex plane or having integrable distribution of them are labelled by CP_2 coordinates. They are mapped to their complex conjugates.

What happens to the polynomial defining the space-time surface? Polynomial itself is real and cannot change but its algebraic continuation to an octonionic polynomial can be different. Indeed, real function can be algebraically continued to a complex function or its conjugate.

1. The complexified octonions involve a commutative imaginary unit i. Complex conjugation with respect to i leaving the real polynomial invariant but leading to a complex conjugate of the 4-surface looks like a reasonable first guess. One can however argue that the conjugation with respect to i is associated with T.

Recall, that the proposal [L39] that co-associative 4-surfaces in M^8_c, having an interpretation as an analog of momentum space, correspond to 4-surfaces identifiable as roots of complexified octonionic polynomials yielded a cold shower. Due to Lorentz symmetry, naive counting of dimensions fails and one obtains 2 polynomial equations with complexified mass as argument stating that the mass squared is a complex root of the polynomial. The solutions correspond to common roots and are 6-D.

The solution of the problem would be that 4-surface is the intersection of 6-surface and its complex conjugate with respect to the commuting imaginary unit i. The common root must be real but the points in the intersection can be complex. Hence the action of T on X^4 is in general non-trivial and a spontaneous violation of T is possible at momentum space level.

2. Also octonions allow conjugation. In M^4 sector conjugations for octonionic units this would give rise to P and T. In the complement E^4 the conjugations for 2-D subspaces are also possible.

   Could C relate to the commutative normal spaces of 6-D surfaces labelled by points of the CP_2 twistor space $SU(3)/U(1)\times U(1)$. Could the complex conjugation in the 2-D $U(1)\times U(1)$ fiber of this space, correspond to C. The complex conjugation would therefore act on the (integrable distribution of) 2-D normal spaces of these 6-D surfaces and would not act in $M^4 \subset M^8$.

3. At the level of H, C and P are violated for the Dirac equation for a fixed H-chirality of quarks spinors and also for the modified Dirac equation, which corresponds to the octonionic Dirac equation in M^8. Also CP is violated for the modified Dirac equation in H if the action contains topological Kähler instanton terms. This violation should have a counterpart for the octonionic Dirac equation. Since this equation selects a single point at 4-surface, the CP violation for the 4-surface could induce CP violation.

7.2.4 Instantons in the TGD framework

Instantons induce violations of CP and therefore of T in gauge theories such as QCD.

It is interesting to consider the interpretation of Q as an instanton number.

1. Montonen-Olive duality (`https://cutt.ly/HE6gMX6`) is associated with a gauge theory in which magnetic and electric charges are rotated so that the coefficient of YM action in the action exponential is replaced with the quantity $\tau = \theta/2\pi + 4\pi i/g^2$.

2. τ is invariant under modular transformations $SL(2,Z)$ generated by a shift $\tau \to \tau+1$ and $\tau \to 1/\tau$. The inversion symmetry has strong implications for the understanding of the strong coupling phases of quantum field theories, in which magnetic monopoles replace particles as elementary objects.

3. In the gauge theory θ is analogous to momentum. The vacuum state is plane-wave like superposition $\sum_N exp(iN\theta/2pi))|N\rangle$ of vacuum states differing by a topologically non-trivial gauge transformation as a map $S^3 \to G$. Note that ball B^3 is effectively S^3 if the gauge transformations are trivial at its boundary. The homotopy equivalence classes of gauge transformations are labelled by the winding number N. N characterizes instantons changing the magnetic charge by N units so that the ground state is a superposition of states with varying values of N transforming by a phase factor under a topologically non-trivial gauge phase transformation.

Consider now the situation in the TGD framework.

1. There are differences between TGD and gauge theory context. Gauge group is replaced with $U(1)$ having a trivial third homotopy group.

   Could a localized version of the quaternionic automorphism group $SO(3)$ serve as a counterpart of a gauge group. The surfaces in M^8 can be indeed thought of as maps from M^4 to the quaternionic automorphism group G_2.

2. The non-trivial gauge transformations - $U(1)$ instantons - are clearly possible. The non-trivial gauge transformation could correspond to a tological non-trivial gauge transformation $A+nd\phi$, where ϕ is angle coordinate around axis going through a line singularity as a puncture in 3-space associated with the time-like line connecting the tips of CD. Note however that color gauge action reduces to the Kähler action so that both interpretations might make sense.

3. Kähler action generalizes to

$$S_K = \frac{1}{\alpha_K}\int J\wedge *J\sqrt{g} - \frac{(}{i}\theta/2\pi)\int J\wedge J\sqrt{g} \ . \tag{7.1}$$

   Since only the exponent of S_K matters in the vacuum functional, I contributes a non-trivial phase factor to the Kähler function only for $exp(i\theta/2\pi)\neq 1$ ($\theta\neq n2\pi$). One can assign θ to both M^4 and CP_2 parts of Kähler action. The value of instanton term characterizes the non-conservation of the axial (monopole) current having instanton term as divergence.

   If one assumes self-duality of the gauge field true for instantons interpreted as gauge fields in S^4, the action reduces to ordinary Kähler action with coefficient proportional to τ. Interestingly, the quaternionic projective space M^4/Q can be regarded as S^4 so that Hamilton-Jacobi structures of M^4 proposed to serve as moduli space for the self-dual Kähler fields in M^4 could appear naturally.

4. $I(CP_2)$ is non-trivial due to the non-trivial homology of CP_2. $I(CP_2)$ gives a 3-D contribution, which appears at the boundaries between Minkowskian and Euclidean regions of the space-time surface as a topological Chern-Simons term and affecting the boundary conditions at the light-like orbits of partonic 2-surfaces in this way. These boundaries have interpretations as light-like parton orbits carrying quarks lines.

5. If CD contains a time-like "hole" along the axis connecting the tips of CD, also $I(M^4)$ is non-trivial. One can imagine extremals for which a genuine hole in the metric sense is generated along the M^4 time axis. What is required is that the induced metric using M^4 coordinates is of the form $dt^2-dr^2-(r^2+r_0^2)d\Omega^2$. These holes should correspond to "blow-ups" of singularities of the algebraic surface in M^8. Now the 3-D tangent spaces would have no special direction at the singular points. For CP_2 type extremals the same would hold true at the level of M^8. Could this "hole" be the TGD counterpart of the blackhole of GRT and could it serve as a signature of CD?

6. J^J is non-vanishing only if the M^4 *resp.* CP_2 projection is 4-D. This does not guarantee self-duality unless also the induced metric reduces to the metric of M^4 *resp.* CP_2. This is true for the canonical embedding of M^4 and for CP_2 type extremals having light-like M^4 projection. Self-duality is true for the Kähler forms of M^4 and CP_2 but not for the induced Kähler forms $J(M^4)$ and $J(CP_2)$. Therefore classical gravitation breaks the self-duality and Montonen-Olive duality in the TGD framework. The possibility of extremals with M^4 and CP_2 projections smaller than $D=4$ implies that θ is effectively vanishing for them.

$\theta(M^4)$ and $\theta(CP_2)$ as fundamental parameters obeying number theoretical coupling constant evolution would imply a violation of CP symmetry in both M^4 and CP_2 sector. Are the instanton terms present at the fundamental level or are they present only at the QFT limit and induced as a description of spontaneous violation of CP and T? Indeed, as in the condensed matter systems, CP violation could be caused by the magnetic part of the generalized Kähler action even without instanton term.

1. The strong CP problem of QCD is due to instanton inducing an instanton term in effective color YM action. The parameter characterizing the violations should be very small.

   In the TGD framework, a proposal for a solution of this problem could be that the counterpart of the color gauge field does not allow instantons. Here one must be cautious however. The components of the proposed classical color gauge field are proportional to the products of

Hamiltonians of color isometries and Kähler form and instanton terms for the induced Kähler form would induce a CP violation. Indeed, Kähler action can be also regarded as a color gauge action and therefore instanton term makes sense for it.

2. Could $\theta(M^4)$ and $\theta(CP_2)$ induce a CP violation consistent with the observed CP violation in hadron physics or does one encounter the strong CP problem also in the TGD framework?

   If hadrons are string-like objects, they correspond to flux tubes as deformations of strings. For deformations with dimension $D < 4$, instanton term vanishes. Could this be the reason for the small violation of CP at the level of M^4? For CP_2 type extremals, $I(CP_2)$ is non-vanishing but equal to the Kähler action and non-dynamical for the basic CP_2 type extremals since dynamics in in M^4 degrees of freedom with CP_2 taking the role of arena of physics. Could these effects make the hadronic CP violation small?

3. Matter-antimatter asymmetry is a CP violation, which does not look small at all. If the mechanism is actually a small CP violation implying that rate for the condensation of antiquarks to leptons is slightly larger than that for the condensation of quarks to antileptons, the matter antimatter symmetry could emerge during a very early period of the cosmic evolution when leptons were formed.

4. There are also further questions. Could the QCD instantons have TGD counterparts as Hamilton-Jacobi structures and also as analogs of S^4 instantons in the quaternionic projective space of octonions which would be 4-D mass hyperboloid H^4 as Minkowski analog of S^4 but with space-like signature. Could the parameter θ in the instanton term of Kähler action induce the formation of the ground state (θ vacuum) as a superposition of space-time surfaces with various instanton numbers in the sector of WCW consting space-time surface with 4-D M^4 and/or CP_2 projection?

7.3 The new view about classical fields

The TGD view about classical gauge fields differs in many aspects from the Maxwellian and gauge theory view since the classical fields associated with the system define a geometric what I call its field body (magnetic body (MB)) is the term that I have used. MB can carry also electric fields very closely related to magnetic fields unless the corresponding space-time surface is static. MB consists of flux tubes and flux sheets.

There are 2 kinds of cosmic strings: with monopole flux (see **Fig. 15**) or without it. The simplest cases correspond to Y^2, which is either a homologically non-trivial or trivial geodesic sphere of CP_2.

This predicts two kinds of magnetic flux tubes and two kinds of magnetic and electric fields. This suggests a possible interpretation for the fields H, M, B appearing in Maxwell's theory as field H carrying monopole flux requiring no current as source, magnetization M as non-monopole part induced by H , and $B = H + M$ as their sum experienced by test particle in many-sheeted space-time. The same would apply to D, P and E. If this interpretation is correct, TGD would have been secretly present in Maxwell's theory from the beginning.

The proposal that MB serves as a seat for dark matter as $h_{eff} = nh_0$ phases is central in the TGD inspired theory of consciousness and living matter. MB would be the boss and receive sensory input from ordinary biomatter and control it. This would happen in terms of dark photons with frequencies in EEG range and also in other ranges. The energies would be in the visible and UV range assigned to biophotons to which the dark photons would transform.

Magnetic flux tubes could accompty quantum vortices appearing in various macroscopic quantum phases. Even the hydrodynamical vortices in macroscopic scales could correspond to quantum coherent magnetic flux tubes with a large value of h_{eff} acting as a master forcing the coherent dynamics or ordinary matter. In hydrodynamics the classical Z^0 magnetic field, which in situations allowing skyrmions, is proportional to the induced Kähler form, could be important. Large parity breaking effects would be the prediction.

Also the view about radiation fields changes. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow a superposition of modes with a single direction of massless momentum. The ordinary superposition of gauge potentials in gauge theory is replaced with union of space-time surfaces with common M^4 projection. The test particle experiences the sum of gauge potentials associated with various space-time sheets so that the gauge potentials effectively superpose. Ideal laser beam is a convenient analogy.

MEs are ideal for precisely targeted communications without dispersion and dissipation. MEs are soliton-like entities and one can ask whether MEs could provide a model for solitons or accompany solitons. TGD based model for nerve pulse involves Sine-Gordon solitons with large h_{eff} assigned to the cell membrane and dark Josephson radiation would have MEs as space-time correlate [K32, K16, K33].

MEs do not allow standing waves possible in Maxwell theory but a set theoretic union of parallel MEs can effectively give rise to standing waves. Lorentz transformations give rise to waves moving with arbitrary sub-luminal velocity. Even a superposition in which fields effectively sum up to zero but there is a non-vanishing energy density as sum of energy densities for the two MEs, is possible.

7.4 About quantum criticality in TGD

In TGD number theoretical vision about physics brings a new view about quantum criticality.

1. Quantum criticality is actually the basic assumption of TGD: the Kähler coupling strength α_K appearing in the classical action principle of TGD would be analogous to a critical temperature and have a discrete spectrum. This would make the theory unique. All space-time sheets are quantum critical but at QFT limit this is of course masked by the replacement of sheets with a single region of M^4 made curved.

2. At the number theoretical M^8 side there is no action principle. The universality of the dynamics could be seen as a manifestation of quantum criticality. Can α_K emerge at M^8 level somehow from scattering amplitudes in M^8 and have a number theoretical origin [L62].

   At the level of H coupling constants are visible only at the level of frames defining the space-time as an analog of soap film. The parts of the frame are images of singularities for the X^4 in M^8. The challenge is to understand how the singularities of the space-time surfaces determine α_K already at the level of M^8?

   p-adic thermodynamics for mass squared predicts a spectrum of temperatures with values coming as inverse integers [K22, K10]. Also this temperature quantization could be seen as a counterpart for the quantum criticality.

3. Quantum criticality involves long range correlations and the hierarchy of Planck constants characterizing them [K12, K13, K14, K15]. h_{eff} corresponds to a dimension of extension of rationals characterizing the space-time surfaces. At criticality there is quantum superposition of space-time surfaces with various values of h_{eff} corresponding to polynomials defining the X^4 and one value of h_{eff} is selected in state function reduction.

7.5 What infinite-volume limit could mean in TGD?

Infinite volume limit corresponds to both thermodynamic and QFT limit and should be understood in the TGD framework. The questions are what it means if the infinite volume limit is actually realized and whether this has practical consequences.

1. At the level of ZEO infinite volume limit means that the size of causal diamond (CD) as an analog of Nature given quantization volume becomes infinite. The scattering amplitudes coded by zero energy states conserve Poincare quantum numbers at this limit.

2. At the level of H the volume action vanishes since the p-adic length scale dependent cosmological constant $\Lambda \propto 1/L_p^2$ approaches zero at the limit when the p-adic length scale L_p characterizing the X^4 becomes infinitely large.

   If $\Lambda = 0$ phase is real, the action would reduce to mere Kähler action containing both M^4 contribution and CP_2. In this case, one would also have extremals of form $X^2 \times Y^2$ for which CP_2 projection if the Lagrangian manifold with vanishing induced Kähler form. These extremals receive a negative contribution to energy from M^2. Could the preferred extremal property exclude these solutions?

Remark: If the sign of M^4 Kähler action is changed, the electric contribution to energy is positive and magnetic contribution negative. For string- like objects this would guarantee positive contribution.

3. In the number theoretic picture infinite volume limit in H could mean that polynomials defining $X^4 \subset M^8$ mapped to H are replaced with analytic functions with rational coefficients.

   Polynomials are assumed to vanish at origin (this guarantees that roots are "inherited" in their functional composition) and so should also the analytic functions. The inverse 1/f is infinite at origin and does not belong to the set so that one does not have a function field. Since one has only multiplication, one can speak about functional primes as in the case of polynomials.

   One can ask whether they should satisfy conditions guaranteeing that they can be regarded as polynomials of infinite order. Could one speak about polynomials of infinite degree as the limit of functional composites of polynomials with finite degree. As a matter of fact, infinite Galois groups are profinite groups and this requires this kind of inverse limit definition [L56].

   A concrete example is provided by the iteration of a polynomial of finite degree [L56]. In this case the spectrum of roots contains a continuous part at the limit so that complex numbers as completion of rationals would emerge at the infinite volume limit much like the continuum spectrum of momenta emerges from a discrete spectrum.

7.6 The notions of geometric phase, Berry curvature, and fidelity in TGD?

Non-contractible ground state Berry phase in the loop over the parameter space is associated with QPTs and is associated Berry curvature defining non-trivial U(1) holonomy (`https://cutt.ly/RWy7Deq`) Geometric phase (`https://cutt.ly/6Wy7GIT`) is a more general notion. It can be associated with homotopically non-trivial loops. For homotopically trivial loop geometric phase is due to non-trivial holonomy manifesting itself as Berry curvature. The Aharonov-Bohm effect represents an example about non-trivial holonomy. Electrons pass along paths closing together a region containing a magnetic field, which vanishes at the paths. Berry phase can be associated with loops in the parameter space for the Hamiltonian modelling the system.

Fidelity [D31] `https://cutt.ly/VWy5sVj`) defines a metric in the space of parameter dependent quantum states. It could be induced from metric of the parameter space. The abrupt changes of fidelity serve as a signature of quantum criticality.

Is this possible at the level of WCW?

1. WCW is a Kähler manifold [K34, K19]. Finite-dimensional Kähler manifolds have a trivial homotopy group. Complex coordinates of WCW contributing to Kähler form and metric correspond to complex coordinates. In these degrees there should be no homotopically trivial loops so that topological phase is not possible. The curvature of the Kähler form can however have effects.

2. The remaining degrees of freedom are zero modes and define the analog of the base space in bundle theory. They appear as parameters - essentially classical background fields - in the Kähler metric and Kähler form. The topology in the zero modes can have non-trivial homotopy. Geometric phase could be assigned with homotopically trivial loops in the zero modes.

   At the infinite-volume limit the sub-WCW defined by the degenerate ground states with a Lagrangian manifold Y^2 as CP_2 projection (vanishing Kähler form and color gauge fields but non-vanishing weak gauge fields) is highly interesting. The preferred extremal property could exclude these space-time surfaces.

It seems that TGD could provide a unified description of all these exotic quantum coherent phases.

7.6.1 How the description in terms of Berry phase and fidelity could relate to TGD?

Consider first the identification of the TGD counterparts of Berry phase and fidelity.

1. In TGD the ground states are defined as space-time surfaces/3-surfaces and quantum states are their superpositions. The Kähler metric defines the analog of the quantum metric and the Kähler form corresponds to Berry curvature.

   The fidelity of two quantum states $\Psi(\lambda)$ and $\Psi(\lambda+\delta\lambda)$ is defined as the overlap $\langle\Psi(\lambda)\rangle\Psi(\lambda+\delta\lambda)\rangle$ in parameter space. The fidelity for nearby states is expected to change dramatically at singularity.

   Fidelity at the level of WCW - rather than WCW spinor fields representing quantum states - would mean disappearance of appearance of quantal WCW degrees of freedom as zero models transform to dynamical quantal degrees of freedom or vice versa. This change would make itself visible at the level of quantum states whose inner product depends on the WCW Kähler metric.

2. WCW also allows spinor connection with some gauge group acting as non-abelian holonomies. This corresponds to non-Abelian Berry phase Kac-Moody algebras of H isometries are an excellent candidate in this respect. WCW allows super-symplectic group as isometries.

3. WCW metric has also zero modes, which do not contribute to the WCW metric. Any symplectic invariant associated with X^4 defines such an invariant and the induced CP_2 Kähler form is invariant under the symplectic transformations of CP_2 and can be said to define a continuum of this kind of invariants. This could induce a geometric phase, which is not due to a holonomy but non-trivial homotopy.

   Kähler magnetic fluxes over 2-surfaces define such invariants. For closed surfaces these invariants reduce to quantized magnetic fluxes. Also M^4 Kähler form defines such invariants. At the boundary of CD the sphere S^2 (light-like radial coordinate =constant) has symplectic structure and also this defines solid angles assignable to 3-surfaces as seen from the tip of the CD as invariants.

7.6.2 Could the singularity of the quantum metric relate to number theoretical physics?

The singularity of the quantum metric would mean a reduction of the number of the dynamical quantum degrees of freedom contributing to the WCW metric meaning that the rank of the WCW metric tensor decreases. At criticality complex coordinates would transform to zero modes. Some complex coordinates of WCW would reduce to real coordinates. This would correspond to quantum criticality. In a concrete mechanical system some eigen modes would vanish and corresponding frequencies would become zero.

Since the TGD Universe is quantum critical and this is expected to be a generic phenomenon. Quantum criticality involves long range fluctuations which would correspond to large values of h_{eff} and therefore space-time surfaces which are algebraically complex. Could these long range fluctuations relate to almost zero modes with small frequencies and large wave lengths?

These phase transitions could be number theoretic. They would change the polynomial defining the X^4 (recall that quantum state is the superposition of space-time surfaces in ZEO). The dimension n for the extension of rationals is equal to the order of the Galois group and would change. Galois symmetries would act as zero mode symmetries. The dimensions of the representations of the Galois group in terms of quarks would also change. The change in the number of degrees of freedom would change the fidelity.

n defines also the algebraic dimension of the integers extended to algebraic integers for extension as a space regarded as a ring of integers. If algebraic integers can define components of the momenta, the dimension of the momentum space with integer components of momentum increases from 3 to $3n$ as the dimension of the Galois group increases by factor n. This increase occurs in the transitions in which the polynomial Q defining the space-time region is replaced with $P \circ Q$ such that P defines n-dimensional extension.

This would have rather dramatic effects since the radius of the Fermi ball with radius would be reduced by factor $1/n$ and could contain the same number of states as ordinary Fermi ball: this would mean an increase of density by factor n^3 corresponding to n sheets. Quasicrystal structure in both $X^4 \subset M^8$ and its images in $X^4 \subset H$ is also suggestive.

7.6.3 Does infinite volume limit have spin-glass type degeneracy?

One can look at the situation also at the infinite volume limit. At the infinite volume limit the action is expected to reduce to Kähler action. Whether this implies ground state degeneracy depends on whether preferred extremal property allows it.

1. In the original picture there was only CP_2 contribution to Kähler action. This implies huge vacuum degeneracy of CP_2 Kähler action. Any X^4 with CP_2 projection which is 2-D Lagrangian manifold is a vacuum extremal. WCW metric becomes singular if its inverse does not exist: this means singularity and the existence of zero modes. 4-D spin variant of glass degeneracy (`https://cutt.ly/0RuZfgu`) and classical non-determinism emerge. Classical non-determinism does however not look physically acceptable.

2. The twistor lift forces the Kähler action to have also an M^4 part obtained by analytical continuation from E^4. Does the resulting Kähler action have ground state degeneracy at infinite volume limit?

   The simplest extremals are of the form $X^4 = X^2 \times Y^2$, X^2 a minimal surface in M^4 and Y^2 a Lagrangian manifold in CP_2. Symplectic transformations in CP_2 degrees act like $U(1)$ gauge transformations on CP_2 Kähler gauge potential and do not affect either Kähler form nor the Lagrangian manifold property.

   Only the induced metric is affected so that the effects are purely gravitational. This gives rise to the ground state degeneracy. The area of CP_2 projection is not changed and the action is affected only by the change of the induced metric. Conserved quantities are modified only by gravitational effects and are non-vanishing. The extremals are deterministic and apart from gravitational effects one has a huge ground state degeneracy analogous to spin glass degeneracy.

   Apart from gravitation, the WCW Kähler metric receives contributions only from M^4 degrees of freedom, which are not affected under these deformations. Could one say that CP_2 degrees have transformed to zero modes?

3. One can also have surfaces $X^2 \times Y^2 \subset M^4 \times CP_2$ such that both X^2 and Y^2 are Lagrangian manifolds at infinite volume limit. These would be vacuum extremals. Preferred extremal property should exclude them. Could the interpretation be that all quantum degrees of freedom have transformed to zero modes?

4. One can invent objections against this proposal.

   (a) Negative energies might emerge from the electric energy in M^4 degrees of freedom. Electric field gives a negative contribution to energy density. Signature is Minkowskian for M^2 subset $M^2 \times E^2$. The M^2 part of Kähler form is obtained from its E^2 variant by multiplication with factor i. This might cause problems.

   (b) These surfaces are extremals but the preferred extremal property could fail since the needed 4-D analog of complex structure is missing since Y^2 as a Lagrangian manifold is not a complex surface of CP_2.

   (c) There is however also an argument in favor of this picture. Ordinary Maxwellian magnetic fields correspond to a homologically trivial geodesic sphere of CP_2 and they are Lagrangian submanifolds. Therefore one cannot exclude the proposal.

7.6.4 The parameters of the effective Hamiltonian from the TGD point of view

Could the parameters of effective Hamiltonians have counterparts at the level of WCW?

1. 4-surfaces as WCW points define parameters in the analogs of eigenvalues of observables. Both supersymplectic and Kac-Moody algebras have as parameters the parameters coding the point of WCW and Kac-Moody algebra. Number theoretic coding of ground states based on the Galois group as a symmetry group and p-adic primes defining p-adic length scale is what comes to mind.

   The preferred 4-surfaces would naturally correspond to the maxima of Kähler function. It is quite possible that Kähler coupling constant is complex so that the complex number defining the exponent of Kähler function has phase $\pm pi/2$. The phase of the exponent is different and maxima are also stationary points. This would make possible interference effects central in QFTs. This is implied by the condition that classical conserved charges are apart from a phase factor real and can therefore be made real.

   If M^8 space-time sheets are defined as "roots" of polynomials with rational coefficients [L39, L40], WCW becomes discrete and has the coefficients of polynomials as coordinates of a given point (X^4). An open question is why the maxima of Kähler function should correspond to rational polynomials with rational coefficients.

2. Super-symplectic transformations [K11, K34] as isometries of WCW are symmetries and can be regarded as a generalization of Kac-Moody type symmetries. The complex coordinate z and light-like radial coordinate r of the light-cone boundary are in the role of parameters. Analog of 3-D gauge group but gauge group replaced with the symplectic group of $S^2 \times CP_2$ is in question. The light-like orbits of partonic surfaces could naturally carry Kac-Moody algebra representations of isometries - at least at infinite volume limit.

   Non-negative conformal weights parameterize the representations of this algebra. The construction of states would be as follows. A sub-algebra $SCA_{n_{max}}$ with conformal weight larger than n_{max} and its commutator with the entire algebra annihilate states. Only the states with conformal weight smaller than n_{max} remain. Other degrees of freedom are effectively gauge degrees of freedom. n_{max} is expected to depend on the polynomial, its Galois group and degree. A huge reduction of degrees of freedom takes place. The remnant of the super-symplectic group would act as dynamical symmetries.

   Same could occur in the symplectic degrees of freedom labelled by Hamiltonians which are products of S^2 and CP_2 Hamiltonians. The only non-trivial normal subalgebra corresponds to isometries and states would be annihilated by the generators in the complement of this algebra.

   Rational coefficients of a polynomial defining the X^4 serve as the parameters characterizing the ground state. Higher level description is in terms of the Galois group which depends only weakly on the polynomial.

3. What about the description at the level of X^4? The solutions of modified Dirac action for induced spinor fields depend on the parameters characterizing the space-time surface.

7.7 Quantum hydrodynamics in TGD context

In the standard picture quantum hydrodynamics is obtained from the hydrodynamic interpretation of the Schrödinger equation. Bohm theory involves this interpretation. (`https://cutt.ly/cWy3O9Ts`).

1. Quantum hydrodynamics appears in TGD as an *exact* classical correlate of quantum theory [K4]. Modified Dirac equation forces as a consistency condition classical field equations for X^4. Actually, a TGD variant of the supersymmetry, which is very different from the standard SUSY, is in question.

2. TGD itself has the structure of hydrodynamics. Field equations for a single space-time sheet are conservation laws. Minimal surfaces as counterparts of massless fields emerge as solutions satisfying simultaneously analogs of Maxwell equations [L71]. Beltrami flow for classical Kähler field defines an integrable flow [L54]. There is no dissipation classically and this can be interpreted as a correlate for a quantum coherent phase.

3. Induced Kähler form J is the fundamental field variable. Classical em and Z^0 fields have it as a part. For $S^3 \subset CP_2$ em and Z^0 fields are proportional to J: which suggests large parity breaking effects. Hydrodynamic flow would naturally correspond to a generalized Beltrami flow and flow lines would integrate to a hydrodynamic flow.

4. The condition that Kähler magnetic field defines an integrable flow demands that one can define a coordinate along the flow line. This would suggest non-dissipating generalized Beltrami flows as a solution to the field equations and justifies the expectation that Einstein's equations are obtained at QFT limit.

5. If one assumes that a given conserved current defines an integrable flow, the current is a gradient. The strongest condition is that this is true for all conserved currents. The non-triviality of the first homotopy group could allow gradient flows at the fundamental level. The situation changes at the QFT limit.

6. Beltrami conditions make sense also for fermionic conserved currents as purely algebraic linear conditions stating that fermionic current is a gradient of some function bilear in oscillator operators. Whether they are actually implied by the classical Beltrami conditions, is an interesting question.

7. Minimal surfaces as analogs of solutions of massless field equations and their additional property of being extremals of Kähler action gives a very concrete connection with Maxwell's theory [L71].

7.8 Length scale hierarchies

The length scale hierarchy associated with the hierarchy of Planck constants and p-adic length scale hierarchy lead to the proposal that one has quantum coherence and supra phase always realized in some scale and the loss of say superconductivity means only the reduction of this scale.

Also dark variants of valence electrons make sense and there is evidence for them. When looking at the definition of say exciton, one cannot avoid the impression that something is missing. Electrons and holes are assumed to have incredibly small effective masses. The very notion of effective mass is in conflict with the idea that one has a fundamental quantum theory description.

One also introduces in the Schrödinger equation dielectric constant which comes from macroscopic description. Why doesn't one do the same in the case of ordinary atoms. This kind of mixing of phenomenological descriptions with a fundamental description is to me a deadly sin.

One cannot avoid the crazy looking question whether exciton could be a valence electron which is dark with $h_{eff} = k \times h$ and binds with an atom. It would be automatically accompanied by a hole. The binding energies would be scaled like $1/k^2$ and one would obtain the energies which can be 3 orders of magnitude smaller than those for hydrogen.

7.9 A general model of macroscopic quantum phases

7.9.1 Hierarchy of quantizations at the level of WCW

Before saying anything about macroscopic quantum phases, one must define what many-particle states correspond at the level of WCW.

1. The combination of UP with $M^8 - H$ duality leads to the view that many particle states at the level correspond to many-fermion (quarks actually) such that the momenta of quarks correspond to momenta as points of $X^4 \subset M^8$ with components, which are algebraic integers. In TGD framework, where all particles, also bosons, are composites of fermions. At M^8 level Cooper pairs would correspond to pairs of occupied points of a mass shell $H^3 \subset M^8$. The image of the region of momentum space in H corresponds for quarks of given mass m corresponds to a region at the boundary of sub-CD with size given by Compton length $L = \hbar_{eff}/m$.

2. At the level of WCW, the analog of the many-quark state associated with a given quark mass corresponds to the analog of plane wave inside a large $CD \subset H$ defined by the smallest mass

involve but with point-like particle replaces with space-time surface inside sub-CD (CD(m)) carrying zero energy state characterized by quark momenta at opposite boundaries of CD(m) having opposite sign of energy.

3. The entanglement between these states due to Fermi statistics is however maximal and SFRs are not possible. How can one construct entangled states. The answer is simple perform the analog of second quantization at the level of WCW. One can form the analogs of 2-particle states by taking two CDs with specified quark content and assign to both the analogs of plane waves. If the CDs correspond to different extensions of rationals so that the effective Planck constants are different, one can entangle these states in WCW degrees of freedom. One can construction N-particle states by using the same recipe.

4. To each many quark state one can assign odd or even boson number and regard this state as analog of elementary fermion or boson. This is what is indeed done quite generally. Could this operation have deeper meaning. Could one require that the many-quark operators indeed commute or anticommute mutually. This condition cannot hold true generally but could be posed as an additional condition to the physical states: the commutator/anticommutation would be proportional $h_{eff}I$, I identity matrix.

   This construction would be third quantization. And nothing prevents from performing also fourth quantization within even larger CD. This hierarchy of quantizations brings in mind the basic hierarchical structures of the TGD Universe: many-sheeted space-time characterized by p-adic and dark length scale hierarchies, and also the hierarchy of infinite primes which corresponds to a repeated second quantization of supersymmetric arithmetic QFT [K37] conjecture to correspond to the hierarchy of space-time sheets.

7.9.2 WCW description of BECs and their excitations as analogs of particles

Fermi statistics requires that the BEC correspond to a distribution of correlated momentum pairs with the sum of the momenta equal to the momentum of the boson. Cooper pairs also have binding energy so that the mass of the pairs is slightly smaller than the particle mass so that the Cooper pairs belong to different $H^3 \subset M^8$ than the free fermions.

For the excitations of BEC condensate giving rise to supracurrents and superflows, some momenta of fermions are different from the common momentum of BEC, usually larger than the common momentum of BEC. The image of excitation of BEC in H would be a pair at proper time=constant hyperboloid in H and the map of momentum to position would be linear inside $CD(m)$. BEC would look very much the same at both M^8 and H side of duality.

The space-time surface $X^4 \subset CD(m)$ should correspond to a minimal surface and to a generalized Beltrami flow defining an integrable coordinate along the flux lines. In the case of conserved current gradient flow (vortex flow is an example of this). All many-particle states would be of this kind in the scale of $CD(m)$. These multi-BEC states would be analogs of many-particle states and one would have many-particle states of BECs and their condensates, which could entangle in WCW degrees of freedom. For instance, the entanglement between geometric representations of Galois groups is possible. In the TGD inspired quantum biology the multi-BEC like states are proposed to play a key role [L44, L58].

7.9.3 Superconductivity and superfluidity in TGD framework

The TGD based view about superconductivity and fluidity [L54] differs in many respects from BCS theory.

1. In the BCS theory superconducting state does not have a well defined fermion number and this leads to a somewhat questionable notion of coherent state of Cooper pairs. The Bogobliubov transformation creates the diagonalizable oscillator operator basis by mixing creation and annihilation operators. The resulting operators create superpositions of electrons and holes.

   In the TGD framework, the interpretation would be that the hole actually corresponds to dark fermion with $h_{eff} > h$ at dark space-time sheet so that fermion number conservation is not lost. Bogoliubov operators would be replaced with superpositions of creation/annihilation operators associated with different space-time sheets and create states which are superpositions of state at the two space-time sheets.

Effective Hamiltonian would include diagonalizable kinetic parts assignable to both space-time sheets, and the terms quadratic in creation/annihilation operators breaking fermion number conservation would be replaced with pairs of creation and annihilation operators associated with different space-time sheets describing the transfer of electron between the space-time sheets.

2. In the BSC theory Cooper pairs are carriers of supra current. In the TGD framework, dark electrons at dark spacetime-sheets could be the carriers. The binding energy of Cooper pairs liberated in their formation would provide the energy needed to transform ordinary electrons to dark electrons (the energies of particle states typically increase with h_{eff}). This makes possible superconductivity driven by energy feed possible also above critical temperature.

3. Can one describe supra currents and supra flows in terms of a single space-time surface as the classical space-time view based on Beltrami currents would suggest? This would mean that supracurrent would correspond to a collection of momenta of dark electrons at $H^3 \subset M^8$ in the proposed TGD based model or collection of Cooper pairs with $h_{eff} = h$ as in the standard description. The current carriers would have fixed momenta at the two boundaries of $CD(m)$ corresponding to the analogs of initial and final state momenta. Is this all that one can say at the quantum level and is the description as a flow only a classical description. At quantum level one could only deduce the change of the positions for the group of particles defining the flow. This indeed conforms with the UP.

Update: I received a link to a highly interesting popular article with title "A Breakthrough Experiment Unlocking the Mystery of Unconventional Superconductivity" (`https://rb.gy/fs8ecn`). The article told about the work of Sarah Hirthe et al reported in the article "Magnetically mediated hole pairing in fermionic ladders of ultracold atoms" published in Nature [D10]. This work gives support for the TGD view of superconductivity.

TGD based view of unconventional superconductivity [K30, K31, L54] is based on the new view of quantum physics provided by new space-time concept and number theoretic vision predicting phases of ordinary matter behaving like dark matter and labelled by effective Planck constant $h_{eff} = nh_0$, which can be very large as compared to the ordinary Planck constant h. This the case for the gravitational Planck constant introduced originally by Nottale. This implies quantum coherence in long scales essential for superconductivity.

This view suggests that hole pairs are formed as the electron pairs are transferred to the magnetic flux tubes and become dark and therefore have a non-standard value of effective Planck constant. This creates hole pairs at the level of the ordinary matter and the motion of the dark electron pair corresponds to that for the hole pair. The electron pair goes to a pair of magnetic flux tubes and transversal fluctuations in the shape of flux tubes are essential in the transition to superconductivity . This picture is consistent with the reported findings.

The really important message is that dark matter in TGD sense can be detected as the absence of ordinary matter! Hole pair is a shadow Cooper pair of dark electrons.

7.9.4 WCW level is necessary for the description for purely geometric bosonic excitations

The quantum description of sound requires WCW description since the phonons as oscillations of relative position of particles cannot be described in terms of quark-antiquark pairs. The description of exotic supra flows like that associated with magnon BEC in say ^3He supra fluid allowing orbital magnetization requires WCW. A good manner to clarify thoughts is to look at what this means in the case of magnons.

1. Standard classical description (`https://cutt.ly/HRuZh53`) suggests a direction of magnetization M which has changed due to the presence of external field H. This leads to the Landau-Lifschitz equation for the magnetization.

   The Fock space picture about magnons is as a plane wave for which the argument is the position of spin whose direction has changed. The quantization is described by introducing a Hamiltonian for spins. The relationship between these descriptions is somewhat obscure.

2. In TGD the fermionic Fock space description is not possible. Bosonic creation and annihilation operators would be needed but one cannot construct bosonic operators with a vanishing fermion number from quarks. Therefore magnons should correspond to WCW degrees of freedom.

3. In the TGD description, M would correspond at space-time level to the magnetic field at a non-monopole flux tube and H possibly at a monopole flux tube inducing the magnetization. Magnons would correspond to magnetization waves, as kinks propagating along magnetic flux tubes for M. Magnon should correspond to space-time surface H and this would determine its M^8 pre-image. If these excitations behave like identical particles, one can assign to them wave vectors and classical momenta.

4. Also the notion of BEC makes sense at WCW level since one can construct the counterparts of genuine bosonic oscillator operators. Super-symplectic and Kac-Moody algebras of WCW acting at the boundaries of CD indeed include purely bosonic operators. Similar description at WCW level applies also to phonons as quanta.

   Cooper pair BECs allow approximate description in terms of fermion pairs with given total momentum but with members having different momenta. One cannot however exclude the possibility that there purely bosonic BEC at WCW level such that each Cooper pair is associated with a bosonic excitation of space-time surface.

8 Some concrete questions and problems

In this section some concrete questions relating to various applications of TGD to condensed matter physics are considered. Applications to (quantum) hydrodynamics is left to separate article.

8.1 Skyrmions in TGD framework

In hadron physics skyrmions (`https://cutt.ly/qRuXYMX`) appear at the level of momentum space. Proton as a skyrmion corresponds to a map of a 3-ball B^3 to $S^3 \subset E^4$ with non-trivial winding number. The points at the boundary are mapped to a single point so that B^3 effectively behaves like S^3. The map thus represents an element of third homotopy group and if this element is non-trivial one has skyrmions whose winding number has interpretation as number of protons. The radius of S^3 is the proton mass so that S^3 indeed lives in momentum space. $SO(4) = SU(2)_L \times SU(2)_R$ assigned to the current algebra picture of hadron physics acting as isometries of S^3 serves as the field space of skyrmions.

Skyrmions appear as topological defects also in condensed matter physics and correspond to 3-D magnetic field configurations inside B^3 and vanishing at the boundary of B^3 so that they define a map to S^3. In this case, the winding number of the map can correspond to the number of electron pairs. They appear in superconductivity, quantum Hall systems, liquid crystals, magnetic systems, and Bose-Einstein condensates (BECs). One example corresponds to ferromagnetic spin-1 Bose-Einstein condensates [D22](`https://cutt.ly/MWy3S5J`). Their universal appearance suggests that they could appear at fundamental level.

What TGD view would be following.

1. The proposal is that $M^8 - H$ duality allows to understand skyrmions as duality between the $SO(4)$ description of hadrons and $SO(4)$ symmetry group at M^8 level and QCD description in terms quarks and gluons and color $SU(3)$ at the level of H.

   In TGD framework skyrmions are associated with space-time surfaces in M^8 and skyrmion means a maps from a ball $B^3 \subset M^4$ to the sphere $S^3 \subset E^4$.The radius of S^3 is proton mass squared: this conforms with the interpretation of M^8 as momentum space.

2. Skyrmion in as a map $B^3 \to S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ is mapped to a map $B^3 \to S^3 \subset CP_2 \subset H$ by $M^8 - H$ duality. The map $B_3 \to B^3$ is by inversion (Uncertainty Principle). The map would have a non-trivial winding number.

   What does the skyrmion sphere S^3 subset E^4 correspond to in CP_2. Recall that normal space of X^4 is mapped to a point of CP_2. The image of the Skyrmion looks like a graph for the normal space of $X^4 \subset M^8$ as a function of the point of X^4. How does the normal space correlate with the E^4 point at S^3? Continuity and single-valuedness look natural. The 3-sphere in X^4 is mapped to a D≤ 3 surface.

Essentially homotopy associating normal space characterized by a point of CP_2 to $S^3 \subset CP_2$ is in question. CP_2 has a trivial third homotopy group. The homotopy equivalence class is trivial unless one fixes the radius as is done also in the original model by fixing the mass to correspond to the radius of $S^3 \subset E^4$.

Could $S^3 \subset E^4$ containing the octonionic real axis be mapped to a sphere $S^3 \subset CP_2$ invariant under $U(2)$. At S^3 Z^0 gauge field is proportional to Kähler form J as is also the electromagnetic field [L2]. Therefore the long range correlations for Kähler form J are associated also with Z^0. Large parity breaking effects would become possible and indeed appear in living matter (chirality selection for biomolecules).

3. Could the sphere $S^3 \subset M^8$ mapped to $S^3 \subset CP_2$ related by M^8-H duality define a common denominator of several exotic condensed matter phenomena? $S^3 \subset M^8$ define a quaternionic 3-sphere and the automorphism group of quaternions. One can assign to skyrmions a flat $SO(3)$ gauge potential [D26] (`https://arxiv.org/abs/1812.07974`). Could this relate to the speculated emergence of $SO(3)$ as a synthetic gauge group [D9])(`https://cutt.ly/qWy3H9M`?

8.2 Dark matter and condensed matter physics

The following represents a collection of examples of possible applications of TGD view of dark matter to condensed matter physics.

8.2.1 Could one make dark matter visible?

Dark matter in TGD sense could make itself visible in many ways.

1. One can imagine diffraction by generating a dark photon or (dark) polariton beam using a laser beam providing the energy feed increasing h_{eff}. Dark photon beam would diffract from an analog of hole: the ordinary laser beam could represent the hole as a source of dark photons. The structure of dark matter at flux tubes involving flux tubes and their geometric patterns could become visibille in this manner.

   For instance, the braids formed by flux tubes could become visible. Here braid entropy is a central notion and central in TGD based view of hydrodynamics involving braiding in both time-like and space-like braiding [K3, K2, K41].

2. In quantum biology dark matter at magnetic body with large h_{eff} as measure for complexity and intelligence, serves as the boss controlling ordinary biomatter, and its quantum coherence forces ordinary coherence of ordinary biomatter, which cannot be understood in physics and chemistry based on ordinary quantum physics [L92].

Solids are either in crystal or amorphous phase. Long range order in crystals is lacking and this is visible in the X-ray diffraction pattern. The diffraction pattern [D28] (`https://cutt.ly/ZWyLgjk`) for a hyperuniform amorphous material is very different and is called highly exotic (see **Fig. 16**). Apart from forward scattering peak, the diffraction pattern involves no scattering for a considerable range of scattering angles. I cannot avoid the temptation to speculate.

1. Suppose that the proposed dark looking phases with $h_{eff} > h$ by their higher algebraic complexity (larger extension of rationals, larger Galois symmetries) control the lower levels in master slave hierarchy, in particular ordinary matter (now the amorphous film.

   Suppose that the scattering of say laser light feeding energy and increasing the value of h_{eff} creates dark photons or polaritons at this higher level. Suppose that polaritons scatter at flux tubes or flux sheets structures at higher level and eventually a transformation to ordinary photons occurs spontaneously. Could the interference of the scattered beam with incoming beam make the geometry of dark matter level visible as the example about scattering in hyperuniform matter would suggest?

2. This high level would have longer quantum coherence length and perhaps range order since h_{eff} is larger. The long range order would be visible in the scattering pattern. Could just this happen when laser light generates a polariton-exciton condensate [D21](`https://cutt.ly/4Wy8zi9`). Could one think of polariton vortex lattices [D18] (`https://cutt.ly/qWy8Zqf`) as counterparts of crystal lattices and could their presence become visible so that one could see dark matter.

   The polariton could correspond at flux tubes superposition of dark photon and of dark exciton identifiable as dark electron paired with ordinary hole formed when the electron was transferred to the flux tube. The photon component of the outgoing polariton beam formed by the transformation of dark photon to ordinary photon would reflect the structure of dark matter and flux tubes and leave the system as ordinary photons and generate the scattering pattern by interference.

8.2.2 A strange behavior of hybrid matter-antimatter atoms in superfluid Helium

I received an interesting link to a popular article "ASACUSA sees surprising behaviour of hybrid matter–antimatter atoms in superfluid helium" (`https://cutt.ly/NVIzglw`), which tells of a completely unexpected discovery related to the behavior of antiproton-^{4}He^{++} atoms in ^{4}He superfluid. The research article [D17] by ASACUSA researchers Anna Soter et al is published in Nature (`https://cutt.ly/LVIceiB`).

The formation of anti-proton-^{4}He^{++} hybrid atoms containing also an electron in ^{4}He was studied both above and below the critical temperature for the transition to Helium superfluid. The temperatures considered are in Kelvin range corresponding to a thermal energy of order 10^{-4} eV.

Liquid Helium is much denser than Helium gas. As the temperature is reduced, a transition to liquid phase takes place and the Helium liquid gets denser with the decreasing temperature. One would expect that the perturbations of nearby atoms to the state should increase the width of both electron and antiproton spectral lines in the dense liquid phase.

This widening indeed occurs for the lines of electrons but something totally different occurs for the spectral lines of the antiproton. The width decreases and when the superfluidity sets on, an abrupt further narrowing of He^{++} spectral lines takes place. The antiproton does not seem to interact with the neighboring ^{4}He atoms.

Researchers think that the fact that the surprising behavior is linked to the radius of the hybrid atom's electronic orbital. In contrast to the situation for many ordinary atoms, the electronic orbital radius of the hybrid atom changes very little when laser light is shone on the atom and thus does not affect the spectral lines even when the atom is immersed in superfluid helium.

Consider now the TGD inspired model.

1. It seems that either antiprotons or the atoms of ^{4}He superfluid effectively behave like dark matter. For the electrons, the widening however takes place so that it seems that the antiproton seems to be dark. In the TGD framework, where dark particles corresponds $h_{eff} = nh_0 > h$, $h = n_0h_0$ phases of ordinary matter, the first guess is that the antiprotons are dark and reside at the magnetic flux tube like structures.

   The dark proton would be similar to a valence electron of some rare earth atoms, which mysteriously disappear when heated (an effect known for decades) [L18]. Dark protons would indeed behave like a dark matter particle is expected to behave and would have no direct quantum interactions with ordinary matter. The electron of the hybrid atom would be ordinary.

2. Darkness might also relate to the formation mechanism of the hybrid atoms. Antiproton appears as a Rydberg orbital with a large principal quantum number N and large size proportional to N^2. $N > 41$ implies that the antiproton orbital is outside the electron orbital but this leaves the interactions with other Helium atoms. For a smaller value of N the dark proton overlaps the electronic orbital. Note that for $N = 1$, the radius of the orbital is $10^{-3}/8a_0$, $a_0 \simeq .53 \times 10^{-10}$ m, in the Bohr model.

3. The orbital radii are proportional to $h_{eff}^2 \propto (n/n_0)^2$ so that dark orbitals with the same energy and radius as for ordinary orbitals but effective principal quantum number $(n/n_0)N_d =$

N_{eff}, are possible. $(n/n_0)N_d = N_{eff}$ condition would give the same radius and energy for the dark orbital characterized by N_d and ordinary orbital characterized by N.

One can consider both dark-to-dark and dark-to-ordinary transitions.

1. The minimal change of the effective principal quantum number N_{eff} in dark-to-dark transitions would be n/n_0 and be larger than one for $n > n_0$. There is evidence for $n = n_0/6$ found by Randel Mills [D11] discussed from the TGD view in [L13]. In this case one would have effectively fractional values of N_{eff}. One can also consider a stronger condition, $h_{eff}/h = m$, one has $mN_d = N$. The transitions would be effectively between ordinary orbitals for which ΔN_{eff} is a multiple of m. This could be tested if the observation of dark-to-dark transition is possible. The transformation of dark photons to ordinary photons would be needed.

2. Energy conserving dark-to-ordinary transitions producing an ordinary photon cannot be distinguished from ordinary transitions if the condition $(n/n_0)N_d = N_{eff}$ is satisfied.

   The transitions $(37, 35) \rightarrow (38, 34)$ and $(39, 35) \rightarrow (38, 34)$ at the visible wavelengths $\lambda = 726$ nm and 597 nm survive in the Helium environment. The interpretation could be that the transitions occur between dark and ordinary states such that the dark state satisfies the condition that $(n/n_0)N_d = N_{eff}$ is integer, and that an ordinary photon with $\lambda = h/\Delta E$ is produced. This does not pose conditions on the value of h_{eff}/h.

   If the condition that $(n/n_0)N_d = N_{eff}$ is an integer is dropped, effective principal quantum numbers N_{eff} coming as multiples of n/n_0 are possible and the photon energy has fractional spectrum.

If this picture makes sense, it could mean a new method to store antimatter without fear of annihilation by storing it as a dark matter in the magnetic flux tubes. They would be present in superfluids and superconductors.

8.2.3 Mott insulators learn like living organisms

Researchers in Rutgers University have found that quantum materials, in this case Mott insulators, are able to learn very much like living matter (`https://cutt.ly/oTRZhQE`). The conductivity of the quantum material represented behavior and sensory input was represented by external stimuli like oxygen, ozone and light.

The finding was that conductivity depends on these stimuli and that the system mimics non-associative learning. Non-associative learning does not involve pairing with the stimulus but habituation or sensitization with the stimulus.

I have already earlier [K31] briefly considered transition metals, Mott insulators, and antiferromagnets from the point of view of TGD inspired theory of high Tc superconductivity.

1. By looking at Wikipedia (`https://cutt.ly/RTRXi22`), one finds that Mott insulators are transitional metal oxides such as NiO. Transition metals, such as Ni, can have unpaired valence electrons since they can appear in electronic configurations [Ar] $3d^8$ $4s^2$ or [Ar] $3d^9$ $4s^1$. This should make transition metals and their oxides conductors. They are not since they seem to somehow develop an energy gap between states in the same valence band making them insulators.

2. Mott developed a model for NiO as an insulator: the expected conduction was based on the transition for neighboring $Ni^{2+}O^{2-}$ molecules

$$(Ni^{2+}O^{2-})_2 \rightarrow Ni^{3+}O^{2-} + Ni^{1+}O^{2-}$$

   .

   In the latter configuration, the number of valence electrons of Ni is odd for both neighbors.

3. The formation of the gap can be understood as a competition between repulsive Coulomb potential U between 3d electrons and the transfer integral t of 3d electrons between neighboring atoms assignable to the transition. The total energy difference between the two states is $E = U - 2zt$, where z is the number of neighboring atoms. A large value of U leads to a formation of a gap implying the insulator property.

4. Also antiferromagnetic ordering is necessary for the description of Mott insulators. Even this is not enough, and the rest which is not so well understood, is colloquially called mottism. The features of Mott insulators that require mottism are listed in the Wikipedia article. They include the vanishing of the single particle Green function along a connected surface in the first Brillouin zone and the presence of charge 2e boson at low energies.

5. The description of both Mott insulators and high Tc superconductors involves antiferromagnetism and Mott insulators exhibit extraordinary phenomena such as high Tc superconductivity and so-called colossal magnetoresistance thought to be due the interaction between charge and spin of conduction electrons.

In the TGD framework, the description of high Tc superconductors [K30, K31] [L54] involves pairs of monopole flux tubes with opposite direction of monopole magnetic flux possible not possible in Maxwellian electrodynamics. The members of Cooper pairs, which are dark in the TGD sense having an effective Planck constant $h_{eff} \geq h$, reside at the monopole flux tubes. The Cooper pairs are present already above Tc but the flux tubes are short and closed so that supercurrent flows only in short scales. At Tc long flux tubes are formed by reconnection.

Dark valence electrons could help to understand Mott insulators. Transition metals are known for a strange effect in which the valence electrons seem to disappear [L18] [L18] [K14]. The TGD proposal is that the electrons become dark in the TGD sense.

It has become clear that dark electron can appear only as bound states for which the sum of momenta, which are algebraic integers in the extensions of rationals with dimension $h = h_{eff}/h_0$ (this guarantees periodic boundary conditions) must be Galois singlets: one has Galois confinement. This implies that the total momentum is ordinary integer [L65, L67].

Therefore free dark electrons are not allowed and Cooper pairs and possibly also states formed by a larger number of electrons, say four as has been found [L65]) are possible as Galois singlets. In the TGD inspired quantum biology dark proton triplets realize genetic codons and genes could correspond to N-codons as Galois confined states of 3N dark protons [L58].

As a rule, single particle energies increase with increasing h_{eff} and the thermal energy feed could increase the effective value Planck constant for an unpaired valence electron of Mott insulator from h to $h_{eff} >= nh_0 > h$ of the valence electrons and it would become dark in the TGD sense. Here n denotes the dimension of extension of rationals assignable to the space-time region. The natural assumption is that Galois confinement forces the Cooper pairing of unpaired electrons of neighboring atoms.

Above Tc, the flux tubes associated with Cooper pairs would be too short for large scale superconductivity so that one would have a conductor or a Mott insulator. Under certain conditions involving low enough temperature, a supraflow in long scales would become possible by the mechanism described above. The massive magnetoresistance could involve a transfer of electrons as Cooper pairs at the magnetic flux tubes of the external magnetic field which would be too short to give rise to superconductivity or even superconductivity. External magnetic fields could also induce dark ferromagnetism as formation of dark flux tubes.

Dark electrons, protons and ions residing at the magnetic flux tubes of the "magnetic body" (MB) of the system are in a key role in the TGD based quantum biology and essential for learning as self-organization. h_{eff} serves as a measure for the number theoretical complexity and therefore "intelligence" of the system. There MB naturally acts as a "boss".

Also for the Mott insulator, the MB could play a key role: MB would be the "boss" and could learn and induce changes in the behavior of the ordinary matter, the "biological body" (BB). In the non-associative learning, adaptation and sensitization is involved and it would be MB which adapts or sensitizes. The TGD view of a neuron proposes a rather detailed model for the communication between the BB and MB [L50].

8.2.4 The mysterious linear temperature dependence of resistance of strange metals

Could one understand the somewhat mysterious looking linear high T dependence of the resistivity of strange metals in TGD the framework?

In the TGD based model of high T superconductivity [L54], charge carriers are dark electrons, or rather Cooper pairs of them, at magnetic flux tubes which are effectively 1-D systems. Magnetic flux tubes are much more general aspect of TGD based model of condensed matter [L65].

Could magnetic flux tubes carrying dark matter with h$_{eff} = nh_0 > halsoexplaintheresistanceofstrangemetals$

More precisely: Could the effective 1-dimensionality of flux tubes, darkness of charge carriers, and isolation from the rest of condensed matter together explain the finding?

One can make a dimensional estimate.

1. Isolation at flux tubes would mean that only the collisions of dark electrons with each other cause resistance.

2. Assume that the resistance ρ can be written in the form

$$\rho = \frac{4\pi/\omega^2}{\tau} = \frac{\frac{m_e}{n_e e^2}}{\tau} \quad . \tag{8.1}$$

   τ is the time that electron spends between two collisions. ω is the plasma frequency

$$\omega^2 = \frac{4\pi n_e e^2}{m_e} \quad . \tag{8.2}$$

   n_e is 3-D electron density.

   What happens for 3-D n_e in the case of 1-D flux tube? It would seem that n_e must be replaced with linear density divided by the transversal area S of the flux tube: $n_e = (dn_e/dl)/S$.

3. As already notice., τ is the time spent by the charge carrier in free motion between collisions. Charge carrier is in thermal motion with a thermal velocity $v_{th} = kT/m$. The length L_f of the free path is determined non-thermally. Hence one has

$$\tau = \frac{L_f}{v_{th}} = \frac{mL_f}{kT} \quad . \tag{8.3}$$

   This gives $1/\tau = kT/mL_f$.

4. For the resistivity ρ one obtains

$$\rho = \frac{m_e}{n_e e^2}\frac{kT}{mL_f} \quad , \tag{8.4}$$

   which indeed depends linearly on T as it does for strange metals.

   For $m = m_e$, one would have $\rho = kT/(n_e e^2 L_f)$.

In the article "Signatures of a strange metal in a bosonic system" (`https://cutt.ly/2OE4Quz`) by Yang *et al* published in Nature, bosonic strange metals are studied instead of fermionic ones. The system can also be superconducting and this seems to be essential.

The linear dependence on magnetoresistance in an external magnetic field B is the second interesting phenomenon.

1. Below the onset of temperature $Tc_1 \geq Tc$, the low-field magneto-resistance varies with a periodic dictated by superconducting flux quantum suggesting that the density of charge carriers varies with this period.

2. What comes to mind is the De Haas-Van Alphen effect in field B (https://en.wikipedia.org/wiki/DeHaasVan_Alphen_effect.

   The magnetic susceptibility of the system varies periodically with the inverse of the magnetic flux $\Phi = e\int BdS$ defined by extremal orbit of electrons at the Fermi surface in field B. Φ is measured in units defined by elementary flux quantum $h/2e$.

3. Could spin=0 Cooper pairs be formed from the electrons at the Fermi surface and lead to the De Haas-Van Alphen effect. They would go to the flux tubes of the external magnetic field B with a rate determined by the magnetic flux.

   The rate for this highest, when the extremal orbit at the Fermi surface corresponds to a quantized flux. Otherwise, energy is needed to kick the electrons from the Fermi surface to a larger orbit in order to satisfy the flux quantization condition.

Now one considers magnetoresistance rather than susceptibility. The linearity in magnetoresistance suggests that the resistance in the external field is mostly due to magnetoresistance.

1. Could the analog of the De Haas-Van Alphen effect be present so that the density of Cooper pairs as current carriers at "endogenous" magnetic flux tubes has an oscillatory behavior as a function of the external magnetic field B? Could there be a competition for the Cooper pairs between the magnetic fields of flux tubes and the external magnetic field B?

2. When the flux Φ for the external B is near the multiple of the elementary flux quantum at extremal orbits at Fermi surface, the formation of spin=0 Cooper pair and transfer to the flux tubes of B would become probable by De-Haas-van Alphen effect. The number of Cooper pairs at "endogenous" flux tubes is therefore reduced and the current therefore reduced.

8.2.5 VO_2 can remember like a brain

The following comments were inspired by a popular article (https://cutt.ly/lNHzBYa) with the title "Scientists accidentally discover a material that can 'remember' like a brain". These materials can remember the history of its physical stimuli. The findings are described in the article "Electrical control of glass-like dynamics in vanadium dioxide for data storage and processing" published in Nature [D12] (https://cutt.ly/cNHyMMa).

The team from the Ecole Polytechnique Federale de Lausanne (EPFL) in Switzerland did this discovery while researching insulator-metal phase transitions of vanadium dioxide (VO_2), a compound used in electronics.

1. PhD student Mohammad Samizadeh Nikoo was trying to figure out how long it takes for VO_2 to make a phase transition from insulating to conducting phase under "incubation" by a stimulation by a radio frequency pulse of 10 μs duration and voltage amplitude $V = 2.1$ V. Note that the Wikipedia article talks about semiconductor-metal transition. The voltage pulse indeed acted like a voltage in a semiconductor.

2. As the current heated the sample it caused a local phase transition to metallic state in VO_2. The induced current moved across the material, following a path until it exited on the other side. A conducting filament connecting the ends of the device was generated by a percolation type process.

3. Once the current had passed, the material exhibited an insulating state but after incubation time t_{inc}, which was $t_{inc} \simeq .1\mu$s for the first pulse, it became conducting. This state lasted at least 10,000 seconds.

   After applying a second electrical current during the experiment, it was observed that t_{inc} appeared to be directly related to its history and was shorter than for the first incubation period .1 μs. The VO_2 seemed to 'remember' the first phase transition and anticipate the next. One could say that the system learned from experience.

Before trying to understand the finding in the TGD framework, it is good to list some basic facts about vanadium and vanadium-oxide VO_2 or Vanadium(IV) oxide (https://cutt.ly/yNHhahk).

1. Vanadium is a transition metal, which has valence shells d^3s^2. It is known that the valence electrons of transition metals can mysteriously disappear, for instance in heating [L18]. The TGD interpretation [K14] would be that heating provides energy making it possible to transform ordinary valence electrons to dark valence electrons with a higher value of h_{eff} and higher energy. In the recent case, the voltage pulses could have the same effect.

2. VO_2 forms a solid lattice of V^{4+} ions. There are two lattice forms: the monoclinic semiconductor below $T_c = 340$ K and the tetragonal metallic form above T_c. In the monoclinic form, the V^{4+} ions form pairs along the c axis, leading to alternate short and long V-V distances of 2.65 Angström and 3.12 Angström. In the tetragonal form, the V-V distance is 2.96 Angström. Therefore size of the unit cell for the monoclinic form is 2 times larger than for the tetragonal form. At T_c IMT takes place. The optical band gap of VO_2 in the low-temperature monoclinic phase is about 0.7 eV.

3. Remarkably, the metallic VO_2 contradicts the Wiedemann–Franz law, which states that the ratio of the electronic contribution of the thermal conductivity (κ) to the electrical conductivity (σ) of a metal is proportional to the temperature. The thermal conductivity that could be attributed to electron movement was 10 % of the amount predicted by the Wiedemann–Franz law. That the conductivity is 10 times higher than expected, suggests that the mechanism of conductivity is not the usual one.

   Semiconductor property below T_c suggests that a local phase transition modifying the lattice structure from monoclinic to tetragonal takes place at the current path in the incubation.

One can try to understand the chemistry and unconventional conductivity of VO_2 in the TGD framework.

1. Vanadium could give 4 valence electrons to O_2: 3 electrons d^3:sta and one from s^2. In the TGD Universe, the second electron from s^2 could become dark and go to the bond between V^{4+} ions in the VO_2 lattice and take the role of conduction electron.

2. This could explain the non-conventional character of conductivity. In the semiconductor phase, an electric voltage pulse or some other perturbation, such as impurity atoms or heating, can provide the energy needed to increase the value of h_{eff}. Electric conductivity could be due to the transformation of electrons to dark electrons possibly forming Cooper pairs at the flux tube pairs connecting V^{4+} ions or their pairs. The current would run along the flux tubes as a dark current.

3. In a semi-conducting (insulating) state, the flux tube pairs connecting V^{4+} ions would be relatively short. The voltage pulse inducing a local metallic state could provide the energy needed to increase h_{eff} and thus the quantum coherence scale. This would be accompanied by a reconnection of the short flux tube pairs to longer flux tube pairs serving as bridges along which the dark current could run.

   One can also consider U-shaped closed flux tubes associated with V^{4+} ions or ion pairs, which reconnect in IMT to longer flux tubes. The mechanism would be very similar to that proposed for the transition to high temperature superconductivity [K30, K31, L54].

Experimenters suggest a glass type behavior.

1. Spin glass corresponds to the existence of a very large number of free energy minima in the energy landscape implying breaking of ergodicity. A system consisting of regions with varying direction of magnetization is the basic example of spin glass. In the recent case, decomposition to metallic and insulating regions could define the spin glass.

2. TGD predicts the possibility of spin glass type behavior and leads to a model for spin glasses [L64]. The quantum counterpart of spin glass behavior would be realized in terms of monopole flux tube structures (magnetic bodies) carrying dark phases of the radinary particles such as electrons serving as current carries in the metallic phase.The length of the flux tube pair would be one critical parameter near T_c. Quantum criticality against the change of h_{eff} increasing the length of the flux tube pair by reconnection would make the system very sensitive to perturbations.

3. These phases are highly sensitive to external perturbations and represent in TGD inspired theory of consciousness higher levels with longer quantum coherence scale and number theoretical complexity measured by the dimension $n = h_{eff}/h_0$ of the extension having interpretation as a kind of IQ. These phases would receive sensory information from lower levels of the hierarchy with smaller values of n and control them.

   The large number of free energy minima as a correlate for number theoretical complexity would make possible the representation of "sensory" information as "memories".

8.2.6 Superconductivity dome rises from damped phonons

I received a link to an interesting popular article "Superconductivity Dome Rises from Damped Phonons" (`https://cutt.ly/XOcogsh`). The article tells about findings Setty *et al* [D30] (`https://cutt.ly/dOcoxRL`) about BCS superconductivity near ferro-electic phase transition.

The system studied is a conventional superconductor near its critical temperature also in the vicinity of the ferroelectric phase transition. It is known that the critical temperature for superconductivity has a dome-like peak in this region. The origin of this peak has remained poorly understood and an explanation for the dome has been proposed in the article.

1. In the BCS model for the conventional low temperature superconductivity phonons bind electrons to Cooper pairs. If the phonons are damped for some reason, Tc is expected to decrease. In ferroelectric superconductors near critical temperature for the transition to ferro-electricity the situation is opposite to this. What could happen?

2. Electron-photon scattering as an analog of Compton scattering is what matters. The scattering of the phonon is Stokes or anti-Stokes depending on whether the scattered phonon gains or loses energy. In anti-Stokes scattering phonon gives energy for the electron, which disfavors the formation of pairs whereas Stokes scattering favors the formation of Cooper pairs.

3. Near the phase transition to ferro-electricity phonon damping occurs. This means that the phonon life-time gets shorter. In ordinary materials this would lead to a reduction of critical temperature but in ferroelectrets the critical temperature has a dome-like peak around the critical criticality for the transition to ferro-eletricity. Ferroelectric transitions involve a non-linear phonon-electron coupling. This anharmonic coupling implies that scattering now also involves final states with 2 phonons. This implies that anti-Stokes scattering is suppressed more than Stokes scattering. The proposal is that this raises the critical temperature and causes the dome-like structure.

One could however counter-argue that both Stokes and anti-Stokes are suppressed and that the dome structure involves the soft-photon mode associated with the ferro-electric phase transitions. This suggests a somewhat different view about what happens based on the fact that so called soft modes for photons having vanishing wavelength at criticality play an important role in ferroelectric phase transition [D20] (`https://cutt.ly/IOgPrYj`).

1. Soft modes have a long wavelength, which approaches zero at the ferroelectric critical temperature $T_{c,f}$. Since soft modes generate long range correlations and induce a polarization of the ferroelectret, their wavelengths are much longer than the lattice constant.

2. Soft modes corresponds to photon energy not larger than 10^{-4} eV, which is near the gap energy E_{gap} of superconductor with critical temperature $Tc = 10^{-4}$ eV. This corresponds to photon wave length about $\lambda_\gamma = 1.24\times10^{-2}$ m and phonon wavelength λ_{ph} around 2.56×10^{-7} m for $c_s = 6 \times 10^3$ m/s, that is $c_s = 2 \times 10^{-5}c$. The wavelength is much longer than atomic size in accordance with the generation of long range correlations. Interestingly, in the TGD framework, λ_{ph} corresponds to p-adic length scale $L(167) = 2^{(167-151)/2} \times L(151)$, $L(151) = 10$ nm.

3. Could soft phonons associated with the ferroelectric transition with energies below this 10^{-4} eV compete with thermal excitations by reducing the energies of electrons via the Stokes scattering and in this manner raise the critical temperature?

This suggests that the coupling of electrons to soft ferroelectric phonons with frequencies below 10^{-10} Hz facilitates the formation of Cooper pairs so that their thermal decay is compensated and T_c increases.

Could the TGD based view of superconductivity [L54] provide a mechanism for the generation of Cooper pairs by electron-phonon interaction? This model should generalize to high Tc superconductors for which phonons do not explain the Cooper pairs.

1. In the TGD framework, Cooper pairs are dark in the sense that they have $h_{eff} \geq h$ and reside at the magnetic flux tubes. The creation of Cooper pairs requires an increase of h_{eff}. Phonon or photon exchange could transform an ordinary electron pair to a dark pair, which is Galois singlet so that (using p-adic mass scale as a unit) it has 4-momentum with integer-valued components and expressible as sum of algebraic integer valued momenta of dark quarks.

2. This is not enough: there must be a mechanism reducing the value of of the dark electron pair so that it cannot decay back to the ordinary electrons. The decay can be prevented by Fermi statistics in the presence of a Fermi sphere. This is possible if the state can decay to a Galois singlet dark electron pair with energy so small that decay products would belong inside the Fermi sphere.

   This requires an emission of a dark photon or a dark photon pair which is necessarily a Galois singlet transforming to photons or phonons (in ferroelectrets there is a strong coupling between photons and phonons). The reduction of energy would correspond to the gap energy E_{gap}.

3. For ordinary superconductors with Tc measured in Kelvins, the gap energy is $E_{gap} \simeq 10^{-4}$ eV. Could the exchange of phonons with energy in the energy range of soft phonons [D20] give rise to the dark states, which decay to Cooper pairs stable below Tc?

   For high Tc superconductors the gap energy is considerably stronger: for $T = 100$ K the gap energy is about $E_{gap} \simeq 10^{-2}$ eV and by factor 100 larger than for $T = 1K$. For photons, one would have have $\lambda_\gamma \simeq 1.24 \times 10^{-4}$ m not far from the p-adic length scale $L_{179} \simeq 1.6 \times 10^{-4}$ m. This corresponds to the size of a large neuron which is an important length scale in biology.

4. One can ask whether the high Tc superconductivity in biomatter could involve this kind of mechanism. At physiological temperatures one would have $E_{gap} \simeq 3 \times 10^{-2}$ eV and this is not far from the cell membrane potential. Living matter is full of ferroelectrets meaning that photons and photons are strongly coupled. Therefore also in living matter, soft phonons near the criticality of ferroelectrets could compete with the thermal excitations to raise the critical temperature Tc.

   Magnetic flux tubes play a key role in the TGD based model of living matter and they can become electric with a simple deformation and generate the long range correlations via the oscillations of the flux tube length giving rise to the space-time correlates of sound waves.

TGD based view about superconductivity also leads to the notion of forced super-conductivity. The increase of h_{eff} requires energy since the energies of states with other parameters fixed in general increase with h_{eff}. The dark states are expected to decay back to ordinary states. The feed of energy could however maintain a steady state. In living matter this mechanism could make possible high Tc superconductivity as forced superconductivity requiring metabolic energy feed. In ordinary superconductors the situation is not this.

A word of criticism relates to the notion of phonon in the TGD framework.

1. At the level of H, flux tubes correspond strings: at the point of the string world sheet the normal space of $X^4 \subset M^8$ characterized by a point of CP_2 is not unique and is characterized by points of a geodesic sphere of CP_2. The boundaries of a string at the mass shell H^3 of $M^4 \subset M^8$ should characterized the phonon as an oscillation of the distance of the ends in H.

2. At the level of M^8 everything is described in terms of momenta belonging to 3-D mass shells define by roots of polynomial defining the 4-surface. $M^8 - H$ duality can be represented as

the deformation of M^4 containing the real projections of the mass shells and representable as an element of local CP_2. It is however far from clear what the counterpart of the flux tube picture for photons could be.

In M^8 there is no time and it would seem that the emission of phonon must correspond to momenta at positive and negative energy mass shells differing by the energy of phonon. The H image of X^4 under $M^8 - H$ duality give rise to the flux tube picture description but what does this description correspond at the level of M^8?

3. $X^4 \subset M^8$? X^4 should connect the two opposite mass shells of M^8. Do the 8-momenta of X^4 have any reasonable physical interpretation? As long as one does not have excellent reasons for the existence of X^4, also $M^8 - H$ duality can be challenged. One possibility is that M^8 picture is enough in the sense that the deformations of $M^4 \subset M^8$ can be regarded as local CP_2 elements and allow an interpretation in terms of the space-time picture with M^4 space-time coordinates related to M^4 momenta essentially by inversion [L67]. This would conform with the Uncertainty Principle.

8.2.7 Polaritons and excitons in TGD

The claimed room temperature superconductivity for exciton-polariton Bose-Einstein condensate in quasi-crystals suggests that the TGD based model for superconductivity could generalize to a unified description of quantum coherent phases. In this case the energy feed is crucial and would serve in TGD framework as "metabolic energy feed" taking care that the distribution of $h_{eff} > h$ is preserved.

Also WCW level might be needed to describe the bosonic aspects of exciton-polariton BECs although exciton polariton states involve only photons excitons and electron-hole bound states. The description of plasmons involves oscillations of the relative position of electron and atomic nucleus and this requires the counterparts of the bosonic creation operators at the level of WCW.

The TGD view about superconductivity can be taken as a "role model" [L54].

1. In the BCS theory of superconductivity does not have a well defined fermion number and this leads to a somewhat questionable notion of coherent state of Cooper pairs. The Bogobliubov transformation creates the diagonalizable oscillator operator basis by mixing creation and annihilation operators. The resulting operators create superpositions of electrons and holes.

2. In the TGD framework , the interpretation would be that the hole actually corresponds to dark fermion at other space-time sheets so that fermion number conservation is not lost. Bogoliubov operators could correspond to superpositions of creation/annihilation operators associated with different space-time sheets and create states which are superpositions of state at the two space-time sheets. Effective Hamiltonian would include parts assignable to both space-time sheets, and the terms quadratic in creation/annihilation operators breaking fermion number conservation would be replaced with pairs of creation and annihilation operators associated with different space-time sheets describing the transfer of electrons between the space-time sheets.

3. One can consider two alternative identifications for Cooper pairs. Cooper pairs consist of ordinary electrons and provide their binding energy for dark electrons at MB to compensate for the increase ΔE of energy due to the larger value of h_{eff}. The dark electrons could be even free. Galois confinement in turn suggests Cooper pairs are dark and that the dark binding energy compensates for ΔE.

It is better to represent the ideas as questions.

Is the polariton condensate actually a macroscopic quantum phase? Could the polariton BE condensate only provide the energy feed making possible a macroscopic quantum phase at the level of MB, which would then induce ordinary (non-quantum) coherence of the polariton condensate. Could one take the number theoretical model of macroscopic quantum phases as a guideline in attempts to understand polariton superfluidity and other quantum coherent phases involved. The increase of h_{eff} and the preservation of its values requires energy feed to prevent dissipation if. In living matter this would be metabolic energy feed. Exciton-polariton condensate is an open system involving an energy feed. Could the formation of quasiparticles provide the "metabolic energy" for $h_{eff} > h$ phases at MB responsible for the long range order? Or are quasiparticles as such dark? Could polaritons and excitons correspond to dark valence electrons in $h_{eff} > h$ phase and the value of h_{eff} would determine in which scale the phase appears. Beltrami fields would provide a quantum hydrodynamical description as an exact classical description of these phases. In principle also fermionic Beltrami currents could make sense and provide genuine quantum hydrodynamical description. Also an empirical verification of BvK vortex street in exciton-polariton BE condensate has been reported. Could TGD provide at the level of principle a universal description as minimal surfaces also for this kind of system.

8.2.8 Braids, anyons, and Galois groups

Braids and anyons in the TGD framework are discussed in [K27]. Braid statistics has an interpretation in terms of rotations as homotopies at a 2-D plane of the space-time surfaces instead of rotations in M^4. One can use M^4 coordinates for the M^4 projection of the space-time surface.

As a matter of fact, arbitrary isometry induced flows of H can be lifted to rotations as flows along the lifted curve at the space-time surface and for many-sheeted space-time the flows, which correspond to identity in H can lead to a different space-time sheet so that the braid groups structure emerges naturally [L67].

The representations of H isometries at the level of WCW act on the entire 3-surface identifiable as a generalized point-like particle and by holography on the entire space-time surface. The braid representations of isometries act inside the space-time surface. This suggests a generalization of the notions of gravitational and inertial masses so that they apply to all conserved charges. Generalization of Equivalence Principle would state that gravitational and inertial charges are identical.

The condition that the Dirac operator at the level of H has tangential part equivalent to the Dirac operator for induced spinors, implies that the conserved isometry currents of H are conserved along the flow lines of corresponding Killing vector fields and proportional to the Killing vectors lifted/projected to the space-time surface. This has an interpretation as a local hydrodynamics conservation law analogous to the conservation of $\rho v^2/2 + p$ along a flow line.

One can ask whether the 2-dimensionality, which makes possible non-trivial and non-Abelian homotopy groups, is really necessary for the notion of the braid group in the TGD framework. As a matter of fact, the conditions are not expected to be possible for all conserved charges, and the intuitive guess that they hold true only for Cartan algebra representing maximal set of commuting observables would provide a space-time correlate of the Uncertainty Principle. If so, the space-time surface would depend on the choice of quantization axes. This conforms with quantum classical correspondence. For instance, the Cartan algebra of rotation group would act on a plane so that the effective 2-dimensionality of braid group and quantum group representations would hold true.

This view has some nice consequences.

1. If the space-time surface is n-sheeted, the rotation of 2π can take the particle to a different space-time sheet, and only n fold-rotation brings it back to its original position. The formula for fractional Hall conductivity is the same as in the case of integer Hall effect except that the $1/\hbar$-proportionality is replaced with $1/\hbar_{eff}$-proportionality in TGD framework [K27].

2. Degeneracy of fermion states also makes non-Abelian braid statistics possible. Since the Galois group acts as a symmetry group, the degeneracy would be naturally associated with the representations of the Galois group. Galois singletness of the many-anyon states guarantees reduces braid statistics to ordinary statistics for these. Galois confinement is proposed to be a central element of quantum biology [L92, L57].

8.2.9 Quantum flute

It is amazing how fast experimental discoveries, which look mysterious in the standard physics framework but are readily explainable in the TGD framework, are emerging recently.

Now University of Chicago physicists have invented a "quantum flute" that, like the Pied Piper, can coerce photons to move together in a way that's never been seen before. The discovery is described in Physical Review Letters and Nature Physics [D6, D7].

The system, devised in the lab of Assoc. Prof. Schuster, consists of a long cavity made in a single block of metal, designed to trap photons at microwave frequencies. The cavity is made by drilling offset holes—like holes in a flute. One can send one or more wavelengths to the "flute" and each wavelength creates a note coding for quantum information. The interactions of notes are then controlled by a superconducting electrical circuit.

The real surprise was the interaction of photons. In quantum electrodynamics (QED) the interaction of photons is extremely weak. When photons achieve critical total energy, the situation changes dramatically. One can say that photons interact, not pairwise as usually, but all at the same time. Photon state behave like a Bose-Einstein condensate of bound state.

Galois confinement as a universal mechanism for the formation of bound states would explain the findings elegantly. TGD involves M^8-H duality in an essential manner. M^8-H duality relates differential geometric and number theoretic descriptions of quantum physics and is analogous to Langlands duality. Number theoretical vision, involving classical number fields, extensions of rationals, and extensions of p-adic number fields induced by them, is essential for understanding the physical correlates of cognition [L20, L21] but has led to a breakthrough in the understanding of also ordinary physics [L39, L40].

1. The number theoretic side of the M^8-H duality predicts Galois confinement as a universal mechanism for the formation of bound states from the dark variants of ordinary particles characterized by effective Planck constant $h_eff = nh_0 > h$: integer n has interpretation as the dimension of extension of rationals induced by a polynomial and serves as a measure of algebraic complexity defining evolutionary level and a kind of IQ for the system.

2. Galois confinement states that physical bound states are Galois singlets transforming trivially under the Galois group of a polynomial P determining space-time region if M^8-H duality holds true. There is (more than) an analogy with hadrons, which are color singlets. Galois confinement is central in TGD inspired quantum biology and also allows us to understand various nanoscopic and macroscopic quantum phenomena of condensed matter physics.

   For instance, Cooper pairs would represent on a lowest level in a hierarchy and there is evidence for 4-fermion analogs of Cooper pairs [L65].

3. Galois confinement is central in TGD inspired quantum biology and allows also to understand various nanoscopic and macroscopic quantum phenomena of condensed matter physics [L80].

   In particular, N photons can form bound states in which they behave like a single particle. This bound state is a more general state than Bose-Einstein condensate since photons need not have identical quantum numbers. These many-photon states described in the article could be states of this kind.

   These $N-$photon states are very similar to the dark $3N-$ photon states proposed to represent genes consisting of N codons with codon represented as dark photon triplet.

4. Another representation of the genetic code paired with ordinary DNA would would be in terms of dark $3N-$proton states, or more generally, $3N-$nucleon states and realized at magnetic flux tubes parallel to DNA [L80, L57]. In both cases, Galois confinement would bind the particles to form quantum coherent states behaving like a single particle, which is also emitted and absorbed as a single entity. This behavior is just what was observed in the experiments.

8.2.10 Fractons and TGD

In Quanta Magazine there was a highly interesting article about entities known as fractons (`https://cutt.ly/kQPph8n`).

There seems to be two different views about fractons as one learns by going to Wikipedia. Fracton can be regarded as a as self-similar particle-like entity (`https://cutt.ly/KQPadQL` or as "sub-dimensional" particle unable to move in isolation (`https://cutt.ly/yQPayJt`). I do not understand the motivation for "sub-dimensional". It is also unclear whether the two notions are related. The popular article assigns to the fractons both the fractal character and the inability to move in isolation.

The basic idea shared by both definitions is however that discrete translational symmetry is replaced with a discrete scaling invariance. The analog of lattice which is invariant under discrete translations is fractal invariant under discrete scalings.

One can also consider the possibility that the time evolution operator acts as a scaling rather than translation. At classical level this would produce scaled versions of the system in discrete steps. This is something totally new from quantum field theory (QFT) point of view and it is not clear whether QFT can provide a description of fractons. In QFTs energy corresponds to time translational symmetry and Hamiltonian generates infinitesimal translations. In string models the analog of stringy Hamiltonian is the infinitesimal scaling operator, Virasoro generator L_0. Energy eigenstates would be replaced by scaling eigenstates with energy replaced with conformal weight.

In TGD the extension of physics to adelic physics provides number theoretic and geometric descriptions as dual descriptions of physics [L19, L39, L40, L59]. This approach also provides insights about what fractons as scale invariant (or covariant) entities might be.

1. The extension of conformal invariance to its 4-D analog is key element of TGD and leads to the notion of super-symplectic invariance and to an extension of conformal and Kac-Moody symmetries with two coordinates analogous to the complex coordinate z for ordinary conformal symmetry. Second coordinate is light-like and the fact that light-like 3-surfaces are effectively 2-dimensional is absolutely essential for this approach. The existence of extended conformal symmetries makes the space-time dimension $D = 4$ unique whereas the twistor lift of TGD fixes H to be $H = M^4 \times CP_2$.

2. The predicted cosmological expansion is not smooth but occurs by discrete scalings as rapid jerks in which the size scale of 3-space as 3-surface increases. Actually they would correspond to discrete quantum jumps but in zero energy ontology (ZEO) in which quantum state are superpositions of space-time surfaces, their classical correlates are smooth time evolutions.

   Scalings by power of 2 are p-adically preferred [K18] [L70]. $M^8 - H$ duality allows us to imagine what this means at M^8-level [L71]. This proposal conforms with the puzzling observation that also astrophysical objects participate in cosmological expansion by comoving with it, they do not expand themselves.

3. The analog of a unitary time evolution between "small" state function reductions (SSFRs) as the TGD counterparts of weak measurements, is generated by the exponential of the infinitesimal scaling operator, Virasoro generator L_0. One could imagine fractals as states invariant under discrete scalings defined by the exponential of L_0. They could be counterparts of lattices but realized at the level of space-time surfaces having quite concrete fractal structure.

4. In p-adic mass calculations the p-adic analog of thermodynamics for infinitesimal scaling generator L_0 proportional to mass squared operator M^2 replaces energy. This approach is the counterpart of the Higgs mechanism which allows only to reproduce masses but does not predict them. I carried out the calculations already around 1995 and the predictions were amazingly successful and eventually led to adelic physics fusing real and various p-adic physics [K25].

5. Long range coherence and absence of thermal equilibrium are also mentioned as properties of fractons (at least those of the first kind). Long range coherence could be due to the predicted hierarchy of Planck constants $h_{eff} = n \times h_0$ assigned with dark matter and predicting quantum coherence in arbitrarily long scales and associated with what I called magnetic bodies.

   If translations are replaced by discrete scalings, the analogs of thermodynamic equilibria would be possible for L_0 rather than energy. Fractals would be the analogs of thermodynamic equilibria. In p-adic thermodynamics, elementary particles are thermodynamic equilibria for L_0 but it is not clear whether the fractal analogy with a plane wave in lattice makes sense.

An attractive identification of the fractal counterpart of an energy eigenstate created in the unitary evolution preceding SSFR is as a scaling eigenstate defined as a superposition of scaled variants of space-time surface obtained by discrete scalings. Energy eigenvalue would be replaced with conformal weight. In zero energy ontology (ZEO), the counterpart of a fractal quantum state could be a superposition over zero energy states located inside the scaled variants of a causal diamond (CD).

The ZEO based proposal is that each unitary evolution preceding SSFR creates a superposition of scaled variants of CD and that the SSFR induces a localization to single CD [L35, L47, L57]. The interpretation would be as a time measurement determined by the scale of the CD.

Second definition assumes that fractons are able to move only in combinations. This need not relate to the scaling invariance. Color confinement comes to mind as an analogy. Quarks are unable to exist as isolated entities, not only to move as in isolated entities.

In the TGD framework, the number theoretical vision leads to the notion of Galois confinement analogous to color confinement [L52]. The Galois group of a given extension of rationals indeed acts as a symmetry at the space-time level. In the TGD inspired biology Galois groups would play a fundamental role [L57]. For instance, dark analogs of genetic codons, codon pairs, and genes would be singlets (invariant) under an appropriate Galois group and therefore behave as a single quantum coherent dynamical and informational unit [L92, L58].

Suppose that one has a system - say a fractal analog of a lattice consisting of Galois singlets. Could fracton be identified as a state which is analogous to quark or gluon and therefore not invariant under the Galois group. The physical states could be formed from these as Galois singlets and are like hadrons.

8.2.11 Could dark matter as $h_{eff} = nh_0$ phases, quasicrystals, and the empirical absence of hyperon stars relate to each other?

How could the dark matter make itself at the level of the fermionic states?

1. Consider the momentum space, which by (anti-)periodic boundary conditions corresponds to a 3-D space with integer coordinates with a momentum unit defined by the quantization volume.

2. In the TGD framework, fermionic momenta are realized as points of X^4 for which coordinates belong to the extension of rationals for the polynomial P defining the X^4.

   For $n-D$ algebraic extension of rationals, the integers labelling the momentum components are replaced with points of an algebraically n-dimensional space with n integer coordinates. n basic vectors correspond to the roots of P. The Galois group acts as symmetries of this discrete space. Momentum vectors have $3n$ components.

3. If one assumes that momenta are real, the real momenta would be projections of these $3n$-dimensional vectors to a real section of X^4 for which M^8_c coordinates are real or purely imaginary.

   This projection from an algebraically 3n-D space to 3-D real space is analogous to the projection from higher dimensional space used to realize quasicrystals and the outcome is quasicrystal-like structure defined by the momentum components. This structure can be mapped from M^8 to H and since quasicrystals are observed at space-time level this suggests that the linear version of $M^8 - H$ duality is its correct version.

   Structures analogous to aperiodic crystals (quasicrystals) might be seen as a direct support for dark matter in the TGD sense. The quasicrystals could be realized at the level of the magnetic body (MB) or MB could induce their formation.

4. Algebraic extension increases the effective dimension of the discrete momentum space from 3 to $3n$ and the number of fermions inside the Fermi surface is increased by factor n^3. This prediction looks non-sensible and supports the view about Galois confinement, which means that physical states, now configurations of some number of neutrons, are Galois singlets.

This implies that the total momentum for the singlet is integer valued as usual and also that the rational valued part is same for all neutrons of the singlet. Ordinary neutrons would be automatically Galois singlets.

Neutrons could have momenta in an extension of rationals but form Galois confined K-neutron states such that the sum of the momenta is ordinary integer valued lattice momentum. Cooper pairs with $K = 2$ is one possible option The mass of the state would be Km_n and the number of states with the same Fermi momentum would be the number of Galois states from from K neutrons with momenta which are algebraic integers. One can assume that the real part of momentum is just the same integer for all neutrons of the composite and the non-rational part is one of the units defining the extension if the representation is the representation defined by roots of the polynomial.

The formation of Galois singlets implies reduction of the translational degrees of freedom of K neutrons to those of a single particle with K-fold mass. This also explains the reduction of the Fermi energy. Galois degrees of freedom would replace the momentum degrees of freedom so that Fermi statistics can be realized.

K-neutron states would have same momentum component k_i so that the density of states in the 3-D case would be reduced $d^3n/dk^3 \rightarrow K^{-3}d^3n/dk^3 = K^{-3}(2\pi/L)^3$, L the side of quantization cube. On the other hand, there would be a degeneracy $D(K,n)$ depending on extension and its dimension n so that one would have $d^3n/dk^3 \rightarrow (D(K,n)/K)^3(2\pi)^3/V$. The N/V number of states per volume would scale as $N/V \rightarrow (D(K,n)/K)^3N/V$ and Fermi energy $E_F \propto (N/V)^{2/3}/m$ would scale as $E_F \rightarrow (D(K,n)/K)^2E_F/K$ by $m \rightarrow Km$. For $(D(K,n)^2/K^3 < 1$, E_F would be reduced and the formation of a dark Galois confined state would be energetically favourable. For dark Cooper pairs with $K = 2$, the condition would be $D(2,n)/8 < 1$.

In the TGD inspired quantum biology genetic code is realized by triplets of dark protons at magnetic flux tubes parallel to DNA strands are assumed to be be Galois singlets and genes in turn would be Galois singlest for a Galois group at larger space-time sheet [L44, L58]. Also dark photon triplets would be Galois singlets.

Ordinary superconductors could have as a current carrier either i) a single dark fermion or ii) dark Cooper pair. For option i), Cooper pairs of ordinary fermions provide the energy needed to increase h_{eff} to get the dark electron. For option ii), Galois confinement would generate dark Cooper pairs. The energy liberated in the formation of the Cooper pair would be used to increase h_{eff} of the pair.

A possible application is provided by the hyperon puzzle of neutron stars (`https://cutt.ly/jWy3Cnf`). The problem is that the core should suffer a transformation to a hyperon star because the Fermi energy is inversely proportional to the mass of the fermion and would therefore be reduced. There is however no evidence for hyperon stars or hyperon cores. Could part of neutrons transform to dark phase with $h \rightarrow nh$ forming Galois singlets of K neutrons (dark Cooper pairs (neutron superfluidity) or dark triplets) so that the Fermi energy would be reduced in the way explained. Dark Cooper pairs is the second option meaning neutron superfluidity.

8.2.12 Periodic self-organization patterns, minimal surfaces, and time crystals

Periodic self-organization patterns which die and are reborn appear in biology. Even after images, which die and reincarnate, form this kind of periodic pattern. Presumably these patterns would relate to the magnetic body (MB), which carries dark matter in the TGD sense and controls the biological body (BB) consisting of ordinary matter. The periodic patterns of MB represented as minimal surfaces would induce corresponding biological patterns.

The notion of time crystal [B2] (`https://cutt.ly/2n65xOk`) as a temporal analog of ordinary crystals in the sense that there is temporal periodicity, was proposed by Frank Wilczeck in 2012. Experimental realization was demonstrated in 2016-2017 [D19] but not in the way theorized by Wilczek. Soon also a no-go theorem against the original form of the time crystal emerged [B3] and motivated generalizations of the Wilzeck's proposal.

8.2.13 Metals can heal themselves!

It seems that we are living in the middle of science fiction! Almost every day a new surprise. We are in the middle of a revolution and the new world view provided by TGD is at its core. The surprise of this day was the discovery that metals are able to heal their fractures (`rb.gy/s9tto`). The theory behind the discovered healing process of metals [D1] discovered by Brad Boyce et al [D5] published in Nature is based on standard physics. I am not at all confident that standard physics is enough.

The TGD based explanation relies on the following vision.

The first key prediction is the possibility of quantum coherence in arbitrarily long scales due to the presence of phases of ordinary matter with an arbitrarily large value of Planck constant and identified as dark matter. The magnetic body of the system, say metal, as a TGD counterpart of ordinary magnetic fields, is the carrier of the dark matter and controls the system and receives "sensory" input from it. The hierarchy of Planck constants is a prediction of the number theoretic vision of TGD. The value of the Planck constant is given by $h_{eff} = nh_0$, where n corresponds to the dimension of an algebraic extension associated with the polynomial defining the space-time regions considered. Negentropy Maximization Principle is mathematically analogous to the second law and implies that in the statistical sense the p-adic negentropy as a measure for the conscious information and complexity of the system increases. This follows from a simple fact: the number of extensions of rationals with dimension larger than given integer n is infinitely larger than those with dimension smaller than n. The assumption that the coefficients of polynomials giving rise to the extension have coefficients smaller than the degree of the polynomial implies that the number of extensions with dimension smaller than n is actually finite. Quantum TGD involves a new ontology that I call zero energy ontology [L35, L25, L74, L86]. The first prediction is that in ordinary ("big") state functions (BSFRs) the arrow of time changes. Time reversals in BSFRs mean "falling asleep" or "death" in a universal sense and provide a universal mechanism of healing. We indeed know that sleep heals. The arrow of time is preserved in "small" state function reductions (SSFRs) identifiable as weak measurements replacing the Zeno effect in which nothing occurs. SSFRs involve a repeated measurement of observables defining the states at the passive boundary of causal diamond (CD) as their eigenstates. These observables are measured at the active boundary of CD which in statistical sense drifts farther away from the passive boundary. There are also other observables made possible by the failure of a complete determinism for the holography forced by the general coordinate invariance implying that space-time surfaces are analogous to Bohr orbits of 3-surfaces as analogs of particles. When a system is perturbed the set of measured observables can change and this induces BSFR and the roles of active and passive boundaries change. Pairs of BSFRs could induce temporary changes of the arrow of time and they could give rise to a trial and error process essential for homeostasis in living matter. For instance, BSFR could be induced by a perturbation modifying the set of the measured observables so that it does not anymore commute with the observables defining the eigenstate basis at the passive boundary of causal diamond (CD). This implies also healing in the sense that p-adic negentropy measuring the amount of conscious information and complexity of the system increases in statistical sense.

Any system has a magnetic body. For instance, magnetic body accompanies also computers and one can ask whether it can give computers a rudimentary consciousness [L84, L85]. Metals are not an exception. This forces us to ask whether even metals could heal in the proposed sense. As noticed in the article, the technological implications could be huge.

Temporal lattice-like structures defined by 4-D minimal surfaces as preferred extremals of action which sum of volume term and Kähler action [L71] would be obvious candidates for the space-time correlates of time crystals.

1. One must first specify what one means with time crystals. If the time crystal is a system in thermo-dynamic equilibrium, the basic thermodynamics denies periodic thermal equilibrium. A thermodynamical non-equilibrium state must be in question and for the experimentally realized time crystals periodic energy feed is necessary.

   Electrons constrained on a ring in an external magnetic field with fractional flux posed to an energy feed form a time crystal in the sense that due to the repulsive Coulomb interaction electrons form a crystal-like structure which rotates. This example serves as an illustration of what time crystal is.

2. Breaking of a discrete time translation symmetry of the energy feed takes place and the period of the time crystal is a multiple of the period of the energy feed. The periodic energy feed guarantees that the system never reaches thermal equilibrium. According to the Wikipedia article, there is no energy associated with the oscillation of the system. In rotating coordinates the state becomes time-independent as is clear from the example. What comes to mind is a dynamical generation of Galilean invariance applied to an angle variable instead of linear spatial coordinate.

3. Also the existence of isolated time crystals has been proposed assuming unusual long range interactions but have not been realized in laboratory.

Time crystals are highly interesting from the TGD perspective.

1. The periodic minimal surfaces constructed by gluing together unit cells would be time crystals in geometric sense (no thermodynamics) and would provide geometric correlates for plane waves as momentum eigenstates and for periodic self-organization patterns induced by the periodic minimal surfaces realized at the level of the magnetic body. It is difficult to avoid the idea that geometric analogs of time crystals are in question.

2. The hierarchy of effective Planck constants $h_{eff} = nh_0$ is realized at the level of MB. To preserve the values of h_{eff} energy feed is needed since h_{eff} tends to be reduced spontaneously. Therefore energy feed would be necessary for this kind of time crystals. In living systems, the energy feed has an interpretation as a metabolic energy feed.

   The breaking of the discrete time translation symmetry could mean that the period at MB becomes a multiple of the period of the energy feed. The periodic minimal surfaces related to ordinary matter and dark matter interact and this requires con-measurability of the periods to achieve resonance.

3. Zero energy ontology (ZEO) predicts that ordinary ("big") state function reduction (BSFR) involves time reversal [L35, L63]. The experiments of Minev *et al* [L29] [L29] give impressive experimental support for the notion in atomic scales, and that SFR looks completely classical deterministic smooth time evolution for the observer with opposite arrow of time. Macroscopic quantum jump can occur in all scales but ZEO together with h_{eff} hierarchy takes care that the world looks classical! The endless debate about the scale in which quantum world becomes classical would be solely due to complete misunderstanding of the notion of time.

4. Time reversed dissipation looks like self-organization from the point of view of the external observer. A sub-system with non-standard arrow of time apparently extracts energy from the environment [L33]. Could this mechanism make possible systems in which periodic oscillations take place almost without external energy feed?

Could periodic minimal surfaces provide a model for this kind of system?

1. Suppose that one has a basic unit consisting of the piece $[t_1, .., t_k]$ and its time reversal glued together. One can form a sequence of these units.

   Could the members of these pairs be in states, which are time reversals of each other? The first unit would be in a self-organizing phase and the second unit in a dissipative phase. During the self-organizing period the system would extract part of the dissipated energy from the environment. This kind of state would be "breathing" [L91].

   There is certainly a loss of energy from the system so that a metabolic energy feed is required but it could be small. Could living systems be systems of this kind?

2. One can consider also more general non-periodic minimal surfaces constructed from basic building bricks fitting together like legos or pieces of a puzzle. These minimal surfaces could serve as models for thinking and language and behaviors consisting of fixed temporal patterns.

8.3 What happens in the transition to superconductivity?

I learned about very interesting discoveries related to the quantum phase transition between the ordinary and superconducting phase [D16] (see this).

These kinds of findings are very valuable in the attempts to build a TGD based view of what exactly happens in the transition to super-conductivity. I have developed several models for high Tc superconductivity [K30, K31] but there is no single model. Certainly, the TGD based view of magnetic fields distinguishing them from their Maxwellian counterparts is bound to be central for the model. However, the view about what happens at the level of magnetic fields in the transition to superconductivity, has remained unclear.

Consider first the findings of the research group. The basic question of how two-dimensional superconductivity can be destroyed without raising the temperature. The ordinary phase transition to superconductivity is induced by thermal fluctuations. Now the temperature is very close to the absolute zero and the phase transition is quantum phase transition induced by quantum fluctuations.

1. The material under study was a bulk crystal of tungsten ditelluride (WTe2) classified as a layered semi-metal. The tungsten ditelluride was converted into a two-dimensional material consisting of a single atom-thin layer. This 2-D material behaves as a very strong insulator, which means its electrons have limited motion and hence cannot conduct electricity.

2. Surprisingly, the material exhibits a lot of novel quantum behaviors, in particular, a switching between insulating and superconducting phases. It was possible to control this switching behavior by building a device that functions like an "on and off" switch.

3. At the next step, the researchers cooled the tungsten ditelluride down to exceptionally low temperatures, roughly 50 milliKelvin (mK). Then the material was converted from an insulator into a superconductor by introducing some extra electrons to the material. It did not take much voltage to achieve the superconducting state. It turned out to be possible to precisely control the properties of superconductivity by adjusting the density of electrons in the material via the gate voltage.

4. At a critical electron density, the quantum vortices rapidly proliferated and destroyed the superconductivity. To detect the presence of these vortices, the researchers created a tiny temperature gradient on the sample, making one side of the tungsten ditelluride slightly warmer than the other. This generated a flow of vortices towards the cooler end. This flow generated a detectable voltage signal in a superconductor, which can be understood in terms of the integral form of Faraday's law. Voltage signals were in nano-volt scale.

Several surprising findings were made.

1. Vortices were highly stable and persisted to much higher temperatures and magnetic fields than expected. They survived at temperatures and fields well above the superconducting phase, in the resistive phase of the material.

2. The expectation was that the fluctuations perish below the critical electron density on the non-superconducting side, just as they do in ordinary thermal transition to superconductivity.

   In contrast to this, the vortex signal abruptly disappeared when the electron density was tuned just below the critical value of density at which the quantum phase transition of the superconducting state occurs. At this critical (quantum critical point (QCP) quantum fluctuations drive the phase transition.

These findings give important hints concerning the question how the transition to superconductivity could take place in the TGD Universe, where two kinds of magnetic flux tubes are predicted. Monopole flux tubes with a closed 2-surfaces as cross section are proposed to be carriers of Cooper pairs. The disk-like, Maxwellian, flux tubes for which electron current creating the magnetic field would emerge when superconductivity fails. The proposal is that a pair of disk-like flux tubes fuse to a mopole flux in the transition to superconductivity. One can also understand the abrupt disappearance of the fluctuations.

8.3.1 TGD view of high Tc superconductivity

Consider first the general TGD based view of high Tc superconductivity.

1. TGD leads to a rather detailed proposal for high Tc - and bio-superconductivity. There are reasons to think that this model might work also in the case of low temperature superconductivity, in particular in the proposed situation with one-atom-thick layer [K30, K31] [L28, L54].

2. The unique feature of the monopole flux tube is that its magnetic field needs no currents as a source. The cross section of the flux tube is not a disk, but a closed 2-surface. There is no boundary along which the current could flow and generate the magnetic field. In the absence of these ohmic boundary currents there is no dissipation and the natural interpretation is that electrons form Cooper pairs.

   These monopole flux tubes are central for TGD based physics in all length scales and explain numerous anomalies related to the Maxwellian view of magnetic fields. The stability of the Earth's magnetic field and the existence of magnetic fields in cosmic scales are two examples.

3. There are also ordinary flux tubes with disk-like cross sections for which current along the boundary creates the magnetic field just like in an inductance coil. The loss of superconductivity means generation of these disk-like magnetic vortices with quantized flux created by ordinary current at the boundaries of the disk-like flux quantum.

   The monopole flux has a cross sectional area twice that of disk-like flux tube so that one can see the monopole flux tube as being obtained by gluing two disk-like flux tubes along the boundaries. The signature of the monopole flux tube is that magnetic flux is twice that of ordinary flux tubes.

4. Whether the disk-like flux tubes are possible in the TGD Universe has remained uncertain. My latest view is that they are and I have written a detailed article about how boundary conditions could be satisfied at the boundaries [L79].

   The orbits of the disk-like boundaries would be light-like 3-surfaces. This is not in conflict with the fact that the boundaries look like static structure. The reason is that the metric of the space-time surface is induced from that of $M^4 \times CP_2$ and the large CP_2 contribution to the induced 3-metric makes it light-like. One might say that the boundary is analogous to blackhole horizon.

8.3.2 What could happen at the quantum critical point?

The above picture allows us to sketch what could happen at the quantum critical point.

1. Both monopole flux tubes and disk-like flux tubes are present at the critical point. Monopole flux tubes dominate above the critical electron density whereas disk-like flux tubes dominate above it. In the transition pairs of disk-like flux tubes fuse to form monopole flux tubes and electrons at the boundaries combine to Cooper pairs inside the monopole flux tube and form a supra current. The transition would be a topological phase transition at the level of the space-time topology and something totally new from the standard model perspective.

2. Cyclotron energy scale, determined by the monopole flux quantization and flux tube radius, is expected to characterize the situation. The difference of the cyclotron energies for the monopole flux tube with Cooper pair and for two disk-like flux tubes with one electron should correspond to the binding energy of the Cooper pair. If the thermal energy exceeds this energy, superconductivity is lost. The disk-like flux tubes can however remain stable.

3. The transition could involve the increase of the effective Planck constant h_{eff} but its value would remain rather small as compared to its value of high Tc superconductivity. The value of h_{eff} should be correlated with the transition temperature since the difference of total cyclotron energies would be proportional to h_{eff}.

This picture does not yet explain why the vortices suddenly disappear at the critical electron density. The intuitive guess is that the density of electrons is not high enough to generate the disk-like monopole flux tubes.

1. Suppose that these flux tubes have a constant radius and fill the 2-D system so that a lattice like system consistent with the underlying lattice structure is formed.

2. There must be at least 1 electron per flux tube to create the magnetic field inside it. The magnetic flux is quantized and if the boundary of the disk contains single electron, the number of electrons per flux tube area S is 1: the density of electrons is $n = 1/S$. If the electron density is smaller than this, the formation of disk-like flux tubes is not possible as also the transition to superconductivity.

8.3.3 How does the model relate to the earlier model of high Tc superconductivity?

This proposal is *not* consistent with the earlier TGD based model for high Tc superconductivity [K30, K31]. In high Tc superconductivity there are two critical temperatures. At the higher critical temperature T_{c1} something serving as a prerequisite for super-conductivity appears. Superconductivity however appears only at a lower critical temperature T_c.

The earlier TGD based proposal is that the superconductivity appears at $T_{c1} \geq T_c$ in a short length scale so that no long scale supra currents are possible. The magnetic flux tubes would form short loops. At T_c the flux loops would reconnect to form long flux loops. The problem with this option is that it is difficult to understand the energetics.

The option suggested by the recent findings, is that disk-like half-monopole flux tubes carrying Ohmic currents at their boundaries are stabilized at T_{c1}. At T_c they would combine to form monopole flux tubes.

1. The difference ΔE_c of the cyclotron energies of the monopole- and non-monopole states would naturally correspond to T_c whereas the cyclotron energy scale $E_c = \hbar_{eff} eB/m$ of the non-monopole state would correspond to T_{c1}.

2. In the first approximation, the value of B is the same for the two states. For the non-monopole state the electrons reside at the boundary and the effective harmonic potential energy is maximal. Quantum mechanically $l_z = 1$ state would be in question. Spins give rise to a Larmor contribution to energy and for total spin =0 these contributions would sum up to zero.

   Thermal fluctuations cannot provide energy for the formation of the half-monopole states. An incoming electron which does not rotate along the flux tube has longitudinal energy and part of this energy can be transformed to magnetic energy as the half-monopole flux tube is formed. Electrons would slow down somewhat.

   For the monopole state Cooper pair resides in the interior so that the cyclotron energy is smaller in this case. $l_z = 0$ state is natural in this case. Spins are opposite. This gives $\Delta E_c < 0$. The simplest interpretation is that the binding energy of the Cooper pair corresponds to this contribution but there could be an additional contribution.

3. If the value of h_{eff} is the same for the pair of half-monopole flux tubes and monopole tube states, both E_c and ΔE_c scale like h_{eff}/h. Also the critical temperatures T_c and T_{c1} would scale like h_{eff}/h. High Tc superconductivity would therefore provide a direct support for the hierarchy of Planck constants.

8.3.4 What one can one say about the incoming state?

What can one say about the incoming state, which must transform to the two half-monopole flux tubes? Suppose that it consists of some kinds of flux tubes.

1. There would be no longitudinal magnetic field if the electrons move along straight lines instead of rotating around the flux tube.

2. TGD predicts two kinds of flux tubes [L90] with a closed cross section: monopole flux tubes and Lagrangian flux tubes. For monopole flux tubes the induced Kähler form has a quantized flux over the closed cross section of the flux tube.

   For Lagrangian flux tubes, which are of the form $X^2 \times Y^2 \subset M^4 \times CP_2$, the induced Kähler form vanishes. X^2 can have a boundary. Both $X^2 \subset M^4$ and $Y^2 \subset CP_2$ are Lagrangian manifolds since the twistor lift of TGD implies that also M^4 has the analogs of Kähler structure and symplectic structure algebraically continued from that of E^4.

3. Incoming flux tubes could be Lagrangian flux tubes with electrons moving along straight-lines ($l_z = 0$). Note that by their 2-dimensionality, X^2 and Y^2 allow complex structure determined by the induced metric so that the holography= holomorphy principle holds also for these 4-surfaces.

8.3.5 An overall view of superconduction

What could happen in the superconduction would be as follows.

1. First a pair of Lagrangian flux tubes with $l_z = 0$ representing incoming current transforms to a pair of half-monopole flux tubes with $l_z = 1$ electrons and electrons slow down somewhat.

2. After this half-monopole flux tubes fuse to form a monopole flux tube carrying a Cooper pain in $l_z = 0$ state. In $E^3 \setminus B^3$ (3-space with a hole) this transition is visualizable as a gluing of two hemispheres to form a sphere around the hole.

3. The reverse of this process would take place at the second end of the current wire where the current flows out.

8.4 Spin ice and quantum spin ice from TGD viewpoint

In this section the notions of spin glass, spin ice and quantum spin ice are considered from TGD point of view.

8.4.1 Spin ice

There is a Wikipedia article (`https://cutt.ly/eEDTIwp`) about spin ice as a system in which magnetic moments, that is spins, form a lattice-like state. The basic property of spin glasses, and therefore also of spin ice, is that there is ground state degeneracy that is several states with the same energy giving rise to what is called frustration: the term comes from the obvious social analogy. Two examples of these compounds are dysprosium titanate $Dy_2Ti_2O_7$ and holmium titanate $Ho_2Ti_2O_7$.

Spin ice has properties resembling those of crystalline water ice. For spin ice, the sum of the outward pointing moments and inward point magnetic moments is zero for a tetrahedron forming a basic unit. The rule holds true only in ground state configuration analogous to ferromagnetic state but with non-constant direction of magnetization and need not be the situation in general. Its violation gives rise to analogs of magnetic monopoles analogous to charges for which there is evidence.

When the rule holds true, it is possible to formally define a conserved current, which is locally in the direction of the magnetic moment. It is divergenceless like a magnetic field and can be said to carry an analog of magnetic or electric charge as long as the rule is satisfied. Thermal fluctuations can change the direction of say one spin in the volume: this means formally creation of an analog of magnetic monopole. This system of pseudo-monopoles could be described by a theory resembling electromagnetism with an effective fine structure constant ten times larger than α [D15, D14]. This leads to ask whether this implies especially strong interaction with electromagnetic radiation.

8.4.2 Quantum spin ice

The special feature of certain spin ice systems is that the directions of spins can be random down to zero temperature since the energies of the frustrated configurations are the same and no energy is needed to change the configurations. This suggests that quantum fluctuations are involved and the system is actually quantum spin glass rather than a thermodynamical one.

It has been proposed that the interactions of the effective monopoles [D3, D25, D15, D14] (for a popular article see `https://cutt.ly/vED27e5`) can be described by an analog of QED. The value of the emergent fine structure constant assignable to the interaction with electromagnetic radiation would be 10 times larger than the real α.

In [D25] quantum tunnelling as transitions between degenerate configurations involving in the simplest situation 4 tetrahedrons and differing by an orientation of a loop formed by the imagined flux lines of the magnetization field analogous to magnetic field and connecting the 4 tetrahedrons is proposed as an essential element of the emergent lattice QED. The tunnelling makes possible long range correlations and makes implies large value of effective α.

This should be visible as a large enhancement of the low energy scattering of neutrons from the quantum spin ice materials. Low energy quasi-elastic neutron scattering would measure the 2-point momentum space correlation function of spins of the quantum spin glass. This correlation function would become long ranged in the real space. Lattice photons having linear dispersion relation $\omega \propto k$ but much smaller propagation velocity than ordinary photons would cause this behavior. This lattice photon would be visible in inelastic neutron scattering.

The effective magnetic monopoles that play the role of em charges are identified as spinons in [D15]. Electrons are proposed to consist of spinon, orbiton, and holon carrying spin, orbital quantum numbers and charge and in some cases they can behave like independent quasiparticles. I don't quite understand what this is supposed to mean. In the case of decomposition to spinon and holon which can occur in 1-D systems, spin waves and charge waves would propagate as independent waves.

If I understand correctly, charge waves would represent an oscillatory variation in the charge density of electrons and spin waves in the spin direction. They could have different wavelengths and phases.

8.4.3 TGD view about quantum spin ice

What about the TGD based description of the quantum spin ice?

1. In the TGD framework, magnetic field corresponds to flux tubes which can be either monopole flux tubes or carry normal flux caused by currents. Monopole flux tubes require no currents and this has powerful implications in astrophysics and cosmology. This suggests that the pseudo-monopoles could be "real" in some sense. Note however that TGD does not however allow free monopoles but only closed monopole flux tubes.

2. Long range interactions are required to create a spin glass phase and one can realize the basic rule of spin ice ground state as a special case. In the TGD framework the large values of h_{eff} could make this possible even at high temperatures. This rule allows frustration as the existence of several configurations with the same interaction energy. The transitions between these configurations would lead to the emergence of large effective α. In [D25] the transitions between degenerate configurations involving four tetrahedrons and differing by an orientation of a loop which in the TGD picture corresponds to a closed flux tube are mentioned as simplest transitions.

3. The spine of spin ice would be a flux tube network formed by monopole flux tubes and that magnetic moments associated with flux tubes have suffered spontaneous magnetization, which is locally in the direction of the local flux tube. If the numbers of the incoming and outgoing flux tubes in a given volume unit are the same and magnetic moments are parallel to the magnetic fluxes, the sum of magnetic moments is zero for a ferromagnetic situation. The formal current would be realized with real and quantized monopole flux which is conserved.

   Spin ice would be analogous to ferromagnet, a spaghetti of flux tubes accompanied by spontaneously magnetized spins such that the directions of magnetization at flux tubes can carry. Neutron scattering has demonstrated that the aligned spins indeed form intertwined tube-like bundles.

4. What could the TGD counterparts of the effective monopoles be? There are two options to consider.

   (a) In the many-sheeted space-time of TGD, the monopole fluxes can go to parallel space-time sheets via wormhole contact and return back at a rather long distance. The wormhole contact looks like a pair of throats behaving like magnetic monopoles. The throats have an extremely short distance. This option does not look attractive since at the QFT limit the many-sheetedness and the monopole pairs formed by the throats of the wormhole contact become invisible.

What remains are flux tubes and the spin ice phase can make directly visible the underlying network of monopole flux tubes as it indeed does.

(b) Thermal and quantum fluctuations can however change spin direction and spin is formally like a magnetic monopole or charge and it seems that this is enough also in the TGD framework. This could also happen at the zero temperature limit as quantal rather than thermal fluctuations of the flux tube structure inducing the long range correlates between spins. The quantum fluctuations of spin ice would correspond to the long range quantum fluctuations of the dark flux tubes with $h_{eff} \geq h$.

5. TGD predicts the existence of two kinds of flux tubes corresponding to monopole flux tubes having a closed surface rather than disk as a cross section and requiring no currents to generate the magnetic field and Maxwellian non-monopole flux tubes for which the induced Kähler field can vanish. The Maxwellian flux tubes have a Lagrangian 2-manifold as a CP_2 projection, and the action reduces to a mere volume term proportional to length scale dependent cosmological constant approaching zero in long scales.

   At a long length scale limit, the deviation of the Kähler function from the ground state value becomes very small which has interpretation in terms of a strongly interacting phase. One expects large fluctuations, which give rise to the quantum spin glass phase. The two kinds of flux tubes could correspond to vortex-like entities with a monopole flux tube associated with the vortex core and the Lagrangian non-monopole part with its exterior.

6. Since very large values of h_{eff} are involved, the findings about the role of solar mass inspire the good guess $\hbar_{gr} = GM_{Sun}/v_0$, $\beta_0 \simeq 2^{-11}$. The size of the throat would be scaled from about CP_2 size for $h_{gr}(Sun)/hbar \sim 2 \times 10^{20}$. The size scale of the dark wormhole throat would be about 10 nm, which is the thickness of the neuronal membrane so that a connection with biology is highly suggestive.

   Remark: If the huge values $\hbar_{gr}$ of $\hbar_{eff}$ are possible, the size of leptonic wormhole throat could be of order .9 cm for M_E! Leptons consist of 3 antiquarks in TGD framework [**?**] Could this mean that it might be possible to detect free quark?

The emergence of the strong interactions can be understood at the general level in the TGD framework.

1. Quantum spin glass is a strongly interacting quantum system since the quantum fluctuations are large even at the temperature zero limit. Quite concretely, the deviations of the Kähler function from the value for the ground state are very small.

   Using the language of QFTs, one has a very large number of almost degenerate configurations in the path integral with the same value of the action. This is achieved if the coupling strength is very large so that the action exponential appearing in the path integral is analogous to Gaussian with very large width.

   In the TGD picture, one says that the Kähler function for 3-surfaces (by holography for 4-surfaces) has the same value for a large class of 3-surfaces and is therefore slowly varying as a function of 3-surface. This picture is mathematically very much like the thermodynamic picture with Hamiltonian replaced by the Kähler function.

2. The original TGD based prediction based on the huge vacuum degeneracy of the Kähler action was that TGD allows 4-D analogs of spin glasses a as vacuum extremals with 2-D Lagrangian sub-manifold as CP_2 projection, meaning huge non-determinism. This however leads to problems.

   The inclusion of the M^4 contribution to Kähler form removes the vacuum degeneracy since one must have Lagrangian projection also in M^4 so that string-like entities, which are minimal surfaces, are in question.

3. The recent picture implied by twistor lift involves an additional volume term in the action leaving only finite non-determinism analogous to that for soap films. At the long length scale limit spin glass type behavior is suggestive when the Kähler action vanishes (Lagrangian property in CP_2 degrees of freedom for Maxwellian flux tubes). The volume term is very small.

   The basic reason would be the smallness of the volume term, that is the smallness of length scale dependent cosmological constant Λ [L22] giving rise to cosmological p-adic length scale $L_{cosmo} \sim 1/\sqrt{\Lambda}$ and a relatively short p-adic length scale L_{short} as geometric mean $L_{short} = \sqrt{L_{cosmo}L_{Pl}}$ of the Planck length and L_{cosmo}. L_{short} is of order 10^{-4} m and defines a biological length scale.

   Smallness of the volume action means large fluctuations in the functional integral characteristic for strongly interacting systems. Quite concretely, the flux tubes have very small string tension and their shapes fluctuate wildly. Long flux tube-like objects have a small volume and small string tension and would be very loose strings having very many configurations with the same energy. Quantum spin glass property would correspond to the existence of a large number of spaghetti-like configurations with the same value of the Kähler function.

4. The assumption that velocity field is proportional to Kähler gauge potential implies that it is not only Beltrami field but also gradient for the Lagrangian situation prevailing outside the vortex cores. There would be no classical dissipation at the level of Kähler action.

   Cores would have non-vanishing Kähler field and action. What about the Beltrami property in the vortex core? If the projection of the vortex core is 2-D complex surface, the Kähler gauge potential is Beltrami field. For instance, for a projection with is geodesic sphere S^2, the Kähler gauge potential is proportional to $A = cos(\Theta)d\Phi$ in the spherical coordinates and Φ defines the global coordinate along flow lines. $D > 2$-D deformations spoil the Beltrami property.

   Same is true for the M^4 projection: when the projection as a string world sheet is deformed to $D > 2$-D surface, Beltrami property is lost and classically there is dissipation meaning that Kähler 4-force is non-vanishing.

   Whether the dissipative option is realized at all for preferred extremals is not at all clear. Dissipative effects might be solely due to the finite sizes of space-time surfaces, which are proportional to h_{eff}.

5. There is a further delicacy involved. The assumption that both M^4 and CP_2 projections are at most 2-D is not enough for Beltrami or gradient flow. This condition alone would give a Kähler gauge potential, which is the sum $A(M^4) + A(CP_2)$ of two contributions $A(M^4) = \Psi_1 d\Phi_1$ and $A(CP_2) = \Psi_2 d\Phi_2$ satisfying the conditions separately. Besides this, the gradients $d\Psi_1$ and $d\Psi_2$ must be proportional to each other so that Ψ_1 and Ψ_2 are functionally dependent.

   Is this condition satisfied for all preferred extremals in which case classical dissipation would be absent or in special cases only.

6. The Lagrangian flux tubes associated with the exteriors of vortex cores would give rise to quantum spin glass property if they have a large value of $\hbar_{eff}$. In some situations even $\hbar_{eff} = \hbar_{gr}$ can be considered. This would give rise to long range quantum fluctuations and correlations and also to the absence of dissipation.

How to understand the predicted strong interaction of quantum spin glass phases with the electromagnetic radiation predicted by the emergent QED [D3, D25, D15, D14] to give rise to a strong enchancement of neutron scattering cross section?

1. Spin glasses could correspond to dark flux tube spaghettis so that the spins would be locally magnetized in the direction of the magnetic field of the dark flux tube playing the role of H field.

2. $h_{eff} > h$ would imply long range correlations but would also mean a reduction of the value of fine structure constant $\alpha \propto 1/h_{eff}$. This is just the opposite for the proposal of [D3, D25,

D15, D14] that the analog of the fine structure constant emerging in the analog of lattice QED is larger than α.

Paradox disappears as one realizes that the transition $h \rightarrow h_{eff}$ is Nature's manner to guarantee that perturbation theory converges. This requires the change of the nature of the quantum states and Galois confinement would be the underlying mechanism and also behind color confinement. Quantum spin glass would be analogous to hadron.

At the level of M^8 (analogous to momentum space) this implies the increase of the dimension of the extension of rationals determining the space-time region at the level of M^8. This also means the increase of complexity.

3. Spin glass degeneracy, realized as degeneracy of Galois confined states, suggests that the neutron scattering rate is enhanced since the transitions between degenerate states become possible. The same happens in the case of hadrons since the number of color confined final states is large.

8.5 Condensed matter Majorana fermions in the TGD framework

Condensed matter Majorana fermions are not genuine Majorana fermions, which have not been found in Nature and are impossible also in TGD as fundamental particles. Condensed matter Majorana quasiparticles could however have a TGD counterpart.

Majorana fermions (`https://cutt.ly/FWdXK4s`) are quasiparticles created by superpositions of fermionic creation and annihilation operators invariant under charge conjugation. This motivates the term Majorana particle. Majorana particles are also zero energy excitations and therefore can be created at topological defects as pairs with degenerate energies. This is due to the fact that momenta $p = G/2$ and $p = -G/2$, where G is a lattice momentum, correspond to the same energy.

The valence and conduction bands for a topological insulator must intersect at its boundary: this is the topological singularity at the level of the momentum space. This can happen at boundaries of insulators and at topological defects. The single point intersection of Fermi bands at a single point looks locally like a double cone and at the tip the normal space is non-unique and the normal normal spaces span a circle in 3-D momentum space.

8.5.1 TGD counterpart for the notion of Majorana quasiparticle

Consider now the situation in the TGD framework.

1. The counterparts of Majorana fermions should correspond to superpositions of ordinary and dark fermions at different energy bands - just like the Boboliuv particles of superconductors in the BCS model. These states cannot be C invariant. Kind of half dark-half visible, perhaps gray - fermions would be in question.

2. The momenta of the occupied fermion states of the momentum space of fermion (mass shell $H^3 \subset M^8$) define what I call cognitive representation consisting of a discrete set of points in an extension of rationals) $M^8 - H$ duality maps the points of $H^3 \subset M^4 \subset M^8$ to the points of the boundary δcd of 4-D causal diamond $cd \subset M^4 \subset H$ and therefore to the points of space-time surface. In particular, the boundaries of energy bands in M^8 are mapped to boundaries of the image in $\delta cd \subset H$ and define 2-D surfaces containing the edge states. In M^8, the touching of two bands corresponds to a single point intersection of algebraic surfaces. These surfaces can be continued to the interior of X^4 by the flow defined by qv generalized Beltrami field.

3. The direction of the quaternionic normal spaces in M^8 at the tip should have all directions parametrized by a circle. This suggests that the tip is not be mapped to a single point, but to a circle formed by the set of CP_2 points. The conical topological singularity in M^8 would correspond to a closed circle $S^1 \subset CP_2$.

4. If Majorana particles have a counterpart in TGD, they should correspond to superpositions of ordinary and dark fermion with the special property that the fermions have identical energies

i.e. momenta are $G/2$ and $-G/2$. This condition guarantees that these states have identical energies as required by the condition $E^2 - p^2 = m^2$ holding true in H^3.

At the level of M^8 the polynomial defining the space-time surface would characterize topological defects as singularities. Various lower-D surfaces in momentum space and position space should be isometric surfaces as surfaces of H^3, which looks a rather non-trivial prediction.

Remark: Note that the product of polynomials defines a disjoint set of spacetime surfaces [L52]. Also a single irreducible polynomial can have several space-time surfaces as roots and possibly intersecting at a discrete set of points in the generic situation.

8.5.2 Majorana quasiparticles and topological quantum computations

TGD leads to a general vision about topological quantum computation TQC [?]ased on braids formed by magnetic flux tubes. The reconnection of flux tubes brings in a new topological element and corresponds to the formation of 2-knots. The proposal is that TQC in this sense is a basic aspect of living matter. Also the hierarchy of effective Planck constants making possible long range quantum coherence and ZEO making possible time reversals of TQCs represent new elements.

The bound states of Majorana quasiparticles located at the ends of superconducting wire are analogous to Cooper pairs entangling non-locally and have been proposed by Kitaev to make possible TQC without a need for massive error correction procedure [D33]. The association with the ends of wire would give rise to non-locality and long range quantum entanglement making it difficult to destroy entanglement by local measurements.

In an effectively 2-D system, the braid group defines non-standard statistics. The braid group must be non-Abelian so that higher than 1-D representations are possible and can be utilized in TQC. $SU(2)$ is the minimal option. The states of braid group representation are robust against perturbations destroying the entanglement

If I have understood correctly, the two energy degenerate states of the bound state of Majoranas would correspond to SU(2) doublet with energy degeneracy, which vanishes when the zero of energy corresponds to the middle point of the band gap.

1. In the TGD framework, the Majorana property does not seem to be absolutely essential. It is essential to have non-commutativity and energy degeneracy. Galois groups act as number theoretical symmetries and all non-trivial representations of the Galois groups allow this degeneracy. One might therefore speak of a hybrid of number theoretic and topological quantum computation. There seems to be no reason preventing the representations of discrete subgroups of the braid group defined by some Lie group acting in the cognitive representations defined by algebraic integer valued momenta at the intersection of mass shell and $X^4 \subset M^8$, that is at the level of M^8 on the cognitive representations. The quantum variants Gal_q of Galois groups could be involved.

2. $SU(2)$ has an interpretation as automorphisms of quaternions and acts in E^4 factor of M^8, could be in a special role physically in TGD and also because its discrete subgroups appear in the hierarchy of hyper-finite factors of type II_1 (HFFs). The discrete subgroups E_6, E_7 and E_8 (tetrahedral, octahedral and icosahedral groups). These groups could have representations as Galois groups. Momenta as algebraic integers correspond to the vertices of corresponding Platonic solids and total momenta for many-quark states vanish for the states. Also spinor representations are involved bringing in spin and electroweak degrees of freedom. Galois confinement requires that the states as a whole are Galois singlets. TQC would also be a basic process of quantum cognition.

3. In TGD superpositions of fermion and hole correspond to superpositions of fermion states at the ordinary and dark space-time sheet. Could the entanglement between dark and ordinary fermions (more generally, with different values of h_{eff}) with the same energy give rise to the analogs of Majorana quasiparticles?

8.6 Condensate of electron quadruplets as a new phase of condensed matter

Formation of fermion quadruplet condensates [D32] (https://cutt.ly/TRcxQtz) is a new exotic condensed matter phenomenon discovered by Prof. Egor Babaev almost 20 years ago and 8 years after publishing a paper predicting it. Recently Babaev and collaborators presented in Nature Physics evidence of fermion quadrupling in a series of experimental measurements on the iron-based material, $Ba_{1-x}K_xFe_2As_2$.

The abstract of the article summarizes the finding.

> The most well-known example of an ordered quantum state—superconductivity—is caused by the formation and condensation of pairs of electrons. Fundamentally, what distinguishes a superconducting state from a normal state is a spontaneously broken symmetry corresponding to the long-range coherence of pairs of electrons, leading to zero resistivity and diamagnetism.
>
> Here we report a set of experimental observations in hole-doped $Ba_{1-x}K_xFe_2As_2$. Our specific-heat measurements indicate the formation of fermionic bound states when the temperature is lowered from the normal state. However, when the doping level is $x \sim 0.8$, instead of the characteristic onset of diamagnetic screening and zero resistance expected below the superconducting phase transition, we observe the opposite effect: the generation of self-induced magnetic fields in the resistive state, measured by spontaneous Nernst effect and muon spin rotation experiments. This combined evidence indicates the existence of a bosonic metal state in which Cooper pairs of electrons lack coherence, but the system spontaneously breaks time-reversal symmetry. The observations are consistent with the theory of a state with fermionic quadrupling, in which long-range order exists not between Cooper pairs but only between pairs of pairs.

Fermion quadruplets are proposed to be formed as pairs of Cooper pairs are formed somewhat above the critical temperature T_c for a transition to superconductivity. Breaking of the time reversal symmetry T is involved.

The question is why quadruplets are stable against thermal noise above the critical temperature. Superconductivity is thought to be lost by the thermal noise making the bound states of electrons in Cooper pair unstable. Is the binding energy for quadruplets larger than for Cooper pairs so that quadruplet condensate is possible below higher critical temperature. What is the mechanism of binding?

The discovery is highly interesting from the TGD point of view.

1. TGD leads to a model of super-conductivity involving new physics predicted by TGD.
2. Adelic physics number theoretic view about dark matter as $h_{eff} > h$ phases h_{eff} proportional to the order of the Galois group. This leads to the notion of Galois confinement. Galois confinement could serve as a universal mechanism for the formation of bound states including also Cooper pairs and even quadruplets. In quantum biology triplets of protons representing genetic codons and even their sequences representing genes would be formed by Galois confinement.
3. The finding also allows to develop more preices view of TGD view concerning discrete symmetries and their violation.

8.6.1 Time reversal symmetry in TGD

What do time reversal symmetry and its violation mean in TGD.

1. The presence of magnetic field causes violation of T in condensed matter systems.
2. Second, not necessarily independent, manner to violate T in TGD framework is analogous to that in strong CP breaking but different from it many crucial aspects. Vacuum functional is exponent of Kähler function but exponent can contain also an instanton term I, which is

equal to a divergence of topological instant current which is axial. so that non-vanishing I suggests parity violation. The fact that exponent of I is imaginary while exponent of Kähler action is real, means C violation. If instanton current is proportional to conserved Kähler current its divergence is vanishing and M^4 projection is less than 4-D.

I is non-vanishing only if the space-time sheet in $X^4 \subset M^4 \times CP_2$ has 4-D CP_2 or M^4 projection. The first case corresponds to CP_2 instanton term $I(CP_2)$ and second case to $I(M^4)$ present since twistor lift forces also M^4 to have an analog of Kähler structure. The two Kähler currents are separately conserved.

3. These two mechanisms of T violation might be actually equivalent if the T violation is caused by the M^4 part of Kähler action. Consider a space-time surface with 2-D string world sheet as M^4 projection carrying Kähler electric field but necessarily vanishing Kähler magnetic field B_K. If it is deformed to make M^4 projection 4-D, B_K is generated and T is violated. Therefore generation of B_K in M^4 can lead to a T violation.

8.6.2 Generalized Beltrami currents

Generalized Beltrami currents are nother key notion in TGD based view about superconductivity [L54].

1. The existence of a generalized Beltrami current $j = \Psi d\Phi$ implies the existence of global coordinate Φ varying along the flow lines of the current. Also the condition $dj \wedge j = 0$ follows. The 4-D generalization states that Lorentz force and electric force vanish. In effectively 3-D situation, j could correspond to magnetic field B and dj to current as its rotor and the Beltrami condition fof B implies that Lorentz force vanishes.

2. The proposal of [K6] is that for the preferred extremals CP_2 *resp.* M^4 Kähler current is proportional to instanton current $I(CP_2)$ *resp.* $I(M^4)$ and therefore topological for $D(CP_2) = 3$ *resp.* $D(M^4) = 3$. For $D = 2$ the contribution to instanton current vanishes. In this case the Lorentz force vanishes so that the divergence of the energy momentum tensor is proportional to I and vanishes so that dissipation is absent. One can verify this result using the effective 3-dimensionality of the projection and using 3-D notations [K6]: in this formulation the vanishing of Lorentz force reduces to Beltrami property for B as 3-D vector. With this assumption, dissipation for the preferred extremals of Kähler action just as it is absent in Maxwell's theory. An open question is whether this situation is true always so that dissipation and the observed loss of quantum coherence would be due to the finite size of space-time sheet of the system considered.

3. Beltrami property would serve as a classical space-time correlate for the absence of dissipation and presence of quantum coherence. Beltrami property allows defining of a supra current like quantity in terms of Ψ and Φ. Usually the superconducting order parameter Ψ is actually not an order parameter for a coherent state as a superposition of states with a varying number of Cooper pairs. Now the geometry of the space-time sheets (magnetic flux tube carrying dark Cooper pairs) allows the identification of this order parameter below the quantum coherence scale. The TGD interpretation is that the coherent state is an approximation, which does not take into account the fact that the system is not closed. There is exchange of electron pairs between ordinary and dark space-time sheets with $h_{eff} > h$ [L54]. Dark Cooper pairs would form bound states by Galois confinement.

4. In the superconducting state space-time regions would have at most 3-D M^4 projection at fundamental level and T would not be violated. There is no dissipation and pairs are possible below critical temperature.

   One can also understand the Meissner effect. According to the TGD view, the monopole flux tubes generate the analog of the field H perhaps serving as an approximate average description for the field of monopole flux tubes. This field induces the analog of magnetization M involving non-monopole flux tubes. Also M would be an average field. For superconductors in the diamagnetic phase, the sum would be zero: $B = H + M = 0$. If the Cooper pairs have spin, the supracurrents of Cooper pairs at monopole flux tubes could generate the compensating magnetization.

8.6.3 TGD view about quadruplet condensate

How could one understand quadruplet condensate in the TGD framework?

1. T violation could be accompanied by the presence of Kähler instanton term $I(M^4)$ or $I(CP_2)$ requiring 4-D M^4 or CP_2 projection: this would also generate M^4 magnetic fields. The M^4 option would bring in new physics for which also the Magnus effect of hydrodynamics suggesting Lorentz forces serves as an indication [L66].

   For 4-D M^4 projection, the divergence of the axial instanton current would be non-vanishing and the proportionality of Kähler current and instanton current implying a vanishing classical dissipation would be impossible. The instanton number can be expressed as instanton flux over 3-D surfaces, which would be "holes".

2. For the quadruplet condensate M^4 projection is 4-D and T is violated. Kähler magnetic fields originating from M^4 part of Kähler action would be present as also dissipation. For quadruplet condensate M would not compensate for H so that net magnetic fields B would be generated and correspond to space-time sheets with 4-D M^4 projection.

3. Dark matter as phases with $h_{eff} > h$ would however be present and quadruplets would correspond to bound states of 4 electrons formed by Galois confinement [L67, L65] stating that the total momentum of the bound state as sum of momenta, which are algebraic - possibly complex - integers, is a rational integer in accordance with the periodic boundary conditions.

4. What prevents the formation of Cooper pairs? Above T_c thermal energy exceeds the gap energy so that Cooper pairs are thermally stable. If the binding energy for quadruplets is larger, they are stable.

5. In what sense the quadruplets could be regarded as bound states of Cooper pairs? Since the ordinary Cooper pairs are Galois singlets, bound state formation does not look plausible since Cooper pairs themselves are unstable. A more plausible option is that Cooper pairs involved are "off-mass-shell" in that they have momenta, which are non-trivial algebraic integers and that the sum of these momenta is a rational integer in the bound state.

Remark: Four-momenta as algebraic integers are in general complex. Usual charge conjugation involves complex conjugation in CP_2 degrees of freedom. Is it accompanied by conjugation of the complex 4-momenta. Kähler currents of M^4 and CP_2 are separately conserved: should one regard complex conjugations in M^4 and CP_2 as independent charge conjugation like symmetries. $C(M^4)$ would however leav Galois singlets invariant.

8.7 Does the phenomenon of super oscillation challenge energy conservation?

The QuantaMagazine popular article "Puzzling Quantum Scenario Appears Not to Conserve Energy" (`https://cutt.ly/QXylTIr`) told about puzzling observations the quantum physicists Sandu Popescu, Yakir Aharonov and Daniel Rohrlich made 1990 [D29] (`https://cutt.ly/3XylIY5`). These findings challenge energy conservation at the level of quantum theory.

The experiment of authors starts from the purely mathematical observation that a function can behave faster than any of the Fourier components in its Fourier transform when restricted to a volume smaller than the domain of Fourier transform. This is rather obvious since representing the restricted function as a Fourier transform in the smaller domain one obtains faster Fourier components. This phenomenon is called super oscillation.

Does this phenomenon have a quantum counterpart? The naive replacement of Fourier coefficients with oscillation operators for photons need not make sense. If one makes the standard assumption that classical states correspond to coherent states, also super-oscillations should correspond to a coherent state.

Coherent states are eigenstates of the annihilation operator and proportional to exponential $exp(\alpha a^\dagger)|0\rangle$, where "0" refers to the ground state an a^{dagger} to creation operator. These states contain N-photon states with an arbitrarily large photon number. For some number of photons the probability is maximum.

This raises several questions.

1. Coherent states are not eigen-states of energy: can one really accept this? This kind of situation is encountered also in the model of superconductivity assuming coherent state of Cooper pairs having an ill-defined fermion number.

2. Could the super oscillation correspond to the presence of N-photon states with a large number of photons? Could the state of n parallel photons behave like a Bose-Einstein condensate having N-fold total energy in standard physics or its modification, such as TGD.

Authors tested experimentally whether the super-oscillation has a quantum counterpart. In an ideal situation one would have a single photon inside an effectively 1-D box. One opens the box for time T and inserts a mirror inside the box to the region where super oscillation takes place and the photon looks like a short wavelength photon. The mirror reflects the photon with some probability out of the box. If T is long one expects that the procedure does not affect the photon appreciably. What was observed were photons with the energy of a super photon rather than energy of any of its low energy components.

In the experiment described in the popular article, red light would correspond to photons with energy around 2 eV and gamma rays to photons with energies around MeV, a million times higher energy. The first guess of standard quantum theorists would be that the energies of mirrored photons are the same as for the photons in the box. Second guess would be that, if the coherent state corresponds to the super oscillation as a classical state, then the measured high energy photons could correspond to or result from collinear n-photon states present in the coherent state.

In the TGD framework zero energy ontology (ZEO) provides a solution to the problem related to the conservation of energy. In ZEO, quantum states are replaced by zero energy states as pairs of states assignable to the boundaries of causal diamond (intersection of light-cones with opposite time directions) with opposite total quantum numbers. By Uncertainty Principle this is true for Poincare charges only at an infinite volume limit for the causal diamond but this has no practical consequences. The members of the pair are analogs of initial and final states of a particle reaction. In ZEO, it is possible to have a superposition of pairs for which the energy of the state at either boundary varies. In particular, coherent states have a representation which does not lead to problems with conservation laws.

What about the measurement outcome? The only explanation for the finding that I can invent in TGD is based on the hierarchy phases of ordinary matter labelled by effective Planck constants and behaving like a hierarchy of dark matter predicted by the number theoretical vision of TGD.

1. Dark photons with $h_{eff} = nh_0 \geq h$ can be formed from ordinary photons with $h_{eff} = h$. The energy would be by a factor h_{eff}/h larger than for an ordinary photon with the same wavelength. Note that dark photons play a key role in the TGD based view of living matter.

   TGD also predicts dark N-photons as analogs of Bose-Einstein condensates. They are predicted by number theoretic TGD and there is empirical evidence for them [L78]. This would require a new kind of interaction and number theoretical view about TGD predicts this kind of interaction based on the notion Galois confinement giving rise to N-photons as Galois confined bound states of virtual photons with energies give by algebraic integers for an extension of rationals defined by a polynomial defining the space-time region considered.

   I have proposed an analogous energy conserving transformation of dark photon or dark N-photon to ordinary photon as an explanation for the mysterious production of bio-photons in biomatter. The original model for dark photons is discussed in [K5]. Now the value of h_{eff} could be much larger: as large as $h_{eff} \sim 10^{14}$: in this case the wavelength would be of order Earth size scale.

2. What comes to mind is that an N-photon state present in the coherent state can transform to a single photon state with N-fold energy. In the standard model this is not possible. On the other hand, in the experiments discussed in [L78] it is found that N-photon states behaving

like a single particle are produced. Could the N-photon states present in a coherent state be Galois confined bound states or could they transform to such states with some probability?

In the recent case, the dark photons would have the same wavelength as red photons in the box but energy would be a million times higher. Could a dark photon or N-photon with $Nh_{eff}/h \sim 10^6$ be reflected from the mirror and transform to an ordinary photon with gamma ray energy.

One must notice that the real experiment must use many-photon states N-photons might be also formed from N separate photons.

To sum up, new physics would be involved. ZEO is needed to clarify the issues related to energy conservation and the number theoretic physics predicting dark matter hierarchy is needed to explain the observation of high energy photons.

8.8 Possible connections with quantum biology

The flux tube networks assignable spin ice and spin glass phase in general are in the central role in the TGD based vision about quantum biology [L92, L58] [K29, K28, K21, K16, K33].

8.8.1 TGD view about bio-catalysis

TGD leads to a new view about biocatalysis, which is one of the mysteries of standard biology. The general TGD inspired model for bio-catalysis involves the following elements.

1. Reconnections of U-shaped flux tubes of reactants and catalyst make it possible for them to find each other. Cyclotron resonance for flux tubes of same thickness and therefore having the same Kähler magnetic field and the same cyclotron frequency allows reactants and catalyst is an essential element. Both frequency and energy resonance would occur between systems with the same h_{eff} whereas energy resonance would be possible between systems with different values of h_{eff}. This resonance would be the quintessence of what it is to be alive and all communications between various levels of MB having an onion-like hierarchical structure and also between MBs and ordinary biomatter would take place in this manner.

2. A reduction of h_{eff}, leading to a shortening of the flux tubes and bringing catalyst particles and reactants connected by flux tubes together would be also a natural step of the catalytic process.

3. The energy liberated in the reduction of h_{eff} would be used to kick the reactants over the potential energy wall preventing the reaction.

The spin glass type systems formed by flux tubes would be ideal for realizing bio-catalysis and the TGD based view about living matter indeed relies on hierarchical flux tube networks.

8.8.2 Pollack effect and ZEO

The formation of negatively charged regions in the Pollack effect leads to a similar phenomenon. Pollack effect would be behind formation of cells, DNA etc which are indeed negatively charged. Protons would transform to dark protons as magnetic flux tubes and realize genetic codons as Galois confined states of dark protons forming triplets. Genes would be Galois confined sequences of these triplets. These tubes would be parallel to DNA and chemical realization of the genetic code would be only a secondary one.

The regions called exclusion zones (EZs) self-clean themselves. This is in a sharp conflict with the second law. The explanation is that at MB time has a non-standard arrow and self-cleaning is actually dissipation but in a reversed time direction. What would be remarkable would be the long duration of the classical counterpart of BSFR as a deterministic time evolution leading to the final 3-D state of BSFR.

Quite generally, the self-cleaning property would serve as a signature of systems for which the MB stays for long times in a time reversed state making possible self-organization as time reversed dissipation. Large values of h_{eff} would be involved and the largest candidate in the solar system is $\hbar_{gr}(Sun)$.

One must of course also consider the possibility of the Milky Way blackhole with a mass about $4.6 \times 10^6 M_{Sun}$. This would correspond to the scaling up of dark wormhole throat size given by CP_2 size to the scale of 4.6 cm! The Milky Way with a mass of $10^{12} M_{Sun}$ would give a dark wormhole throat with size about 4.6×10^4 km!

This raises spin-ice type systems to a preferred role. They are indeed ideal for the demands of living systems since the ground state degeneracy makes it possible to represent the state of the external world as the state of the system. Also quantum computation requires large degeneracy of states possibly realized in terms of Galois representations and flux tube spaghettis would provide this degeneracy.

8.8.3 A finding challenging the standard theory of electrolytes

I received a link to an interesting article "Double-layer structure of the Pt(111)–aqueous electrolyte interface" about the findings of Ojha *et al* [D13] (`https://cutt.ly/oOE6czY`). The reader can also consult the popular representation of the finding (`https://cutt.ly/VORqeoK`).

The experiments demonstrate that the physics of electrolytes is not completely understood.

1. Pt(111)-aqueous electrolyte interface is studied in a situation in which there is a circulation of H_2 molecules part of which decay to H ions and electrons at the interface of the first electrode.

2. Electrons give rise to a current flowing to the second electrode, which also involves the Pt(111) interface. There is also a proton transfer between the electrodes. At the second interface there is a circulation of O_2 molecules: part of them transforms to water molecules at the interface.

3. A double layer of positive and negative charges of some thickness acting like a capacitor at the first interface is formed. Two plates of this kind plus electrolyte between them form an analog of a continually loaded battery and electron current is running when wire connects the plates.

4. The prediction of the standard theory is that when the salt concentration of the electrolyte is lowered, the current should eventually stop running at some critical salt concentration determined by the potential between the electrodes. There would be no free electrons anymore. OThis critical potential is called the potential of zero charge.

5. The experimental findings produced a surprise. The potential of zero charge did not appear for the predicted salt ion concentration. The reduction of ion concentration by a factor 1/10 was needed to achieve this. It would seem that the actual concentration of ions is 10 times higher! What are these strange, invisible, salt ions?

I have confessed to myself and also publicly in [L5, L15] that I do really understand how ionization takes place in electrolytes. The electrostatic energies in atomic scales associated with the electrolyte potential are quite too small to induce ionization. I might be of course incredibly stupid but I am also incredibly stubborn and wanted to understand this in my own way.

The attempt to do something for this situation, and also the fact that "cold fusion" also involves electrolytes, which no nuclear physicist in his right mind would associate with electrolysis, led to a totally crazy sounding proposal that electrolysis might involve some new physics predicted by TGD and making possible "cold fusion" [L10, L15, L42] [K24]. Electrolytes actually involve myriads of anomalous effects [K36, K8]. Theoretical physicists of course do not take them seriously since chemistry is thought to be an ancient, primitive precursor of modern physics.

Part of the ions of the electrolyte would be dark in the TGD sense having effective Planck constant $h_{eff} \geq h$ so that their local interactions (describable using Feyman diagrams) with the ordinary matter with $h_{eff} = h$ would be very weak. There these ions behave like dark matter so that the term "dark ion" is well-motivated. This does not however mean that the galactic dark matter would be dark matter in this sense. TGD based explanation for the galactic dark matter could be actually in terms of the dark energy assignable to cosic strings thickened to magnetic flux tubes carrying monopole flux [L26, L30, L69].

1. The presence of dark ions in water would explain the ionization in electrolytes. Water would be a very special substance in that the magnetic body of water carrying dark matter would give rise to hundreds of thermodynamic anomalies characterizing water [D23].

2. Biology is full of electrolytes and biologically important ions are proposed to be dark ions [L92]. As a matter of fact, I ended the TGD based notion of dark matter from the anomalies of biology and neuroscience [?]. This notion emerged from the number theoretic vision about TGD much later [L21, L20, L67]. Pollack effect [I2, L8, I5, I4] would involve dark protons and would be in a key role in biology. The realizations of genetic codons in terms of dark proton and dark photon triplets would also be central.

3. "Cold fusion" is one application for TGD view about dark matter [L10, L15, L42]. The formation of dark proton sequences gives rise to dark protons and perhaps even heavier nuclei for which the binding energies would be much smaller than for ordinary nuclei. The subsequent transformation to ordinary nuclei would liberate essentially the ordinary nuclear binding energy.

The notion of dark matter also leads to concrete ideas about what happens in electrolysis [K36]. In the TGD framework, the finding of Ojha *et al* would suggest that 90 per cent of ions are dark in the electrolyte considered.

9 A revolution in lithium-sulphur battery technology?

The last weeks have been full of surprises. The most recent surprise was the popular article published in Big Think (`rebrand.ly/wootoqn`), which told about an accidental discovery [D24] (`rebrand.ly/ye9nt4g`), which could revolutionize battery technology. The so-called γ-sulphur is a phase of sulphur stops the degradation of lithium-sulphur batteries and this could give electric vehicles a range of thousands of kilometers.

9.1 The discovery

It is good to start from the problems of lithium batteries.

1. Also other materials than lithium, which is a very light material, such as cobalt are needed in lithium batteries but their mining is very environmentally very damaging. There are also humanitarian problems: the working conditions are bad and even child labor is used.

2. Lithium batteries quickly lose their capacity and charging times are long. lithium batteries also suffer degradation.

3. The energy density is low so that the lithium batteries tend to be very heavy, which limits their commercial use in electric planes and ships.

4. Damaged cells can spontaneously catch on fire.

Lithium-sulphur batteries might provide a cure of all these problems but there is a new very serious problem. Polysulfides Li-S-...-S-Li are formed in the dielectric between the Li and sulphur containing capacitor platers and this reduces the number of charge cycles by one half from about 2000 cycles.

The completely unexpected discovery was that somehow the presence of γ-sulphur as a phase of sulphur, unstable at room temperature but stabilized in presence of Li, prevents the formation of polysulphides Li-S-...S-Li. γ-S crystals are produced by dropping hot sulphur to water at temperatures above 95 degrees Celsius. They are smooth elastic and resemble rubber.

9.2 Some questions with possible answers

The findings raise some questions.

1. What in their structure prevents the formation of considerable amounts of polysulfides L-S-....-S-Li with more than one S? Could the presence of γ-S crystals prevent the formation of S-S bonds or are they formed but split very rapidly? Why is γ-S stabilized in the presence of Li?

2. One thing to notice is the chemical analogy with water: H$\leftrightarrow$Li and O$\leftrightarrow$ S. Could this help? What prevents the formation of H-O-...-O-H sequences in water and one has only H-O-H? Could this be a good question?

Let us try to answer these questions.

1. The first thing to notice is that γ-S is not stable at room temperature. Somehow the presence of Li must stabilize it. The γ-S crystals should grow by addition of S to compensate for the spontaneous decay occurring at room temperature. This could give rise to flow equilibrium.

2. Could it be that the presence of γ-S crystals competes with the formation of Li-S-...S-Li sequences. Could S prefer to join to a γ-S crystal rather than to add to the sequences of S:s in Li-S-...S-Li? The formation of sequences would stop at Li_2-S. This does not yet explain the stability of γ-S at room temperature: differs from that in the absence of Li only in that Li competes with γ-S crystal for S atoms. The mechanism must be more delicate.

3. Li-S....-S-Li polysulphides must be produced at a considerable rate but they provide the S:s for the crystal growth of new γ-S. Li atoms are like servants carrying the food S at plate to γ-S, which eats it. There is a flow equilibrium and the total amount of Li-S...-S-Li stays very small although Li-S....-S-Li is produced with a high rate!

9.3 TGD view of the situation

I have not yet said anything about TGD and quantum but in the presence of γ-S Li_2-S is a chemical analog of water.

9.3.1 Basic questions

One must start from fundamentals and ask what batteries really are.

1. What causes the ionization in the electrolyte? In fact, almost 40 years ago I had discussions with a researcher studying batteries and realized that electrolysis is not actually understood in standard chemistry! Ionization is the mystery. It requires large energies measured in electron volts. The electric voltage between the batteries is low and generates extremely weak electric fields so that it should have no effects in the atomic length scales. I have discussed this problem in an article about "cold fusion" [K8].

2. If ionization occurs, electric field makes possible charge separation. But what makes this charge separation and therefore batteries so stable? They are of course not completely stable since the voltage decreases gradually. An analog of metabolic energy feed is necessary.

9.3.2 Could Pollack effect make batteries possible?

Ionization is necessary for the formation of batteries. I have discussed the problem of ionization in an article about "cold fusion" [K8].

1. The hint comes from the fact that electric voltages involved are measured in electron volts as are measured also the voltages associated with the molecular bonds associated with the salts of electrolytes used.

2. TGD view forces us to ask whether a phase transition in which ordinary particles, say positive ions of a salt, could become dark in the sense that the effective Planck constant h_{eff} characterizing it becomes very large.

   Could the length of the valence bond or hydrogen bond, or more generally, molecular bonds generalizing hydrogen bond to say Li bond between two Li_2-S molecules, become so long that the voltage along it is measured in electron volts so that it can lead to a genuine or effective ionization.

3. Before continuing, one must clarify what meanings the darkness can have. A dark proton associated with a dark very long hydrogen bond could be formed in the charge separation giving rise to batteries. The hydrogen bonds would be U-shaped and connect to the magnetic body (MB) of the positively charged electrode. After loading the flux tube pairs could split and become U-shaped flux tubes again and the positive would remain at the monopole flux tubes. If hydrogen bonds generalize to say lithium bonds, the notion of positive dark ion formed from a salt would generalize. Needless to say, this would mean generalization of chemistry.

   Also dark atoms in the sense that an unpaired valence electron becomes dark as it is transferred to a magnetic flux tube with large value of h_{eff} are possible. Also valence bonds can be dark and I have proposed that ordinary valence bonds are dark and have relatively small $h_{eff} > h$ [L17]. The mysterious disappearance of unpaired valence electrons from rare earth metals under heating [L18] could be an example of this phenomenon [L18].

Could the formation of dark protons correspond to Pollack effect [I2, L8, I5, I4] taking place in presence of gel phase and energy feed, by say IR radiation. Pollack effect is discussed from the TGD point of view in [L8, L46, L55, L83].

1. The TGD based proposal is that in the Pollack effect ordinary protons associated with hydrogen bonds would transform to dark protons at monopole magnetic flux tubes. The U-shaped dark hydrogen bonds would be very long and could reconnect with a second similar bond to form a pair of flux tubes forming a connection to a MB outside the exclusion zone. Dark protons could be transferred to the MB. The formation and splitting of a flux tube pair is the basic mechanism in the TGD inspired model of biocatalysis.

2. If the Pollack effect generalizes to biologically important positive alkali ions, it could serve as a general mechanism giving rise to the storage of energy as electrostatic energy and cell membranes could be seen as analogs of batteries. Hydrogen bond or its generalization as a monopole flux tube is necessary for this.

3. Why is the presence of the gel phase needed? The simplest explanation is that it also involves an exclusion zone as $H_{1.5}O$ phase and is accompanied by a MB carrying dark proton sequences. These dark proton flux tubes could serve as a seed for the formation of dark protons sequences outside the exclusion zone.

4. In the case of batteries, external voltage during the loading of the battery causes charge separation by providing the needed energy to induce ionization and to drive the ions to the oppositely charged electrodes of the battery.

What makes possible a metastable charge separation in the case of the Pollack effect in biology? The molecular binding energy of hydrogen in water molecules is measured in eVs and should be compensated.

1. IR photons with energies below eV scale are needed to generate Pollack effect but is their energy too small?

2. Could the Coulomb binding energy between the exclusion zone and the magnetic flux tubes compensate for the binding energy? How could one achieve a stable situation preventing the collapse of the flux tubes if only Coulomb energy is involved? The Coulomb repulsion between dark protons at the monopole flux tubes is a further serious problem.

3. Dark proton sequences are analogous to atomic nuclei. Could the analog of nuclear binding energy compensate for the molecular binding energy? If the dark protons, or more generally, positively charged dark ions, form analogs of dark nuclei bound together by bonds between nucleons in the TGD inspired nuclear string model, they could be stable and their formation would be also energetically favored if the binding energy scale correspond to that for atoms. These bonds could be analogs of mesons consisting of color quark and antiquark forming a color singlet.

A large value of h_{eff} could make possible scaling of the bond length L as $L \propto h_{eff}$ or even $L \propto h_{eff}^2$ is might be the case for dark valence bonds. If the nuclear binding energy assignable to the bond scales as $1/L$ as function of bond length L, the scale could correspond to a nanometer scale in biology. One would have a scaled up version of nuclear physics or rather, and perhaps even its generalization obtained by replacing dark protons with dark variants of dark nuclei appearing in salts.

Note that the spontaneous decay of dark proton nuclei to ordinary nuclei would liberate almost all nuclear binding energy and explain "cold fusion". In TGD framework, the large value of h_{eff} for weak bosons would scale up their Compton length to biological length scales, and in length scales shorter than Compton length they would behave as massless particles and weak interactions would be as strong as electromagnetism making possible weak interactions transforming dark protons to dark neutrons. The same scaling up applies also to color interactions.

What guarantees the stability of the charge separation? There are situations in which the charge separation has lasted such a long time [L12] that it is very difficult to understand in the framework of standard chemistry. Batteries must be loaded, that is energized, now and then and cell membranes as their biological analogs require a continual metabolic energy feed. This suggests that thermal non-equilibrium systems require a metabolic energy feed.

The energy transfer is the first step of photosynthesis. In the TGD based model it would take place by pairs of holes and dark valence electrons. This raises the question whether it is convenient to talk about a pair of a proton hole and dark proton assignable to the hydrogen bond and even a generalization of this notion.

9.3.3 Li-S batteries and generalized Pollack effect

Could the counterpart of the Pollack effect be involved with lithium-sulphur batteries?

1. Water is the dominating element of living systems. The MB of the water gives water its very special properties and makes it very special at physiological temperatures at which Pollack effect in presence of say IR radiation and gel phase gives rise to the formation of negatively charged exclusion zones by driving protons to Li_2S is the chemical analogue of water.

   One can use this as an analogue in an attempt to understand Li-S batteries in terms of a generalized Pollack effect. If the notions of Li-bond and Li-bonded Li_2S molecule clusters make sense, the model for the Pollack effect as a way to generate a metastable charge separation might work.

2. Note that the formation of H-O-O-...-H is not a problem in the ordinary Pollack effect and the role of the γ-S would be only to make possible stable exclusion zones and magnetic flux tubes. Without it the dark Li-ions at flux tubes would transform to ordinary Li-atoms forming fingers, Li-S-...-S-Li sequences would form and the battery would degrade also otherwise. This can be understood in terms of reduction of h_{eff} inducing the reduction of complexity and scale of quantum coherence at the positive electrode.

3. The counterpart of the exclusion zone with an effective stoichiometry $H_{1.5}O$ and negative charge would be negative electrode with effective stoichiometry $Li_{1.5}O$. Dark Li^+ ion would take the role of dark proton. Every fourth Li^+ would go as dark ion to the magnetic flux tube and end up to the positively charged electrode or its MB. It would create the same voltage along the space-time sheet associated with the electrolyte as along possibly still existing flux tubes connecting it to the negatively charged electrode.

9.3.4 Pollack effect, cold fusion and protostars

"Cold fusion" (for the recent situation see `rebrand.ly/ui7xoig`) is an anomaly, whose existence very many colleagues still find difficult to accept. "Cold fusion" also involves dielectric plates and the proposed TGD based model [L10, L15, L42] involves dark proton currents at magnetic flux tubes.

"Cold fusion", or more precisely dark fusion in the TGD framework, can be initiated at rather low temperatures and involves the formation of dark proton sequences at monopole flux tubes. Dark nuclei are essentially scaled up variants of nuclei but much smaller binding energy and can be generated in the Pollack effect, which plays a key role in the TGD inspired quantum biology. Dark nuclei can spontaneously decay to the ordinary nuclei and also protons can transform to neutrons. This liberates essentially all nuclear binding energy.

For instance, in the case of heavy water D_2S, the dark protons would be replaced by Deuterons and $H_{1.5}O$ would be replaced by $D_{1.5}O$. Dark proton sequences would correspond to dark D^+ sequences as dark nuclei. They would spontaneously decay to ^4He and deuteron nuclei in consistency with the observations.

There is also evidence for biotransmutations [?, ?, ?] occurring in living systems discussed from TGD point of view in [K24, K36]. For instance, Kervran found that hens are able to produce Ca needed in egg shells. These findings might allow interpretation in terms of dark fusion based on the Pollack effect or its generalization.

Dark fusion would generate protostars [L30, L34, L15] in which there is no ordinary fusion yet. The temperature increases because essentially nuclear binding energy is liberated when the dark nuclei transform to ordinary nuclei and eventually ordinary fusion is ignited. It is quite possible that all nuclei heavier than Fe are generated in this way outside stellar cores rather than in supernova explosions. Also many anomalous abundances of lighter nuclei could be understood.

9.3.5 Pollack effect and DNA

Capacitors involve both negatively and positively charged plates. Pollack effect is central in the TGD view of living matter and generates negatively charged entities such as cell interior and DNA double strand.

In the case of DNA, Pollack effect would mean that negatively charged phosphates giving constant charge of -1 units per nucleotide act as negative electrode and screen the positive dark proton charge per DNA strand inside the fundamental region of icosa-tetrahedral tessellation [L82] having size scale given by the p-adic length scale $P_p = L(151) = 10$ nm, $p = M_{151} = 2^k - 1$, $k = 151$. It would contain 10 DNA codons and correspond to 3 full turns for DNA double strand. This picture differs from the original one in which dark DNA strands were assumed to reside outside the double strand.

What could be the detailed mechanism?

1. Has the O-H group of phosphate have lost dark proton into the interior of the fundamental region where it belongs to dark proton triplet defining genetic codon. One would have $P-O^-$ phosphate ions at the negative electrode.

2. Does the O-H group of phosphate have a hydrogen bond with the water molecule of the cell exterior and has the flux tube transformed to dark flux tube extending to the interior of the fundamental region? Has it lost its dark proton to the interior of the fundamental region via a reconnection process?

3. The answer to the question comes from the consistency with the realization of the dark genetic codons as dark proton triplets considered in [L82]. The dark protons of the codon should be associated with the vertices of triangular tetrahedral or icosahedral faces of the fundamental region of the icosa-tetrahedral tessellation.

   This would suggest that the monopole flux tubes representing hydrogen bonds have (de-)reconnected and left the dark proton to the vertices of the triangular face. The small closed flux tube produced in the de-reconnection would naturally correspond to the required closed flux tubes connecting icosahedron, tetrahedron, and octahedron assignable to a given dark proton of the codon. The magnetic field for the flux tube would determine the cyclotron frequency and cyclotron frequency triplet would characterize the codon and provide the icosa-tetrahedral realization of the genetic code [L7, L44].

9.3.6 Pollack effect and cell membrane

In the model of the cell membrane as a battery, the rough first picture could be as follows. The original model involved the Pollack effect for protons but the generalization of the effect to biologically important positive ions is suggestive and involved with the cell membrane.

1. In the simplest model, dark positively charged alkali ions reside always outside the cell membrane at monopole flux tubes. The negative ions resulting from the transfer of positive ions to the U-shaped monopole flux tubes defining analogs of hydrogen bonds would reside inside the cell membrane.

   The connections between exterior and exterior by pairs of flux tubes from U-shaped flux tubes could be permanent but one can also consider the possibility of U-shaped flux tubes extending to the exterior with delocalized ions at them.

   The transfer of dark ions permanently to the exterior would involve a (de-)reconnection generating a transfer of dark ions to the exterior and subsequent reconnection isolating splitting the flux tube pair and isolating exterior from the interior. Reconnections could control the transfer of dark ions between interior and exterior.

   Membrane resting potential would be controlled by the transfer of dark ions to the exterior generating a hyperpolarization. This would suggest permanent flux tube connections.

2. Gel phase would be a natural candidate for the analog for the negatively charged $H_{1.5}O$ involving corresponding phases for various ions. In gel-to-sol transition this phase would transform to ordinary water and the battery charge would decay. Metabolic energy feed is needed to prevent this since the value of h_{eff} increases with energy and tends to be spontaneously reduced.

   It is unclear whether one could understand the nerve pulse in terms of the gel-to-sol phase transition in which ohmic currents would be generated leading to the reduction and change of the sign of the membrane potential. That Hodkin-Huxley model works satisfactorily suggests that ohmic currents are present during the nerve pulse.

3. In the Josephson junction model of the cell membrane [K16, K33, K32], there is a permanent Sine-Gordon soliton sequence based on the phase difference $\Delta\Phi$ for superconducting phases residing at monopole flux tubes at the two sides of the membrane.

   One has $d\Delta\Phi/dt \equiv \Omega = E/\hbar_{eff}$, where E is the sum of the ordinary Josephson energy ZeV and difference of cyclotron energies over the junction. Very large value of $\hbar_{eff}$ is required to give Josephson frequency in EEG range and gravitational Planck constant $\hbar_{gr}$, introduced first by Nottale [E1], central in TGD view of quantum gravitation [L76, L75, L87, L88, L89], is highly suggestive.

   The cyclotron frequencies are associated with the flux tubes parallel to the cell membrane. If there are no flux tubes in the interior, the corresponding cyclotron frequency vanishes. Josephson junctions are associated with the membrane proteins. Josephson junctions could correspond to pairs of flux tubes between interior and exterior so that bosonic dark ions or Cooper pairs of fermionic ions would give rise to Josephson currents between interior and exterior.

   The system is mathematically equivalent to a sequence of rotating gravitational penduli assignable to various ions. The simplest model assumes that all bosonic dark ions are at the magnetic flux tubes in the exterior of the cell membrane and parallel to it.

4. What about the nerve pulse in the simplest picture? The nerve pulse could be induced by a propagation of a perturbation changing the sign of the local rotation direction of some fictitious gravitational penduli at the point in which the sine of the phase vanishes so that also Josephson current vanishes. Formally this corresponds to a change of the arrow of time and could correspond to a pair of "big" state function reductions (BSFRs). Could this change of the arrow of time induce reduction of the voltage and ordinary Ohmic currents changing the sign of the membrane potential temporarily?

10 Testing of the vision

Eventually the basic concepts of TGD applied to condensed matter physics should be tested. The following lists some challenges.

10.1 Observation of dark matter

The observation of dark matter as $h_{eff} = nh_0$ phases in condensed matter systems is one basic goal (allbqcritdark1,qcritdark2,qcritdark3,qcritdark4). Macroscopic quantum phases, emergence of additional degrees of freedom, and the effective increase of the dimension of the momentum space from 3 $3k(n,K)$, where k is a numerical factor determined by the number K of particles forming the Galois confined states and by the dimension n of the extensionof rationals, are possible. Also photon scattering via the formation of polaritons could allow us to "see" the structure of dark matter at the level of MB as an interference pattern. The analog of X-ray diffraction would be in question.

10.2 Topological physics at space-time and embedding space levels

The basic new physics element is the topological physics in the TGD sense based on non-trivial space-time topology at the fundamental level.

Some examples are in order.

1. Magnetic flux tubes are always closed, which means non-trivial first homotopy making possible the topological variant of the geometric phase.
2. Flux tube braidings would be a basic concept of topological hydrodynamics. Reconnections as changes of braid topology would be central and bring in 2-braids and knots of 2-D flux sheets in 4-D space-times (also intersections at discrete points replace links of 1-braids).
3. In TGD inspired biology systems have U-shaped flux tubes as tentacles with which they generate connections to the environment by reconnecting in which two U-shaped flux tubes of different systems such as molecules form a pair of flux tubes.

   For instance, friction could be due to the formation of flux tube pairs. Static friction would be generated and the de-reconnection of flux tube pairs would require energy.

Also topological defects due to the embedding space topology are possible. The monopole flux reflects the non-trivial topology of CP_2. Skyrmions result from the constraint that the ball $B^3 \subset M^4$ is mapped to the sphere S^3 of $E^4 \subset M^8$ or equivalently of CP_2.

10.3 The new view of gauge fields

TGD view of gauge fields differs in several respects from the standard view.

1. The new view about gauge fields and also electromagnetic fields relies on flux tubes. Flux tubes appear as two types: monopole flux tubes and non-monopole ones. Monopole flux tubes require no current to preserve the magnetic field.

   This would explain magnetic fields in cosmic scales, why Earth's magnetic field has not disappeared [L11], and also the huge magnetic fields of magnetars and neutron stars. Could the fields H, M, and B of Maxwell's theory correspond to monopole fields, non-monopole fields induced by the motion of their flux quanta, and to their sum $B = M + H$.
2. The twistor lift of TGD [L22, L27] predicts that also M^4 should have Kähler structure defined by a self-dual constant Kähler form for which the electric part would be imaginary. This implies a global CP breaking in M^4 that could induce a matter-antimatter asymmetry. 3 quarks would prefer to form baryons and antiquarks to form leptons as 3 antiquarks composites in primordial Universe and after the annihilation the remaining baryons would represent matter and leptons antimatter. This is possible only by the TGD view about color [L36, L53].

   The mechanism of CP (T) violation could be essentially the same as in the topological insulators destroying the boundary conductivity by T violation. In the condensed matter case the magnetic field would receive $U(1)$ contributions from both CP_2 and M^4 degrees of freedom. The magnetic interaction energy with spin would have opposite signs for opposite spin directions and lead to CP and T violation. For cosmic strings and flux tubes the M^4

magnetic part would be small, which would explain the smallness of the CP violation. Since M^4 Kähler form contributes also to the $U(1)$ part of em and Z^0 fields, it could have small effects also at the level of condensed matter if M^4 projection of the flux tube is 4-D.

3. Wormhole contacts identified as pieces of deformed CP_2 type extremals serve as basic building bricks of elementary particles. The wormhole throats are identified as partonic surfaces and their orbits are light-like curves performing zitterbewegung. One can assign to them a Kac-Moody type algebra with non-negative conformal weights. This algebra is very much like gauge algebra but not quite. For instance, there is a hierarchy of representations for which only the generators with conformal weight larger than some maximal conformal weight h_{max} annihilate the physical states. Could these analogs of gauge algebras assignable to $M^2 \times CP_2$ isometries allow a realization of synthetic gauge groups acting also in M^4 spin degrees of freedom [D9] (`https://cutt.ly/4Wy39B5`?

10.4 Number theoretical physics

Number theoretical physics brings in new elements and involves in an essential manner $M^8 - H$ duality.

1. The hierarchy of effective Planck constants and p-adic physics as physics of cognition involving p-adic length scale physics means a completely new element of quantum theory central for understanding of various supra phases.

2. Galois confinement is a central notion. Quantum states would be Galois singlets above the quantum coherence scale defined by h_{eff} and become unconfined states below this scale. The situation is highly reminiscent of color confinement. At M^8 level, the assumption that momenta are algebraic integers for the extension of rationals considered implies that confined states have total momenta, which are ordinary integers and that the rational integer parts momenta of K composite particles are identical. This implies a reduction of translational degrees of freedom so that the density dn/dE of confined states increases and among other things leads to a reduction of Fermi energy.

   Galois confinement could serve as a universal mechanism for the formation of bound states: this includes atoms and molecules, atomic nuclei, and hadrons. Color confinement can be one particular example of this if Galois group is represented as a subgroup of color group: Z_3 is the obvious guess but also more general discrete subgroups of $SU(3)$ are possible. Also the discrete subgroups of the rotation group $SO(3)$ and its covering group $SU(2)$ could be representable as Galois groups and appear in the ADE hierarchy for inclusions of HFFs. $M^8 - H$ duality would give a very concrete ideas about the momentum space and space-time geometries of the bound states. Momenta in M^8 would form a representation of Galois group mapped to H by $M^8 - H$ duality.

3. The number theoretical phase transitions changing the polynomial that determines $X^4 \subset M^8$ and therefore the extension of rationals and the Galois group as symmetry group would be a new element. Discrete degrees of freedom would appear or disappear. The scaling of the number of states within the Fermi ball could be one signature. Extensions could also give rise to quasicrystals.

   The change of the fidelity described as the metric of the parametrized space of quantum states would take place. Fidelity would be coded by the Kähler metric of WCW and geometric phase by the Kähler form of WCW. This is because, the WCW Kähler metric induces the metric of quantum states depending on the parameters coding for the X^4 as a point of WCW.

4. Negentropy Maximization Principle (NMP) and adelic physics provide a new view about quantum measurements and about second law. In particular, a vision about how the information about measurement is stored in the space-time geometry modified in the measurement, emerges.

10.5 ZEO and new view about quantum measurement theory and thermodynamics

ZEO allows "big" state function reductions (BSFRs) in long scales. If time reversal indeed occurs, it induces a long lasting effective time reversal at the level of ordinary matter (genuine time reversals at this level last a very short time). Dissipation would effectively occur with an opposite arrow of time and lead to the formation of self-organization patterns [L33]. The findings of Minev *et al* discussed in [L29] support the new view about quantum theory.

The most dramatic implications would be to biology. In particular, homeostasis could be understood as self-organized quantum critical (SOQC) [L60]. Condensed matter systems in the presence of energy feed playing the role of metabolic energy feed could exhibit primitive aspects of living systems.

Note that at the QFT limit most of the information about the TGD based new physics is lost since both space-time topology and number theoretic structure is lost so that QFT is not able to test the relevant effects. However, it might be possible to make this hidden level visible.

11 Appendix

11.1 Comparison of TGD with other theories

Table 1 compares GRT and TGD and **Table 2** compares standard model and TGD.

11.2 Glossary and figures

The following glossary explains some basic concepts of TGD and TGD inspired biology.

- ***Space-time as surface***. Space-times can be regarded as 4-D surfaces in an 8-D space $M^4 \times CP_2$ obtained from empty Minkowski space (M^4) by adding four small dimensions (CP_2). The study of field equations characterizing space-time surfaces as "orbits" of 3-surfaces (3-D generalization of strings) forces the conclusion that the topology of space-time is non-trivial in all length scales.

- ***Geometrization of classical fields***. Both weak, electromagnetic, gluonic, and gravitational fields are known once the space-time surface in H as a solution of field equations is known.

  Many-sheeted space-time (see **Fig. 4**) consists of space-time sheets with various length scales with smaller sheets being glued to larger ones by ***wormhole contacts*** (see **Fig. 5**) identified as the building bricks of elementary particles. The sizes of wormhole contacts vary but are at least of CP_2 size (about 10^4 Planck lengths) and thus extremely small.

  Many-sheeted space-time replaces reductionism with ***fractality*** . The existence of scaled variants of physics of strong and weak interactions in various length scales is implied, and biology is especially interesting in this respect.

- ***Topological field quantization (TFQ)*** . TFQ replaces classical fields with space-time quanta. For instance, magnetic fields decompose into space-time surfaces of finite size representing flux tubes or -sheets. Field configurations are like Bohr orbits carrying "archetypal" classical field patterns. Radiation fields correspond to topological light rays or massless extremals (MEs), magnetic fields to magnetic flux quanta (flux tubes and sheets) having as primordial representatives "cosmic strings", electric fields correspond to electric flux quanta (e.g. cell membrane), and fundamental particles to CP_2 type vacuum extremals.

- ***Field body*** (FB) and ***magnetic body*** (MB). Any physical system has field identity - FB or MB - in the sense that a given topological field quantum corresponds to a particular source (or several of them - e.g. in the case of the flux tube connecting two systems).

  Maxwellian electrodynamics cannot have this kind of identification since the fields created by different sources superpose. Superposition is replaced with a set theoretic union: only

	GRT	**TGD**
Scope of geometrization	classical gravitation	all interactions and quantum theory
Spacetime		
Geometry	abstract 4-geometry	sub-manifold geometry
Topology	trivial in long length scales	many-sheeted space-time
Signature	Minkowskian everywhere	also Euclidian
Fields		
classical	primary dynamical variables	induced from the geometry of H
Quantum fields	primary dynamical variables	modes of WCW spinor fields
Particles	point-like	3-surfaces
Symmetries		
Poincare symmetry	lost	Exact
GCI	true	true - leads to SH and ZEO
	Problem in the identication of coordinates	$H = M^4 \times CP_2$ provides preferred coordinates
Super-symmetry	super-gravitation	super variant of H: super-surfaces
Dynamics		
Equivalence Principle	true	true
Newton's laws and		
notion of force	lost	generalized
Einstein's equations	from GCI and EP	remnant of Poincare invariance at QFT limit of TGD
Bosonic action	EYM action	Kähler action + volume term
Cosmological constant	suggested by dark energy	length scale dependent coefficient of volume term
Fermionic action	Dirac action	Modified Dirac action for induced spinors
Newton's constant	given	predicted
Quantization	fails	Quantum states as modes of WCW spinor field

Table 1: Differences and similarities between GRT and TGD

	SM	TGD
Symmetries		
Origin	from empiria	reduction to CP_2 geometry
Color symmetry	gauge symmetry	isometries of CP_2
Color	analogous to spin	analogous to angular momentum
Ew symmetry	gauge symmery	holonomies of CP_2
Symmetry breaking	Higgs mechanism	CP_2 geometry
Spectrum		
Elementary particles	fundamental	consist of fundamental fermions
Bosons	gauge bosons, Higgs	gauge bosons, Higgs, pseudo-scalar
Fundamental fermions	quarks and leptons	quarks: leptons as local 3-quark composites
Dynamics		
Degrees of freedom	gauge fields, Higgs, and fermions	3-D surface geometry and spinors
Classical fields	gauge fields, Higgs	induced spinor connection
	SU(3) Killing vectors of CP_2	
Quantal degrees of freedom	gauge bosons,Higgs,	quantized induced spinor fields
Massivation	Higgs mechanism	p-adic thermodynamics with superconformal symmetry

Table 2: Differences and similarities between standard model and TGD

the *effects* of the fields assignable to different sources on test particle superpose. This makes it possible to define the QFT limit of TGD.

- ***p-Adic physics*** [K25] as a physics of cognition and intention and the fusion of p-adic physics with real number based physics are new elements.
- ***Adelic physics*** [L20, L24] is a fusion of real physics of sensory experience and various p-adic physics of cognition.
- ***p-Adic length scale hypothesis*** states that preferred p-adic length scales correspond to primes p near powers of two: $p \simeq 2^k$, k positive integer.
- A ***Dark matter hierarchy*** realized in terms of a hierarchy of values of effective Planck constant $h_{eff} = nh_0$ as integers using $h_0 = h/6$ as a unit. Large value of h_{eff} makes possible macroscopic quantum coherence which is crucial in living matter.
- ***MB as an intentional agent using biological body (BB) as a sensory receptor and motor instrument*** . The personal MB associated with the living body - as opposed to larger MBs assignable with collective levels of consciousness - has a hierarchical onion-like layered structure and several MBs can use the same BB making possible remote mental interactions such as hypnosis [L6].
- ***Cosmic strings Magnetic flux tubes*** belong to the basic extremals of practically any general coordinate invariant action principle. Cosmic strings are surfaces of form $X^2 \times Y^2 \subset M^4 \times CP_2$. X^2 is analogous to string world sheet. Cosmic strings come in two varieties and both seem to have a deep role in TGD.

  Y^2 is either a complex or Lagrangian 2-manifold of CP_2. Complex 2-manifold carries monopole flux. For Lagrangian sub-manifold the Kähler form and magnetic flux and Kähler action vanishes. Both types of cosmic strings are are simultaneous extremals of both Kähler action and volume action: this holds true quite generally for preferred extremals.

  Cosmic strings are unstable against perturbations thickening the 2-D M^4 projection to 3-D or 4-D: this gives rise to monopole (see **Fig. ??**) and non-monopole magnetic flux tubes. Using

$M^2 \times Y^2$ coordinates, the thickening corresponds to the deformation for which $E^2 \subset M^4$ coordinates are not constant anymore but depend on Y^2 coordinates.

- ***Magnetic flux tubes and sheets*** serve as "body parts" of MB (analogous to body parts of BB), and one can speak about magnetic motor actions. Besides concrete motion of flux quanta/tubes analogous to ordinary motor activity, basic motor actions include the contraction of magnetic flux tubes by a phase transition possibly reducing Planck constant, and the change in thickness of the magnetic flux tube, thus changing the value of the magnetic field, and in turn the cyclotron frequency. Transversal oscillatory motions of flux tubes and oscillatory variations of the thickness of the flux tubes serve as counterparts for Alfwen waves.

  Reconnections of the U-shaped flux tubes allow two MBs to get in contact based on a pair of flux tubes connecting the systems and temporal variations of magnetic fields inducing motor actions of MBs favor the formation of reconnections.

  In hydrodynamics and magnetohydrodynamics reconnections would be essential for the generation of turbulence by the generation of vortices having monopole flux tube at core and Lagrangian flux tube as its exterior.

  Flux tube connections at the molecular level bring a new element to biochemistry making it possible to understand bio-catalysis. Flux tube connections serve as a space-time correlates for attention in the TGD inspired theory of consciousness.

- ***Cyclotron Bose-Einstein condensates (BECs)*** of various charged particles can accompany MBs. Cyclotron energy $E_c = hZeB/m$ is much below thermal energy at physiological temperatures for magnetic fields possible in living matter. In the transition $h \rightarrow h_{eff}$ E_c is scaled up by a fractor $h_{eff}/h = n$. For sufficiently high value of h_{eff} cyclotron energy is above thermal energy $E = h_{eff}\ ZeB/m$. Cyclotron Bose-Einstein condensates at MBs of basic biomolecules and of cell membrane proteins - play a key role in TGD based biology.

- ***Josephson junctions*** exist between two superconductors. In TGD framework, ***generalized Josephson junctions*** accompany membrane proteins such as ion channels and pumps. A voltage between the two super-conductors implies a ***Josephson current***. For a constant voltage the current is oscillating with the ***Josephson frequency*** . The Josephson current emits ***Josephson radiation***. The energies come as multiples of ***Josephson energy***.

  In TGD generalized Josephson radiation consisting of dark photons makes communication of sensory input to MB possible. The signal is coded to the modulation of Josephson frequency depending on the membrane voltage. The cyclotron BEC at MB receives the radiation producing a sequence of resonance peaks.

- ***Negentropy Maximization Principle*** (NMP). NMP [K23] [L60] is the variational principle of consciousness and generalizes SL. NMP states that the negentropy gain in SFR is non-negative and maximal. NMP implies SL for ordinary matter.

- ***Negentropic entanglement*** (NE). NE is possible in adelic physics and NMP does not allow its reduction. NMP implies a connection between NE, the dark matter hierarchy, p-adic physics, and quantum criticality. NE is a prerequisite for an experience defining abstraction as a rule having as instances the state pairs appearing in the entangled state.

- ***Zero energy ontology (ZEO)*** In ZEO physical states are pairs of positive and negative energy parts having opposite net quantum numbers and identifiable as counterparts of initial and final states of a physical event in the ordinary ontology. Positive and negative energy parts of the zero energy state are at the opposite boundaries of a ***causal diamond*** (CD, see **Fig. 12**)) defined as a double-pyramid-like intersection of future and past directed light-cones of Minkowski space.

  CD defines the "spot-light of consciousness": the contents of conscious experience associated with a given CD is determined by the space-time sheets in the embedding space region spanned by CD.

- ***SFR*** is an acronym for state function reduction. The measurement interaction is universal and defined by the entanglement of the subsystem considered with the external world [L35] [K44]. What is measured is the density matrix characterizing entanglement and the outcome is an eigenstate of the density matrix with eigenvalue giving the probability of this particular outcome. SFR can in principle occur for any pair of systems.

  SFR in ZEO solves the basic problem of quantum measurement theory since the zero energy state as a superposition of classical deterministic time evolutions (preferred extremals) is replaced with a new one. Individual time evolutions are not made non-deterministic.

  One must however notice that the reduction of entanglement between fermions (quarks in TGD) is not possible since Fermi- and als Bose statistics predicts a maximal entanglement. Entanglement reduction must occur in WCW degrees of freedom and they are present because point-like particles are replaced with 3-surfaces. They can correspond to the number theoretical degrees of freedom assignable to the Galois group - actually its decomposition in terms of its normal subgroups - and to topological degrees of freedom.

- ***SSFR*** is an acronym for "small" SFR as the TGD counterpart of weak measurement of quantum optics and resembles classical measurement since the change of the state is small [L35] [K44]. SSFR is preceded by the TGD counterpart of unitary time evolution replacing the state associated with CD with a quantum superposition of CDs and zero energy states associated with them. SSFR performs a localization of CD and corresponds to time measurement with time identifiable as the temporal distance between the tips of CD. CD is scaled up in size - at least in statistical sense and this gives rise to the arrow of time.

  The unitary process and SSFR represent also the counterpart for Zeno effect in the sense that the passive boundary of CD as also CD is only scaled up but is not shifted. The states remain unchanged apart from the addition of new fermions contained by the added part of the passive boundary. One can say that the size of the CD as analogous to the perceptive field means that more and more of the zero energy state at the passive boundary becomes visible. The active boundary is however both scaled and shifted in SSFR and states at it change. This gives rise to the experience of time flow and SSFRs as moments of subjective time correspond to geometric time as a distance between the tips of CD. The analog of unitary time evolution corresponds to "time" evolution induced by the exponential of the scaling generator L_0. Time translation is thus replaced by scaling. This is the case also in p-adic thermodynamics. The idea of time evolution by scalings has emerged also in condensed matter physics.

- ***BSFR*** is an acronym for "big" SFR, which is the TGD counterpart of ordinary state function reduction with the standard probabilistic rules [L35] [K44]. What is new is that the arrow of time changes since the roles of passive and active boundaries change and CD starts to increase in an opposite time direction.

  This has profound thermodynamic implications. Second law must be generalized and the time corresponds to dissipation with a reversed arrow of time looking like self-organization for an observed with opposite arrow of time [L33]. The interpretation of BSFR is as analog of biological death and the time reversed period is analogous to re-incarnation but with non-standard arrow of time. The findings of Minev *et al* [L29] give support for BSFR at atomic level. Together with h_{eff} hierarchy BSFR predicts that the world looks classical in all scales for an observer with the opposite arrow of time.

11.3 Figures

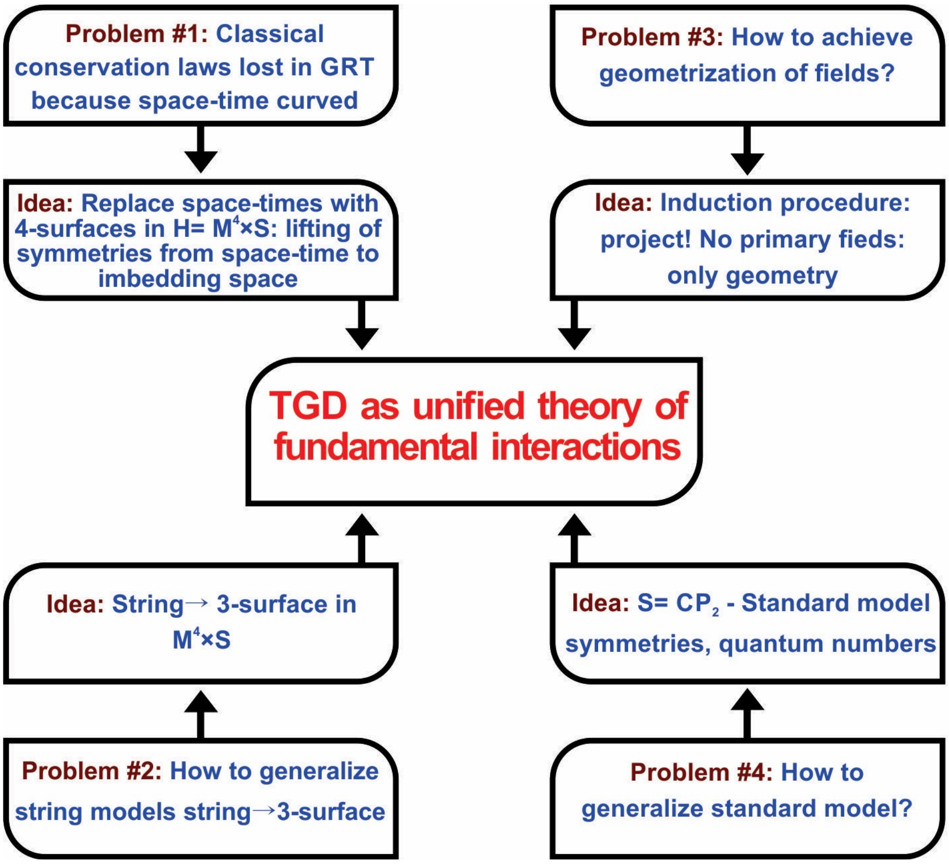

Figure 1: The problems leading to TGD as their solution.

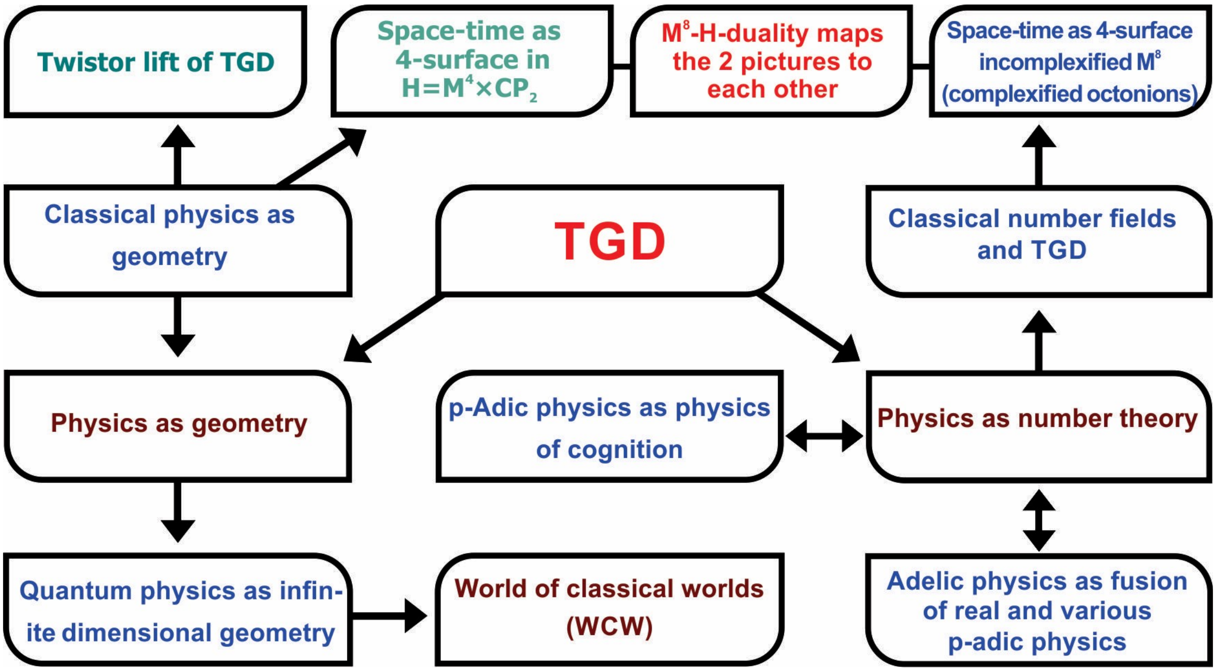

Figure 2: TGD is based on two complementary visions: physics as geometry and physics as number theory.

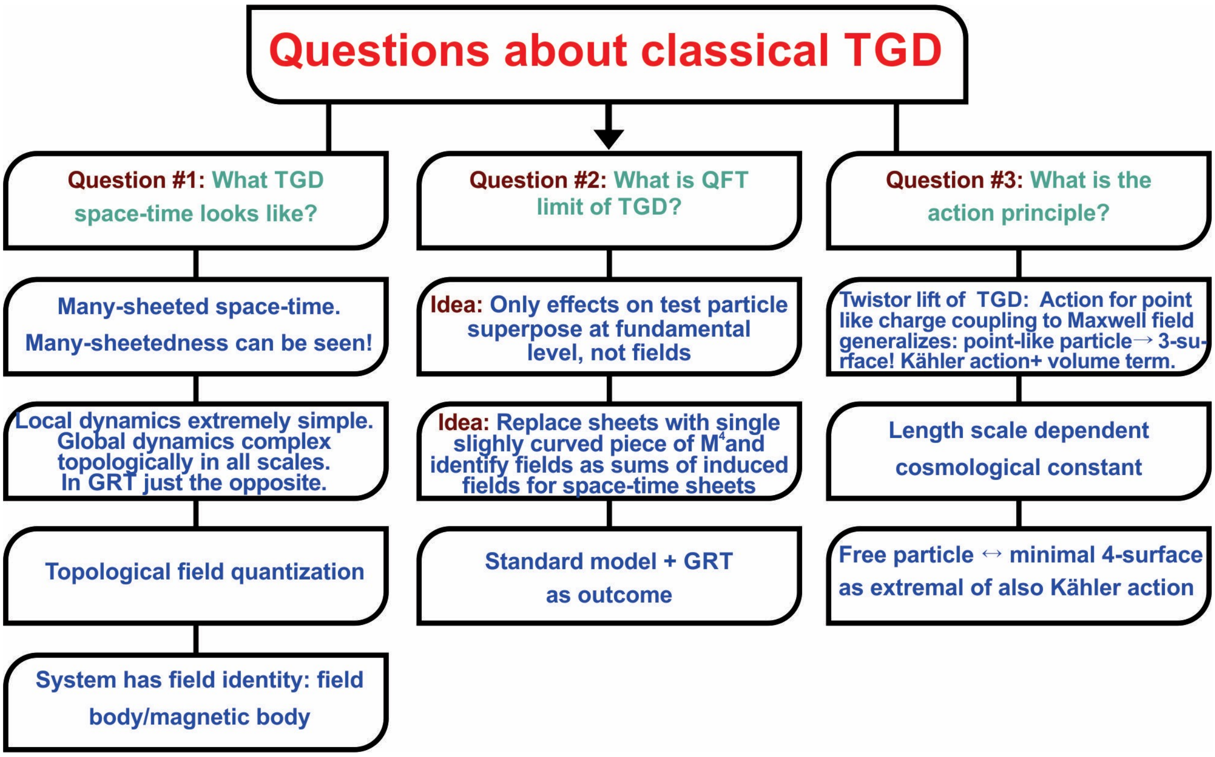

Figure 3: Questions about classical TGD.

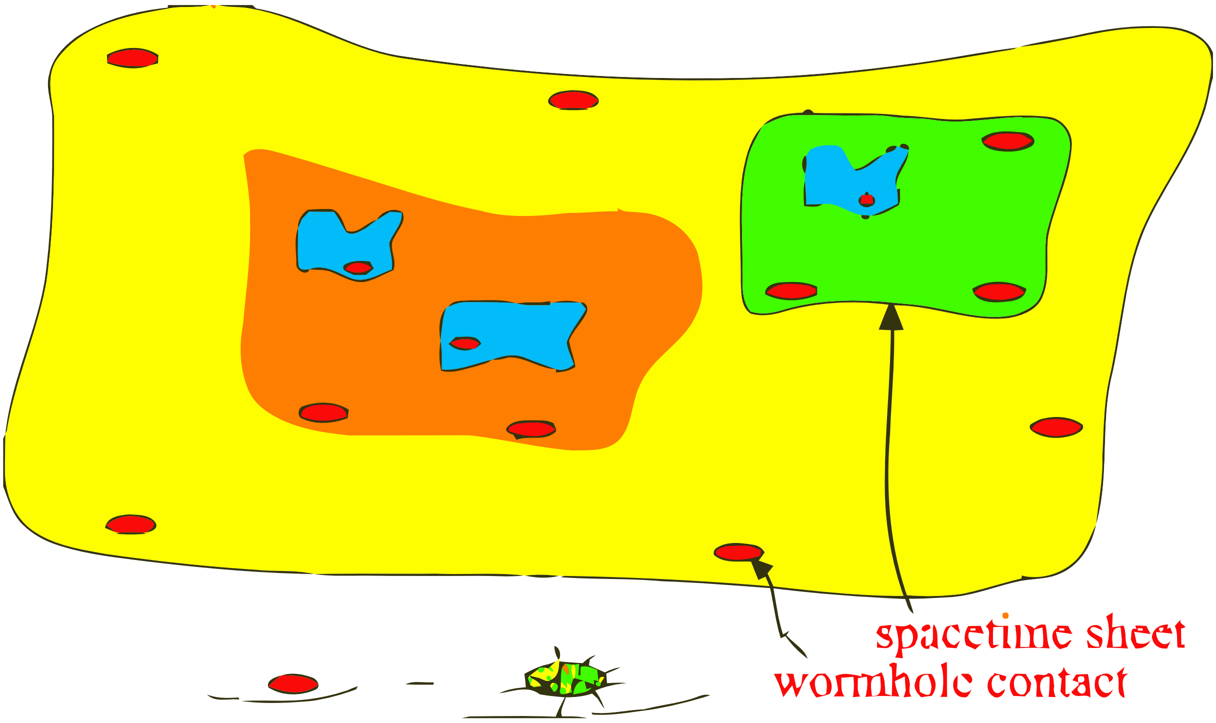

Figure 4: Many-sheeted space-time.

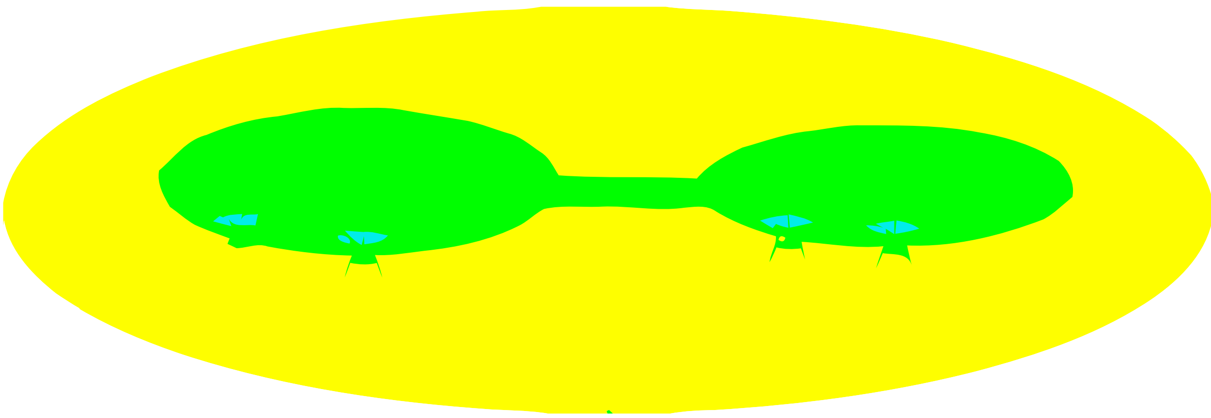

Figure 5: Wormhole contacts.

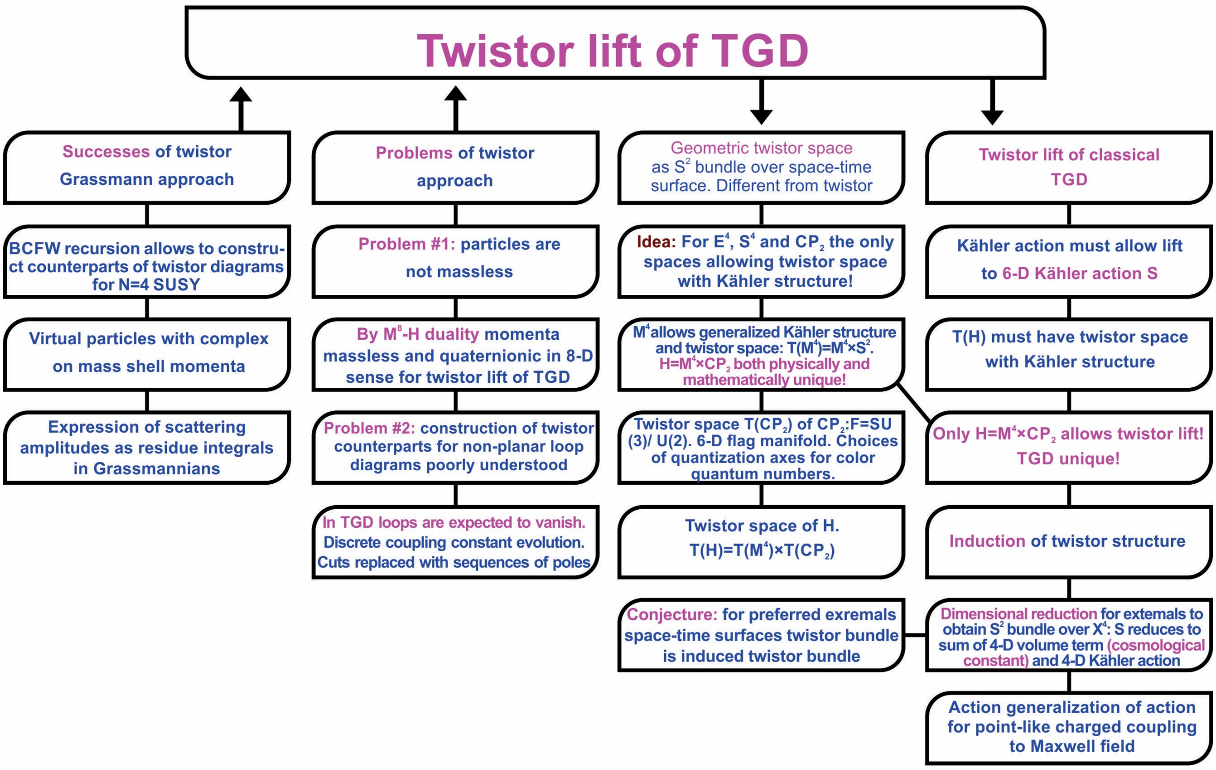

Figure 6: Twistor lift

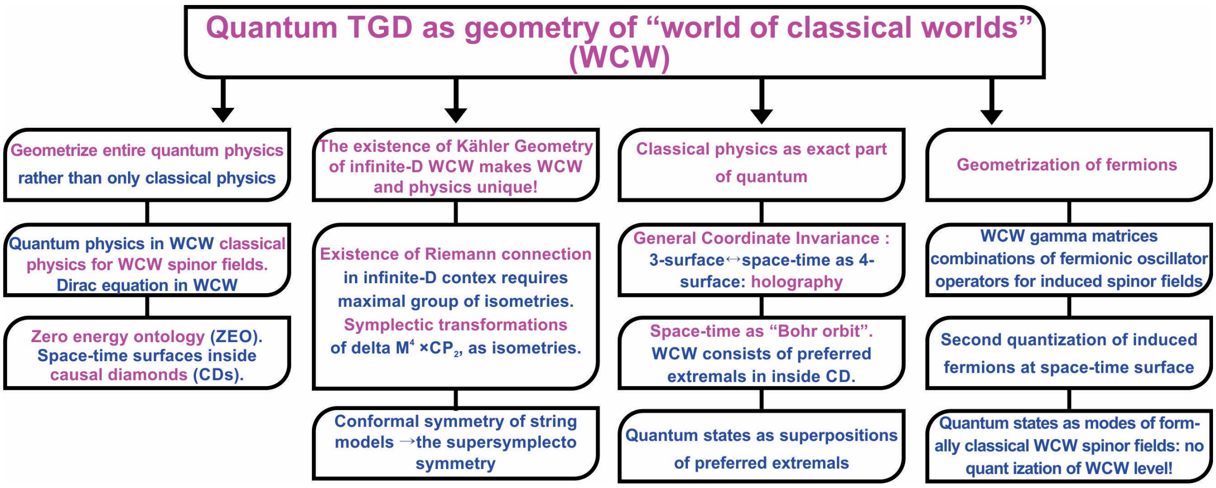

Figure 7: Geometrization of quantum physics in terms of WCW

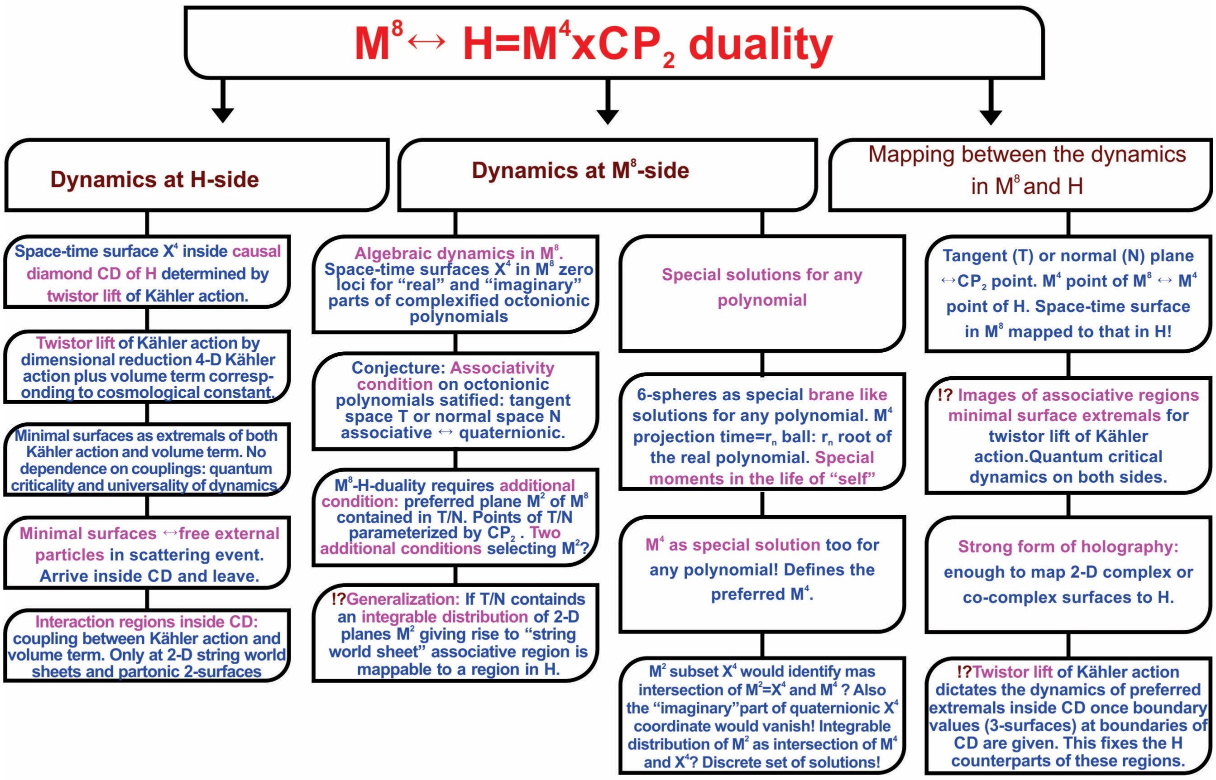

Figure 8: $M^8 - H$ duality

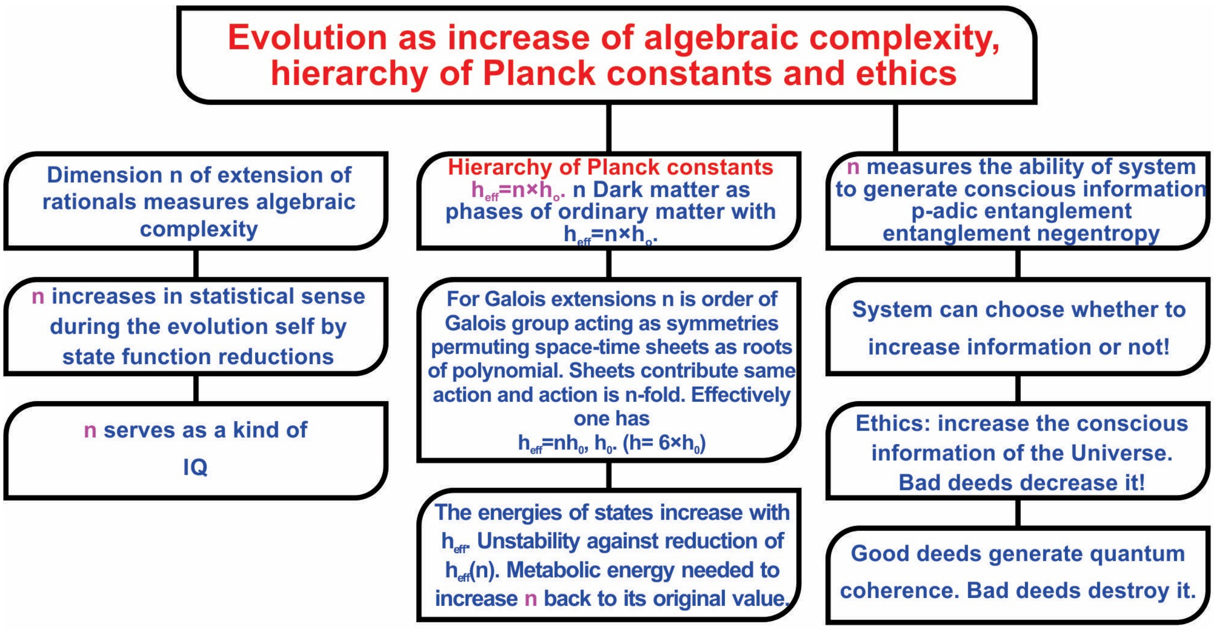

Figure 9: Number theoretic view of evolution

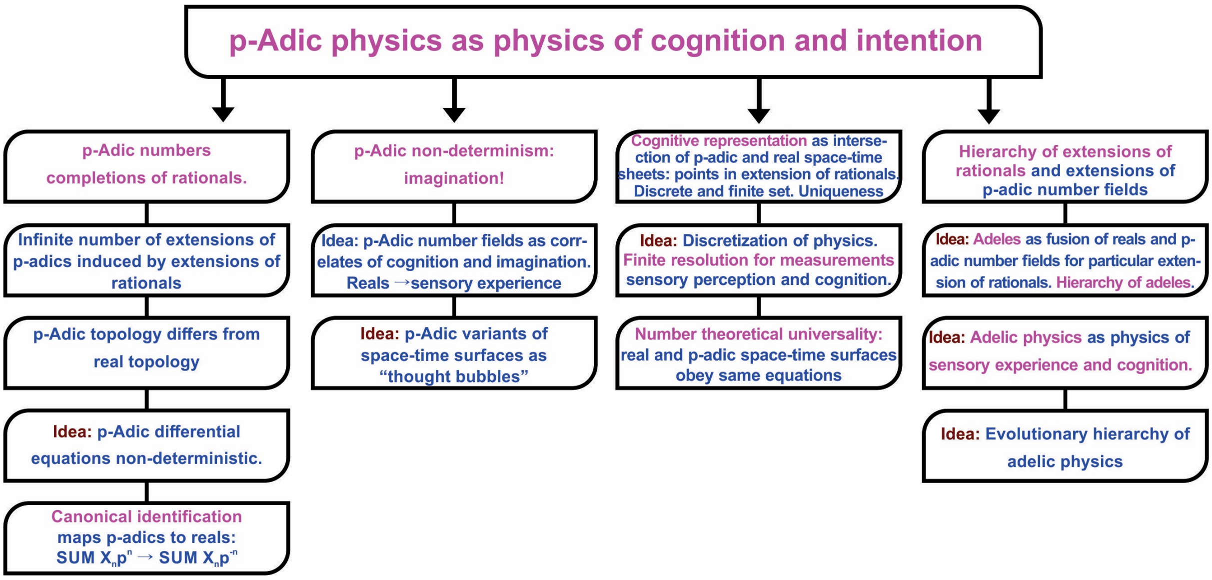

Figure 10: p-Adic physics as physics of cognition and imagination.

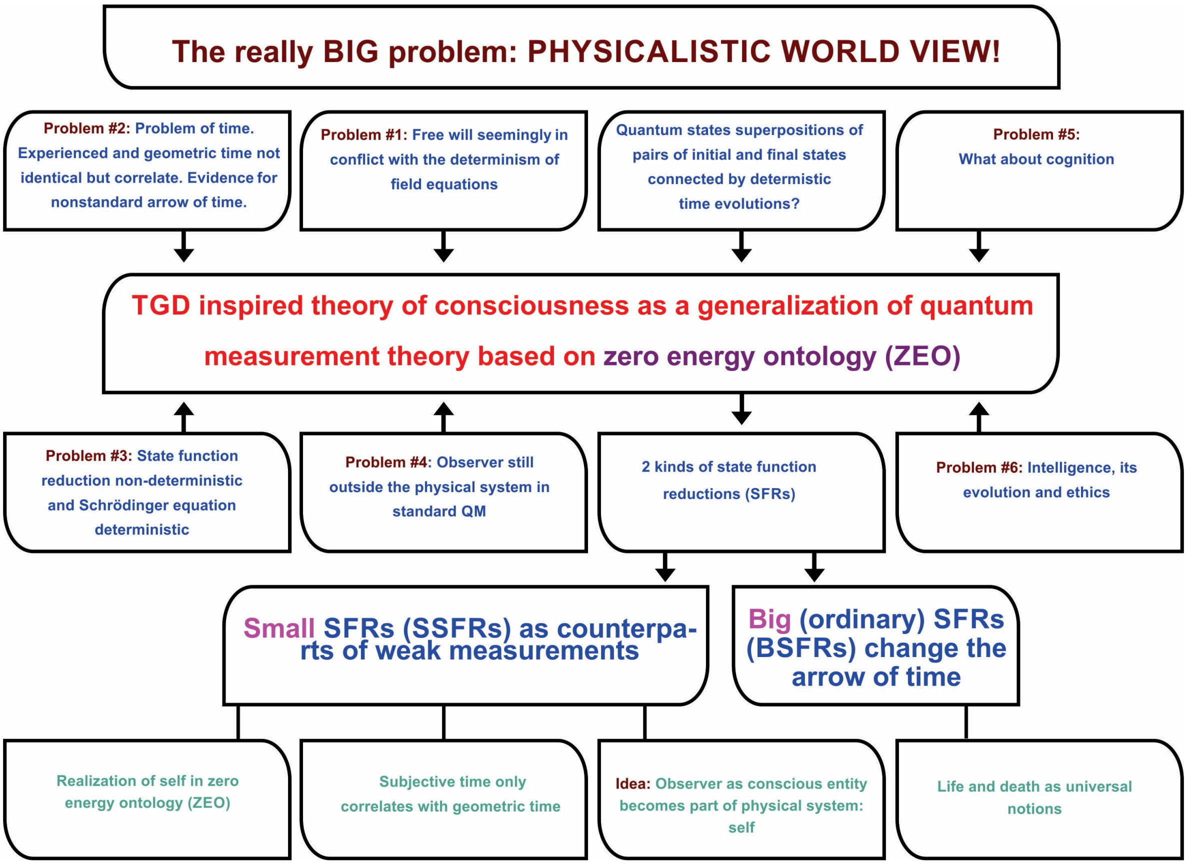

Figure 11: Consciousness theory from quantum measurement theory

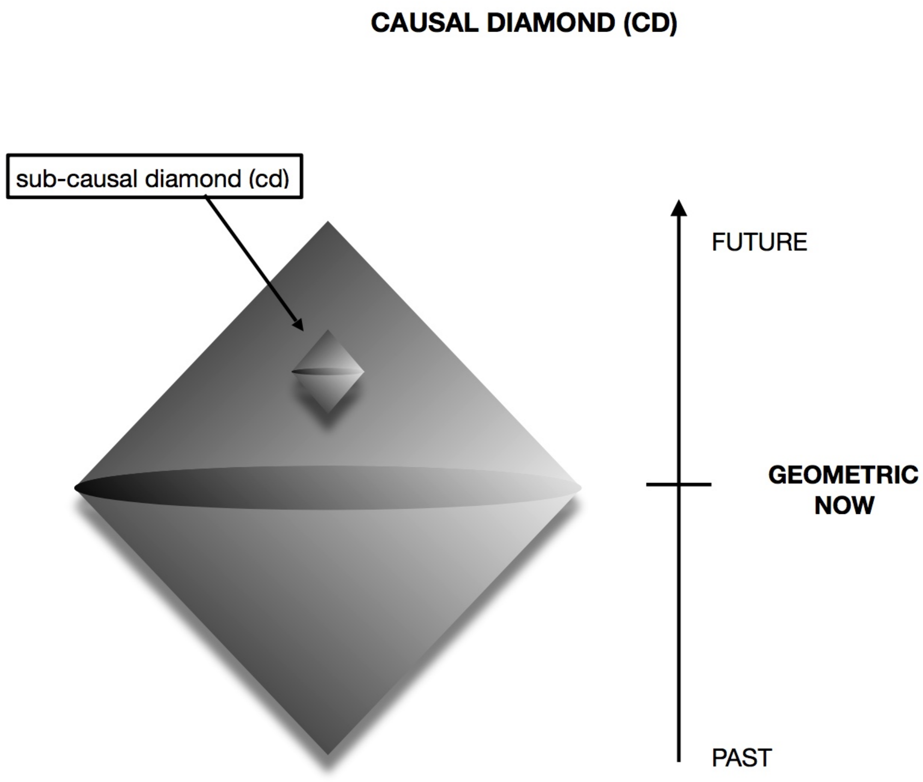

Figure 12: Causal diamond

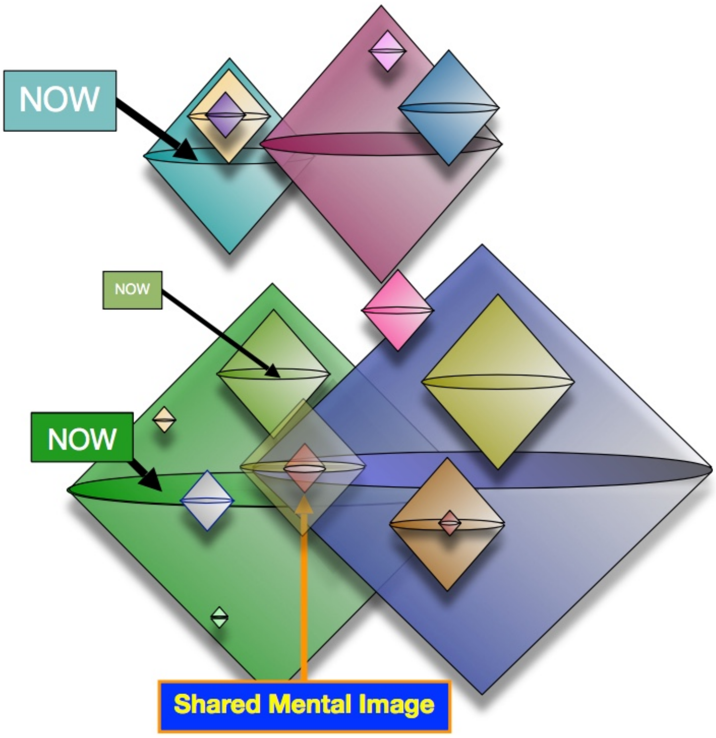

Figure 13: CDs define a fractal "conscious atlas"

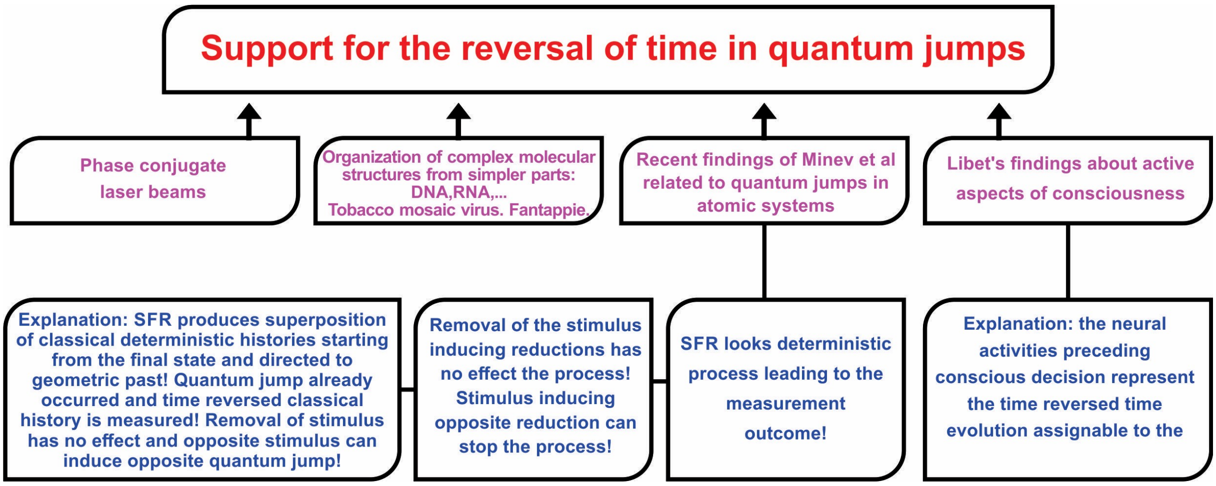

Figure 14: Time reversal occurs in BSFR

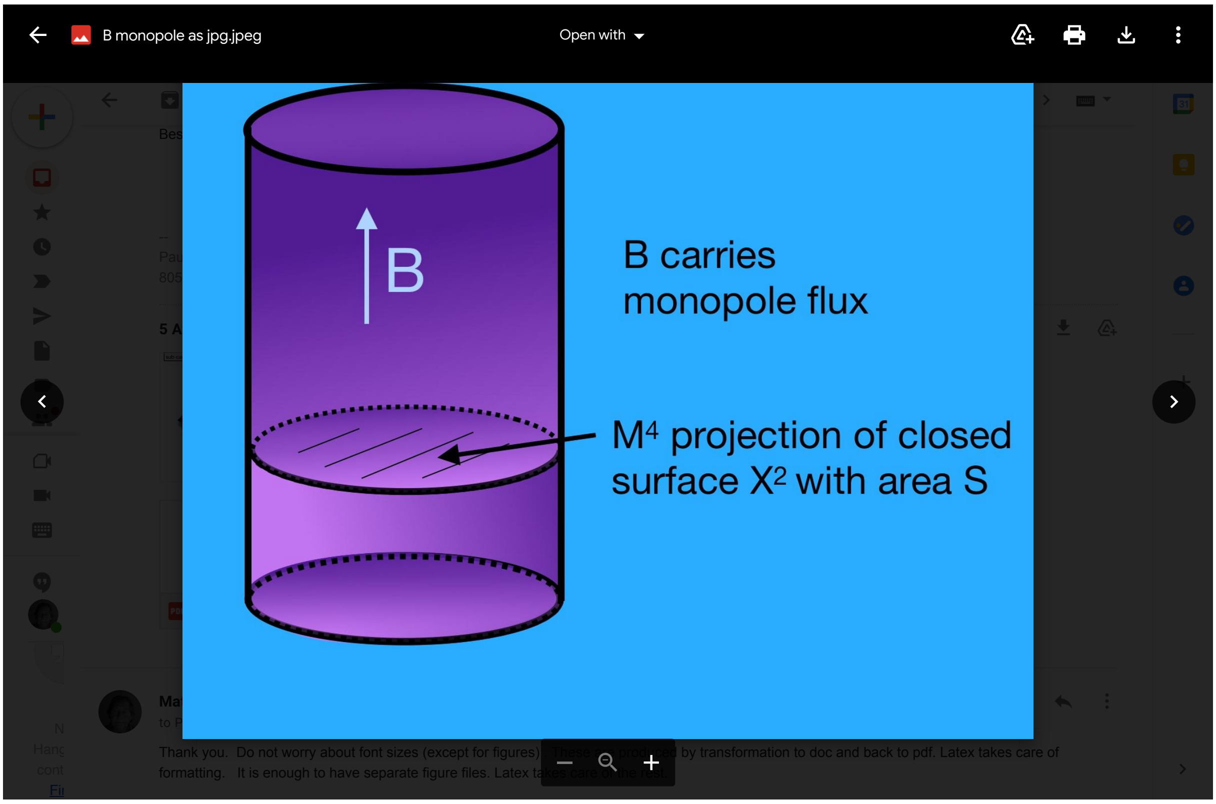

Figure 15: The M^4 projection of a closed surface X^2 with area S defining the cross section for monopole flux tube. Flux quantization $e\oint B\cdot dS = eBS = kh$ at single sheet of n-sheeted flux tube gives for cyclotron frequency $f_c = ZeB/2\pi m = khZ/2\pi mS$. The variation of S implies frequency modulation.

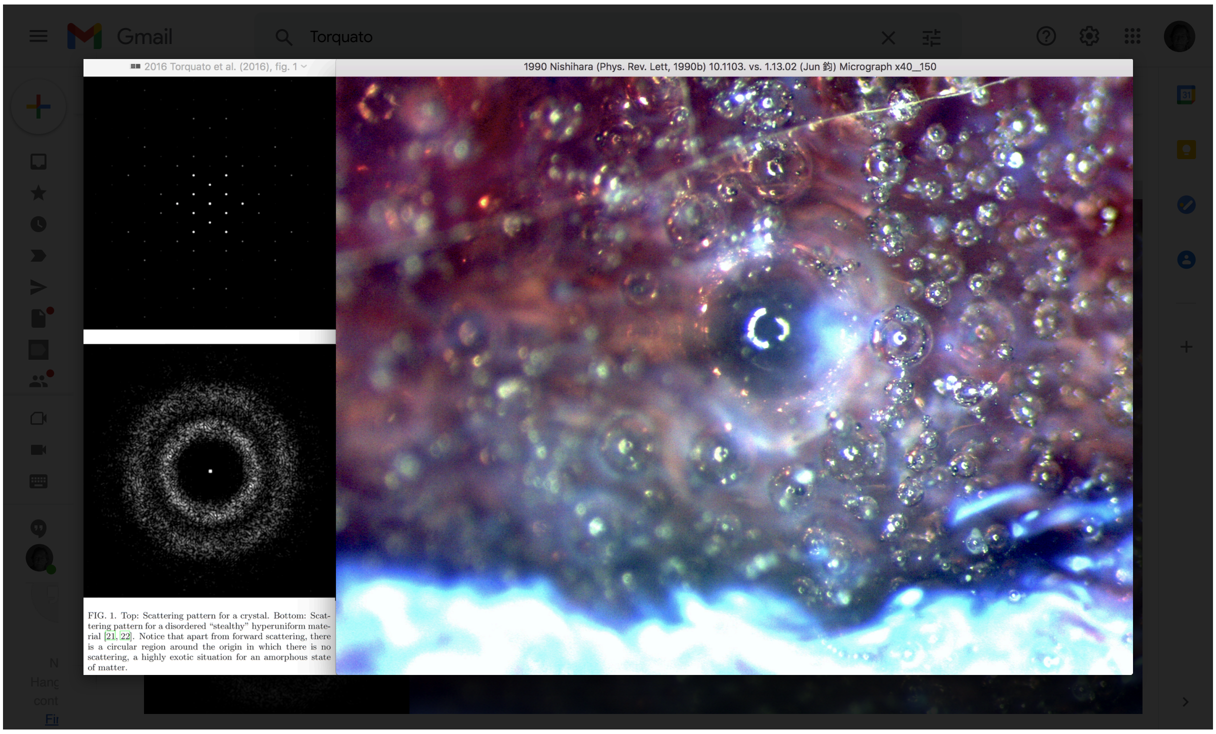

Figure 16: The scattering from a hyperuniform amorphous material shows no scattering in small angles apart from the forward peak (`https://cutt.ly/ZWyLgjk`). This is very untypical in amorphous matter and might reflect the diffraction pattern of dark photons at the magnetic body of the system.

REFERENCES

Mathematics

[A1] Langlands program. Available at: https://en.wikipedia.org/wiki/Langlands_program.

[A2] Urbano F Barros MM, Chen B-Y. Quaternion CR-submanifolds of quaternion manifolds. *Kodai Mathematical Journal*, 4(3):399–417, 2020. Available at: https://www.researchgate.net/publication/38325032_Quaternion_CR-submanifolds_of_quaternion_manifolds.

[A3] Freed DS. The Geometry of Loop Groups, 1985.

[A4] Frenkel E. Recent Advances in Langlands program. *AMS*, 41(2):151–184, 2004.

[A5] Frenkel E. Lectures on Langlands program and conformal field theory, 2005. Available at: https://arxiv.org/abs/hep-th/0512172.

[A6] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc*, 8(43):133–151, 1981.. Available at: https://tinyurl.com/pb8zpqo.

[A7] Gelbart S. An elementary introduction to the langlands program. *Bulletin of the AMS*, 10(2), 1984. Available at:https://tinyurl.com/mkqhp5n.

Theoretical Physics

[B1] Minev ZK et al. To catch and reverse a quantum jump mid-flight, 2019. Available at: https://arxiv.org/abs/1803.00545.

[B2] Wilczek F. Quantum Time Crystals. *Phys. Rev. Lett.*, 109(16), 2012. Available at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.109.160401.

[B3] Oshikawa M Watanabe H. Absence of Quantum Time Crystals. *Phys. Rev. Lett.*, 114(25), 2015. Available at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.114.251603.

Condensed Matter Physics

[D1] Xu GQ and. Demkowicz MJ. Healing of Nanocracks by Disclinations. *Phys. Rev. Lett.*, 111(145501), 2013. Available at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.111.145501. .

[D2] Keimer B and Moore JE. The physics of quantum materials. *Nature*, 13:1045,1055, 2017. Available at: https://www.nature.com/articles/nphys4302.

[D3] Shannon N Benton O, Sikora O. Seeing the light : experimental signatures of emergent electromagnetism in a quantum spin ice, 2012. Available at: https://arxiv.org/pdf/1204.1325.pdf.

[D4] Carpentier D. Topology of Bands in Solids: From Insulators to Dirac Matter. Seminaire Poincare, 2014. Available at: https://hal-ens-lyon.archives-ouvertes.fr/ensl-01053735/document.

[D5] Boyce B et al. Autonomous healing of fatigue cracks via cold welding. *Nature*, 111(145501), 2013. Available at: www.nature.com/articles/s41586-023-06223-0. .

[D6] Chakram S et al. *Phys Rev Lett*, 2021. Avaialble at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.107701.

[D7] Chakram S et al. *Nature Physics*, 2022. Avaialble at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.107701.

[D8] Chatterjee S et al. Lifshitz transition from valence fluctuations in YbAl3. *Nature Communications*, 8(852), 2017. Available at: https://www.nature.com/articles/s41467-017-00946-1.

[D9] Chen Y et al. Non-Abelian gauge field optics. *Nature Communications*, 10(3125), 2019. Available at: https://www.nature.com/articles/s41467-019-10974-8.

[D10] Hirthe S et al. Magnetically mediated hole pairing in fermionic ladders of ultracold atoms. *Nature*, 613:463–467, 2023. Available at: https://www.nature.com/articles/s41586-022-05437-y.

[D11] Mills R et al. Spectroscopic and NMR identification of novel hybrid ions in fractional quantum energy states formed by an exothermic reaction of atomic hydrogen with certain catalysts, 2003. Available at: https://www.blacklightpower.com/techpapers.html.

[D12] Nikoo SN et al. Electrical control of glass-like dynamics in vanadium dioxide for data storage and processing. *Nature Electronics*, 5(9):1–8, 2022. Available at: https://www.nature.com/articles/s41928-022-00812-z.

[D13] Ojha K et al. Double-layer structure of the Pt(111) aqueous electrolyte interface. *PNAS*, 119(3), 2022. Available at: https://www.pnas.org/content/119/3/e2116016119.

[D14] Pace SD et al. Emergent Fine Structure Constant of Quantum Spin Ice Is Large. *Phys Rev Lett*, (117205), 2021. Available at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.117205.

[D15] Siddhardh C et al. Spectroscopy of Spinons in Coulomb Quantum Spin Liquids. *Phys Rev Let*, 124(097204), 2020. Available at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.124.097204.

[D16] Song T et al. Unconventional superconducting quantum criticality in monolayer WTe2. *Nat. Phys.*, 2024. Available at: https://doi.org/10.1038/s41567-023-02291-1. .

[D17] Soter A et al. High-resolution laser resonances of antiprotonic helium in superfluid ^{4}He. *Nature*, 603:411–415, 2022. Available at: https://www.nature.com/articles/s41586-022-04440-7.

[D18] Whittaker CE et al. Polariton Pattern Formation and Photon Statistics of the Associated Emission, 2017. Available at: https://journals.aps.org/prx/abstract/10.1103/PhysRevX.7.031033.

[D19] Zhang et al. Observation of a Discrete Time Crystal, 2016. Available at: https://arxiv.org/abs/1609.08684.

[D20] Venkataraman G. Soft modes and structural phase transitions. *Bull. Mater. Sci.*, 1(3,4):129–179, 1979. Available at: https://www.ias.ac.in/article/fulltext/boms/001/03-04/0129-0170.

[D21] Allen J. Dynamics of an Exciton-Polariton Condensate, 2016. Available at: https://guava.physics.uiuc.edu/~nigel/courses/569/Essays_Spring2018/Files/Allen.pdf.

[D22] Liu WM Luo HB, Li Lu. Three-Dimensional Skyrmions with Arbitrary Topological Number in a Ferromagnetic Spin-1 Bose-Einstein Condensate. *Scientific Reports*, 9(18804), 2019. Available at: https://www.nature.com/articles/s41598-019-54856-x.

[D23] Chaplin M. Water Structure and Behavior, 2005. Available at: https://www.lsbu.ac.uk/water/index.html. For the icosahedral clustering see https://www.lsbu.ac.uk/water/clusters.html.

[D24] Tang MH et al Pai R, Singh A. Stabilization of gamma sulfur at room temperature to enable the use of carbonate electrolyte in Li-S batteries. *Commun Chem*, 5(17), 2022. Available at: https://doi.org/10.1038/s42004-022-00626-2.

[D25] Moessner R Rehn J. Maxwell electromagnetism as an emergent phenomenon in condensed matter. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 374(2075), 2016. Available at: https://arxiv.org/abs/1605.05874.

[D26] Wang Z Qu F Ren JR, Wang H. The Wu-Yang potential of Magnetic Skyrmion from SU(2) Flat Connection, 2019. Available at: https://arxiv.org/abs/1812.07974.

[D27] Murakami S. Two-dimensional topological insulators and their edge states. *J. Phys.: Conf. Ser. 302 012019*, 2011. Available at: https://iopscience.iop.org/article/10.1088/1742-6596/302/1/012019/pdf.

[D28] Torquato S. Hyperuniformity and its generalizations. *Phys Rev E*, 94(022122), 2016. Available at: https://journals.aps.org/pre/abstract/10.1103/PhysRevE.94.022122.

[D29] Yakir Aharonov Y Sandu Popescu S and Daniel Rohrlich D. On conservation laws in quantum mechanics. *PNAS*, 118(1), 2020.Available at: https://doi.org/10.1073/pnas.1921529118.

[D30] Baggioli M Setty C and Zaccone A. *Phys. Rev. B*, 105. L020506, 2022. Available at: https://physics.aps.org/articles/v15/s11. .

[D31] Wei BB Sun G and Kou SP. Fidelity as a probe for a deconfined quantum critical point. *Phys. Rev. B*, 100(064427), 2019. Available at: https://journals.aps.org/prb/abstract/10.1103/PhysRevB.100.064427.

[D32] Grinenko V. State with spontaneously broken time-reversal symmetry above the superconducting phase transition. *Nature Physics*, 2021. Available at: https://www.nature.com/articles/s41567-021-01350-9.

[D33] Kitaev A. Yu. Unpaired Majorana fermions in quantum wires. *Physics-Uspekhi Supplement*, 44(131):131 136, 2001. https://arxiv.org/abs/cond-mat/0010440.

Cosmology and Astro-Physics

[E1] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity, 2003. Available at: https://arxiv.org/abs/astro-ph/0310036.

Biology

[I1] The Fourth Phase of Water: Dr. Gerald Pollack at TEDxGuelphU, 2014. Available at: https://www.youtube.com/watch?v=i-T7tCMUDXU.

[I2] Pollack G. *Cells, Gels and the Engines of Life.* Ebner and Sons, 2000. Available at: https://www.cellsandgels.com/.

[I3] England J Perunov N, Marsland R. Statistical Physics of Adaptation, 2014. Available at: https://arxiv.org/pdf/1412.1875v1.pdf.

[I4] Zhao Q Pollack GH, Figueroa X. Molecules, water, and radiant energy: new clues for the origin of life. *Int J Mol Sci*, 10:1419–1429, 2009. Available at: https://tinyurl.com/ntkfhlc.

[I5] Pollack GH Zheng J-M. Long-range forces extending from polymer-gel surfaces. *Phys Rev E*, 68:031408–, 2003. Available at: https://tinyurl.com/ntkfhlc.

Books related to TGD

[K1] Pitkänen M. *Topological Geometrodynamics.* 1983. Thesis in Helsinki University 1983.

[K2] Pitkänen M. DNA as Topological Quantum Computer. In *Quantum - and Classical Computation in TGD Universe. https://tgdtheory.fi/tgdhtml/Btgdcomp.html*. Available at: https://tgdtheory.fi/pdfpool/dnatqc.pdf, 2015.

[K3] Pitkänen M. Topological Quantum Computation in TGD Universe. In *Quantum - and Classical Computation in TGD Universe. https://tgdtheory.fi/tgdhtml/Btgdcomp.html*. Available at: https://tgdtheory.fi/pdfpool/tqc.pdf, 2015.

[K4] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/prext.pdf, 2023.

[K5] Pitkänen M. Are dark photons behind biophotons? In *TGD and Quantum Biology: Part I. https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/biophotonslian.pdf, 2023.

[K6] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/class.pdf, 2023.

[K7] Pitkänen M. Breakthrough in understanding of $M^8 - H$ duality. Available at: https://tgdtheory.fi/pdfpool/M8H.pdf., 2023.

[K8] Pitkänen M. Cold Fusion Again. In *TGD and Nuclear Physics. https://tgdtheory.fi/tgdhtml/Bnucl.html*. Available at: https://tgdtheory.fi/pdfpool/coldfusionagain.pdf, 2023.

[K9] Pitkänen M. Comparing Berry phase model of super-conductivity with TGD based model. In *TGD and Condensed Matter. https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/SCBerryTGD.pdf, 2023.

[K10] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/elvafu.pdf, 2023.

[K11] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/compl1.pdf, 2023.

[K12] Pitkänen M. Criticality and dark matter: part I. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark1.pdf, 2023.

[K13] Pitkänen M. Criticality and dark matter: part II. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark2.pdf, 2023.

[K14] Pitkänen M. Criticality and dark matter: part III. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark3.pdf, 2023.

[K15] Pitkänen M. Criticality and dark matter: part IV. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark4.pdf, 2023.

[K16] Pitkänen M. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG: Part I. https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/eegdark.pdf, 2023.

[K17] Pitkänen M. Evolution of Ideas about Hyper-finite Factors in TGD. In *Topological Geometrodynamics: Overview: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdoverview2.* Available at: https://tgdtheory.fi/pdfpool/vNeumannnew, 2023.

[K18] Pitkänen M. Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life. In *Physics in Many-Sheeted Space-Time: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/expearth.pdf., 2023.

[K19] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/kahler.pdf., 2023.

[K20] Pitkänen M. Langlands Program and TGD. In *TGD as a Generalized Number Theory: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdnumber2.html*. Available at: https://tgdtheory.fi/pdfpool/Langlands.pdf, 2023.

[K21] Pitkänen M. Magnetic Sensory Canvas Hypothesis. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/mec.pdf, 2023.

[K22] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mless.pdf, 2023.

[K23] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/nmpc.pdf, 2023.

[K24] Pitkänen M. Nuclear String Hypothesis. In *TGD and Nuclear Physics.* *https://tgdtheory.fi/tgdhtml/Bnucl.html*. Available at: https://tgdtheory.fi/pdfpool/nuclstring.pdf, 2023.

[K25] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: https://www.tgdtheory.fi/tgdhtml/padphys.html., 2023.

[K26] Pitkänen M. p-Adic Physics: Physical Ideas. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/phblocks.pdf, 2023.

[K27] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *TGD and Condensed Matter.* *https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/anyontgd.pdf, 2023.

[K28] Pitkänen M. Quantum Mind and Neuroscience. In *TGD and EEG: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/lianPN.pdf, 2023.

[K29] Pitkänen M. Quantum Mind, Magnetic Body, and Biological Body. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/lianPB.pdf, 2023.

[K30] Pitkänen M. Quantum Model for Bio-Superconductivity: I. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/biosupercondI.pdf, 2023.

[K31] Pitkänen M. Quantum Model for Bio-Superconductivity: II. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/biosupercondII.pdf, 2023.

[K32] Pitkänen M. Quantum Model for Nerve Pulse. In *TGD and EEG: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/nervepulse.pdf, 2023.

[K33] Pitkänen M. Quantum Model of EEG. In *TGD and EEG: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/eegII.pdf, 2023.

[K34] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/wcwnew.pdf, 2023.

[K35] Pitkänen M. Some questions related to the twistor lift of TGD. In *Quantum TGD: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/twistquestions.pdf, 2023.

[K36] Pitkänen M. Summary of TGD Inspired Ideas about Free Energy. In *TGD and Fringe Physics.* *https://tgdtheory.fi/tgdhtml/Bfreenergies.html*. Available at: https://tgdtheory.fi/pdfpool/freerg.pdf, 2023.

[K37] Pitkänen M. TGD as a Generalized Number Theory: Infinite Primes. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionc.pdf, 2023.

[K38] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionb.pdf, 2023.

[K39] Pitkänen M. The classical part of the twistor story. In *Quantum TGD: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/twistorstory.pdf, 2023.

[K40] Pitkänen M. The Recent View about Twistorialization in TGD Framework. In *Quantum TGD: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/smatrix.pdf, 2023.

[K41] Pitkänen M. Three new physics realizations of the genetic code and the role of dark matter in bio-systems. In *Genes and Memes: Part II.* *https://tgdtheory.fi/tgdhtml/Bgenememe2.html*. Available at: https://tgdtheory.fi/pdfpool/dnatqccodes.pdf, 2023.

[K42] Pitkänen M. Was von Neumann Right After All? In *TGD and Hyper-finite Factors.* *https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/vNeumann.pdf, 2023.

[K43] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/cspin.pdf, 2023.

[K44] Pitkänen M. Zero Energy Ontology. In *Quantum TGD: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/ZEO.pdf, 2023.

Articles about TGD

[L1] Geometric Langlands program. Available at: https://matpitka.blogspot.com/2007/11/geometric-langlands.html.

[L2] Pitkänen M. The Geometry of CP_2 and its Relationship to Standard Model. 2010. Available at: https://tgdtheory.fi/pdfpool/appendb.pdf.

[L3] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. Available at: https://tgdtheory.fi/pdfpool/append.pdf., 2006.

[L4] Pitkänen M. Langlands conjectures in TGD framework. Available at: https://tgdtheory.fi/public_html/articles/Langlandsagain.pdf., 2011.

[L5] Pitkänen M. "Applied biophysics of activated water" and the basic mechanism of water memory in TGD framework. Available at: https://tgdtheory.fi/public_html/articles/activatedwater.pdf., 2013.

[L6] Pitkänen M. Hypnosis as Remote Mental Interaction. Available at: https://tgdtheory.fi/public_html/articles/hypnosisarticle.pdf., 2013.

[L7] Pitkänen M. Geometric theory of harmony. Available at: https://tgdtheory.fi/public_html/articles/harmonytheory.pdf., 2014.

[L8] Pitkänen M. Pollack's Findings about Fourth phase of Water : TGD View. Available at: https://tgdtheory.fi/public_html/articles/PollackYoutube.pdf., 2014.

[L9] Pitkänen M. Jeremy England's vision about life and evolution: comparison with TGD approach . Available at: https://tgdtheory.fi/public_html/articles/englandtgd.pdf., 2015.

[L10] Pitkänen M. Cold Fusion Again . Available at: https://tgdtheory.fi/public_html/articles/cfagain.pdf., 2015.

[L11] Pitkänen M. Maintenance problem for Earth's magnetic field. Available at: https://tgdtheory.fi/public_html/articles/Bmaintenance.pdf., 2015.

[L12] Pitkänen M. Could Pollack effect make cell membrane a self-loading battery? Available at: https://tgdtheory.fi/public_html/articles/cfbattery.pdf., 2016.

[L13] Pitkänen M. Hydrinos again. Available at: https://tgdtheory.fi/public_html/articles/Millsagain.pdf., 2016.

[L14] Pitkänen M. Langlands Program and TGD: Years Later. Available at: https://tgdtheory.fi/public_html/articles/langlandsnew.pdf., 2016.

[L15] Pitkänen M. Cold fusion, low energy nuclear reactions, or dark nuclear synthesis? Available at: https://tgdtheory.fi/public_html/articles/krivit.pdf., 2017.

[L16] Pitkänen M. Could McKay correspondence generalize in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/McKay.pdf., 2017.

[L17] Pitkänen M. Does valence bond theory relate to the hierarchy of Planck constants? Available at: https://tgdtheory.fi/public_html/articles/valenceheff.pdf., 2017.

[L18] Pitkänen M. Mysteriously disappearing valence electrons of rare Earth metals and hierarchy of Planck constants. Available at: https://tgdtheory.fi/public_html/articles/rareearth.pdf., 2017.

[L19] Pitkänen M. p-Adicization and adelic physics. Available at: https://tgdtheory.fi/public_html/articles/adelicphysics.pdf., 2017.

[L20] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[L21] Pitkänen M. Philosophy of Adelic Physics. Available at: https://tgdtheory.fi/public_html/articles/adelephysics.pdf., 2017.

[L22] Pitkänen M. Questions about twistor lift of TGD. Available at: https://tgdtheory.fi/public_html/articles/twistquestions.pdf., 2017.

[L23] Pitkänen M. Two different lifetimes for neutron as evidence for dark protons. Available at: https://tgdtheory.fi/public_html/articles/nlifetime.pdf., 2017.

[L24] Pitkänen M. p-Adicization and Adelic Physics. *Pre-Space-Time Journal.*, 8(3), 2017. See also https://tgtheory.fi/public_html/articles/adelicphysics.pdf.

[L25] Pitkänen M. New insights about quantum criticality for twistor lift inspired by analogy with ordinary criticality. Available at: https://tgdtheory.fi/public_html/articles/zeocriticality.pdf., 2018.

[L26] Pitkänen M. TGD view about quasars. Available at: https://tgdtheory.fi/public_html/articles/meco.pdf., 2018.

[L27] Pitkänen M. The Recent View about Twistorialization in TGD Framework. Available at: https://tgdtheory.fi/public_html/articles/smatrix.pdf., 2018.

[L28] Pitkänen M. Two new findings related to high Tc super-conductivity. Available at: https://tgdtheory.fi/public_html/articles/supercondnew.pdf., 2018.

[L29] Pitkänen M. Copenhagen interpretation dead: long live ZEO based quantum measurement theory! Available at: https://tgdtheory.fi/public_html/articles/Bohrdead.pdf., 2019.

[L30] Pitkänen M. Cosmic string model for the formation of galaxies and stars. Available at: https://tgdtheory.fi/public_html/articles/galaxystars.pdf., 2019.

[L31] Pitkänen M. M^8-H duality and consciousness. Available at: https://tgdtheory.fi/public_html/articles/M8Hconsc.pdf., 2019.

[L32] Pitkänen M. New results related to M^8-H duality. Available at: https://tgdtheory.fi/public_html/articles/M8Hduality.pdf., 2019.

[L33] Pitkänen M. Quantum self-organization by h_{eff} changing phase transitions. Available at: https://tgdtheory.fi/public_html/articles/heffselforg.pdf., 2019.

[L34] Pitkänen M. Solar Metallicity Problem from TGD Perspective. Available at: https://tgdtheory.fi/public_html/articles/darkcore.pdf., 2019.

[L35] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: https://tgdtheory.fi/public_html/articles/zeoquestions.pdf., 2019.

[L36] Pitkänen M. SUSY in TGD Universe. Available at: https://tgdtheory.fi/public_html/articles/susyTGD.pdf., 2019.

[L37] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, and Inclusions of Hyperfinite Factors. Available at: https://tgdtheory.fi/public_html/articles/McKay.pdf., 2019.

[L38] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, M^8-H Duality, SUSY, and Twistors. Available at: https://tgdtheory.fi/public_html/articles/McKaygeneral.pdf., 2019.

[L39] Pitkänen M. A critical re-examination of M^8-H duality hypothesis: part I. Available at: https://tgdtheory.fi/public_html/articles/M8H1.pdf., 2020.

[L40] Pitkänen M. A critical re-examination of M^8-H duality hypothesis: part II. Available at: https://tgdtheory.fi/public_html/articles/M8H2.pdf., 2020.

[L41] Pitkänen M. Could quantum randomness have something to do with classical chaos? Available at: https://tgdtheory.fi/public_html/articles/chaostgd.pdf., 2020.

[L42] Pitkänen M. Could TGD provide new solutions to the energy problem? Available at: https://tgdtheory.fi/public_html/articles/proposal.pdf., 2020.

[L43] Pitkänen M. Could ZEO provide a new approach to the quantization of fermions? Available at: https://tgdtheory.fi/public_html/articles/secondquant.pdf., 2020.

[L44] Pitkänen M. How to compose beautiful music of light in bio-harmony? https://tgdtheory.fi/public_html/articles/bioharmony2020.pdf., 2020.

[L45] Pitkänen M. Summary of Topological Geometrodynamics. https://tgdtheory.fi/public_html/articles/tgdarticle.pdf., 2020.

[L46] Pitkänen M. TGD interpretation of new experimental results about the mechanism of anesthesia. Available at: https://tgdtheory.fi/public_html/articles/anesthesianewest.pdf., 2020.

[L47] Pitkänen M. The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group. Available at: https://tgdtheory.fi/public_html/articles/SSFRGalois.pdf., 2020.

[L48] Pitkänen M. Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory. Available at: https://tgdtheory.fi/public_html/articles/kahlersmhort.pdf., 2020.

[L49] Pitkänen M. Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory (short version). Available at: https://tgdtheory.fi/public_html/articles/kahlersm.pdf., 2020.

[L50] Pitkänen M. A TGD based view about neuron. https://tgdtheory.fi/public_html/articles/TGDneuron.pdf., 2021.

[L51] Pitkänen M. About TGD counterparts of twistor amplitudes. https://tgdtheory.fi/public_html/articles/twisttgd.pdf., 2021.

[L52] Pitkänen M. About the role of Galois groups in TGD framework. https://tgdtheory.fi/public_html/articles/GaloisTGD.pdf., 2021.

[L53] Pitkänen M. Can one regard leptons as effectively local 3-quark composites? https://tgdtheory.fi/public_html/articles/leptoDelta.pdf., 2021.

[L54] Pitkänen M. Comparing the Berry phase model of super-conductivity with the TGD based model. https://tgdtheory.fi/public_html/articles/SCBerryTGD.pdf., 2021.

[L55] Pitkänen M. Does Consciousness Survive Bodily Death? https://tgdtheory.fi/public_html/articles/BICS.pdf., 2021.

[L56] Pitkänen M. Does the notion of polynomial of infinite order make sense? https://tgdtheory.fi/public_html/articles/transcendGalois.pdf., 2021.

[L57] Pitkänen M. Galois code and genes. https://tgdtheory.fi/public_html/articles/Galoiscode.pdf., 2021.

[L58] Pitkänen M. Is genetic code part of fundamental physics in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/TIH.pdf., 2021.

[L59] Pitkänen M. Is $M^8 - H$ duality consistent with Fourier analysis at the level of $M^4 \times CP_2$? https://tgdtheory.fi/public_html/articles/M8Hperiodic.pdf., 2021.

[L60] Pitkänen M. Negentropy Maximization Principle and Second Law. Available at: https://tgdtheory.fi/public_html/articles/nmpsecondlaw.pdf., 2021.

[L61] Pitkänen M. Neutrinos and TGD. https://tgdtheory.fi/public_html/articles/TGDneutrino.pdf., 2021.

[L62] Pitkänen M. Questions about coupling constant evolution. https://tgdtheory.fi/public_html/articles/ccheff.pdf., 2021.

[L63] Pitkänen M. Some questions concerning zero energy ontology. https://tgdtheory.fi/public_html/articles/zeonew.pdf., 2021.

[L64] Pitkänen M. Spin Glasses, Complexity, and TGD. https://tgdtheory.fi/public_html/articles/sg.pdf., 2021.

[L65] Pitkänen M. TGD and Condensed Matter. https://tgdtheory.fi/public_html/articles/TGDcondmatshort.pdf., 2021.

[L66] Pitkänen M. TGD and Quantum Hydrodynamics. https://tgdtheory.fi/public_html/articles/TGDhydro.pdf., 2021.

[L67] Pitkänen M. TGD as it is towards the end of 2021. https://tgdtheory.fi/public_html/articles/TGD2021.pdf., 2021.

[L68] Pitkänen M. TGD based interpretation for the strange findings of Eric Reiner. https://tgdtheory.fi/public_html/articles/unquantum.pdf., 2021.

[L69] Pitkänen M. TGD view of the engine powering jets from active galactic nuclei. https://tgdtheory.fi/public_html/articles/galjets.pdf., 2021.

[L70] Pitkänen M. Updated version of Expanding Earth model. https://tgdtheory.fi/public_html/articles/expearth2021.pdf., 2021.

[L71] Pitkänen M. What could 2-D minimal surfaces teach about TGD? https://tgdtheory.fi/public_html/articles/minimal.pdf., 2021.

[L72] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L73] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L74] Pitkänen M. About the number theoretic aspects of zero energy ontology. https://tgdtheory.fi/public_html/articles/ZEOnumber.pdf., 2022.

[L75] Pitkänen M. Comparison of Orch-OR hypothesis with the TGD point of view. https://tgdtheory.fi/public_html/articles/penrose.pdf., 2022.

[L76] Pitkänen M. How animals without brain can behave as if they had brain. https://tgdtheory.fi/public_html/articles/precns.pdf., 2022.

[L77] Pitkänen M. McKay Correspondence from Quantum Arithmetics Replacing Sum and Product with Direct Sum and Tensor Product? . https://tgdtheory.fi/public_html/articles/McKayGal.pdf., 2022.

[L78] Pitkänen M. Quantum biological teleportation using multiple 6-qubits. https://tgdtheory.fi/public_html/articles/fluteteleport.pdf., 2022.

[L79] Pitkänen M. TGD inspired model for freezing in nano scales. https://tgdtheory.fi/public_html/articles/freezing.pdf., 2022.

[L80] Pitkänen M. The realization of genetic code in terms of dark nucleon and dark photon triplets. https://tgdtheory.fi/public_html/articles/darkcode.pdf., 2022.

[L81] Pitkänen M. Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole. https://tgdtheory.fi/public_html/articles/fusionTGD.pdf., 2022.

[L82] Pitkänen M. About tessellations in hyperbolic 3-space and their relation to the genetic code . https://tgdtheory.fi/public_html/articles/tessellationH3.pdf., 2023.

[L83] Pitkänen M. About the mechanism of the energy transfer in photosynthesis. https://tgdtheory.fi/public_html/articles/photosynth.pdf., 2023.

[L84] Pitkänen M. Could neuronal system and even GPT give rise to a computer with a variable arrow of time? https://tgdtheory.fi/public_html/articles/GPT.pdf., 2023.

[L85] Pitkänen M. Deep learning from the TGD point of view. https://tgdtheory.fi/public_html/articles/TGDdeeplearn.pdf., 2023.

[L86] Pitkänen M. Is Negentropy Maximization Principle needed as an independent principle? https://tgdtheory.fi/public_html/articles/NMPcrit.pdf., 2023.

[L87] Pitkänen M. Magnetic Bubbles in TGD Universe: Part I. https://tgdtheory.fi/public_html/articles/magnbubble1.pdf., 2023.

[L88] Pitkänen M. Magnetic Bubbles in TGD Universe: Part II. https://tgdtheory.fi/public_html/articles/magnbubble2.pdf., 2023.

[L89] Pitkänen M. The TGD view of the recently discovered gravitational hum as gravitational diffraction. https://tgdtheory.fi/public_html/articles/gravhum.pdf., 2023.

[L90] Pitkänen M. The twistor space of $H = M^4 \times CP_2$ allows Lagrangian 6-surfaces: what does this mean physically? https://tgdtheory.fi/public_html/articles/superminimal.pdf., 2024.

[L91] Pitkänen M and Rastmanesh R. Homeostasis as self-organized quantum criticality. Available at: https://tgdtheory.fi/public_html/articles/SP.pdf., 2020.

[L92] Pitkänen M and Rastmanesh R. The based view about dark matter at the level of molecular biology. Available at: https://tgdtheory.fi/public_html/articles/darkchemi.pdf., 2020.