TGD as it is towards end of 2021

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Contents

Abstract

This article tries to give a rough overall view about Topological Geometrodynamics (TGD) as it is towards the end of 2021. The two views about TGD and their relationship are discussed at the general level.

1. The first view generalizes Einstein's program for the geometrization of physics. Entire quantum physics is geometrized in terms of the notion of "world of classical worlds" (WCW), which by its infinite dimension has unique Kähler geometry.
2. Second vision reduces physics to number theory. Classical number fields (reals, complex numbers, quaternions, and octonions) are central as also p-adic number fields and extensions of rationals. The physics is classically coded by algebraic 4-surfaces in complexified M^8 having octonionic structure and "roots" of octonionic polynomials obtained as algebraic continuations of real polynomials with rational coefficients. M_c^8 has an interpretation as an analog of momentum space.

The preparation of this summary led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.
2. The understanding of the details of the M^8-H duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. M^8-H duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H. Mass shell (its positive and negative energy parts) is mapped to a light-like boundary of CD with size $T = h_{eff}/m$, m the mass associated with momentum.
3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.
4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

Article includes also a section about neutrinos and TGD. The motivation is that the recent results related to neutrino mixing led to a dramatic progress in the understanding of the role of right-handed neutrino solving long-standing problems of quantum TGD.

1 Introduction

The purpose of this article is to give a rough overall view about Topological Geometrodynamics (TGD) as it is now. It must be emphasized that TGD is only a vision, not a theory able to provide precise rules for calculating scattering amplitudes. A collective theoretical and experimental effort would be needed to achieve this.

It is perhaps good to explain what TGD is not and what it is or hoped to be. The article [L29] gives an overview of various aspects of TGD and is warmly recommended.

1. "Geometro-" refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with

sub-manifold geometry. The basic motivation is the loss of classical conservation laws in General Relativity Theory (GRT)(see **Fig. 1**). Also the interpretation as a generalization of string models by replacing string with 3-D surface is natural.

Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [L2]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely so that TGD is unique [L11, L13](see **Fig. 6**). The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "world of classical worlds" (WCW) - becomes the basic object endowed with Kähler geometry (see **Fig. 7**). General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. Given 3-surface fixes the space-time surface almost completely as analog of Bohr orbit (preferred extremal).This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces.

2. Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$.

   The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 3**).

   Practically any GCI action has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correletes for massless particles. World as a set or particles is replaced with a network having particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

   "Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 10**).

3. Adelic physics fusing real and various p-adic physics are part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical" orders. Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in all scales.

   The concrete realization of the number theoretic vision is based on $M^8 - H$ duality (see **Fig. 8**). The physics in the complexification of M^8 is algebraic - field equations as partial differential equations are replaced with algebraic equations associating to a polynomial with rational coefficients a X^4 mapped to H by $M^8 - H$ duality. The dark matter hierarchy corresponds to a hierarchy of algebraic extensions of rationals inducing that for adeles and has interpretation as an evolutionary hierarchy (see **Fig. 9**).

   $M^8 - H$ duality provides two complementary visions about physics (see **Fig. 2**), and can be seen as a generalization of the q-p duality of wave mechanics, which fails to generalize to quantum field theories (QFTs).

4. In Zero energy ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 12** and **Fig. 13**).

   Quantum jumps occur between these and the basic problem of standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 14**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions.

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary of CD and states at it remain unchanged (Zeno effect).

TGD develops by explaining what TGD is and also this work led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

2. The understanding of the details of the M^8-H duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. M^8-H duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H.

3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

2 Physics as geometry

The following provides a sketchy representation of TGD based on the vision about physics as geometry which is complementary to the vision of physics as number theory. M^8-H duality relates these two visions. A longer representation can be found in [L29].

2.1 Space-time as 4-surface in $H = M^4 \times CP_2$

1. The energy problem of GRT means that since space-time is curved, one cannot define Poincare charges as Noether charges (see **Fig. 1**). If space-time X^4 is a surface in $H = M^4 \times CP_2$, the situation changes. Poincare symmetries are lifted to the level of $M^4 \subset H$.

2. Generalization of the notion of particle is in question: point-like particle $\rightarrow$ 3-surface so that TGD can be seen also as a generalization of string model. String $\rightarrow$ 3-surface. String world sheet $\rightarrow X^4$. The notions of the particle and space are unified.

3. Einstein's geometrization program is extended to standard model interactions. CP_2 codes for standard model symmetries and gauge fields. Isometries $\leftrightarrow$ color SU(3). Holonomies of spinor connection $\leftrightarrow$ electroweak U(2) [L2]. Genus-generation correspondence provides a topological explanation of the family replication phenomenon of fermions [K2]: 3 fermion families are predicted.

4. Induction of spinors structure as projection of components of spinor connection from CP_2 to X^4 is central for the geometrization. The projections of Killing vectors of color isometries yield color gauge potentials. Parallel translation at X^4 using spinor connection of H. Also spinor structure is induced and means projection of gamma matrices.

5. Dynamics for X^4 is determined by an action S consisting of Kähler action plus volume term (cosmological constant) following from the twistor lift of TGD [K19, L13].

6. The dynamics for fermions at space-time level is determined by modified Dirac action determined by S being super-symmetrically related to it. Gamma matrices are replaced with modified gamma matrices determined by the S as contractions of canonical momentum currents with gamma matrices. Preferred extremal property follows as a condition of hermiticity for the modified Dirac operator.

   Second quantized H-spinors, whose modes satisfy free massless Dirac equation in H restricted to X^4: this induces second quantization to X^4 and one avoids the usual problems of quantization in a curved background. This picture is consistent with the modified Dirac equation satisfied by the induced spinors in X^4.

   Only quarks are needed if leptons are 3-quark composites in CP_2 scale: this is possible only if one accepts the TGD view about color symmetries. This also provides a new view about matter antimatter asymmetry [L20, L34]. CP violation is forced by the M^4 part of Kähler form forced by the twistor lift.

2.1.1 Basic extremals of classical action

Practically any GCI action allows the same basic extremals (for basic questions related to classical TGD see **Fig. 3**).

1. CP_2 type extremals having light-like geodesic as M^4 projection and Euclidian signature of the induced metric serve as building bricks of elementary particles. If the volume term is absent as it might be at infinite volume limit, the geodesics become light-like curves [L45]. Wormhole contacts connecting two Minkowskian space-time sheets can be regarded as a piece of a deformed CP_2 type extremal. Monopole flux through contact stabilizes the wormhole contact.

2. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow superposition of modes with single direction of ligth-like momentum. Ideal laser beam is a convenient analogy here.

3. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and their thickenings to flux tubes are also a central notion.

2.1.2 QFT limit of TGD

The induced gauge fields and gravitational field are expressible in terms of only 4 H- coordinates. Locally the theory is too simple to be physical.

1. Many-sheeted space-time means that X^4 is topologically extremely complex. CP_2 coordinates are many-valued functions of M^4 coordinates or vice versa or both. In contrast to this, the space-time of EYM theory is topologically extremely simple.

2. Einsteinian space-times have 4-D projection to M^4. Small test particle experiences the sum of the classical gauge potentials associated with various space-time sheets. At QFT limit the sheets are replaced with a single region of M^4 made slightly curved and gauge potentials are defined as the sums of gauge potentials from different space-time sheets having common M^4 projection. Topological complexity and local simplicity are replaced with topological simplicity and local complexity. (see **Fig. 3**).

2.2 World of classical worlds (WCW)

The notion of WCW emerges as one gives up the idea about quantizing by path integral.

2.2.1 The failure of path integral forces WCW geometry

The extreme non-linearity implies that the path integral for surfaces space-time surfaces fails. A possible solution is generalize Einstein's geometrization program to the level of the entire quantum theory.

1. "World of classical worlds" (WCW) can be identified as the space of 3-surfaces endowe with a metric and spinor structure (see **Fig. 7**). Hermitian conjugation must have a geometrization. This requires Kähler structure requiring also complex structure. WCW has Kähler form and metric.

2. WCW spinors are Fock states created by fermionic oscillator operators assignable to spinor modes of H basically [L27]. WCW gamma matrices as linear combinations of fermionic (quark) oscillator operators defining analog of vielbein.

   WCW has also spinor connection and curvature in WCW. correspond The quantum states of world correspond formally to *classical* spinor fields in WCW. Gamma matrices of WCW expressinble in terms of fermionic oscillator operators are also purely classical objects.

2.2.2 Implications of General Coordinate Invariance

General Coordinate Invariance (GCI) in 4-D sense forces to assign to 3-surface X^3 a 4-surface $X^4(X^3)$, which is as unique as possible. This gives rise to Bohr orbitology and quantum classical correspondence (QCC), and holography. Also zero energy ontology (ZEO) emerges.

Quantum states quantum superpositions of space-time surfaces as analogs of Bohr orbits. QCC means that the classical theory is an exact part of quantum theory (QCC).

A solution to the basic paradox of quantum measurement theory emerges [L19]: superposition of deterministic time evolutions is replaced with a new one in state function reduction (SFR): SFR does not force any failure of determinism for individual time evolutions.

2.2.3 WCW Kähler geometry from classical action

WCW geometry is determined by a classical action defining Kähler function $K(X^3)$ for a preferred extremal $X^4(X^3)$ defining the preferred extremal/Bohr orbit [K9] (see **Fig. 7**).

1. QCC suggests that the definition of Kähler function assigns a more or less unique 4-surface $X^4(X^3)$ to 3-surface X^3. Finite non-uniqueness is however possible [L45].

2. $X^4(X^3)$ is identified as a *preferred* extremal of some general coordinate invariant (GCI) action forcing the Bohr orbit property/holography/ZEO. This means a huge reduction of degrees of freedom.

   Remark:: Already the notion of induced gauge field and metric eliminates fields as primary dynamical variables and GCI leaves locally only 4 H-coordinates as dynamical variables.

3. Twistor lift [L11, L13] of TGD geometrizes the twistor Grassmann approach to QFTs. The 6-D extremal X^6 of 6-D Kähler action as a 6- surface in the product $T(M^4) \times T(CP_2)$ of twistor spaces of M^4 and CP_2 represents the twistor space of X^4.

   The condition that X^6 reduces to an S^2 bundle with X^4 as base space, forces a dimensional reduction of 6-D Kähler action to 4-D Kähler action + volume term, whose value for the preferred extremal defines the Kähler function for $X^4(X^3)$.

4. The volume term corresponds to a p-adic length scale dependent cosmological constant Λ approach zero at long p-adic length scale so that a solution of the cosmological constant problem emerges. Preferred extremal/Bohr orbit property means a simultaneous extremal property for *both* Kähler action and volume term. This forces X^4 to have a generalized complex structure (Hamilton-Jacobi structure) so that field equations trivialize and there is no dependence on coupling parameters. Universality of dynamics follows and the TGD Universe is quantum critical. In particular, Kähler coupling strength is analogous to a critical temperature and is quantized [L41].

5. Soap film analogy is extremely useful [L45]: the analogs of soap film frames are singular surfaces of dimension $D < 4$. At the frame the space-time surface fails to be a simultaneous extremal of both actions separately and Kähler and volume actions couple to each other. The corresponding contributions to conserved isometry currents diverge but sum up to a finite contribution. The frames define the geometric analogs for the vertices of Feynman diagrams.

2.2.4 WCW geometry is unique

WCW geometry is fixed by the existence of Riemann connection and requires maximal symmetries.

1. Dan Freed [A3] found that loop space for a given Lie group allows a unique Kähler geometry: maximal isometries needed in order to have a Riemann connection. Same expected to be true now [K3, K16].

2. Twistor lift of TGD [L11, L13] means that one can replace X^4 with its twistor space $X^6(X^4)$ in the product $T(M^4) \times T(CP_2)$ of the 6-D twistor spaces $T(M^4)$ and $T(CP_2)$. $X^6(X^4)$ is 6-surface with the structure of S^2 bundle.

   Dimensionally reduced 6-D Kähler action gives sum of 4-D Kähler action and volume term. Twistor space must however have a Kähler structure and only the twistor spaces of M^4,E^4, and CP_2 have Kähler structure [A6]. TGD is unique both physically and mathematically!

2.2.5 Isometries of WCW

What can one say about the isometries of WCW? Certainly, they should generalize conformal symmetries of string models.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [K3].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

2.2.6 A possible problem related to the twistor lift

The twistor lift strongly suggests that the Kähler form of M^4 exists. The Kähler gauge potential would be the sum of M^4 and CP_2 contributions. The definition of M^4 Kähler structure is however not straightforward [L23, L24]. The naive guess would be that J represents an imaginary unit as the square root of -1 represented by the metric tensor. This would give the condition $J^2 = -g$ for the tensor square but this leads to problems.

To understand the situation, notice that the analogs of symplectic/Kähler structures in $M^4 \subset H$ have a moduli space, whose points correspond to what I have called Hamilton-Jacobi structures defined by integrable distributions of orthogonal decompositions $M^4 = M^2(x) \times E^2(x)$: $M^2(x)$ is analogous to string world sheet and Y^2 to partonic 2-surface. This means the presence of slicing by string world sheets $X^2(x)$, where x labels a point of Y^2. $X^2(x)$ is orthogonal to Y^2 at x. One can interchange the roles X^2 ad Y^2 in the slicing.

The induced Kähler form has an analogous decomposition. The decomposition is completely analogous to the decomposition of polarizations to non-physical time-like ones and physical space-like ones. This decomposition allows a natural modification of the definition of the symplectic structure so that the problem caused by $J^2 = -g$ conditions is avoided.

Consider first the problem. The $E^2(x)$ part of M^4 Kähler metric produces no problems since the signature of the metric is Euclidean. For $M^2(x)$ part, the Minkowskian signature produces problems. If one assumes that the $M^2(x)$ part of the Kähler form is non-vanishing, it should be imaginary in order to satisfy $J^2(M^2(x)) = -g(M^2(x))$. This implies that Kähler gauge potential is imaginary and this spoils the hermiticity of the modified Dirac equation [K22]. Also the electric contribution to the Kähler energy is negative.

The solution of the problem turned out to be ridiculously simple and I should have noticed it a long time ago.

1. $M^2(x)$ has a hypercomplex structure, which means that the imaginary unit e satisfies $e^2 = 1$ rather than $e^2 = -1$. Hamilton-Jacobi structure allows one to decompose J locally into two parts $J = J(M^2(x)) + J(E^2(x))$ such that $J^2 = g(M^2(x)) - g(E^2(x))$. This gives $J^4 = g(M^4)$. The Kähler energy of the canonically embedded M^4 is non-vanishing and positive whereas Kähler action vanishes by self-duality. Situation is identical to that in Maxwell's electrodynamics.

2. Kähler action for the canonically embedded M^4 vanishes and it is possible to define also Lagrangian 2-surfaces as surfaces for which the induced Kähler form vanishes. These are of special interest since they would guarantee small CP violation: string world sheets could be examples of these surfaces. Note that since the magnetic part of J induces violation of CP, the violation is vanishing for CP_2 type extremals and cosmic strings and also small for flux tubes.

If the notion of symplectic/canonical transformation generated by Hamiltonian preserving J generalizes, one could generate an infinite number of slicings.

Consider first ordinary symplectic transformations.

1. For the ordinary symplectic transformations, the closedness of the symplectic for J is essential ($dJ = 0$ corresponds to topological half of Maxwell's equations).

2. Second essential element is that symplectic transformation is generated as a flow for some Hamiltonian H: $j_H = i_{dH}J$ or more explicitly: $j_H^l = J^{kl}\partial_l H$. It is essential that one has $i_{j_H}J = -dH$: having a vanishing exterior derivative. In other words, $J_{kl}j_H^l = -\partial_k H$ is a gradient vector field and has therefore a vanishing curl. Together with $dJ = 0$, this guarantees the vanishing of the Lie derivative of J: $d_{j_H}J = d(i_{j_H}J) + i_{j_H}dJ = ddH + dJ(j_H) = 0$ so that J is preserved.

Could one talk about symplectic transformations in M^4?

1. The analogs of symplectic/canonical transformations should map the Hamilton-Jacobi structure to a new one and leave $J(M^2(x))$ and $J(E^2(x))$ invariant. The induced metrics of X^2 and Y^2 need not be preserved since only the diagonal metric $g_l^k(X^2/Y^2)$ appears in the conditions $J^2 = g(X^2) - g(Y^2)$.

2. The symplectic transformation generated by the Hamiltonian H would be a flow defined by the vector field $j_H = i_{dH}J$ and one would have $i_{j_H}J = -d_1H + d_2H$, where d_1 and d_2 are gradients operators in X^2 and Y^2. Usually one would have $J_{kl}j^l = dH$ satisfying $d^2H = 0$.

   The condition $ddH = 0$ satisfied by the ordinary symplectic transformations is replaced with the condition $d(-d_1H + d_2H) = 0$. This can be written as $-d_1^2H + d_2^2H + [d_2, d_1]H = 0$, and is satisfied. Therefore this part is not a problem.

3. Also the orthogonality of $M^2(x)$ and $E^2(x)$ must be preserved. This is a highly non-trivial condition since the metrics are induced and the symplectic transformations change the slicing and the metrics. An arbitrary Hamiltonian flow f, which depends on the coordinates of Y^2 only, maps Y^2 to itself but takes the tangent space $E^2(x)$ to $E^2(f(x))$. Unless the slicing satisfies special conditions, $E^2(f(x))$ is not orthogonal to $M^2(x)$.

4. The orthogonality is expressed as orthogonality of the projectors $P(X^2)$ and $P(Y^2)$: $P(X^2)P(Y^2)=0$. This condition must be respected by the Hamiltonian flow. The product involves 4 components giving 4 conditions which turn out to be partial differential equations for Hamiltonian. The naive expectation is that there are very few solutions. The Lie-derivative of the product must therefore vanish:

$$L_{j_H}[P(X^2)P(Y^2)] = L_{j_H}(P(X^2))P(Y^2) + P(X^2))L_{j_H}(P(Y^2)) = 0 \ . \tag{2.1}$$

The projector $P_{mn}(X^2)$ can be expressed as

$$P^{mn} = g^{\alpha\beta}\partial_\alpha m^k \partial_\beta m^l \ . \tag{2.2}$$

Here $g_{\alpha\beta} = m_{kl}\partial_\alpha m^k \partial_\beta m^l$ is the induced metric of X^2 or Y^2. m_{kl} is Minkowski metric and one can use linear Minkowski coordinates so that m_{kl} is constant.

The Lie derivative of $P^{mn}(X^2) \equiv P$ can be written as

$$L_j P^{mn} = L_j(g^{\alpha\beta})\partial_\alpha m^k \partial_\beta m^l + g^{\alpha\beta})(\partial_r j^k \partial_\alpha m^r \partial_\beta m^l + \partial_r j^l k \partial_\alpha m^r \partial_\beta m^k \ . \tag{2.3}$$

The Lie derivative of the induced metric is

$$\begin{aligned} L_j g^{\alpha\beta} &= g^{\alpha\mu} g^{\beta\nu} L_J g_{\mu\nu} \ , \\ L_j g_{\alpha\beta} &= m_{kl}(\partial_\alpha j^k \partial_\beta m^l + \partial_\alpha m^k \partial_\beta j^l \ . \end{aligned} \tag{2.4}$$

Although the existence of symplectic transformations in the general case seems implausible, one can construct special slicings for which symplectic transformations are possible.

1. One can start from a trivial slicing defined by $M^2 \times E^2$ decomposition and perform slicings of M^2 and E^2. The orthogonality is trivially true for all slicings of this kind since $Y^2(y)$ is orthogonal to X^2 not only at y but at every point x. Symplectic transformations of M^2 and Y^2 produce new slicings of this kind. Even symplectic flowqs defined by general Hamiltonians respect the orthogonality.

2. Second example is provided by the slicing of the light-one boundary by light-like 2-surfaces Y^2_v labelled by the value of light-like radial coordinate v with metrics differing by r^2 factor. The surfaces X^2 would be planes $X^2(y)$ orthogonal to Y^2 at y with light-like coordinates u and v. The orthogonality would be preserved by symplectic transformations.

   The open question is whether these slicings are the only possible slicings allowing symplectic transformations. Although the construction of these slicings looks trivial, they are not trivial physically.

2.3 Should unitarity be replaced with the Kähler-like geometry of the fermionic state space?

Physical states correspond to WCW spinor fields and in ZEO. WCW spinors at a given point of WCW correspond to pairs of Fock states assignable to the 3-surfaces at the opposite boundaries of CD defining space-time surface. These pairs of many-fermion states in fermionic degrees of freedom define the TGD counterpart of the S-matrix.

Unitary is a natural notion in non-relativistic wave-mechanics but already in quantum field theory it becomes problematic. In the twistor approach to the scattering amplitudes of massless gauge theories both unitarity and locality are problematic. Whether TGD can give rise to a unitary S-matrix has been a continual head-ache. This leads to a heretic question.

Is unitarity possible at all in TGD framework and should it be replaced with some deeper principle? I have considered these questions several times and in [L31] a rather radical solution was proposed. The implications of this proposal for the construction of scattering amplitudes are discussed in [L32].

Assigning an S-matrix to a unitary time evolution works in non-relativistic theory but fails already in the generic QFT and correlation functions replace S-matrix.

1. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space could replace the unitary S-matrix.

2. An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of the Teichmüller matrix.

   Teichmüller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong form of holography (SH), the most natural candidate would be Cartesian product of Teichmüller spaces of partonic 2 surfaces with punctures and string world sheets.

3. Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

4. In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmüller condition guaranteeing non-negative probabilities.

5. Equivalence Principle generalizes to the level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

6. There is also an objection. The transition probabilities would be given by $P(A,B) = g^{A,\overline{B}}g_{\overline{B},A}$ and the analogs for unitarity conditions would be satisfied by $g^{A,\overline{B}}g_{\overline{B},C} = \delta^A_C$.

The problem is that $P(A,B)$ is not real without further conditions. Can one imagine any physical interpretation for the imaginary part of $Im(P(A,B))$?

In this framework, the twistorial scattering amplitudes as zero energy states define the covariant Kähler metric $g_{A\overline{B}}$, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric. The problem with the unitarity would disappear.

This view is developed in detail in [L32] and one ends up with a very concrete and surprisingly simple number theoretic view about scattering amplitudes.

2.4 About Dirac equation in TGD framework

2.4.1 Three Dirac equations

In TGD spinors appear at 3 levels:

1. At the level of embedding space $H = M^4 \times CP_2$ the spinor field embedding space $M^4 \times CP_2$spinor fields (quark field) is a superposition of the harmonics of the Dirac operator. In the complexified M^8 having interpretation as complexified octonions, spinors are octonionic spinors. In accordance with the fact that M^8 is analogous to momentum space, the Dirac equation is purely algebraic and its solutions correspond to discrete points analogous to occupied points of Fermi ball.

2. The spinors at the level of 4-surfaces $X^4 \subset H$ are restrictions of the second quantized embedding space spinor field in X^4 so that the problematic second quantization in curved background is avoided. At the level of M^8 the restriction selects the points of M^8 belonging to 4-surface and carrying quark. The simplest manner to realize Fermi statistics is to assume that there is at most a single quark at a given point.

3. The third realization is at the level of the "world of classical worlds" (WCW) assigned to H consisting of 4-surfaces as preferred extremals of the action. Gamma matrices of WCW are expressible as superpositions of quark oscillator operators so that anti-commutation relations are geometrized. The conditions stating super-symplectic symmetry are a generalization of super-Kac-Moody symmetry and of super-conformal symmetry and give rise to the WCW counterpart of the Dirac equation [K16] [L29].

4. What the realization of WCW at the level of M^8 is, has remained unclear. The notion of WCW geometry does not generalize to his level and should be replaced with an essentially number theoretic notion.

   Adelic physics as a fusion of real and p-adic physics suggests a possible realization. Given extension of rationals induces extensions of various p-adic number fields. These can be glued to a book-like structure having as pages real numbers and the extensions of p-adic number fields.

   The pages would intersect along points with coordinates in the extension of rationals. These points form a cognitive representation. The additional condition that the active points are occupied by quarks guarantees that this makes sense also for octonions, quaternions and 4-surface in M^8. The p-adic sector could consist of discrete and finite cognitive representations continued to the p-adic surface and define the counterpart of WCW at the level of M^8?

2.4.2 The relationship between Dirac operator of H and modified Dirac operator

At the level of $X^4 \subset H$, the proposal is that modified Dirac action for the induced spinor fields defines the dynamics somehow. Modified Dirac equation or operator should be also consistent with the second quantization of induced spinor fields performed at the level of H and inducing the second quantization at the level of X^4.

1. The modified gamma matrices Γ^α are defined by the contractions of H gamma matrices Γ_k and canonical momentum currents $T^{k\alpha}$ associated with the action defining space-time

surface. The modified Dirac operator $D = \Gamma^\alpha D_\alpha$, where D_α is X^4 projection of the vector defined by the covariant derivative operators of H ($D_\alpha = \partial_\alpha h^k D_k$). Hermiticity requires $D_\alpha \Gamma^\alpha = 0$ implying that classical field equations are satisfied.

2. Can one assume that the modified Dirac equation is satisfied? Or is it enough to assume that this is not the case so that the modified Dirac operator defines the propagator as its inverse as the QFT picture would suggest?

   In fact, the propagators in H allow to compute N-point functions involving quarks and at the level of H the theory is free and the restriction to the space-time surface brings in the interactions. Therefore the notion of space-time propagator is not absolutely necessary. One can however ask whether some weaker condition could be satisfied and provide new insights.

   One can also ask whether the solutions of the modified Dirac equation correspond to external particles, which correspond to space-time surfaces for which the solution of the modified Dirac equation is consistent with the solution of the Dirac equation in H. Are these kinds of space-time surfaces possible?

3. The intuitive picture is that the solutions of the modified Dirac equation correspond to the external particles of a scattering diagram having an interpretation on mass shell states and are possible only for a very special kind of preferred extremals. Intuitively they should correspond to singular surfaces in M^8 and their mapping to H would involve blow-up due to the non-uniqueness of the normal space along lower than 4-D surface. String like objects and CP_2 type extremals would be basic entities of this kind. Could the modified Dirac equation or its weakened form hold true for these surfaces.

The strong form of equivalence of modified Dirac equation and ordinary Dirac equation would mean the equivalence of the actions of two Dirac operators acting on the second quantized induced spinor field.

1. The modified Dirac operator is given by $\Gamma_k T^{\alpha k} \partial_\alpha h^k D_k$ and its action should be same as H Dirac operator $\Gamma^k D_k$. This would require

$$\Gamma_k T^{\alpha k} \partial_\alpha h^k D_k \Psi = \Gamma^k D_k \Psi \ . \tag{2.5}$$

   Not surprisingly, it turns out that this condition is too strong.

2. One can express Γ_k using an overcomplete basis defined by the Killing vector fields j_A^k for H isometries. In the case of M^4 it is enough to use translations by using the identity $\sum_A j_A^k j_A^l = h^{kl}$. This allows to define gamma matrices $\Gamma_A = \Gamma_k j_A^k$ and to write the equation in the form

$$\Gamma_A T^{A\alpha} \partial_\alpha h^k D_k \Psi = \Gamma_A j_A^k D_k \Psi \ . \tag{2.6}$$

   Here $T^{A\alpha}$ is the conserved isometry current associated with the Killing vector j_A^k. Is it possible to satisfy the condition

$$T^{A\alpha} \partial_\alpha h^k = j_A^k \tag{2.7}$$

   or its suitably weakened form?

   The strong form of the condition cannot be satisfied. The left hand side of the equation is determined by the gradients of H coordinates and parallel to X^4 whereas the right hand side also involves the component normal to X^4. Therefore the condition cannot be satisfied in the general case.

3. By projecting the condition to the tangent space, one obtains a weaker condition stating that the tangential parts of two Dirac operators are proportional to each other with a position dependent proportionality factor $\Lambda(x)$:

$$\begin{aligned} T^{A\alpha} &= \Lambda(x) j^{\alpha}_A \\ j^{\alpha}_A &= j^k_A \partial^{\alpha} h_k = j^k_A h_{kl} g^{\alpha\beta}\partial_{\beta} h^l \ . \end{aligned} \tag{2.8}$$

   The conserved isometry current is proportional to the projection of the Killing vector to the tangent space of X^4. $\Lambda(x)$ is proportionality constant depending on the point of X^4. Isometry current is analogous to a Hamiltonian vector field being parallel to the Killing vector field.

4. If the action were a mere cosmological volume term, the isometry currents would be proportional to j^{α} so that the conditions would be automatically satisfied. The contribution to $\Lambda(x)$ is proportional to the p-adic length scale dependent cosmological constant.

   Kähler action receives contributions from both M^4 and CP_2. Both add to $T^{A\alpha}$ a term of form $T^{\alpha\beta} j_{A\beta}$ coming from the variation of the Kähler action with respect to $g_{\alpha\beta}$. $T^{\alpha\beta}$ is the energy momentum tensor with a form similar to that for Maxwell action.

   Besides this, M^4 *resp.* CP_2 contribute a term proportional to $J^{\alpha\beta} J_{kl} \partial_{\beta} h^k j^k_A$ coming from the variation of the Kähler action with respect to $J_{\alpha\beta}$ contributing only to M^4 *resp.* CP_2 isometries. These contributions make the conditions non-trivial. The Kähler contribution to $\Lambda(x)$ need not be constant. Note that the Kähler contributions to the energy momentum tensor vanish if X^4 is (minimal) surface of form $X^2 \times Y^2 \subset M^4 \times CP_2$ so that both X^2 and Y^2 are Lagrangian.

5. The vanishing of the divergence of $T^{A\alpha}$ using the Killing property $D_l j_{Ak} + D_k j_{Al} =$ of j_{Ak} gives

$$j^{A\alpha} \partial_{\alpha} \Lambda = 0 \ . \tag{2.9}$$

   Λ is constant along the flow lines of $j^{A\alpha}$ and is therefore analogous to a Hamiltonian. The constant contribution from the cosmological term to Λ does not contribute to this condition.

6. An attractive hypothesis, consistent with the hydrodynamic interpretation, is that the proposed condition is true for all preferred extremals. The conserved isometry current along the X^4 projection of the flow line is proportional to the projection of Killing vector: this conservation law is analogous to the conservation of energy density $\rho v^2/2 + p$ along the flow line). One can say that isometries as flows in the embedding space are projected to flows along the space-time surface. One could speak of projected or lifted representation.

7. The projection to the normal space does not vanish in the general case. One could however ask whether a weaker condition stating that the second fundamental form $H^k_{\alpha\beta} = D_{\alpha} h^k$, which is normal to X^4, defines the notion of the normal space in terms of data provided by space-time surface. If X^4 is a geodesic submanifold of H, in particular a product of geodesic submanifolds of M^4 and CP_2, one has $H^k_{\alpha\beta} = 0$.

2.4.3 Gravitational and inertial representations of isometries

The lift/projection of the isometry flows to X^4 strongly suggests a new kind of representation of isometries as analog of the braid representation considered earlier.

1. Projected/lifted representation would clarify the role of the classical conserved charges and currents and generalize hydrodynamical conservation laws along the flow lines of isometries. In particular, quark lines would naturally correspond to time-like flow lines of time translations. In the case of CP_2 type extremals, quark momenta for the lifted representations would be light-like.

2. The conservation conditions along the flow lines are very strong, and one can wonder if they might provide a new formulation of the preferred extremal property. It is quite possible that the conditions apply only to a sub-algebra. Quantum classical correspondence (QCC) suggests Cartan algebra for which the quantum charges can have well-defined eigen values simultaneously. In accordance with QCC, the choice of the quantization axes would affect the space-time surfaces considered and could be interpreted as a higher level quantum measurement.

3. Projected/lifted representation provides a new insight also to the Equivalence Principle (EP) stating that gravitational and inertial masses are identical. At the level of scattering amplitudes involving isometry charges defined at the level of H, the isometries affect the entire space-time surface, and one could see EP as an almost trivial statement. QCC however forces us to consider EP more seriously.

   I have proposed that QCC could be seen as the identification of the eigenvalues of Cartan algebra isometry charges for quantum states with the classical charges associated with the preferred extremals. EP would follow from QCC: gravitational charges would correspond to the representation of the flows defined by isometries as their projections/lifts to X^4 whereas inertial charges would correspond to the representation at the level of H with isometries affecting the entire space-time surfaces.

4. The lifted/projected/gravitational representation of isometries, which seems possible in 4-D situation, is analogous to braid group representation making sense only in 2-D situation. Indeed, for the many-sheeted space-time surfaces assignable to $h_{eff} > h_0$, it can happen that rotation by 2π leads to a new space-time sheet and that the $SO(2)$ subgroup of the rotation group associated with the Cartan algebra is lifted to n-fold covering. Same can can happen in the case of color rotations. This leads to a fractionation of quantum numbers usually assigned with quantum group representations suggested to correspond to $h_{eff} > h$ [K15].

   Also for the quantum groups, Cartan algebra plays a special role. In the case of the Poincare group, the 2-D nature of braid group representations would correspond to the selection $M^2 \times SO(2)$ as a Cartan subgroup implying effective 2-dimensionality in the case rotation group. Gravitational representations could therefore correspond to quantum group representations.

5. The gravitational representation provides also a new insight on $M^8 - H$ duality. The source of worries has been whether Uncertainty Principle (UP) is realized if a given 4-surface in M^8 is mapped to a single space-time surface in M^8. It seems that UP can be realized both in terms of inertial and gravitational representations.

   (a) In the case of the "inertial" representation of H-isometries at the level of H, one must regard $X^4 \subset H$ representing images of particle-like 4-surface in M^8 analog of Bohr orbit (holography) and map it to an analog of plane wave define as superposition of its translates and by the total momentum associated with the either boundary of CD associated with the particle. The same applies to the transforms to other Cartan algebra generators.

       In a cognitive representation based on extension of rationals, the shifts for Cartan algebra would be discrete: the values of the plane wave would be roots of unity belonging to the extension and satisfy periodic boundary conditions at the boundary of larger CD.

       Periodic boundary conditions pose rather strong conditions on the time evolution by scaling between two SSFRs. The scaling must respect the boundary conditions. If the momenta assignable to the plane waves of massive particles are conserved and h_{eff} is conserved, the scaling must multiply CD size by integers. The iterations of integer scalings, in particular $n = 2$ scalings (period doubling), are in a preferred position.

   (b) If one replaces the inertial representation of isometries with the gravitational representation, the quantum states can be realized at the level of a single space-time surface. One would have two representations: gravitational and inertial -subjective and objective, one might say.

(c) Gravitational representations make also sense for the super-symplectic group acting at the boundary of light-cone as well as for the Kac-Moody type algebra associated with the isometries of H realized the light-like orbits of partonic 2-surfaces.

2.5 Different ways to understand the "complete integrability" of TGD

There are several ways to see how TGD could be a completely integrable theory.

2.5.1 Preferred extremal property

Preferred extremal property requires Bohr orbit property and holography and is an extremely powerful condition.

1. Twistor lift of TGD implies that X^4 in H is simultaneous extremal of volume action and Kähler action. Minimal surface property is counterpart for massless field equations and extremality for Kähler action gives interpretation for massless field as Kähler form as part of induced electromagnetic field.

   The simultaneous preferred extremal property strongly suggests that 2-D complex structure generalizes for 4-D space-time surfaces and so called Hamilton-Jacobi structure [L16] meaning a decomposition of M^4 to orthogonal slicings by string world sheets and orthogonal partonic 2-surfaces would realize this structure.

2. Generalized Beltrami property [L35] implies that 3-D Lorentz force and dissipation for Kähler form vanish. The Kähler form is analogous to the classical Maxwell field. Energy momentum tensor has vanishing divergence, which makes it plausible that QFT limit is analogous to Einstein-Maxwell theory.

   The condition also implies that the Kähler current defines an integrable flow so that there is global coordinate varying along flow lines. This is a natural classical correlate for quantum coherence. Quantum coherence would be always present but broken only by the finite size of the region of the space-time considered.

   Beltrami property plus current conservation implies gradient flow and an interesting question is whether conserved currents define gradient flows: non-trivial space-time topology would allow this at the fundamental level. Beltrami condition is a very natural classical condition in the models of supraphases.

3. The condition that the isometry currents for the Cartan algebra of isometries are proportional to the projections of the corresponding Killing vectors is a strong condition and could also be at least an important aspect of the preferred extremal property.

2.5.2 Supersymplectic symmetry

The third approach is based on the super-symplectic symmetry of WCW. Isometry property would suggest that an infinite number of super-symplectic Noether charges are defined at the boundaries of CD by the action of the theory. They need not be conserved since supersymplectic symmetries cannot be symmetries of the action: if they were, the WCW metric would be trivial.

The gauge conditions for Virasoro algebra and Kac-Moody algebras suggest a generalization. Super-symplectic algebra (SSA) involves only non-negative conformal weights n suggesting extension to a Yangian algebra (this is essential!). Consider the hierarchy of subalgebras SSA_m for which the conformal weights are m-tiples of those of entire algebra. These subalgebras are isomorphic with the entire algebra and form a fractal hierarchy.

Assume that the sub-algebra SSA_m and commutator $[SSA_m, SSA]$ have vanishing classical Noether charges for $m > m_{max}$. These conditions could fix the preferred extremal. One can also assume that the fermionic realizations of these algebras annihilate physical states. The remaining symmetries would be dynamical symmetries.

The generators are Hamiltonians of $\delta M^4_+ \times CP_2$. The symplectic group contains Hamiltonians of the isometries as a normal sub-algebra. Also the Hamiltonians of and one could assume that only the isometry generators correspond to non-trivial classical and quantal Noether charges. Could the actions of SSA and Kac-Moody algebras of isometries be identical if a similar construction applies to Kac-Moody half-algebras associated with the light-like partonic orbits. Super-symplectic symmetry would reduce to a hierarchy of gauge symmetries.

3 Physics as number theory

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L9, L10], and classical number fields [K18].

3.1 p-Adic physics

The motivation for p-adicization came from p-adic mass calculations [K11, K2].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.
2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Boltzmann weights to exist. Conformal invariance guarantees this.
3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1 + i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).
4. p-Adic physics [K13] is interpreted as a correlate for cognition. Motivation comes from the observation that piecewise constant functions depending on a finite number of pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

Unlike the Higgs mechanism, p-adic thermodynamics provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry which guarantees the existence of p-adic Boltzmann weights.

The number theoretic picture leads to a deeper understanding of a long standing objection against p-adic thermodynamics [K11] as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra.

If one requires super-Virasoro symmetry and identifies mass squared with a scaling generator L_0, one can argue that only massless states are possible since L_0 must annihilate these states! All states of the theory would be massless, not only those of fundamental particles as in conformally invariant theories to which twistor approach applies! This looks extremely beautiful mathematically but seems to be in conflict with reality already at single particle level!

The resolution of the objection is that *thermodynamics* is indeed in question.

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and describes approximately the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors. The excited states of the entire system indeed are massless [?]
2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment and the entire mass squared would remain vanishing. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

3. In the p-adic mass calculations [K11, K2], the effective environment for quarks and leptons would in a good approximation consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons). The many-quark state many-quark state associated with the wormhole throat (single quark state for quarks and 3-quark-state for leptons [L34].

4. In M^8 picture [L23, L24], tachyonicity is unavoidable since the real part of the mass squared as a root of a polynomial P can be negative. Also tachyonic real but algebraic mass squared values are possible. At the H level, tachyonicity corresponds to the Euclidean signature of the induced metric for a wormhole contact.

   Tachyonicity is also necessary: otherwise one does not obtain massless states. The super-symplectic states of quarks would entangle with the tachyonic states of the wormhole contacts by Galois confinement.

5. The massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-symplectic representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Galois confinement is satisfied. The tachyonic mass squared is assigned with wormhole contacts with the Euclidean signature of the induced metric, whose throats in turn carry the fermions so that the wormhole contact would form the nearby environment.

   The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state remains massless and massive single particle states receive a compensating negative mass squared from the wormhole contact. Thermal mass squared corresponds to a single particle mass squared and does not take into account the contribution of wormhole contacts except for the ground state.

6. There is a further delicate number theoretic element involved [L40, L45]. The choice of $M^4 \subset M^8$ for the system is not unique. Since M^4 momentum is an M^4 projection of a massless M^8 momentum, it is massless by a suitable choice of $M^4 \subset M^8$. This choice must be made for the environment so that both the state of the environment and the single particle ground state are massless. For the excited states, the choice of M^4 must remain the same, which forces the massivation of the single particle excitations and p-adic massivation.

These arguments strongly suggest that pure states, in particular the state of the entire Universe, are massless. Mass would reflect the statistical description of entanglement using the density matrix. The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural. This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K14] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and $M^8 - H$ duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L46, L47].

3.2 Adelic physics

Adelic physics fuses real and various p-adic physics to a single structure [L10].

1. One can combine real numbers and p-adic number fields to a product: number fields would be like pages of a book intersecting along rationals acting as the back of the book.

2. Each extension of rational induces extensions of p-adic number fields and extension of the basic adele. Points in the extension of rationals are now common to the pages. The infinite hierarchy of adeles defined by the extensions forms an infinite library.

3. This leads to an evolutionary hierarchy (see **Fig. 9**) . The order n of the Galois group as a dimension of extension of rationals is identified as a measure of complexity and of evolutionary level, "IQ". Evolutionary hierarchy is predicted.

4. Also a hierarchy of effective Planck constants interpreted in terms of phases of ordinary matter is predicted. X^4 decomposes to n fundamental regions related by Galois symmetry. Action is n times the action for the fundamental region. Planck constant h is effectively replaced with $h_{eff} = nh$. Quantum coherence scales are typically proportional to h_{eff}. Quantum coherence in arbitrarily long scales is implied. Dark matter at the magnetic body of the system would serve as controller of ordinary matter in the TGD inspired quantum biology [L48].

   $h_{eff} = nh_0$ is a more general hypothesis. Reasons to believe that h/h_0 could be the ratio R^2/L_p^2 for CP_2 length scale R deduced from p-adic mass calculations and Planck length L_P [L41]. The CP_2 radius R could actually correspond to L_P and the value of R deduced from the p-adic mass calculations would correspond to a dark CP_2 radius $\sqrt{h/h_0}l_P$.

3.3 Adelic physics and quantum measurement theory

Adelic physics [L10] forces us to reconsider the notion of entanglement and what happens in state function reductions (SFRs). Let us leave the question whether the SFR can correspond to SSFR or BSFR or both open for a moment.

1. The natural assumption is that entanglement is a number-theoretically universal concept and therefore makes sense in both real and various p-adic senses. This is guaranteed if the entanglement coefficients are in an extension E of rationals associated with the polynomial Q defining the space-time surface in M^8 and having rational coefficients.

   In the general case, the diagonalized density matrix ρ produced in a state function reduction (SFR) has eigenvalues in an extension E_1 of E. E_1 is defined by the characteristic polynomial P of ρ.

2. Is the selection of one of the eigenstates in SFR possible if E_1 is non-trivial? If not, then one would have a number-theoretic entanglement protection.

3. On the other hand, if the SFR can occur, does it require a phase transition replacing E with its extension by E_1 required by the diagonalization?

Let us consider the option in which E is replaced by an extension coding for the measured entanglement matrix so that something also happens to the space-time surface.

1. Suppose that the observer and measured system correspond to 4-surfaces defined by the polynomials O and S somehow composed to define the composite system and reflecting the asymmetric relationship between O and S. The simplest option is $Q = O \circ S$ but one can also consider as representations of the measurement action deformations of the polynomial $O \times P$ making it irreducible. Composition conforms with the properties of tensor product since the dimension of extension of rationals for the composite is a product of dimensions for factors.

2. The loss of correlations would suggest that a classical correlate for the outcome is a union of uncorrelated surfaces defined by O and S or equivalently by the reducible polynomial defined by the $O \times S$ [L37]. Information would be lost and the dimension for the resulting extension is the sum of dimensions for the composites. O however gains information and quantum classical correspondence (QCC) suggests that the polynomial O is replaced with a new one to realize this.

3. QCC suggests the replacement of the polynomial O the polynomial $P \circ O$, where P is the characteristic polynomial associated with the diagonalization of the density matrix ρ. The final state would be a union of surfaces represented by $P \circ O$ and S: the information about the measured observable would correspond to the increase of complexity of the space-time surface associated with the observer. Information would be transferred from entangled Galois

degrees of freedom including also fermionic ones to the geometric degrees of freedom $P \circ O$. The information about the outcome of the measurement would in turn be coded by the Galois groups and fermionic state.

4. This would give a direct quantum classical correspondence between entanglement matrices and polynomials defining space-time surfaces in M^8. The space-time surface of O would store the measurement history as kinds of Akashic records. If the density matrix corresponds to a polynomial P which is a composite of polynomials, the measurement can add several new layers to the Galois hierarchy and gradually increase its height.

   The sequence of SFRs could correspond to a sequence of extensions of extensions of..... This would lead to the space-time analog of chaos as the outcome of iteration if the density matrices associated with entanglement coefficients correspond to a hierarchy of powers P^k [L25, L36].

Does this information transfer take place for both BSFRs and SSFRs? Concerning BSFRs the situation is not quite clear. For SSFRs it would occur naturally and there would be a connection with SSFRs to which I have associated cognitive measurement cascades [**?**]

1. Consider an extension, which is a sequence of extensions $E_1 \rightarrow ..E_k \rightarrow E_{k+1}.. \rightarrow E_n$ defined by the composite polynomial $P_n \circ .... \circ P_1$. The lowest level corresponds to a simple Galois group having no non-trivial normal subgroups.

2. The state in the group algebra of Galois group $G = G_n$ having G_{n-1} as a normal subgroup can be expressed as an entangled state associated with the factor groups G_n/G_{n-1} and subgroup G_{n-1} and the first cognitive measurement in the cascade would reduce this entanglement. After that the process could but need not to continue down to G_1. Cognitive measurements considerably generalize the usual view about the pair formed by the observer and measured system and it is not clear whether $O - S$ pair can be always represented in this manner as assumed above: also small deformations of the polynomial $O \times S$ can be considered.

   These considerations inspire the proposal the space-time surface assigned to the outcome of cognitive measurement G_k, G_{k-1} corresponds to polynomial the $Q_{k,k-1} \circ P_n$, where $Q_{k,k-1}$ is the characteristic polynomial of the entanglement matrix in question.

3.4 Entanglement paradox and new view about particle identity

A brain teaser that the theoretician sooner or later is bound to encounter, relates to the fermionic and bosonic statistics. This problem was also mentioned in the article of Keimer and Moore [D1] discussing quantum materials `https://cutt.ly/bWdTRj0`. The unavoidable conclusion is that both the fermions and bosons of the entire Universe are maximally entangled. Only the reduction of entanglement between bosonic and fermionic states of freedom would be possible in SFRs. In the QFT framework, gauge boson fields are primary fields and the problem in principle disappears if entanglement is between states formed by elementary bosons and fermions.

In the TGD Universe, all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) so that if Fock space the Fock states of fermions and bosons express everything worth expressing, SFRs would not be possible at all!

Remark: In the TGD Universe all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) localized at the points of space-time surface defining a number theoretic discretization that I call cognitive representation. Besides this there are also degrees of freedom associated with the geometry of 3-surfaces representing particles. These degrees of freedom represent new physics. The quantization of quarks takes place at the level of H so that anticommutations hold true over the entire H.

Obviously, something is entangled and this entanglement is reduced. What these entangled degrees of freedom actually are if Fock space cannot provide them?

1. Mathematically entanglement makes sense also in a purely classical sense. Consider functions $\Psi_i(x)$ and $\Psi_j(y)$) and form the superposition $\Psi(x) = \sum_{ij} c_{ij} \Psi_i(x) \Psi_j x)$. This function is completely analogous to an entangled state.

2. Number theoretical physics implies that the Galois group becomes the symmetry group of physics and quantum states are representations of the Galois group [L30, L33]. For an extension of extension of ...., the Galois group has decomposition by normal subgroups to a hierarchy of coset groups.

   The representation of a Galois group can be decomposed to a tensor product of representations of these coset groups. The states in irreps of the Galois group are entangled and the SFR cascade produces a product of the states as a product of representations of the coset groups. Galois entanglement allows us to express the asymmetric relation between observer and observed very naturally. This cognitive SSFR cascade - as I have called it - could correspond to what happens in at least cognitive SFRs.

   If so, then SFR would in TGD have nothing to do with fermions and bosons (consisting of quarks too) since the maximal fermionic entanglement remains. For instance, when one for instance talks about long range entanglement the entanglement that matters would correspond to entanglement between degrees of freedom, which do not allow Fock space description.

In the TGD framework, the replacement of particles with 3-surfaces brings in an infinite number of non-Fock degrees of freedom. Could it make sense to speak about the reduction of entanglement in WCW degrees of freedom? There is no second quantization at WCW level so that one cannot talk about Fock spaces WCW level but purely classical entanglement is possible as observed.

1. In WCW unions of disjoint 3-surfaces correspond to classical many-particle states. One can form single particle wave functions for 3-surfaces with a single component, products of these single particle wave functions, and also analogs of entangled states as their superposition realized as building bricks of WCW spinor fields.

   If one requires that these wave functions are completely symmetric under the exchange of 3-surfaces, maximal entanglement in this sense would be realized also now and SFR would not be possible. But can one require the symmetry? Under what conditions one can regard two 3-surfaces as identical? For point-like particles one has always identical particles but in TGD the situation changes.

2. Here theoretical physics and category theory meet since the question when two mathematical objects can be said to be identical is the basic question of category theory. The mathematical answer is they are isomorphic in some sense. The physical answer is that the two systems are identical if they cannot be distinguished in the measurement resolution used.

4 $M^8 - H$ duality

There are several observations motivating $M^8 - H$ duality (see **Fig. 8**).

1. There are four classical number fields: reals, complex numbers, quaternions, and octonions with dimensions $1, 2, 4, 8$. The dimension of the embedding space is $D(H) = 8$, the dimension of octonions. Spacetime surface has dimension $D(X^4) = 4$ of quaternions. String world sheet and partonic 2-surface have dimension $D(X^2) = 2$ of: complex numbers. The dimension $D(string) = 1$ of string is that of reals.

2. Isometry group of octonions is a subgroup of automorphism group G_2 of octonions containing $SU(3)$ as a subgroup. $CP_2 = SU(3)/U(2)$ parametrizes quaternionic 4-surfaces containing a fixed complex plane.

Could M^8 and $H = M^4 \times CP_2$ provide alternative dual descriptions of physics (see **Fig. 8**)?

1. Actually a complexification $M^8_c \equiv E^8_c$ by adding an imaginary unit i commuting with octonion units is needed in order to obtain sub-spaces with real number theoretic norm squared. M^8_c fails to be a field since $1/o$ does not exist if the complex valued octonionic norm squared $\sum o_i^2$ vanishes.

2. The four-surfaces $X^4 \subset M^8$ are identified as "real" parts of 8-D complexified 4-surfaces X^4_c by requiring that $M^4 \subset M^8$ coordinates are either imaginary or real so that the number theoretic metric defined by octonionic norm is real. Note that the imaginary unit defining the complexification commutes with octonionic imaginary units and number theoretical norm squared is given by $\sum_i z_i^2$ which in the general case is complex.

3. The space H would provide a geometric description, classical physics based on Riemann metric, differential geometric structures and partial differential equations deduced from an action principle. M^8_c would provide a number theoretic description: no partial differential equations, no Riemannian metric, no connections...

   M^8_c has only the number theoretic norm squared and bilinear form, which are real only if M^8_c coordinates are real or imaginary. This would define "physicality". One open question is whether all signatures for the number theoretic metric of X^4 should be allowed? Similar problem is encountered in the twistor Grassmannian approach.

4. The basic objection is that the number of algebraic surfaces is very small and they are extremely simple as compared to extremals of action principle. Second problem is that there are no coupling constants at the level of M^8 defined by action.

   Preferred extremal property realizes quantum criticality with universal dynamics with no dependence on coupling constants. This conforms with the disappearance of the coupling constants from the field equations for preferred extremals in H except at singularities, with the Bohr orbitology, holography and ZEO. $X^4 \subset H$ is analogous to a soap film spanned by frame representing singularities and implying a failure of complete universality.

5. In M^8, the dynamics determined by an action principle is replaced with the condition that the *normal* space of X^4 in M^8 is associative/quaternionic. The distribution of normal spaces is always integrable to a 4-surface.

   One cannot exclude the possibility that the normal space is complex 2-space, this would give a 6-D surface [L23, L24]. Also this kind of surfaces are obtained and even 7-D with a real normal space. They are interpreted as analogs of branes and are in central role in TGD inspired biology.

   Could the twistor space of the space-time surface at the level of H have this kind of 6-surface as M^8 counterpart? Could $M^8 - H$ duality relate these spaces in 16-D M^8_c to the twistor spaces of the space-time surface as 6-surfaces in 12-D $T(M^4) \times T(CP_2)$?

6. Symmetries in M^8 number theoretic: octonionic automorphism group G_2 which is complexified and contains $SO(1,3)$. G_2 contains $SU(3)$ as M^8 counterpart of color $SU(3)$ in H. Contains also $SO(3)$ as automorphisms of quaternionic subspaces. Could this group appear as an (approximate) dynamical gauge group?

   $M^8 = M^4 \times E^4$ as $SO(4)$ as a subgroup. It is not an automorphism group of octonions but leaves the octonion norm squared invariant. Could it be analogous to the holonomy group $U(2)$ of CP_2, which is not an isometry group and indeed is a spontaneously broken symmetry.

   A connection with hadron physics is highly suggestive. $SO(4) = SU(2)_L \times SU(2)_R$ acts as the symmetry group of skyrmions identified as maps from a ball of M^4 to the sphere $S^3 \subset E^4$. Could hadron physics $\leftrightarrow$ quark physics duality correspond to $M^8 - H$ duality. The radius of S^3 is proton mass: this would suggest that M^8 has an interpretation as an analog of momentum space.

7. What is the interpretation of M^8? Massless Dirac equation in M^8 for the octonionic spinors must be algebraic. This would be analogous to the momentum space Dirac equation. Solutions would be discrete points having interpretation as quark momenta! Quarks pick up discrete points of $X^4 \subset M^8$.

   States turn out to be massive in the M^4 sense: this solves the basic problem of 4-D twistor approach (it works for massless states only). Fermi ball is replaced with a region of a mass shell (hyperbolic space H^3).

   M^8 duality would generalize the momentum-position duality of the wave mechanics. QFT does not generalize this duality since momenta and position are not anymore operators.

4.1 Associative dynamics in M_c^8

How to realize the associative dynamics in M_c^8 [L23, L24]?

1. Number theoretical vision requires hierarchy of extensions of rationals and polynomials with rational coefficients would realize them. Rational coefficients make possible the interpretation as a polynomial with p-adic argument and therefore number theoretical universality.

   One cannot exclude the possibility that also real argument is allowed and that number theoretic universality and adelization applies only for the space-time surfaces defined by polynomials with rational coefficients.

2. Algebraic physics suggests that X^4 is in some sense a root of a M_c^8 valued polynomial. One can continue polynomials P with rational coefficients to M_c^8 by replacing the real argument with a complexified octonion.

3. The algebraic conditions should imply that the normal space of X^4 is quaternionic/associative. One can decompose octonions to sums $q_1 + I_4 q_2$, or "real" and "imaginary" parts q_i, which are quaternions and I_4 is octonion unit orthogonal to quaternions. The condition is that the "real" part of the octionic polynomial vanishes. Complexified 4-D surface whose projection to a real section (M^8 coordinates imaginary or real so that complexified octonion norm squared is real) is 4-D.

4. $M^8 - H$ duality requires an additional condition. The normal space contains also a complex plane M^2 which is commutative. This guarantees that normal spaces correspond to a point of CP_2. This is necessary in order to define $M^8 - H$ duality mapping X^4 from M^8 to H. M^2 can be replaced with an integrable distribution of M^2s if the assignment of the CP_2 point to tangent space can be made unique. This is the case if the spaces $M^2(x)$ are obtained from $M^2(y)$ by a unique G_2 automorphism $g(x, y)$.

4.1.1 Associativity condition at the level of M^8

Associativity condition for polynomials allows to characterize space-time surfaces in terms of polynomials with rational coefficients and possibly also analytic functions with rational Taylor coefficients at M^8 level. $M^8 - H$ duality would map $X^4 \subset M^8$ to $X^4 \subset H$. In M_c^8 the space-time surfaces could be also seen as graphs of local (complex) G_2 gauge transformations.

Remark: Even non-rational coefficients can be considered. In this case polynomials with rational coefficients would define a unique discretion of WCW and allow p-adicization and adelization.

In the generic case the set of points in the extension of rationals defining cognitive representation is discrete and finite. The surprise was that the "roots" can be solved explicitly and that the discrete cognitive representation is dense so that momentum quantization due to the finite volume of CD must be assumed to obtain finite cognitive representation inside CD. Cognitive representation could be defined by the points which correspond to the 8-momenta solving octonionic Dirac equation. This is excellent news concerning practical applications.

The outcome of a detailed examination of the "roots" of the octonionic polynomial having real part $X = Re_Q(P)$ and imaginary part $Y = Im_Q(P)$ in quaternionic sense, yielded a series of positive and negative surprises and demonstrated the failure of the naive arguments based on dimension counting.

1. Although no interesting associative space-time surfaces are possible, every distribution of normal associative planes (co-associativity) is integrable. Note that the distribution of normal spaces must have an integrable distribution of commutative planes in order to guarantee the existence of $M^8 - H$ duality. Generic arguments fail in the presence of symmetries.

2. Another positive surprise was that Minkowski signature is the only possible option. Equivalently, the image of M^4 as real co-associative subspace of O_c (complex valued octonion norm squared is real valued for them) by an element of local $G_{2,c}$ or its subgroup $SU(3, c)$ gives a real co-associative space-time surface.

3. The conjecture based on naive dimensional counting, which was not correct, was that the polynomials P determine these 4-D surfaces as roots of $Re_Q(P)$. The normal spaces of these surfaces possess a fixed 2-D commuting sub-manifold or possibly their distribution allowing the mapping to H by $M^8 - H$ duality as a whole.

   If this conjecture were correct, strong form of holography (SH) would not be needed and would be replaced with extremely powerful number theoretic holography determining space-time surface from its roots and selection of real subspace of O_c characterizing the state of motion of a particle.

4. One of the cold showers during the evolution of the ideas about $M^8 - H$ duality was that the naive expectation that one obtains complex 4-D surfaces as solutions is wrong. The equations for $Re_Q(P) = 0$ ($ImQ_P = 0$) reduce to roots of ordinary real polynomials defined by the odd (even() parts of P and have interpretation as complex values of 8-D mass squared. These surfaces have complex dimension 7. 4 complex dimensions should be eliminated in order to have a complex 4-D surface, whose real parts would give a real 4-surface X^4. The explanation for the unexpected result comes from the symmetries of the octonionic polynomial implying that generic arguments fail.

4.1.2 How does one obtain 4-D space-time surfaces?

Contrary to the naive expections, the solutions of the vanishing conditions for the $Re_Q(P)$ ($Im_Q(P)$) (real (imaginary) part in quaternionic sense) are 7-D complex mass shells $r^2 = r_{n,1}$ as roots of $P_1(r) = 0$ or $r^2 = r_{n,2}$ of $P_2(r) = 0$ rather than 4-D complex surfaces (for a detailed discussion see [K1]) A solution of both conditions requires that P_1 and P_2 have a common root but the solution remains a 7-D complex mass shell! This was one of the many cold showers during the development of the ideas about $M^8 - H$ duality! It seems that the adopted interpretation is somehow badly wrong. Here zero energy ontology (ZEO) and holography come to the rescue.

1. Could the roots of P_1 or P_2 define only complex mass shells of the 4-D complex momentum space identifiable as M_c^4? ZEO inspires the question whether a proper interpretation of mass shells could be as pre-images of boundaries of cd:s (intersections of future and past directed light-cones) as pairs of mass shells with opposite energies. If this is the case, the challenge would be to understand how X_c^4 is determined if P does not determine it.

   Here holography, considered already earlier, suggests itself: the complex 3-D mass shells belonging to X_c^4 would only define the 3-D boundary conditions for holography and the real mass shells would be mapped to the boundaries of cds. This holography can be restricted to X_R^4. Bohr orbit property at the level of H suggests that the polynomial P defines the 4-surface more or less uniquely.

2. Let us take the holographic interpretation as a starting point. In order to obtain an X_c^4 mass shell from a complex 7-D light-cone, 4 complex degrees of freedom must be eliminated. $M^8 - H$ duality requires that X_c^4 allows M_c^4 coordinates.

   Note that if one has $X_c^4 = M_c^4$, the solution is trivial since the normal space is the same for all points and the H image under $M^8 - H$ duality has constant $CP_2 = SU(3)/U(2)$ coordinates. X_c^4 should have interpretation as a non-trivial deformation of M_c^4 in M^8.

3. By $M^8 - H$ duality, the normal spaces should be labelled by $CP_2 = SU(3)/U(2)$ coordinates. $M^8 - H$ duality suggests that the image $g(p)$ of a momentum $p \in M_c^4$ is determined essentially by a point $s(p)$ of the coset space $SU(3)/U(2)$. This is achieved if M_c^4 is deformed by a local $SU(3)$ transformation $p \rightarrow g(p)$ in such a way that each image point is invariant under $U(2)$ and the mass value remains the same: $g(p)^2 = p^2$ so that the point represents a root of P_1 or P_2.

   Remark: I have earlier considered the possibility of G_2 and even $G_{2,c}$ local gauge transformation. It however seems that that local $SU(3)$ transformation is the only possibility since G_2 and $G_{2,c}$ would not respect $M^8 - H$ duality. One can also argue that only real $SU(3)$

maps the real and imaginary parts of the normal space in the same manner: this is indeed an essential element of $M^8 - H$ duality.

4. This option defines automatically $M^8 - H$ duality and also defines causal diamonds as images of mass shells $m^2 = r_n$. The real mass shells in H correspond to the real parts of r_n. The local $SU(3)$ transformation g would have interpretation as an analog of a color gauge field. Since the H image depends on g, it does not correspond physically to a local gauge transformation but is more akin to an element of Kac-Moody algebra or Yangian algebra which is in well-defined half-algebra of Kac-Moody with non-negative conformal weights.

The following summarizes the still somewhat puzzling situation as it is now.

1. The most elegant interpretation achieved hitherto is that the polynomial P defines only the mass shells so that mass quantization would reduce to number theory. Amusingly, I started to think about particle physics with a short lived idea that the d'Alembert equation for a scalar field could somehow give the mass spectrum of elementary particles so that the issue comes full circle!

2. Holography assigns to the complex mass shells complex 4-surfaces for which $M^8 - H$ duality is well-defined even if these surfaces would fail to be 4-D co-associative. These surfaces are expected to be highly non-unique unless holography makes them unique. The Bohr orbit property of their images in H indeed suggests this apart from a finite non-determinism [L45]. Bohr orbit property could therefore mean extremely powerful number theoretical duality for which the roots of the polynomial determine the space-time surface almost uniquely. $SU(3)$ as color symmetry emerges at the level of M^8. By $M^8 - H$ duality, the mass shells are mapped to the boundaries of CDs in H.

3. Do we really know that X^4_r co-associative and has distribution of 2-D commuting subspaces of normal space making possible $M^8 - H$ duality? The intuitive expectation is that the answer is affirmative [A2]. In any case, $M^8 - H$ duality is well-defined even without this condition.

4. The special solutions to $P = 0$, discovered already earlier, are restricted to the boundary of CD_8 and correspond to the values of energy (rather than mass or mass squared) coming as roots of the real polynomial P. These mass values are mapped by inversion to "very special moments in the life of self" (a misleading term) at the level of H as special values of light-cone proper time rather than linear Minkowski time as in the earlier interpretation [L15]. The new picture is Lorenz invariant.

4.1.3 Octonionic Dirac equation requires co-associativity

The octonionic Dirac equation allows a second perspective on associativity [L24].

1. Everything is algebraic at the level of M^8 and therefore also the octonionic Dirac equation should be algebraic. The octonionic Dirac equation is an analog of the momentum space variant of ordinary Dirac equation and also this forces the interpretation of M^8 as momentum space.

2. Fermions are massless in the 8-D sense and massive in 4-D sense. This suggests that octonionic Dirac equation reduces to a mass shell condition for massive particle with $q \cdot q = m^2 = r_n$, where $q \cdot q$ is octonionic norm squared for quaternion q defined by the expression of momentum p as $p = I_4 q$, where I_4 is octonion unit orthogonal to q. r_n represents mass shell as a root of P.

3. For the co-associative option, the co-associative octonion p representing the momentum is given in terms of quaternion q as $p = I_4 q$. One obtains $p \cdot p = q\overline{q} = m^2 = r_n$ at the mass shell defined as a root of P. Note that for M^4 subspace the space-like components of p p are proportional to i and the time-like component is real. All signatures of the number theoretic metric are possible.

4. For associative option, one would obtain $qq = m^2$, which cannot be satisfied: q reduces to a complex number $zx + Iy$ and one has analog of equation $z^2 = z^2 - y^2 + 2Ixy = m_n^2$, which cannot be true. Hence co-associativity is forced by the octonionic Dirac equation.

This picture combined with zero energy ontology leads also to a view about quantum TGD at the level of M^8. Local $SU(3)$ element g has properties suggesting a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P. The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

4.1.4 Hamilton-Jacobi structure and Kähler structure of $M^4 \subset H$ and their counterparts in $M^4 \subset M^8$

The Kähler structure of $M^4 \subset H$, forced by the twistor lift of TGD, has deep physical implications and seems to be necessary. It implies that for Dirac equation in H, modes are eigenstates of only the longitudinal momentum and in the 2 transversal degrees of freedom one has essentially harmonic oscillator states [L43, L40], that is Gaussians determined by the 2 longitudinal momentum components. For real longitudinal momentum the exponents of Gaussians are purely imaginary or purely real.

The longitudinal momentum space $M^2 \subset M^4$ and its orthogonal complement E^2 is in a preferred role in gauge theories, string models, and TGD. The localization of this decomposition leads to the notion of Hamilton-Jacobi (HJ) structure of M^4 and the natural question is how this relates to Kähler structures of M^4. At the level of H spinors fields only the Kähler structure corresponding to constant decomposition $M^2 \oplus E^2$ seems to make sense and this raises the question how the H-J structure and Kähler structure relate. TGD suggests the existence of two geometric structure in M^4: HJ structure and Kähler structure. It has remained unclear whether HJ structure and Kähler structure with covariantly constant self-dual Kähler form are equivalent notions or whether there several H-J structures accompaning the Kähler structure.

In the following I argue that H-J structures correspond to different choices of symplectic coordinates for M^4 and that the properties of $X^4 \subset H$ determined bt M^-H duality make it natural to to choose particular symplectic coordinates for M^4.

Consider first what H-J structure and Kähler structure could mean in H.

1. The H-J structure of $M^4 \subset H$ would correspond to an integrable distribution of 2-D Minkowskian sub-spaces of M^4 defining a distribution of string world sheets $X^2(x)$ and orthogonal distribution of partonic 2-surfaces $Y^2(x)$. Could this decomposition correspond to self-dual covariantly Kähler form in M^4?

   What do we mean with covariant constancy now? Does it mean a separate covariant constancy for the choices of $M^2(x)$ and $Y^2(x)$ or only of their sum, which in Minkowski coordinates could correspond to a constant electric and magnetic fields orthogonal to each other?

2. The non-constant choice of $M^2(x)$ ($E^2(x)$) cannot be covariantly constant. One can write $J(M^4) = J(M^2(x)) \oplus J(E^2(x)$ corresponding to decomposition to electric and magnetic parts. Constancy of $J(M^2(x)$ would require that the gradient of $J(M^2(x)$ is compensated by the gradient of an antisymmetric tensor with square equal to the projector to $M^2(x)$. Same condition holds true for $J(E^2(x))$. The gradient of the antisymmetric tensor would be parallel to itself implying that the tensor is constant.

3. H-J structure can only correspond to a transformation acting on J but leaving $J_{kl}dm^kdm^l$ invariant. One should find analogs of local gauge transformations leaving J invariant. In the case of CP_2, these correspond to symplectic transformations and now one has a generalization of the notion. The M^4 analog of the symplectic group would parameterize various decompositions of $J(M^4)$.

Physically the symplectic transformations define local choices of 2-D space $E^2(x)$ of transversal polarization directions and longitudinal momentum space M^2 emerging in the construction of extremals of Kähler action.

4. For the simplest Kähler form for $M^4 \subset H$, this decomposition in Minkowski coordinates would be constant: orthogonal constant electric and magnetic fields. This Kähler form extends to its number theoretical analog in M^8. The local $SU(3)$ element g would deform M^4 to $g(M^4)$ and define an element of local CP_2 defining $M^8 - H$ duality. g should correspond to a symplectic transformation of M^4.

Consider next the number theoretic counterparts of H-J- and Kähler structures of $M^4 \subset H$ in $M^4 \subset M^8$.

1. In M^4 coordinates H-J structure would correspond to a constant $M^2 \times E^2$ decomposition. In M^4 coordinates Kähler structure would correspond to constant E and B orthogonal to each other. Symplectic transformations give various representations of this structure as H-J structures.

2. The number theoretic analog of H-J structure makes sense also for $X^4 \subset M^8$ as obtained from the distribution of quaternionic normal spaces containing 2-D commutative sub-space at each point by multiplying then by local unit $I_4(x)$ orthogonal to the quaternionic units $\{1, I_1 = I_2 = I_3\}$ with respect to octonionic inner product. There is a hierarchy of CDs and the choices of these structures would be naturally parameterized by G_2.

   This would give rise to a number theoretically defined slicing of $X_c^4 \subset M_c^8$ by complexified string world sheets X_c^2 and partonic 2-surfaces Y_c^2 orthogonal with respect to the octonionic inner product for complexified octonions.

3. In $M^8 - H$ duality defined by $g(p) \subset SU(3)$ assigns a point of CP_2 to a given point of M^4. $g(p)$ maps the number theoretic H-J to H-J in $M^4 \subset M^8$. The space-time surface itself - that is $g(p)$ - defines these symplectic coordinates and the local $SU(3)$ element g would naturally define this symplectic transformation.

4. For $X^4 \subset M^8$ g reduces to a constant color rotation satisfying the condition that the image point is $U(2)$ invariant. Unit element is the most natural option. This would mean that g is constant at the mass and energy shells corresponding to the roots of P and the mass shell is a mass shell of M^4 rather than some deformed mass shell associated with images under $g(p)$.

   This alone does not yet guarantee that the 4-D tangent space corresponds to M^4. The additional physically very natural condition on g is that the 4-D momentum space at these mass shells is the same. $M^8 - H$ duality maps these mass shells to the boundaries of these cd:s in M^4 (CD= $cd \times CP_2$). This conforms with the identification of zero energy states as pairs of 3-D states at the boundaries of CD.

This generalizes the original intuitive but wrong interpretation of the roots r_n of P as "very special moments in the life of self" [L15].

1. Since the roots correspond to mass squared values, they are mapped to the boundaries of cd with size $L = \hbar_{eff}/m$ by $M^8 - H$ duality in M^4 degrees of freedom. During the sequence of SSFRs the passive boundary of CD remains does not shift only changes in size, and states at it remain unaffected. Active boundary is shifted due to scaling of cd.

   The hyperplane at which upper and lower half-cones of CD meet, is shifted to the direction of geometric future. This defines a geometric correlate for the flow of experienced time.

2. A natural proposal is that the moments for SSFRs have as geometric correlates the roots of P defined as intersections of geodesic lines with the direction of 4-momentum p from the tip of CD to its opposite boundary (here one can also consider the possibility that the geodesic lines start from the center of cd). Also energy shells as roots $E = r_n$ of P are predicted. They decompose to a set of mass shells $m_{n.,k}$ with the same $E = r_n$: similar interpretation applies to them.

3. What makes these moments very special is that the mass and energy shells correspond to surfaces in M^4 defining the Lorentz quantum numbers. SSFRs correspond to quantum measurements in this basis and are not possible without this condition. At $X^4 \subset M^8$ the mass squared would remain constant but the local momentum frame would vary. This is analogous to the conservation of momentum squared in general relativistic kinematics of point particle involving however the loss of momentum conservation.

4. These conditions, together with the assumption that g is a rational function with real coefficients, strongly suggest what I have referred to as preferred extremal property, Bohr orbitology, strong form of holography, and number theoretical holography.

In principle, by a suitable choice of M^4 one can make the momentum of the system light-like: the light-like 8-momentum would be parallel to M^4. I have asked whether this could be behind the fact that elementary particles are in a good approximation massless and whether the small mass of elementary particles is due to the presence of states with different mass squares in the zero state allowed by Lorentz invariance.

The recent understanding of the nature of right-handed neutrinos based on M^4 Kähler structure [L40] makes this mechanism un-necessary but poses the question about the mechanism choosing some particular M^4. The conditions that $g(p)$ leaves mass shells and their 4-D tangent spaces invariant provides this kind of mechanism. Holography would be forced by the condition that the 4-D tangent space is same for all mass shels representing inverse images for very special moments of time.

4.2 Uncertainty Principle and $M^8 - H$ duality

The detailed realization of $M^8 - H$ duality involves still uncertainties. The quaternionic normal spaces containing fixed 2-space M^2 (or an integrable distribution of M^2) are parametrized by points of CP_2. One can map the normal space to a point of CP_2.

The tough problem has been the precise correspondence between M^4 points in $M^4 \times E^4$ and $M^4 \times CP_2$ and the identification of the sizes of causal diamonds (CDs) in M^8 and H. The identification is naturally linear if M^8 is analog of space-time but if M^8 is interpreted as momentum space, the situation changes. The option discussed in [L23, L24] maps mass hyperboloids to light-cone proper time =constant hyperboloids and it has turned out that this correspondence does not correspond to the classical picture suggesting that a given momentum in M^8 corresponds in H to a geodesic line emanating from the tip of CD.

4.2.1 $M^8 - H$ duality in M^4 degrees of freedom

The following proposal for M^8-H duality in M^4 degrees of freedom relies on the intuition provided by UP and to the idea that a particle with momentum p^k corresponds to a geodesic line with this direction emanating from the tip of CD.

1. The first constraint comes from the requirement that the identification of the point $p^k \in X^4 \subset M^8$ should classically correspond to a geodesic line $m^k = p^k\tau/m$ ($p^2 = m^2$) in M^8 which in Big Bang analogy should go through the tip of the CD in H. This geodesic line intersects the opposite boundary of CD at a unique point.

   Therefore the mass hyperboloid H^3 is mapped to the 3-D opposite boundary of $cd \subset M^4 \subset H$. This does not fix the size nor position of the CD ($= cd \times CP_2$) in H. If CD does not depend on m, the opposite light-cone boundary of CD would be covered an infinite number of times.

2. The condition that the map is 1-to-1 requires that the size of the CD in H is determined by the mass hyperboloid M^8. Uncertainty Principle (UP) suggests that one should choose the distance T between the tips of the CD associated with m to be $T = \hbar_{eff}/m$.

   The image point m^k of p^k at the boundary of $CD(m, h_{eff})$ is given as the intersection of the geodesic line $m^k = p^k\tau$ from the origin of $CD(m, h_{eff})$ with the opposite boundary of $CD(m, h_{eff})$:

$$m^k = \hbar_{eff} X \frac{p^k}{m^2} \quad , X = \frac{1}{1+p_3/p_0} \quad . \tag{4.1}$$

Here p_3 is the length of 3-momentum.

The map is non-linear. At the non-relativistic limit ($X \to 1$), one obtains a linear map for a given mass and also a consistency with the naive view about UP. m^k is on the proper time constant mass shell so the analog of the Fermi ball in $H^3 \subset M^8$ is mapped to the light-like boundary of $cd \subset M^4 \subset H$.

3. What about massless particles? The duality map is well defined for an arbitrary size of CD. If one defines the size of the CD as the Compton length $\hbar_{eff}/m$ of the massless particle, the size of the CD is infinite. How to identify the CD? UP suggests a CD with temporal distance $T = 2\hbar_{eff}/p_0$ between its tips so that the geometric definition gives $p^k = \hbar_{eff} p^k/p_0^2$ as the point at the 2-sphere defining the corner of CD. p-Adic thermodynamics [K11]) strongly suggests that also massless particles generate very small p-adic mass, which is however proportional to $1/p$ rather than $1/\sqrt{p}$. The map is well defined also for massless states as a limit and takes massless momenta to the 3-ball at which upper and lower half-cones meet.

4. What about the position of the CD associated with the mass hyperboloid? It should be possible to map all momenta to geodesic lines going through the 3-ball dividing the largest CD involved with T determined by the smallest mass involved to two half-cones. This is because this 3-ball defines the geometric "Now" in TGD inspired theory of consciousness. Therefore all CDs in H should have a common center and have the same geometric "Now".

   $M^8 - H$ duality maps the slicing of momentum space with positive/negative energy to a Russian doll-like slicing of $t \geq 0$ by the boundaries of half-cones, where t has origin at the bottom of the double-cone. The height of the $CD(m, h_{eff})$ is given by the Compton length $L(m, h_{eff}) = \hbar_{eff}/m$ of quark. Each value of h_{eff} corresponds its own scaled map and for $h_{gr} = GMm/v_0$, the size of $CD(m, h_{eff}) = GM/v_0$ does not depend on m and is macroscopic for macroscopic systems such as Sun.

5. The points of cognitive representation at quark level must have momenta with components, which are algebraic integers for the extension of rationals considered. A natural momentum unit is $m_{Pl} = \hbar_0/R$, h_0 is the minimal value of $h_{eff} = h_0$ and R is CP_2 radius. Only "active" points of $X^4 \subset M^8$ containing quark are included in the cognitive representation. Active points give rise to active CD:s $CD(m, h_{eff})$ with size $L(m, h_{eff})$.

   It is possible to assign $CD(m, h_{eff})$ also to the composites of quarks with given mass. Galois confinement suggest a general mechanism for their formation: bound states as Galois singlets must have a rational total momentum. This gives a hierarchy of bound states of bound states of ..... realized as a hierarchy of CDs containing several CDs.

6. This picture fits nicely with the general properties of the space-time surfaces as associative "roots" of the octonionic continuation of a real polynomial. A second nice feature is that the notion of CD at the level H is forced by this correspondence. "Why CDs?" at the level of H has indeed been a longstanding puzzle. A further nice feature is that the size of the largest CD would be determined by the smallest momentum involved.

7. Positive and negative energy parts of zero energy states would correspond to opposite boundaries of CDs and at the level of M^8 they would correspond to mass hyperboloids with opposite energies.

8. What could be the meaning of the occupied points of M^8 containing fermion (quark)? Could the image of the mass hyperboloid containing occupied points correspond to sub-CD at the level of H containing corresponding points at its light-like boundary? If so, $M^8 - H$ correspondence would also fix the hierarchy of CDs at the level of H.

It is enough to realize the analogs of plane waves only for the actualized momenta corresponding to quarks of the zero energy state. One can assign to CD as total momentum and passive *resp.*

active half-cones give total momenta $P_{tot,P}$ *resp.* $P_{tot,A}$, which at the limit of infinite size for CD should have the same magnitude and opposite sign in ZEO.

The above description of $M^8 - H$ duality maps quarks at points of $X^4 \subset M^8$ to states of induced spinor field localized at the 3-D boundaries of CD but necessarily delocalized into the interior of the space-time surface $X^4 \subset H$. This is analogous to a dispersion of a wave packet. One would obtain a wave picture in the interior.

4.2.2 Does Uncertainty Principle require delocalization in H or in X^4?

One can argue that Uncertainty Principle (UP) requires more than the naive condition $T = \hbar_{eff}/m$ on the size of sub-CD. I have already mentioned two approaches to the problem: they could be called inertial and gravitational representations.

1. The inertial representations assigns to the particle as a space-time surface (holography) an analog of plane wave as a superposition of space-time surfaces: this is natural at the level of WCW. This requires delocalization space-time surfaces and CD in H.

2. The gravitational representation relies on the analog of the braid representation of isometries in terms of the projections of their flows to the space-time surface. This does not require delocalization in H since it occurs in X^4.

Consider first the inertial representation. The intuitive idea that a single point in M^8 corresponds to a discretized plane wave in H in a spatial resolution defined by the total mass at the passive boundary of CD. UP requires that this plane wave should be realized at the level of H and also WCW as a superposition of shifted space-time surfaces defined by the above correspondence.

1. The basic observation leading to TGD is that in the TGD framework a particle as a point is replaced with a particle as a 3-surface, which by holography corresponds to 4-surface.

   Momentum eigenstate corresponds to a plane wave. Now planewave could correspond to a delocalized state of 3-surface - and by holography that of 4-surface - associated with a particle.

   A generalized plane wave would be a quantum superposition of shifted space-time surfaces inside a larger CD with a phase factor determined by the 4-momentum. $M^8 - H$ duality would map the point of M^8 containing an object with momentum p to a generalized plane wave in H. Periodic boundary conditions are natural and would force the quantization of momenta as multiples of momentum defined by the larger CD. Number theoretic vision requires that the superposition is discrete such that the values of the phase factor are roots of unity belonging to the extension of rationals associated with the space-time sheet. If momentum is conserved, the time evolutions for massive particles are scalings of CD between SSFRs are integer scalings. Also iterated integer scalings, say by 2 are possible.

2. This would also provide WCW description. Recent physics relies on the assumption about single background space-time: WCW is effectively replaced with M^4 since 3-surface is replaced with point and CP_2 is forgotten so that one must introduce gauge fields and metric as primary field variables.

As already discussed, the gravitational representation would rely on the lift/projection of the flows defined by the isometry generators to the space-time surface and could be regarded as a "subjective" representation of the symmetries. The gravitational representation would generalize braid group and quantum group representations.

The condition that the "projection" of the Dirac operator in H is equal to the modified Dirac operator, implies a hydrodynamic picture. In particular, the projections of isometry generators are conserved along the lifted flow lines of isometries and are proportional to the projections of Killing vectors. QCC suggests that only Cartan algebra isometries allow this lift so that each choice of quantization axis would also select a space-time surface and would be a higher level quantum measurement.

4.2.3 Exact ZEO emerges only at the limit of CD with infinite size

At the limit when the volume of CD becomes infinite, the sum of the momenta associated with opposite boundaries of CD should automatically vanish and one would obtain ideal zero energy states. The original assumption that ideal zero energy states are possible for finite size of CD, is not strictly true. The situation is the same for quantization in a finite volume.

1. Denote the sum of the total momenta with positive energy associated with passive boundaries of all CDs by $P_{tot,P} = \equiv P_{tot}$. For finite size of CD, $P_{tot,P}$ need not be the same as the total momentum $P_{tot,A}$ associated with the active boundary which can change during the sequence of SSFRs. Denote the difference $P_{tot,P} - P_{tot,A}$ by ΔP.

   This momentum is P_{tot} is large for large CDs, and naturally defines the spatial resolution. Denote by $M^k = nXh_{eff}P^k_{tot}/\cdot P^2_{tot}$, $X = 1/(1+P_3/P_0)$, the shift defined by P_{tot}. The analogs of plane waves for the sub-CDs should be discretized with this spatial resolution and at the limit of large total mass the discretization improves.

2. The image of X^4 in H for a given mass hyperboloid H^3 should define a geometric analog of a plane wave in WCW for the total momentum $P^k = \sum_i p_i^k$, $p_i^2 = m^2$ of H^3, associated with the CD(M) in M^8. It is also possible to include the momenta with different masses since they have images also at the boundaries of all CDs in the Russian doll hierarchy. For $\hbar_{gr}$ there is a common CD for all particle masses with size Λ_{gr}.

   The WCW plane wave would not be a superposition of points but of shifted space-time surfaces. The argument of the plane wave would correspond to the shift of the $X^4 \subset CD(M) \subset H$.

   Maximal spatial resolution is achieved if one shifts the X^4 and corresponding CD(m) in H inside the large CD by nM^k, $M^k = nh_{eff}XP^k_{tot}/\cdot P^2_{tot}$ and forms the WCW spinor field as a superposition of shifted space-time surfaces $X^4(m)$ with $U_n = exp(i\Delta P\cdot nM)$ appearing as plane wave phase factor.

3. At the limit when the size of the largest CD becomes infinite (the mass M defining Λ_{gr} becomes very large), the sum $\sum_n U_n$ obtained as integral over the identical shifted copies of the space-time surfaces is non-vanishing only for $\Delta P = 0$ and one obtains an momentum conserving ideal zero energy state.

These states would be analogs of single particle states as plane waves, with particle replaced with many-quark state inside $CD(m)$. The generalization is obvious: perform the analog of second quantization by forming N-particle states in which one has N $CD(m)$ plane waves.

4.2.4 The revised view about $M^8 - H$ duality and the "very special moments in the life of self"

The polynomial equations allow at M^8 level also highly unique brane-like solutions having the topology of 6-sphere S^6 and intersecting M^4 along $p^0 = E = constant$ hyperplane. These quantizd values of energy E correspond to the roots of the polynomial defining the solution and are algebraic numbers and algebraic integers for monic polynomials of form $P(x) = x^n + p_{n-1}x^{n-1} + ...$

The TGD inspired theory of consciousness motivated the interpretation of these hyperplanes as "very special moments in the life of self": this interpretation [L15] emerged before the realization that M^8 corresponds to momentum space. The images of these planes under $M^8 - H$ duality should however allow this interpretation also in the new picture. Is this possible?

To answer the question one must understand what the image of S^6 under $M^8 - H$ duality is.

1. The image must belong to $M^4 \times CP_2$. The 2-D normal space of the point of S^6 is a complex commutative plane of octonions. Since 4-D normal planes of space-time surface containing complex plane correspond to points of CP_2, the natural proposal is that the image now corresponds to point of CP_1 identified as homologically trivial geodesic sub-manifold S^2_G of CP_2 carrying Kähler magnetic charge.

2. The first thing to notice about the H-image of the 3-D $E = constant$ surface $X^3(E) \subset M^4$ is that it is indeed 3-D rather than 4-D. In M^4 the map has the form $m^k = X\hbar_{eff}/m^2$, $X = 1/(1 + p_3/p_0)$ already discussed.

   The value of $m^2 = E^2 - p_3^2$ decreases as p_3^2 increases so that the values of light-cone proper time $a = t^2 - r^2$ for the image are larger than $a_{min} = \hbar_{eff}/m$. "Fermi-spheres" $S^2_F(p_3)$ are mapped to 2-spheres $S^2(r) \subset M^4 \subset H$ with an increasing radius $r(t) = \sqrt{t^2 - a^2_{min}}$. 2-sphere is born at $t = a_{min}$ and starts to increase in size and the expansion velocity approaches light velocity asymptotically. This expanding sphere would be magnetically charged.

   The sequence a_n of "very special moments in the life of self" in the life of self would mean the birth of this kind of expanding sphere and a_n would correspond to the roots of the polynomial considered identified as quantized energies. The dispersion relation $E = constant$ means that energy does not depend on the momentum: plasmons provide the condensed matter analogy.

3. There are interesting questions to be answered. Do the surfaces $X^3(E)$ intersect the 4-D space-time surface $X^4 \subset H$? At the level of M^8 the intersections of 4-D and 6-D surfaces are 2-D. The proposal is that these 2-surfaces M^8 are mapped to partonic vertices identified as 2-surfaces $X^2 \subset X^4 \subset H$ at which 4-D surfaces representing particles meet. This should happen also for the new identification of $M^8 - H$ duality.

   However, in the generic case the intersections of 3-surfaces and 4-surfaces in H are empty. The recent situation is however not a generic one since the S^6 solutions are non-generic (one would expect only 4-D solutions) and 4-D and 6-D solutions are determined by the same polynomial. Therefore the points to which the 2-spheres contract for $t = a_{min}$ should be mapped to partonic 2-surfaces in H. Single point should correspond to the geodesic sphere S^2_G.

   Does this conform with the view that 4-D CP_2 type extremals in H correspond to "blow-ups" of 1-D line singularities of $X^4 \subset M^8$ for which the quaternionic tangent spaces at singularity are not unique and define 3-D surface as points of CP_2. Now the 2-D normal spaces of S^2_F would span $S^2_G \subset CP_2$ and at the limit of S^2_F contracting to a point, one would have a 2-D singularity having an interpretation as a partonic vertex.

4. Cosmic strings $X^4 = X^2 \times S^2_G \subset M^4 \times CP_2$ carrying monopole charge are basic solutions of field equations. Could these cosmic strings relate to the images of $X^3(E)$? For instance, could $X^3(E_1)$ and $X^3(E_2)$ correspond to the ends of a cosmic string thickening to a monopole flux tube? Thickening would correspond to the growth of M^4 projection $S^2(r(t))$ of the flux tube having $r(t) = \sqrt{t^2 - a^2_{min}}$. The interpretation would be as a pair of magnetic poles connected by a monopole flux tube. Cosmic strings would be highly dynamical entities if this is the case.

4.2.5 An objection against $M^8 - H$ duality

Objections are the best manner to proceed. $M^8 - H$ duality maps the point M^8 at mass shell m to points of CD corresponding to the Compton length $\hbar_{eff}/m$ obtained as intersection of line with momentum p starting at the center point of CD and intersecting either boundary of CD. Each quaternionic normal space contains a commuting subspace (in octonionic sense) such that the distribution of the latter spaces is integrable. These normal spaces are parameterized by CP_2. This implies a complete localization in CP_2 so that the restriction of the induced quark field does not have well-defined color quantum numbers.

How to circumvent this objection? The proposed identification of string-like and particle-like space-time surfaces suggests a solution to the problem. Consider first CP_2 type extremals.

1. Consider first CP_2 type extremals as analogs of particles proposed to correspond to line singularities of algebraic 4-surfaces in M^8 with the property that the normal co-quaternionic space is not unique and the normal spaces at given point of the line are parametrized by a 3-D surface of CP_2 at each point of the light-like curve. Algebraic geometers speak of blow-up singularity. This kind of singularity is analogous to the tip of a cone.

   For polynomials the M^4 projection is a light-like geodesic. Also the octonionic continuations of analytic functions of real argument with rational Taylor coefficients can define space-time surfaces and in this case more general light-like curves are expected to be possible. This gives rise to a 4-D surface of H, which has the same Euclidean metric and Kähler form as CP_2 and only the induced gamma matrices are different.

2. The induced spinor field as restriction of the second quantized spinor field of H decomposes into modes, which are modes of H d'Alembertian. The modes have well-defined color quantum numbers so that one can speak of color quarks. This would mean that one can speak about colored quarks only inside CP_2 type extremals and possibly also inside string-like objects. This would trivialize the mysteries of quark and color confinement.

   Gluons would correspond to pairs of quark and antiquark associated with distinct wormhole throats or even - contacts. The mass squared for a given mode is well-defined but at the level of H only the right-handed neutrino is massless. Other states have mass of order CP_2 mass.

3. One can argue that the average momenta associated with these kinds of states have M^4 projection parallel to the light-like geodesic so that the momentum is light-like. There are several justifications for the claim.

   (a) The gravitational representation of isometries already discussed as lift/projection of the corresponding flows in H to X^4 restricts the action of M^4 isometries to a light-like geodesic and implies that the states are massless in this sense.

   (b) The claim conforms with an earlier intriguing observation that the restriction of a massive quark propagator to a pair of space-time points with light-like M^4 distance is essentially a massless propagator irrespective of the value of the mass.

   (c) With a suitable choice of $M^4 \subset M^8$ the ground state mass can be chosen to vanish. The reason is that the 8-D momentum is light-like and if M^4 contains the momentum, then also the M^4 mass vanishes. This choice can be made only for a single mode in the superposition. p-Adic thermodynamics would describe the contribution of higher modes in the quantum superposition of states to the mass squared having interpretation as thermal mass squared.

   (d) One can look at the situation also at the space-time level. If one has a light-like curve or a curve consisting of segments, which are light-like geodesic lines, the situation changes. Since the average velocity for this kind of zigzag (zitterbewegung) curve is below light velocity, the intuitive expectation is that this represents the TGD analog of the Higgs mechanism having interpretation as massivation.

   This finding was the original motivation for p-adic thermodynamics. The conditions stating the light-likeness of the projection are nothing but Virasoro conditions. p-Adic thermodynamics involves also the inclusion of supersymplectic symmetries.

   $H(M^4)$ is orthogonal to the space-time surface and has an interpretation as a local acceleration of the space-time surface as an extended particle. The CP_2 part of H was the original proposal for the Higgs field considered in my thesis. Indeed, $H(CP_2)$ behaves like a complex doublet in complex coordinates. The physical interpretation is that the minimal surface property forces zitterbewegung with acceleration $H(M^4) = H(CP_2)$, which in turn means that light-like curve looks in the average sense like time-like geodesic for a massive particle.

   The problem is that the proposed Higgs field vanishes in the interiors of space-time surfaces. However, the general field equations do not imply minimal surface property and also for preferred extremals it fails at singularities analogous to frames of soap films. At these point one can have non-vanishing $H(CP_2)$. 8-D light-likeness suggests that at these points $H(H)$ is light-like.

What happens to string like-objects corresponding to 2-D singularities such that the normal spaces at a given point correspond to a 2-D surface of CP_2, which in the most general situation can be either complex 2-surface of CP_2 or a minimal Lagrangian 2-manifold? One cannot exclude 1-D singularities associated with surfaces $X^3 \times X^1 \subset M^4 \times CP_2$ for which CP_2 projection is 1-D, presumably a geodesic circle.

(a) The simplest string-like objects come in 2 variants corresponding to CP_2 projection, which is a geodesic sphere, which can be homologically non-trivial or non-trivial. M^4 projection is in the simplest situation 2-D plane M^2.

These two options correspond to the reduction of $SU(3)$ to $U(2)$ or $SO(3)$. The interpretation in terms of spontaneous symmetry breaking is highly suggestive. The representations of $SU(3)$ decompose to those of $U(2)$ or $SO(3)$. Color confinement could weaken to that for $U(2)$ or $SO(3)$ so that the total color quantum numbers I_3 and Y would still vanish but color multiplets would allow these kinds of states.

(b) The simplest symmetry breaking to $U(1)$ could correspond to extremals of form $M^3 \times S^1$ and only $U(1)$ confinement would hold true. In the case of M^4 it does not make sense to speak of color quantum numbers.

4.3 Generalizations related to $M^8 - H$ duality

It has become clear that $M^8 - H$ duality generalizes and there is a connection with the twistorialization at the level of H.

4.3.1 M^8-H duality at the level of WCW and p-adic prime as the maximal ramified prime of polynomial

The vacuum functional as an exponent of the Kähler function determines the physics at WCW level. $M^8 - H$ duality suggests that it should have a counterpart at the level of M^8 and appear as a weight function in the summation. Adelic physics requires that weight function is a power of p-adic prime and ramified primes of the extension are the natural candidates in this respect.

1. The discriminant D of the algebraic extension defined by a polynomial P with rational coefficients (`https://en.wikipedia.org/wiki/Discriminant`) is expressible as a square for the product of the non-vanishing differences $r_i - r_j$ of the roots of P. For a polynomial P with rational coefficients, D is a rational number as one can see for polynomial $P = ax^2 + bx + c$ from its expression $D = b^2 - 4ac$. For monic polynomials of form $x^n + a_{n-1}x^{m-1} + ...$ with integer coefficients, D is an integer. In both cases, one can talk about ramified primes as prime divisors of D.

   If the p-adic prime $p$is identified as a ramified prime, D is a good candidate for the weight function since it would be indeed proportional to a power of p and have p-adic norm proportional to negative power of p. Hence the p-adic interpretation of the sum over scattering amplitudes for polynomials P is possible if p corresponds to a ramified prime for the polynomials allowed in the amplitude.

   p-Adic thermodynamics [K11] suggest that p-adic valued scattering amplitudes are mapped to real numbers by applying to the Lorentz invariants appearing in the amplitude the canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adics to reals in a continuous manner

2. For monic polynomials, the roots are powers of a generating root, which means that D is proportional to a power of the generating root, which should give rise to some power of p. When the degree of the monic polynomial increases, the overall power of p increases so that the contributions of higher polynomials approach zero very rapidly in the p-adic topology. For the p-adic prime $p = M_{127} = 2^{127} - 1 \sim 10^{38}$ characterizing electrons, the convergence is extremely rapid.

   Polynomials of lowest degree should give the dominating contribution and the scattering amplitudes should be characterized by the degree of the lowest order polynomial appearing in it. For polynomials with a low degree n the number of particles in the scattering amplitude could be very small since the number n of roots is small. The sum $x_i + p_i$ cannot belong to the same mass shell for timelike p_i so that the minimal number of roots r_n increases with the number of external particles.

3. $M^8 - H$ duality requires that the sum over polynomials corresponds to a WCW integration at H-side. Therefore the exponent of Kähler function at its maximum associated to a given

polynomial should be apart from a constant numerical factor equal to the discriminant D in canonical identification.

The condition that the exponent of Kähler function as a sum of the Kähler action and the volume term for the preferred extremal $X^4 \subset H$ equals to power of D apart from a proportionality factor, should fix the discrete number theoretical and p-adic coupling constant evolutions of Kähler coupling strength and length scale dependent cosmological constant proportional to inverse of a p-adic length scale squared. For Kähler action alone, the evolution is logarithmic in prime p since the function reduces to the logarithm of D.

$M^8 - H$ duality suggests that the exponent $exp(-K)$ of Kähler function has an M^8 counterpart with a purely number theoretic interpretation. The discriminant D of the polynomial P is the natural guess. For monic polynomials D is integer having ramified primes as factors.

There are two options for the correspondence between $exp(-K)$ at its maximum and D assuming that P is monic polynomial.

1. In the real topology, one would naturally have $exp(-K) = 1/D$. For monic polynomials with high degree, D becomes large so that $exp(-K)$ is large.

2. In a p-adic topology defined by p-adic prime p identified as a ramified prime of D, one would have naturally $exp(-K) = I(D)$, where one has $I(x) = \sum x_n p^n = \sum x_n p^{-n}$.

   If p is the largest ramified prime associated with D, this option gives the same result as the real option, which suggests a unique identification of the p-adic prime p for a given polynomial P. P would correspond to a unique p-adic length scale L_p and a given L_p would correspond to all polynomials P for which the largest ramified prime is p.

   This might provide some understanding concerning the p-adic length scale hypothesis stating that p-adic primes tend to be near powers of integer. In particular, understanding about why Mersenne primes are favored might emerge. For instance, Mersennes could correspond to primes for which the number of polynomials having them as the largest ramified prime is especially large. The quantization condition $exp(-K) = D(p)$ could define which p-adic primes are the fittest ones.

The condition that $exp(-K)$ at its maximum equals to D via canonical identification gives a powerful number theoretic quantization condition.

4.3.2 Space-time surfaces as images of associative surfaces in M^8

$M^8 - H$ duality would provide an explicit construction of space-time surfaces as algebraic surfaces with an associative normal space [L23, L24]. M^8 picture codes space-time surface by a real polynomial with rational coefficients. One cannot exclude coefficients in an extension of rationals and also analytic functions with rational or algebraic coefficients can be considered as well as polynomials of infinite degree obtained by repeated iteration giving rise algebraic numbers as extension and continuum or roots as limits of roots.

$M^8 - H$ duality maps these solutions to H and one can consider several forms of this map. The weak form of the duality relies on holography mapping only 3-D or even 2-D data to H and the strongest form maps entire space-time surfaces to H. The twistor lift of TGD allows to identify the space-time surfaces in H as base spaces of 6-D surfaces representing the twistor space of space-time surface as an S^2 bundle in the product of twistor spaces of M^4 and CP_2. These twistor spaces must have Kähler structure and only the twistor spaces of M^4 and CP_2 have it [A6] so that TGD is unique also mathematically.

An interesting question relates to the possibility that also 6-D commutative space-time surfaces could be allowed. The normal space of the space-time surface would be a commutative subspace of M^8_c and therefore 2-D. Commutative space-time would be a 6-D surface X^6 in M^8.

This raises the following question: Could the inverse image of the 6-D twistor-space of 4-D space-time surface X^4 so that X^6 would be M^8 analog of twistor lift? This requires that $X^6 \subset M^8_c$ has the structure of an S^2 bundle and there exists a bundle projection $X^6 \to X^4$.

The normal space of an associative space-time surface actually contains this kind of commutative normal space! Its existence guarantees that the normal space of X^4 corresponds to a point of CP_2. Could one obtain the M_c^8 analog of the twistor space and the bundle bundle projection $X^6 \to X^4$ just by dropping the condition of associativity. Space-time surface would be a 4-surface obtained by adding the associativity condition.

One can go even further and consider 7-D surfaces of M^8 with real and therefore well-ordered normal space. This would suggest dimensional hierarchy: $7 \to 6 \to 4$.

This leads to a possible interpretation of twistor lift of TGD at the level of M^8 and also about generalization of $M^8 - H$ correspondence to the level of twistor lift. Also the generalization of twistor space to a 7-D space is suggestive. The following arguments representa vision about "how it must be" that emerged during the writing of this article and there are a lot of details to be checked.

4.3.3 Commutative 6-surfaces and twistorial generalization of $M^8 - H$ correspondence

One can generalize the notion of complex 4-surface $X_c^6 \subset M_c^8$ to that of complex 6-surface $X_c^6 \subset M^8$ with a complexified commutative normal space. The 6-surface would correspond to a surface obtained by a local $SU(3)$ element invariant under $U(1) \times U(1) \subset SU(2)$. In complete analogy with 4-D case, these 6-surfaces would contain 5-D mass shells determined by the roots of P. The space $F = SU(3)/U(1) \times U(1)$ of points is nothing but the twistor space of CP_2!

The deformed M^6 defining $X^6 \subset M^8$ regarded as surface in M^8 suggests an interpretation as an analog of 6-D twistor space of M^4. Maybe one could identify the M^6 as the projective space $C^4/C_\times$ obtained from C^4 by dividing with complex scalings? This would give the twistor space $CP_3 = SU(4)/U(3)$ of M^4. This is not obvious since one has (complexified) octonions rather than C^4 or its hypercomplex analog. This would be analogous to using several (4) coordinate charts glued together as in the case of sphere CP_1.

The map $M^6 \to F$ obtained in this manner would define mapping of the twistor spaces of M^4 and CP_2 to each other. The twistor lift of TGD indeed defines this kind of map. The twistor lift involves the additional assumption that the S^2 fibers of these twistor spaces correspond to each other isometrically. This could correspond to a choice of Hamilton-Jacobi structure defining a local decomposition of $M^6 = M^2 \oplus E^4$ such that M^2 defines the analog of the Riemann sphere for M^6.

It might be also possible to identify the octonionic analog of the projective space $CP_3 = C^4/C_\times$. Could the octonionic M^8 momenta be scaled down by dividing with the momentum projection in the commutative normal space so that one obtains an analog of projective space? Could one use these as coordinates for M^6? The scaled 8-momenta would correspond to the points of the octonionic analog of CP_3. The scaled down 8-D mass squared would have a constant value.

A possible problem is that one must divide either from left or right and results are different in the general case. Could one require that the physical states are invariant under the automorphisms generated $o \to gog^{-1}$, where g is an element of the commutative subalgebra in question?

4.3.4 Physical interpretation of the counterparts of twistors at the level of M_c^8

What about the physical interpretation at the level of M_c^8. The twistor space allows a geometrization of spin so that momentum and spin would combine to a purely geometric entity with 6 components. The active points would correspond to fermions (quarks) with a given momentum and spin.

1. The first thing to notice is that in the twistor Grassmannian approach twistor space provides an elegant description of spin. Partial waves in the fiber S^2 of twistor space representation of spin as a partial wave. All spin values allow a unified treatment.

   The problem is that this requires massless particles. In the TGD framework 4-D masslessness is replaced with its 8-D variant so that this difficulty is circumvented. This kind of description in terms of partial waves is expected to have a counterpart at the level of the twistor space $T(M(^4) \times T(CP_2)$. At level of M^8 the description is expected to be in terms of discrete points of M_c^8.

2. Consider first the real part of $X_c^6 \subset M_c^8$. At the level of M^8 the points of X^4 correspond to points. The same must be true also at the level of X^6. Single point in the fiber space S^2 would be selected. The interpretation could be in terms of the selection of the spin quantization axis.

Spin quantization axis corresponds to 2 diametrically opposite points of S^2. Could the choice of the point also fix the spin direction? There would be two spin directions and in the general case of a massive particle they must correspond to the values $S_z = \pm 1/2$ of fermion spin. For massless particles in the 4-D sense two helicities are possible and higher spins cannot be excluded. The allowance of only spin 1/2 particles conforms with the idea that all elementary particles are constructed from quarks and antiquarks. Fermionic statistics would mean that for fixed momentum one or both of the diametrically opposite points of S^2 defining the same and therefore unique spin quantization axis can be populated by quarks having opposite spins.

3. For the 6-D tangent space of X^6_c or rather, its real projection, an analogous argument applies. The tangent space would be parametrized by a point of $T(CP_2)$ and mapped to this point. The selection of a point in the fiber S^2 of $T(CP_2)$ would correspond to the choice of the quantization axis of electroweak spin and diametrically opposite points would correspond to opposite values of electroweak spin 1/2 and unique quantization axis allows only single point or pair of diametrically opposite points to be populated.

   Spin 1/2 property would hold true for both ordinary and electroweak spins and this conforms with the properties of $M^4 \times CP_2$ spinors.

4. The points of $X^6_c \subset M^8_c$ would represent geometrically the modes of H-spinor fields with fixed momentum. What about the orbital degrees of freedom associated with CP_2?

   M^4 momenta represent orbital degrees of M^4 spinors so that E^4 parts of E^8 momenta should represent the CP_2 momenta. The eigenvalue of CP_2 Laplacian defining mass squared eigenvalue in H should correspond to the mass squared value in E^4 and to the square of the radius of sphere $S^3 \subset E^4$.

   This would be a concrete realization for the $SO(4) = SU(2)_L \times SU(2)_R \leftrightarrow SU(3)$ duality between hadronic and quark descriptions of strong interaction physics. Proton as skyrmion would correspond to a map S^3 with radius identified as proton mass. The skyrmion picture would generalize to the level of quarks and also to the level of bound states of quarks allowed by the number theoretical hierarchy with Galois confinement. This also includes bosons as Galois confined many quark states.

5. The bound states with higher spin formed by Galois confinement should have the same quantization axis in order that one can say that the spin in the direction of the quantization axis is well-defined. This freezes the S^2 degrees of freedom for the quarks of the composite.

What does the map of the twistor space $T(M^4)$ to $T(CP_2)$ mean physically? Does spin correspond to color isospin or electroweak spin? Color $U(2)$ corresponds to electroweak $U(2)$ as the holonomy group of CP_2 as symmetric space so that the latter option is possible.

Quarks are doublets with respect to spin and electroweak spin but color triplet contains also isospin singlet. This is not a problem since color is not a spin-like quantum number in TGD but corresponds to color partial waves. This leaves spin-ew spin correspondence realized for quarks. Does the map between spin and electroweak degrees of freedom allow all pairings of spin and electroweak isospin doublets? The map between the spheres S^2 is determined only modulo relative rotation so that this might be the case for spin and color isospin. For composites of quarks obtained as Galois singlets, the relation between spin and ew spin could be more complex.

4.3.5 7-surfaces with real normal space and generalization of the notion of twistor space

The next step is to ask whether it makes sense to consider 7-surfaces with a real normal space allowing well-ordering? This would give a hierarchy of surfaces of M^8 with dimensions 7, 6, and 4. The 7-D space would have bundle projection to 6-D space having bundle projection to 4-D space.

One can also consider the complex 7-D surfaces with a complexified normal space for which the real projection is well-ordered so that the hierarchy of number fields would be realized. These surfaces would be realized by local elements of $SU(3)$ invariant under $U(1) \subset SU(3)$ and would define maps to $SU(3)/U(1)$ defining a generalization of twistor space. Now 6-D complex mass shells would take the role of 3-D complex mass shells and would correspond to the roots of P-

For the 7-D surface also the 7:th component of H- momentum should have some physical interpretation. Fermi statistics at the level of M^8 could be expressed purely geometrically: a single point of X^7 can contain only a single fermion (quark).

What could be the physical interpretation of 7-D surfaces of M^8 with real normal space in the octonionic sense and of their H images?

1. The first guess is that the images in H correspond to 7-D surfaces as generalizations of 6-D twistor space in the product of similar 7-D generalization of twistor spaces of M^4 and CP_2. One would have a bundle projection to the twistor space and to the 4-D space-time.

2. $SU(3)/U(1) \times U(1)$ is the twistor space of CP_2. $SU(3)/SU(2) \times U(1)$ is the twistor space of M^4? Could 7-D $SU(3)/U(1)$ *resp.* $SU(4)/SU(3)$ correspond to a generalization of the twistor spaces of M^4 *resp.* CP_2? What could be the interpretation of the fiber added to the twistor spaces of M^4, CP_2 and X^4? S^3 isomorphic to $SU(2)$ and having $SO(4)$ as isometries is the obvious candidate.

3. The analog of $M^8 - H$ duality in Minkowskian sector in this case could be to use coordinates for M^7 obtained by dividing M^8 coordinates by the real part of the octonion. Is it possible to identify $RP_7 = M^8/R_\times$ with $SU(4)/SU(3)$ or at least relate these spaces in a natural manner. It should be easy to answer these questions with some knowhow in practical topology.

   A possible source of problems or of understanding is the presence of a commuting imaginary unit implying that complexification is involved in Minkowskian degrees of freedom whereas in CP_2 degrees of freedom it has no effect. RP_7 is complexified to CP_7 and the octonionic analog of CP_3 is replaced with its complexification.

What could be the physical interpretation of the extended 7-D twistor space?

1. Twistorialization takes care of spin and electroweak spin and correlates them for quarks. The remaining standard model quantum numbers are Kähler and Kähler magnetic charges for M^4 and CP_2. Could the additional dimension allow a geometrization of these quantum numbers in terms of partial waves in the 3-D fiber? The example with the twistorialization suggests that the M^4 and CP_2 Kähler charges are identical apart from the sign.

2. The first thing to notice is that it is not possible to speak about the choice of quantization axis for $U(1)$ charge. It is however possible to generalize the momentum space picture also to the 7-D branes X^7 of M^8 with real normal space and select only discrete points of cognitive representation carrying quarks. The coordinate of 7-D generalized momentum in the 1-D fiber would correspond to some charge interpreted as a $U(1)$ momentum in the fiber of 7-D generalization of the twistor space.

3. One can start from the level of the 7-D surface with a real normal space. For both M^4 and CP_2, a plausible guess for the identification of 3-D fiber space is as 3-sphere S^3 having Hopf fibration $S^3 \to S^2$ with $U(1)$ as a fiber.

   At H side one would have a wave $exp(iQ\phi/2\pi)$ in $U(1)$ with charge Q and at M^8 side a point of X^7 representing Q as 7:th component of 7-D momentum.

   Note that for X^6 as a counterpart of twistor space the 5:th and 6:th components of the generalized momentum would represent spin quantization axis and sign of quark spin as a point of S^2. Even the length of angular momentum might allow this kind representation.

4. Since both M^4 and CP_2 allow induced Kähler field, a possible identification of Q would be as a Kähler magnetic charge. These charges are not conserved but in ZEO the non-conservation allows a description in terms of different values of the magnetic charge at opposite halfs of the light-cone of M^8 or CD.

   Instanton number representing a change of magnetic charge would not be a charge in strict sense and drops from consideration.

One expects that the action in the 7-D situation is analogous to Chern-Simons action associated with 8-D Kahler action, perhaps identifiable as a complexified 4-D Kähler action.

1. At M^4 side, the 7-D bundle would be $SU(4)/SU(3) \to SU(4)/SU(3) \times U(1)$. At CP_2 side the bundle would be $SU(3)/U(1) \to SU(3)/U(1) \times U(1)$.

2. For the induced bundle as 7-D surface in the $SU(4)/SU(3) \times SU(3)/U(1)$, the two $U(1)$:s are identified. This would correspond to an identification $\phi(M^4) = \phi(CP_2)$ but also a more general correspondence $\phi(M^4) = (n/m)\phi(CP_2)$ can be considered. m/n can be seen as a fractional $U(1)$ winding number or as a pair of winding numbers characterizing a closed curve on torus.

3. At M^8 level, one would have Kähler magnetic charges $Q_K(M^4)$, $Q_K(CP_2)$ represented associated with $U(1)$ waves at twistor space level and as points of X^7 at M^8 level involving quark. The same wave would represent both M^4 and CP_2 waves that would correlate the values of Kähler magnetic charges by $Q_{K,m}(M^4)/Q_{K,m}(CP_2) = m/n$ if both are non-vanishing. The value of the ratio m/n affects the dynamics of the 4-surfaces in M^8 and via twistor lift the space-time surfaces in H.

4.3.6 How could the Grassmannians of standard twistor approach emerge number theoretically?

One can identify the TGD counterparts for various Grassmann manifolds appearing in the standard twistor approach.

Consider first, the various Grassmannians involved with the standard twistor approach (`https://cutt.ly/XE3vDKj`) can be regarded as flag-manifolds of 4-complex dimensional space T.

1. Projective space is FP_{n-1} the Grasmannian $F_1(F^n)$ formed by the k-D planes of V^n where F corresponds to the field of real, complex or quaternionic numbers, are the simplest spaces of this kind. The F-dimension is $d_F = n-1$. In the complex case, this space can be identified as $U(n)/U(n-1) \times U(1) = CP_{n-1}$.

2. More general flag manifolds carry at each point a flag, which carries a flag which carries ... so that one has a hierarchy of flag dimensions $d_0 = 0 < d_1 < d_2 ... d_k = n$. Defining integers $n_i = d_i - d_{i-1}$, this space can in the complex case be expressed as $U(n)/U(n_1) \times .....U(n_k)$. The real dimension of this space is $d_R = n^2 - \sum_i n_i^2$.

3. For $n = 4$ and $F = C$, one has the following important Grassmannians.

   (a) The twistor space CP_3 is projective is of complex planes in $T = C^4$ and given by $CP_3 = U(4)/U(3) \times U(1)$ and has real dimension $d_R = 6$.

   (b) $M = F_2$ as the space of complex 2-flags corresponds to $U(4)/U(2) \times U(2)$ and has $d_R = 16 - 8 = 8$. This space is identified as a complexified Minkowski space with $D_C = 4$.

   (c) The space $F_{1,2}$ consisting of 2-D complex flags carrying 1-D complex flags has representation $U(4)/U(2) \times U(1) \times U(1)$ and has dimension $D_R = 10$.

   $F_{1,2}$ has natural projection ν to the twistor space CP_3 resulting from the symmetry breaking $U(3) \to U(2) \times U(1)$ when one assigns to 2-flag a 1-flag defining a preferred direction. $F_{1,2}$ also has a natural projection μ to the complexified and compactified Minkowski space $M = F_2$ resulting in the similar manner and is assignable to the symmetry breaking $U(2) \times U(2) \to U(1) \times U(1)$ caused by the selection of 1-flag.

   These projections give rise to two correspondences known as Penrose transform. The correspondence $\mu \circ \nu_{-1}$ assigns to a point of twistor space CP_3 a point of complexified Minkowski space. The correspondence $\nu \circ \mu_{-1}$ assigns to the point of complexified Minkowski space a point of twistor space CP_3. These maps are obviously not unique without further conditions.

This picture generalizes to TGD and actually generalizes so that also the real Minkowski space is obtained naturally. Also the complexified Minkowski space has a natural interpretation in terms of extensions of rationals forcing complex algebraic integers as momenta. Galois confinement would guarantee that physical states as bound states have real momenta.

1. The basic space is $Q_c = Q^2$ identifiable as a complexified Minkowski space. The idea is that number theoretically preferred flags correspond to fields R, C, Q with real dimensions 1,2,4. One can interpret Q_c as Q^2 and Q as C^2 corresponding to the decomposition of quaternion to 2 complex numbers. C in turn decomposes to $R \times R$.

2. The interpretation $C^2 = C^4$ gives the above described standard spaces. Note that the complexified and compactified Minkowski space is not same as $Q_c = Q^2$ and it seems that in TGD framework Q_c is more natural and the quark momenta in M_c^4 indeed are complex numbers as algebraic integers of the extension.

Number theoretic hierarchy $R \to C \to Q$ brings in some new elements.

1. It is natural to define also the quaternionic projective space $Q_c/Q = Q^2/Q$ `https://cutt.ly/LE3vMOG`, which corresponds to real Minkowski space. By non-commutativity this space has two variants corresponding to left and right division by quaternionic scales factor. A natural condition is that the physical states are invariant under automorphisms $q \to hqh^{-1}$ and depend only on the class of the group element. For the rotation group this space is characterized by the direction of the rotation axis and by the rotation angle around it and is therefore 2-D.

   This space is projective space QP_1, quaternionic analog of Riemann sphere CP_1 and also the quaternionic analog of twistor space CP_3 as projective space. Therefore the analog of real Minkowski space emerges naturally in this framework. More generally, quaternionic projective spaces Q^n have dimension $d = 4n$ and are representable as coset spaces of symplectic groups defining the analogs of unitary/orthogonal groups for quaternions as $Sp(n+1)/Sp(n) \times Sp(1)$ as one can guess on basis of complex and real cases. M_R^4 would therefore correspond to $Sp(2)/Sp(1) \times SP(1)$.

   QP_1 is homeomorphic to 4-sphere S^4 appearing in the construction of instanton solutions in E^4 effectively compactified to S^4 by the boundary conditions at infinity. For Minkowski signature it would be replaced by 4-D hyperboloid $H^4 = SO(1,4)/SO(3)$ known also as anti-de Sitter space AdS(4,1) (`https://cutt.ly/RRuXIBS`). An interesting question is whether the self-dual Kähler forms in E^4 could give rise to M^4 Kähler structure and could correspond to this kind of self-dual instantons and therefore what I have called H-J structures.

2. The complex flags can also contain real flags. For the counterparts of twistor spaces this means the replacement of $U(1)$ with a trivial group in the decompositions.

   The twistor space CP_3 would be replaced $U(4)/U(3)$ and has real dimension $d_R = 7$. It has a natural projection to CP_3. The space $F_{1,2}$ is replaced with representation $U(4)/U(2)$ and has dimension $D_R = 12$.

To sum up, the Grassmannians associated with M^4 as 6-D twistor space and its 7-D extension correspond to a complexification by a commutative imaginary unit i - that is "vertical direction". The Grassmannians associated with CP_2 correspond to "horizontal ", octonionic directions and to associative, commutative and well-ordered normal spaces of the space-time surface and its 6-D and 7-D extensions. Geometrization of the basic quantum states/numbers - not only momentum - representing them as points of these spaces is in question.

4.3.7 How could the quark content of the physical state determine the geometry of the space-time surface?

In the standard quantum field theory, fermionic currents serve as sources of the gauge fields. This correlation must have a counterpart in the TGD framework. Somehow the selection of the active points of the cognitive representation containing quarks must determine the 4-surface of M^8 determined by a polynomial P with rational coefficients. $M8-H$ duality would in turn determine the space-time surface.

This requirement gives a motivation for the earlier assumption that the roots of P defining 6-D surfaces fix P. Two kinds of surfaces appear.

1. The special $E = E_n$ roots of P having interpretation as energy have 3-D hyperplanes as M^4 intersections that I have misleadingly called "special moments in the life of self".

   The proposal [L23, L24] was that quarks are associated with the 2-D intersections of 4-D space-time surfaces with these planes. At the level of H, these 2-D intersections were assigned to partonic 2-surfaces serving as vertices of topological Feynman diagrams represented as space-time surfaces. Knowledge of the values of energy E_n defining 3-D complex planes at which the quarks of the quantum state are located in momentum space fixes the minimal polynomial P and therefore also space-time surface.

2. Besides energy hyper-planes there are also complex mass hyperboloids. The general 4-D solution of co-associativity conditions is 4-D (in real sense) intersection of two complex mass shells with mass squared $m^2_{c,odd}$ *resp.* $m^2_{c,even}$ with complex mass squared equal to a root of the odd *resp.* even part of the polynomial P defining the 4-surface [L23]. The real projection of the 4-D intersection is 2-D and might have interpretation as counterpart of a partonic 2-surface.

   This complex surface has complex dimension 4 and 4-D real projection in the sense that the number theoretic quadratic form is real. The 6-surface defined by the root reduces to a 3-D real mass shell if the imaginary part of m_c^2 can vanishes: this is possible for real roots only. The 4-D intersection of these complex mass shells provide natural seats for the quark momenta as algebraic integers, which in general are complex. This data can fix the roots of the imaginary part of P as complex mass squared values.

3. Interestingly, also 6-D surfaces having these 4-surfaces as sub-manifolds emerge. A good guess is that these are just the surfaces with commutative normal space and serve as M^8 counterparts of twistor space.

4.3.8 How to understand leptons as bound states of 3 quarks?

A benchmark test for the view about the twistorial aspects of M^8 is the challenge of describing leptons as bound states of 3 quarks assignable to single wormhole contact, single throat, or even single point. The assumption that wormhole contacts correspond to blow-ups of line singularities in M^8 containing quarks favors the strongest option.

1. At the level of H , quarks with different colors (color partial waves in CP_2) could have exactly the same M^4 location inside a single wormhole throat but different CP_2 locations to realize statics. Color can be realized as H partial waves and this would require that the oscillator operators act at the level of M^8 allowing to put several oscillators at a single M^4 point at the level of H.

2. At the level of M^8 the Fermi statistics would state that only a single quark corresponds to a given point. If one works at the level of 4-surface so that only momentum is taken into account, this is not possible. Could the 3 quarks be at different points in the 7-D extension of the twistor space bringing in quark spin and Kähler magnetic charge?

   The total spin of lepton is 1/2 so that two spins are opposite. Kähler magnetic charges of quarks are proposed to be proportional to color hypercharge (2,-1,-1) for quarks to realize Fermi statistics topologically. The points (p,1/2,-1),(p,1/2,-1) and (p,-1/2,2) and the states obtained by permuting Kähler charges would allow arealization of lepton as a 3 quark state with identical momenta.

4.4 Hierarchies of extensions for rationals and of inclusions of hyperfinite factors

TGD suggests 3 different views of finite measurement resolution.

1. At the space-time level, finite measurement resolution is realized in terms of cognitive representations at the level of M^8 actualized in terms of fermionic momenta with momentum components identifiable as algebraic integers. Galois group has natural action on the momentum components.

2. The inclusion $N \subset M$ of group algebras of Galois groups is proposed to realize finite measurement resolution for which the number theoretic counterpart is Galois singlet property of N with respect to the Galois group of M relative to N identifiable as the coset group of Galois groups of M and N. If the origin serves as a root of all polynomials considered, the composite $P \circ Q$ inherits the roots of Q.

   The idea generalizes to infinite-D Galois groups [L36, L33]. The HFF in question would be infinite-D group algebra of infinite Galois group for a polynomial R obtained as a composite $R = P_{infty} \circ Q$ of an infinite iterate P_{infty} of polynomial P and of some polynomial Q of finite degree (inverse limit construction). The roots of R at the limit correspond to the attractor basin associated with P_∞, which is bounded by the Julia set so that a connection with fractals emerges.

3. The inclusions $N \subset M$ of hyperfinite factors of type II_1 (HFFs) [K21, K8] is a natural candidate for the representation of finite measurement resolution. N would represent the degrees of freedom below measurement resolution mathematically very similar to gauge degrees of freedom except that gauge algebra would be replaced with the super-symplectic algebra and analogs of Kac Moody algebra with non-negative conformal weights and gauge conditions would apply to sub-algebra with conformal weights larger than the weight h_{max} defining the measurement resolution.

For HFFs, the index $[M : N]$ of the inclusion defines the quantum dimension $d(N \subset M) \leq 1$ as a quantum trace of the projector $P(M \to N)$ (the identify operator of M has quantum trace equal to one). $d(N \subset M)$ is defined in terms of quantum phase q and serves as a dimension for the analog of factor space M/N representing the system with N regarded as degrees of freedom below the measurement resolution and integrated out in "quantum algebra" M/N. Quantum groups and quantum spaces are closely related notions [K21, K8].

Galois confinement would suggest that $N \subset M$ corresponds to the algebra creating Galois singlets with respect to the Galois group of N relative to M whereas M includes also operators which are not this kind of singlets. In the above example $R = P \circ Q$, the Galois group of P would be represented trivially and the Galois group of Q or its subgroup would act non-trivially. In the case of hadrons, color degrees of freedom perhaps assignable to the Galois group Z^3 in the case of quarks would correspond to the degrees of freedom below the measurement resolution.

The universality of the quantum dimension and its expressibility in terms of quantum phase suggests that the integer m in $q = rxp(i2\pi/m)$ is closely related to the dimension for the extension of rationals $n = h_{eff}/h_0$ and depends therefore only very weakly on the details of the extension. The simplest guess is $m = n$. This conforms with the concrete interpretation of charge fractionation as being due to the many-valuedness of the graphs of space-time surfaces as maps from $M^4 \to CP_2$ or vice versa.

4.5 Galois confinement

The notion of Galois confinement emerged in TGD inspired biology [L48, L28, L33, L38]. Galois group for the extension of rationals determined by the polynomial defining the space-time surface $X^4 \subset M^8$ acts as a number theoretical symmetry group and therefore also as a physical symmetry group.

1. The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L28, L38, L37]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units.

2. The idea generalizes also to nuclear physics and suggests an interpretation for the findings claimed by Eric Reiter [L44] in terms of dark N-gamma rays analogous to BECs and forming Galois singlets. They would be emitted by N-nuclei - also Galois singlets - quantum coherently [L44]. Note that the findings of Reiter are not taken seriously because he makes certain unrealistic claims concerning quantum theory.

4.5.1 Galois confinement as a number theoretically universal manner to form bound states?

It seems that Galois confinement might define a notion much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Why not also hadrons could be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L18, L26, L7].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states. The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action inside the space-time surface is not well-defined. At the level of M^8 octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H. In contrast to this, weak gauge transformations acting as holonomies act in the tangent space of H.

   One can argue that the symmetries of H and even of WCW should/could have a reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (momenta) of M^8_c with coordinates, which are algebraic integers for the extension.

   Momenta as points of M^8_c would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of (in general complex) momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are algebraic, possibly complex, integers.

   Also other quantum numbers, in particular color, would correspond to representations of the Galois group. In the case of angular moment Galois confinement would allow algebraic half-integer valued angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential factors which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

4.5.2 Explicit conditions for Galois confinement

It is interesting to look more explicitly at the conditions for the Galois confinement.

Single quark states have momenta, which are algebraic integers generated by so called integral basis (`https://cutt.ly/SRuZySX`) spanning algebraic integers as a lattice and analogous to unit vectors of momentum lattice but for single component of momentum as a vector in extension. There is also a theorem stating that one can form the basis of extension as powers of a single root. It is also known that irreducible monic polynomials have algebraic integers as roots.

1. In its minimal form Galois confinement states that only momenta, which are rational integers are allowed by Galois confinement. Note that for irreducible polynomials with rational coefficients one does not obtain any rational roots. Monic polynomials with integer coefficients can allow integer roots. If one assumes that single particle states can have arbitrary algebraic integer as momentum, one obtain also rational integers for momentum values. These states are not at mass - or energy shell associated with the single particle momenta.

2. A stronger condition would be that also the inner products of the momenta involved are real so that one has $Re(p_i) \cdot Im(p_j) = 0$. For $i = j$ this gives a condition is possible only for the real roots for the real polynomials defining the space-time surface.

To see that real roots are necessary, some facts about the realization of the co-associativity condition [L23] are necessary.

1. The expectation is that that the vanishing condition for the real part (in quaternionic sense) of the octonionic polynomial gives a co-associative surface. By the Lorentz symmetry one actually obtains as a solution a 6-D complex mass shell $m_c^2 \equiv m_{Re}^2 - m_{Im}^2 + 2iRe(p) \cdot Im(p) = r_1$, where the real and imaginary masses are defined are $m_{Re}^2 = Re(p)^2$ and $m_{Im}^2 = Im(p)^2$ and r_1 is some root for the odd part of the polynomial P assumed to determining the 4-surface.

2. This surface can be co-associative but would be also co-commutive. Maximally co-associative surface requires quaternionic normal space. The first proposal is that the space-time surface is the intersection of the surface defined by the polynomial and its conjugate with respect to i. This gives 4-D surface as the intersection of the two 6-D surfaces.

   Second proposal is that the 6-surface having a structure of S^2 bundle defines as its base space quaternionic 4-surface. This space would correspond to a gauge choices selecting point of S^2 at very point of M^4. To a given polynomial one could assign entire family of 4-surfaces mapped to different space-time surfaces in H. A possible interpretation of gauge group would be as quaternionic automorphisms acting on the 2-sphere.

   These proposals are equivalent if the base base is the intersection of the 6-D bundle spaces. One could say that the fibers are conjugates of each other. This might be relevant for ZEO.

Concerning Galois confinement, the basic result is that for complex roots r_1 the conditions $Re(p_i) \cdot Im(p_i) = 0$ cannot be satisfied unless one requires that r_1 is real. Therefore the stronger option makes sense for real roots only.

1. Galois confinement allows the momenta p_i forming the bound state to be in an extension of rationals defined by the polynomial defining the space-time surface. Galois confinement condition states that the total momentum is rational integer when a suitable unit defined by the size of CD is used (periodic boundary conditions).

2. Another natural condition is the vanishing of the inner products between the real part $Re(p)$ and imaginary part $Im(p)$ of p. This guarantees that the number theoretical norm squared for the momentum is real. For time-like p, this means that $Im(p)$ belongs to the 3-D orthogonal complement E^3 of $Re(p)$. For light-like p, $Im(p)$ belongs to 2-D orthogonal complement E^2.

3. Suppose one has several number theoretic momenta p_i such that $\sum p_i = p$ is rational integer and $p_i \propto p$ holds true. Also in this case, the number theoretic inner products must be real. The orthogonality conditions read as

$$Re(p_i) \cdot Im(p_j) = 0 \quad . \tag{4.2}$$

   For a given pair (i, j), one has several conditions corresponding to algebraically independent imaginary momentum components and it is quite possible that very few solutions exist besides $Im(p_i) = 0$. If $Re(p_i)$ is not a rational integer, the number of conditions still increases.

4. The proposal for Galois confinement is that the real parts of p_i are parallel or even identical: $Re(p_i) \propto Re(\sum p_i) = p$, which is a rational integer. In this case the conditions reduce to $Re(p) \cdot Im(p_i) = 0$ and their number is much smaller.

5. For a given momentum component, the basis $p_{i,k}$ has the dimension n of extension. The basis contains m complex elements e_k and their conjugates $\overline{e}_k$ plus $n-2m-1$ real but algebraically trivial elements r_k besides the real unit 1. The sums $E_k = e_k + \overline{e}_k$ are algebraic integers and give m real basis elements. Note that $F_k = e_k - \overline{e}_k$ are purely imaginary algebraic integers.

   r_k and E_i give $n-m-1$ algebraically non-trivial real momenta. The momentum components $p_{i,k}$ formed as linear combinations of r_k, E_i, and 1 are real. This gives $n-m$-dimensional real subspace and momenta formed in this way satisfy the reality conditions for the inner products.

6. One can also construct complex momenta such that $Im(p_i)$ is a linear combination $Im(p_i) = \sum n_{i,k} F_k$. If $Re(p_i)$ are parallel and rational integers and $p_i \propto p$ holds true, the reality conditions reduce to

$$p \cdot Im(p_i) = \sum_k p^i n_{i,k} F_k = 0 \ . \tag{4.3}$$

   One can construct a maximal set of complex momenta P_K characterized by matrices n_{ik}^K satisfying these conditions. Also linear combinations of P_K satisfy the reality conditions and one obtains a lattice of momenta.

This looks like nice construction but it seems that mere Galois confinement is more realistic.

4.6 $M^8 - H$ duality at the level of WCW

WCW emerges in the geometric view of quantum TGD. $M^8 - H$ duality should lso work for WCW. What is the number theoretic counterpart of WCW? What is the geometric counterpart of the discretization characteristic to the number theoretic approach?

In the number theoretic vision in which WCW is discretized by replacing space-time surfaces with their number theoretical discretizations determined by the points of $X^4 \subset M^8$ having the octonionic coordinates of M^8 in an extension of rationals and therefore making sense in all p-adic number fields? How could an effective discretization of the real WCW at the geometric H level, making computations easy in contrast to all expectations, take place?

1. The key observation is that any functional or path integral with integrand defined as exponent of action, can be *formally* calculated as an analog of Gaussian integral over the extrema of the action exponential $exp(S)$. The configuration space of fields would be effectively discretized. Unfortunately, this holds true only for the so called integrable quantum field theories and there are very few of them and they have huge symmetries. But could this happen for WCW integration thanks to the maximal symmetries of the WCW metric?

2. For the Kähler function K, its maxima (or maybe extrema) would define a natural effective discretization of the sector of WCW corresponding to a given polynomial P defining an extension of rationals.

   The discretization of the sector defined by P should be equivalent with the number theoretical discretization induced by the number theoretical discretization of space-time surfaces. Various p-adic physics and corresponding discretizations should emerge naturally from the real physics in WCW.

3. The physical interpretation is clear. The TGD Universe is analogous to the spin glass phase [?] The discretized WCW corresponds to the energy landscape of spin glass having an ultrametric topology. Ultrametric topology of WCW means that discretized WCW decomposes to p-adic sectors labelled by polynomials P. The ramified primes of P label various p-adic topologies associated with P.

4.7 Some questions and ideas related to $M^8 - H$ duality

In the following some questions and ideas, which do not quite fit under the titles of the previous sections, are considered.

4.7.1 A connection with Langlands program

Langlands correspondence [A7, K10, A5, A4], which I have tried to understand several times [K10] [L1, L3, L6] relates in an interesting manner to $M^8 - H$ duality and Galois confinement.

1. Global Langlands correspondence (GLC) states that there is connection between representations of continuous groups and Galois groups of extensions of rationals.

2. Local LC states (LLC) states this in the case of p-adics.

There is a nice interpretation for the two LCs in terms of sensory experience and cognition in TGD inspired theory of consciousness.

1. In adelic physics real numbers and p-adic number fields define the adele. Sensory experience corresponds to reals and cognition to p-adics. Cognitive representations are in their discrete intersection and for extensions of rationals belonging to the intersection.

   (a) Sensory world, "real" world corresponds to representation of continuous groups/Galois groups of rationals. GLC.

   (b) "p-Adic" worlds correspond to cognition and representations of p-adic variants of continuous groups and Galois groups over p-adics. Local LLC.

   (c) One could perhaps talk also about Adelic LC: ALC in the TGD framework. Adelic representations would combine real and p-adic representations for all primes and give as complete information about reality as possible.

   TGD provides a geometrization for the identification of Galois groups as discrete subgroups of Lie groups, not only of the isometry (automorphism) groups of H (M^8) but perhaps also as discrete sub-groups of more general Lie groups to which the action of super-symplectic representations could reduce. A naive guess is that these groups correspond to the ADE groups appearing in the McKay correspondence [L8, L21, L22].

   The representation of real continuous groups assignable to the real numbers as a piece of adele [L9, L10] would be related to the representations of Galois groups GLC. Also p-adic representations of groups are needed to describe cognition and these p-adic group representations and representations of p-adic Galois groups would be related by LLC.

4.7.2 Could the notion of emergence of space-time have some analog in the TGD Universe?

The idea about the emergence of space-time from entanglement is as such not relevant for TGD. One can however ask whath the emergence of *observed* space-time could mean in TGD. Space-time surface as a continuum exists in TGD but they are not directly observable due to a finite measurement resolution. One can ask what a body with an outer boundary means physically. The space-time regions defined by solid bodies have boundaries. What makes the boundaries of the bodies "hard"?

1. In momentum space Fermi statistics does not allow fermions to get through the boundary of Fermi ball. This is a good guideline.

2. Second feature of a spatial object such as an atom is that it is a bound state quantum mechanically. If it has parts they stay together. In QFT theory the notion of a bound state is however poorly understood.

3. Quantum coherence is a further property considered in the article. Spatial objects correspond to quantum coherent structures. Quantum coherence reduces to entanglement. Quantum coherence length and time determine the size of a quantum object. Somehow one must have stable entanglement in long scales.

Let us see what these guidelines could give in the framework of $M^8 - H$ duality which generalizes the wave particle duality of wave mechanics.

1. In adelic physics space-times can be seen as either surfaces in M^8 or $H = M^4 \times CP_2$. $X^4 \subset M^8$ is analogous to momentum space cognitive representations consist of points of $X^4 \subset M^8$, whose points are algebraic integers in the extension of rationals defined by the polynomial defining the space-time surface and are algebraic integers as roots of monic polynomials of form $x^n + ...$. This defines a unique discretization of the space-time surface. The discretization guarantees number theoretical universality: the cognitive representation makes sense also p-adically and space-time has also p-adic variants.

   Cognitive representations give rise to "cognitive emergence" of the space-time in cognitive sense and since cognitive representations are intersection of reality and p-adicities they must closely related to the "sensory emergence"".

2. $X^4 \subset M^8$ is mapped to H by $M^8 - H$ duality determined by the condition that it momentum is mapped to a geodesic with a direction of momentum and starting from either tip of CD: the image point is its intersection with the opposite light-like boundary of CD and selects a point of space-time surface. The size of CD is $T = h_{eff}/m$ for quark with mass m to satisfy Uncertainty Principle. The map generalizes to bound states of quarks (whatever they are).

Consider the problem of "sensory emergence" in this framework.

1. What makes a point of a cognitive representation "hard"? Quarks are associated with points (not necessarily all) of a cognitive representation: one can say that the point is activated when there is a quark at it. Fermi ball corresponds to a discrete set of activated points at the level of momentum space. These points define activated points also in $X^4 \subset H$ by $M^8 - H$ duality. One could perhaps say that these activated points in M^8 and their H-image containing fermions define the spatial objects as something "hard" and having a boundary. Another fermion knows that there is a space-time point there because it cannot get to this point. The presence of a fermion (quark) would make a space-time point "hard".

2. What about the role of entanglement? The size and duration of the space-time surface (inside a causal diamond CD) defines quantum coherence length and time. Fermionic statistics makes fundamental fermions - to be distinguished from elementary fermions - maximally entangled. One cannot reduce fermionic entanglement in SFR and quantum measurements would be impossible. The entanglement in the WCW degrees of freedom comes to the rescue. This entanglement can be reduced in SFRs since the particles as surfaces are identical under very special - naturally number theoretical - conditions.

   Negentropy Maximization Principle and hierarchy of $h_{eff} = n \times h_0$ phases favor the generation of stable entanglement in the TGD Universe. Also, if the coefficients of the entanglement matrix belong to extension of rationals, entanglement probabilities in general belong to its extension and the density matrix is not diagonalizable without going to a larger extension. This might require "big" SFR increasing the extension: only after this state function reduction to an eigenstate could occur. This leads to a concrete proposal for how the information about the diagonal form of the density matrix expressed by its characteristic polynomial is coded into the geometry of the space-time surface [L33].

3. Bound state formation is third essential element. Momenta are points of the space-time surface $X^4 \subset M^8$ with components which are algebraic integers. Physical momenta are however ordinary integers for a particle in a finite volume defined by causal diamond (CD). This means that one can allow only composites of quarks with rational integer valued momenta which correspond to Galois singlets.

   Galois confinement would be the universal mechanism behind formation of all bound states and also give rise to stable entanglement. One would obtain a hierarchy of bound states corresponding to a hierarchy of polynomials and corresponding Galois groups and extensions of rationals. By $M^8 - H$ duality, bound states of quarks and higher structures formed from them in M^8 would give rise to spatial objects.

5 Zero energy ontology (ZEO)

ZEO [K23] forms the cornerstone of the TGD inspired quantum theory extending to a theory of consciousness. ZEO has so far reaching consequences that it would have deserved a separate section. Since it involves in an essential manner the notion of CD, it is natural to include it to the section discussing $M^8 - H$ duality.

5.1 The basic view about ZEO and causal diamonds

The following list those ideas and concepts behind ZEO that seem to be rather stable.

1. GCI for the geometry of WCW implies holography, Bohr orbitology and ZEO [L19] [K23].

2. X^3 is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. Finite failure of determinism is however possible [L45]. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD) defined as intersection of future and past directed light cones ($\times CP_2$) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. 12 Fig. 13**).

4. The opposite light-like boundaries of CD correspond for fermions dual vacuums (bra and ket) annihilated by fermion annihilation *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD. This seems to be the correct view in H. At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing would be by mass shells (hyperboloids) at the level of M^8 and by CDs with same center point at the level of H.

5. Zeno effect can be understood if the states at either cone of CD do not change in "small" state function reductions (SSFRs). SSFRs are analogs of weak measurements. One could call this half-cone call as a passive half-cone. I have earlier used a somewhat misleading term passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate - geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change. The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like deterministic time evolution leading to the final state [L14] as observed by Minev *et al* [L14].

7. h_{eff} hierarchy [K4, K5, K6, K7] implied by the number theoretic vision [L23, L24] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

8. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. Provides also a mechanism of self-organization [L17]: dissipation with reversed arrow of time looks like self-organization [L48].

5.2 Open questions related to ZEO

There are many unclear details related to the time evolution in the sequence of SSRs. Before discussing these unclear details let us make the following assumptions.

1. The size of CDs increases at least in a statistical sense in the sequence of CD and the second boundary remains stationary apart from scaling (note that one can also consider the possibility that the entire CD is scaled and temporal shift occurs in both directions).
2. Mental mentals (say after images) are in kind of Karma's cycle: they are born and die roughly periodically.
3. I do not experience directly mental images with the opposite arrow of time.
4. I can have memories only about states of consciousness with the same arrow of time that I have. This explains why I do not have memories about periods of sleep if sleep is interpreted as a time reversed state of some subself of me responsible for self-ness.

One can use three empirical inputs in an attempt to fix the model.

1. After images appear and disappear roughly periodically. Also I fall asleep and wake up with a standard arrow of time roughly periodically.
   (a) The first interpretation is that as a sequence of wake up-sleep periods I am a time crystal-like structure consisting of nearly copies of the mental image, such that each mental image - including me as mental images of higher level self - continues Karma's cycle in my geometric past. How "me" is transferred to a new almost copy of my biological body? Does my MB just redirect its attention?
   (b) The second interpretation is that me and my mental images somehow drift towards my geometric future, while performing the Karma's cycle so that my mental images follow me in my time travel. This would require that the sub-CDs of mental images drift towards the geometric future.

       Also sleep could be a "small" death at some layer of the personal hierarchy of MBs. I do not however wake-up in BSFR at the moment of geometric time defined by the moment of falling asleep but later. So it seems that my CD must drift to the geometric future with the same speed that those of other living beings in the biosphere.
2. There is however an objection. In cosmology the observation of stars older than the Universe would have a nice solution if the stars evolve forth and back in time in our distant geometric past rather than drifting towards the future so that they could age by continuing their Karma's cycle with a constant center of mass value of time. Can these three observations be consistent?

5.2.1 Could the scaling dynamics CD induce the temporal shifting of sub-CDs as 4-D perceptive fields?

Suppose that the sub-CDs within a bigger CD "follow the flow". How the dynamics of the bigger CD could induce this flow?

1. The scalings of bigger CD in unitary evolutions between SSFRs induce the scaling of sub-CS. This would not be shifting but scaling and the distance between given CD and larger CDs would gradually scale up.

   This would remove the objection. The astrophysical objects in distant geometric past would move towards the geometric future but with much smaller velocity as the objects with cosmic scale so that the temporal distance to future observers would increase. These objects would be aging in their personal Karma's cycle, and the paradox would disappear.

2. The flow would be defined by the scalings of a larger CD containing our CDs and those of others at my level. Each CD would define a shared time for its sub-CDs. If the CDs form a hierarchy structure with a common center, this is indeed true of the time evolutions as scalings of CDs. There would be scalings induced by scalings at higher levels and "personal" scalings.

3. It however seems that the common center is too strong an assumption and shifted positions for the sub-CDs and associated hierarchy inside a given CD are indeed possible for the proposed realization of $M^8 - H$ duality and actually required by Uncertainty Principle.

A further open question is what happens to the size of CD in the BSFR. Does it remain the same so that the size of the CD would increase indefinitely? Or is the size reduced in the sense that there would be scaling, reducing the size of the CD in which the passive boundary of the CD would be shifted towards the active one. After every BSFR, the self would experience a "childhood".

5.2.2 Are we sure about what really occurs in BSFR?

It has been assumed hitherto that a time reversal occurs in BSFR. The assumption that SSFRs correspond to a sequence of time evolutions identified as scalings, forces to challenge this assumption. Could BSFR involve a time reflection T natural for time translations or inversion $I : T \rightarrow 1/T$ natural for the scalings or their combination TI?

I would change the scalings increasing the size of CD to scalings reducing it. Could any of these options: time reversal T, inversion I, or their combination TI take place in BSFRs whereas arrow would remain as such in SSFRs? T (TI) would mean that the active boundary of CD is frozen and CD starts to increase/decrease in size.

There is considerable evidence for T in BSFRs identified as counterparts of ordinary SFRs but could it be accompanied by I?

1. Mere I in BSFR would mean that CD starts to decrease but the arrow of time is not changed and passive boundary remains passive boundary. What comes to mind is blackhole collapse.

   I have asked whether the decrease in size could take place in BSFR and make it possible for the self to get rid of negative subjective memories from the last moments of life, start from scratch and live a "childhood". Could this somewhat ad hoc looking reduction of size actually take place by a sequence of SSFRs? This brings into mind the big bang and big crunch. Could this period be followed by a BSFR involving inversion giving rise to increase of the size of CD as in the picture considered hitherto?

2. If BSFR involves TI, the CD would shift towards a fixed time direction like a worm, and one would have a fixed arrow of time from the point of view of the outsider although the arrow of time would change for sub-CD. This modified option does not seem to be in conflict with the recent picture, in particular with the findings made in the experiments of Minev *et al* [L14] [L14].

   This kind of shifting must be assumed in the TGD inspired theory of consciousness. For instance, after images as a sequence of time reversed lives of sub-self, do not remain in the geometric past but follow the self in travel through time and appear periodically (when their arrow of time is the same as of self). The same applies to sleep: it could be a period with a reversed arrow of time but the self would shift towards the geometric future during this period: this could be interpreted as a shift of attention towards the geometric future. Also this option makes it possible for the self to have a "childhood"".

3. However, the idea about a single arrow of time does not look attractive. Perhaps the following observation is of relevance. If the arrow of time for sub-CD correlates with that of sub-CD, the change of the arrow of time for CD, would induce its change for sub-CDs and now the sub-CDs would increase in the opposite direction of time rather than decrease.

To sum up, TI or T can be considered as competing options for what happens in BSFR. T should however be able to explain why sub-selves (sub- CDs) drift to the direction of the future. If the time evolutions between SSFRs correspond to scalings rather than time translations, and if the scalings occur also for sub-CDs this can be understood. The dynamics of spin glasses strongly suggests that SSFRs correspond to scalings [L42].

5.3 What happens in quantum measurement?

According to the proposed TGD view about particle identity, the systems for which mutual entanglement can be reduced in SFR must be non-identical in the category theoretical sense.

When SFR corresponds to quantum measurement, it involves the asymmetric observer-system $O-S$ relationship. One cannot exclude SFRs without this asymmetry. Some kind of hierarchy is suggestive.

The extensions of rationals realize this kind of $O-S$ hierarchy naturally. The notion of finite measurement resolution strongly suggests discretization, which favors number theoretical realization. The hierarchies of effective Planck constants and p-adic length scale hierarchies reflect this hierarchy. What about the topological situation: can one order topologies to a hierarchy by their complexity and could this correspond to $O-S$ relationship?

The intuitive picture about many-sheeted space-time is as a hierarchical structure consisting of sheets condensed at larger sheets by wormhole contacts, whose throats carry fermion number. Intuitively, the larger sheet serves as an observer. p-Adic primes assignable to the space-time sheet could arrange them hierarchically and one could have entanglement between wavefunctions for the Minkowskian regions of the space-time sheets and the surface with a larger value for p would be in the role of O

5.3.1 Number theoretic view about measurement interaction

Quantum measurement involves also a measurement interaction. There must be an interaction between two different levels O and S of the hierarchy.

One can look at the measurement interaction from a number theoretic point of view.

1. For cognitive measurements the step forming the composite $O \circ S$ of polynomials would represent the measurement interaction. Before measurement interaction systems would be represented by O and S and measurement interaction would form $O \circ S$ and after the measurement the situation would be as proposed.

   Could one think that in BSFR the pair of uncorrelated surface defined by $O \times S$ with degree $n_O + n_S$ (analog for the additivity of classical degrees of freedom) is replaced with $O \circ S$ with degree $n_O \times n_S$ (analog for multiplicativity of degrees of freedom in tensor product) in BSFR? This would mean that the formation of $O \circ S$ is like a formation of an intermediate state in particle reaction or in chemical reaction.

   Could the subsequent SSFR cascade define a cascade of cognitive measurements [L30]. I have proposed that this occurs in all particle reactions. For instance, nuclear reactions involving tunneling would involve formation of dark nuclei with $h_{eff} > h$ in BSFR and a sequence of SSFRs in opposite time direction performing cognitive quantum measurement cascade [L18] and also the TGD based model for "cold fusion" relies on this picture [L7, L26]. After the SSFR cascade, a second BSFR would occur and bring back the original arrow of time and lead to the final state of the nuclear reaction.

   From the point of view of cognition, BSFR would correspond to the heureka moment and the sequences of SSFRs to the cognitive analysis decomposing the space-time surface defined by $O \circ S$ to pieces.

2. One can also consider small perturbations of the polynomials $O \circ S$ as a measurement interaction. For instance, quantum superpositions of space-time surfaces determined by polynomials depending on rational valued parameters are possible. The Galois groups for two polynomials with parameters which are near to each other are the same but for some critical values of the parameters the polynomials separate into products. This would reduce the Galois group effectively to a product of Galois groups. Quantum measurement could be seen as a localization in the parameter space [L33].

5.3.2 Topological point of view about measurement interaction

The measurement interaction can be also considered from the topological point of view.

1. Wormhole contacts are Euclidean regions of $X^4 \subset H$ couples two parallel space-time regions with Minkowskian signature and could give rise to measurement interaction. Wormhole contact carries a monopole flux and there must be a second monopole contact to make flux loop possible. This structure has an interpretation as an elementary particle, for instance a boson. The measurement interaction could correspond to the formation of this structure and splitting by reconnection to flux loops associated with the space-time sheets after the interaction has ceased.

   Remark: Wormhole contacts for $X^4 \subset H$ correspond in $M^8 - H$ duality images of singularities of $X^4 \subset M^8$. The quaternionic normal space at a given point is not unique but has all possible directions, which correspond to all points of CP_2. This is like the monopole singularity of an electric or magnetic field. At the level of CP_2 wormhole contact is the "blow-up" of this singularity.

2. Flux tube pairs connecting two systems serve also as a good candidate for the measurement interaction. U-shaped monopole flux tubes are like tentacles and their reconnection creates a flux tube pair connecting two systems. SFR would correspond geometrically to the splitting of the flux tube pair by inverse re-connection.

5.3.3 Geometric view about SSFR

The considerations of [L32] strongly suggest the following picture about SSFRs.

In the measurement interaction a quantum superposition of functional composites of polynomials P_i defining the space-time surfaces of external states as Galois singlets is formed. A priori all orders for the composites in the superposition are allowed but if one requires that the same SSFR cascade can occur for all of them simultaneously, only single ordering and its cyclic permutations can be allowed.

The SSFR cascade can of course begin with a reduction selection single permutation and its cyclic permutations: localization in S_n/Z_n would take place.

Incoming states at passive boundary of CD correspond to prepared states and outgoing states at active boundary to state function reduced states. The external states could correspond to products of polynomials as number theoretical correlates for the absence of correlations in unentangled states.

Number theoretic existence for the scattering amplitudes [**?**] require that the p-adic primes characterizing the external states correspond to maximal ramified primes of the corresponding polynomials and therefore also to unique p-adic length scales L_p. In the interaction regions this ramified prime is the largest p-adic (that is ramified) prime for particles participating in the reaction. This correlation between polynomial and p-adic length scale allows a rather concrete geometric vision about what happens in the cascade.

SSFR cascade begins with a reduction of the state to a superposition of single composite with its cyclic variants for positive and negative energy parts separately: this kind of cyclic superpositions appear also in the twistor Grassmann picture [L32] and in string models. In the recent situation this makes possible a well-defined state preparation and SFR cascades at the two sides of CD. In ZEO, the cascade could take place for positive energy states only during SSFR.

A number theoretic SFR cascade would take place and decompose the Galois state group of the composite having decomposition to normal sub-groups to a product of states for the relative Galois groups for the composite.

A given step of the cascade would be a measurement of a density matrix ρ producing information coded by its reduction probabilities as its eigenvalues in turn coded by the characteristic polynomial P_M of the density matrix.

The simplest guess is that the final state polynomial is simply the product $\prod P_{i_-}$ of the polynomials P_{i_-} for the passive boundary of CD and product $\prod P_{i_+}$ for the active boundary.

5.3.4 Question of quantum information theorist

Quantum information theorists could however ask what happens to the information yielded by a given step of the measurement cascade.

1. Could the information about the measured ρ coded by P_M as its algebraic roots be stored to the final state coded by the final state polynomials $P_{i,+}$?

   Could the outcome at the active boundary of CD for which the SSFR cascade is actually not the 4-surface determined by the polynomials $P_{i,+}$ but $P_{M_{i_+}} \circ P_i$, or more generally a quantum superposition of $P_{M_{i_+}} \circ P_i$, and $P_i \circ P_{M_{i_+}}$.

   The "unitary time evolution" preceding the next SSFR would correspond to a functional composite of these polynomials so that the space-time surface would evolve during the SSFR sequence. The basic process would be a formation of functional composite followed by SSFR cascade storing the information about the measured density matrices to the space-time surface.

2. There are strong constraints on this proposal. $P_{M_{i_+}}$ should have rational coefficients in the extension of rationals defined by the composite polynomial, or even polynomial P_i. Monic polynomial property would pose even stronger conditions on entanglement coefficients and the representations of the entire Galois group.

   There is also the notion of Galois confinement for physical states. What constraints does this give?

   These conditions pose very strong conditions on the allowed entanglement matrix and could make the proposal unrealistic.

5.4 About TGD based description of entanglement

The general classification of possible quantum entanglements is an interesting challenge and there are many approaches (`https://cutt.ly/iREIg1u`). One interesting approach relies on the irreducible representations of the unitary group $U(n)$ acting as the isometry group of n-D Hilbert space (`https://cutt.ly/ZREIEAT`). The assumption about irreducibility is however not essential for what follows.

1. A system with n-D state space H_n identified as a sub-system of a larger system with N-D state space H_N can entangle with its $M = N - n$-D complement H_M. Suppose $n \leq M$. Entanglement implies that the n-D state space or its sub-space is embedded isometrically into a subspace of the M-D state space. For a non -trivial subspace one can replace H_n with this subspace H_m in what follows. The diagonal form of the density matrix describes this correspondence explicitly. If the subspace is 1-D one has an unentangled situation.

2. $U(n)$ and its subgroups act as automorphism groups of H_n This inspires the idea that the irreducible representations of $U(n)$ define physically very special entanglements $H_n \subset H_M$. The isometric inclusions $H_n \subset H_M$ are parametrized by a flag-manifold $F_{n,M} = U(M)/U(n) \times U(M-n)$. If one allows second quantization in the sense that the wave functions in the space of entanglements make sense, this flag manifold represents additional degrees of freedom for entanglements $H_n \subset H_M$. If the entanglement does not have maximal dimension, the product of flag manifolds $F_{n,M}$ and $F_{m,n}$ characterizes the space of entanglements.

3. Flag manifold has a geometric interpretation as the space of n-D spaces C^n (flags) embedded in C^M. Interestingly,twistor spaces and more general spaces of twistor Grassmannian approach are flag manifolds and twistor spaces are also related to Minkowski space.

4. I have not been personally enthusiastic about the notion of emergence of 3-space or space-time from entanglement but one can wonder whether flag manifolds related naturally to entanglement could lead to the emergence of Minkowski space. Or perhaps better, whether the notion of entanglement and Minkowski space could be natural aspects of a more general description.

5. One can also have flags inside flags inside leading to more complex flag manifolds $F(n_1, n_2, .., n_k = M) = U(M)/U(m_1) \times ... \times U(m_k)$, $m_k = n_k - n_{k-1}$ assuming $n_0 = 0$. In consciousness theories, the challenge is to understand the quantum correlates of attention. Entanglement is the most obvious candidate in this respect. Attention seems to be something with a directed arrow. This is difficult to understand in terms of the ordinary entanglement. Flag hierarchy would suggest a hierarchical structure of entanglement in which the system entangles with a higher-D system, which entangles with a higher-D system. In this picture the state function reduction would be replaced by a cascade starting from the top.

6. The analog of flags inside flags is what happens in what I call number theoretic measurement cascades for wavefunctions [L30] in the Galois groups which are associated with extension of extensions of..... The already mentioned cognitive measurement cascade corresponds to a hierarchy of normal subgroups of Galois group and one can perhaps say that discret Galois group replaces the unitary group. Each normal subgroup in the hierarchy is the Galois group of the extension of the extension below it. This automatically realizes the hierarchical entanglement as an attentional hierarchy. The cognitive measurement cascade can actually start at any level of the hierarchy of extensions of extensions and if it starts from the top all factors are reduced to a pure state.

   If the polynomials defining the 4-surfaces in M^8 satisfy $P(0) = 0$, the the composite polynomial $P_n \circ P_{n-1} ... \circ P_1$ has the roots of $P_1, ..., P_{n-1}$ as its roots. In this case the inclusion of state spaces are unique so that flag manifolds are not needed.

5.5 Negentropy Maximization Principle

Negentropy Maximization Principle (NMP) [L39] is the basic variational principle of TGD based quantum measurement theory giving rise to a theory of consciousness.

1. The adelic entanglement entropy is the sum of the real entanglement entropy and p-adic entropies. The adelic negentropy is its negative.

   The real part of adelic entropy is non-negative but p-adic negentropies can be positive. The sum of p-adic negentropies can be larger than the real entropy for non-trivial extensions of rationals. NMP is expected to take care that this is indeed the case. Second law for the real entropy would still hold true and guarantee NMP.

2. NMP states that SFRs cannot reduce the *overall* entanglement entropy although this can happen to subsystems. In SFRs this local reduction of negentropy would happen. Entanglement is not destroyed in SFRs in general and new entanglement negentropy can be generated.

3. Although real entanglement entropy tends to increase, the positive p-adic negentropies assignable to the cognition would do the same so that net negentropy would increase. This would not mean only entanglement protection, but entanglement generation and cognitive evolution. This picture is consistent with the paradoxical proposal of Jeremy England [I1] [L5] that biological evolution involves an increase of entropy.

4. It should be noticed that the increase of real entanglement entropy as such does not imply the second law. The reduction of real entropy transforms it to ensemble entropy since the outcome of the measurement is random. This entropy is entropy of fermions at space-time sheets. The fermionic entanglement would be reduced but transformed to Galois entanglement.

6 Appendix

6.1 Comparison of TGD with other theories

Table 1 compares GRT and TGD and **Table 2** compares standard model and TGD.

	GRT	**TGD**
Scope of geometrization	classical gravitation	all interactions and quantum theory
Spacetime		
Geometry	abstract 4-geometry	sub-manifold geometry
Topology	trivial in long length scales	many-sheeted space-time
Signature	Minkowskian everywhere	also Euclidian
Fields		
classical	primary dynamical variables	induced from the geometry of H
Quantum fields	primary dynamical variables	modes of WCW spinor fields
Particles	point-like	3-surfaces
Symmetries		
Poincare symmetry	lost	Exact
GCI	true	true - leads to SH and ZEO
	Problem in the identication of coordinates	$H = M^4 \times CP_2$ provides preferred coordinates
Super-symmetry	super-gravitation	super variant of H: super-surfaces
Dynamics		
Equivalence Principle	true	true
Newton's laws and		
notion of force	lost	generalized
Einstein's equations	from GCI and EP	remnant of Poincare invariance at QFT limit of TGD
Bosonic action	EYM action	Kähler action + volume term
Cosmological constant	suggested by dark energy	length scale dependent coefficient of volume term
Fermionic action	Dirac action	Modified Dirac action for induced spinors
Newton's constant	given	predicted
Quantization	fails	Quantum states as modes of WCW spinor field

Table 1: Differences and similarities between GRT and TGD

	SM	TGD
Symmetries		
Origin	from empiria	reduction to CP_2 geometry
Color symmetry	gauge symmetry	isometries of CP_2
Color	analogous to spin	analogous to angular momentum
Ew symmetry	gauge symmery	holonomies of CP_2
Symmetry breaking	Higgs mechanism	CP_2 geometry
Spectrum		
Elementary particles	fundamental	consist of fundamental fermions
Bosons	gauge bosons, Higgs	gauge bosons, Higgs, pseudo-scalar
Fundamental fermions	quarks and leptons	quarks: leptons as local 3-quark composites
Dynamics		
Degrees of freedom	gauge fields, Higgs, and fermions	3-D surface geometry and spinors
Classical fields	gauge fields, Higgs	induced spinor connection
	SU(3) Killing vectors of CP_2	
Quantal degrees of freedom	gauge bosons,Higgs,	quantized induced spinor fields
Massivation	Higgs mechanism	p-adic thermodynamics with superconformal symmetry

Table 2: Differences and similarities between standard model and TGD

6.2 Glossary and figures

The following glossary explains some basic concepts of TGD and TGD inspired biology.

- ***Space-time as surface***. Space-times can be regarded as 4-D surfaces in an 8-D space $M^4 \times CP_2$ obtained from empty Minkowski space (M^4) by adding four small dimensions (CP_2). The study of field equations characterizing space-time surfaces as "orbits" of 3-surfaces (3-D generalization of strings) forces the conclusion that the topology of space-time is non-trivial in all length scales.

- ***Geometrization of classical fields***. Both weak, electromagnetic, gluonic, and gravitational fields are known once the space-time surface in H as a solution of field equations is known.

  Many-sheeted space-time (see **Fig. 4**) consists of space-time sheets with various length scales with smaller sheets being glued to larger ones by ***wormhole contacts*** (see **Fig. 5**) identified as the building bricks of elementary particles. The sizes of wormhole contacts vary but are at least of CP_2 size (about 10^4 Planck lengths) and thus extremely small.

  Many-sheeted space-time replaces reductionism with ***fractality*** . The existence of scaled variants of physics of strong and weak interactions in various length scales is implied, and biology is especially interesting in this respect.

- ***Topological field quantization (TFQ)*** . TFQ replaces classical fields with space-time quanta. For instance, magnetic fields decompose into space-time surfaces of finite size representing flux tubes or -sheets. Field configurations are like Bohr orbits carrying "archetypal" classical field patterns. Radiation fields correspond to topological light rays or massless extremals (MEs), magnetic fields to magnetic flux quanta (flux tubes and sheets) having as primordial representatives "cosmic strings", electric fields correspond to electric flux quanta (e.g. cell membrane), and fundamental particles to CP_2 type vacuum extremals.

- ***Field body*** (FB) and ***magnetic body*** (MB). Any physical system has field identity - FB or MB - in the sense that a given topological field quantum corresponds to a particular source (or several of them - e.g. in the case of the flux tube connecting two systems).

Maxwellian electrodynamics cannot have this kind of identification since the fields created by different sources superpose. Superposition is replaced with a set theoretic union: only the *effects* of the fields assignable to different sources on test particle superpose. This makes it possible to define the QFT limit of TGD.

- ***p-Adic physics*** [K13] as a physics of cognition and intention and the fusion of p-adic physics with real number based physics are new elements.
- ***Adelic physics*** [L9, L12] is a fusion of real physics of sensory experience and various p-adic physics of cognition.
- ***p-Adic length scale hypothesis*** states that preferred p-adic length scales correspond to primes p near powers of two: $p \simeq 2^k$, k positive integer.
- A ***Dark matter hierarchy*** realized in terms of a hierarchy of values of effective Planck constant $h_{eff} = nh_0$ as integers using $h_0 = h/6$ as a unit. Large value of h_{eff} makes possible macroscopic quantum coherence which is crucial in living matter.
- ***MB as an intentional agent using biological body (BB) as a sensory receptor and motor instrument*** . The personal MB associated with the living body - as opposed to larger MBs assignable with collective levels of consciousness - has a hierarchical onion-like layered structure and several MBs can use the same BB making possible remote mental interactions such as hypnosis [L4].
- ***Cosmic strings Magnetic flux tubes*** belong to the basic extremals of practically any general coordinate invariant action principle. Cosmic strings are surfaces of form $X^2 \times Y^2 \subset M^4 \times CP_2$. X^2 is analogous to string world sheet. Cosmic strings come in two varieties and both seem to have a deep role in TGD.

  Y^2 is either a complex or Lagrangian 2-manifold of CP_2. Complex 2-manifold carries monopole flux. For Lagrangian sub-manifold the Kähler form and magnetic flux and Kähler action vanishes. Both types of cosmic strings are are simultaneous extremals of both Kähler action and volume action: this holds true quite generally for preferred extremals.

  Cosmic strings are unstable against perturbations thickening the 2-D M^4 projection to 3-D or 4-D: this gives rise to monopole (see **Fig. ??**) and non-monopole magnetic flux tubes. Using $M^2 \times Y^2$ coordinates, the thickening corresponds to the deformation for which $E^2 \subset M^4$ coordinates are not constant anymore but depend on Y^2 coordinates.
- ***Magnetic flux tubes and sheets*** serve as "body parts" of MB (analogous to body parts of BB), and one can speak about magnetic motor actions. Besides concrete motion of flux quanta/tubes analogous to ordinary motor activity, basic motor actions include the contraction of magnetic flux tubes by a phase transition possibly reducing Planck constant, and the change in thickness of the magnetic flux tube, thus changing the value of the magnetic field, and in turn the cyclotron frequency. Transversal oscillatory motions of flux tubes and oscillatory variations of the thickness of the flux tubes serve as counterparts for Alfwen waves.

  Reconnections of the U-shaped flux tubes allow two MBs to get in contact based on a pair of flux tubes connecting the systems and temporal variations of magnetic fields inducing motor actions of MBs favor the formation of reconnections.

  In hydrodynamics and magnetohydrodynamics reconnections would be essential for the generation of turbulence by the generation of vortices having monopole flux tube at core and Lagrangian flux tube as its exterior.

  Flux tube connections at the molecular level bring a new element to biochemistry making it possible to understand bio-catalysis. Flux tube connections serve as a space-time correlates for attention in the TGD inspired theory of consciousness.
- ***Cyclotron Bose-Einstein condensates (BECs)*** of various charged particles can accompany MBs. Cyclotron energy $E_c = hZeB/m$ is much below thermal energy at physiological temperatures for magnetic fields possible in living matter. In the transition $h \rightarrow h_{eff}$

E_c is scaled up by a fractor $h_{eff}/h = n$. For sufficiently high value of h_{eff} cyclotron energy is above thermal energy $E = h_{eff}\ ZeB/m$. Cyclotron Bose-Einstein condensates at MBs of basic biomolecules and of cell membrane proteins - play a key role in TGD based biology.

- ***Josephson junctions*** exist between two superconductors. In TGD framework, ***generalized Josephson junctions*** accompany membrane proteins such as ion channels and pumps. A voltage between the two super-conductors implies a ***Josephson current***. For a constant voltage the current is oscillating with the ***Josephson frequency*** . The Josephson current emits ***Josephson radiation***. The energies come as multiples of ***Josephson energy***.

  In TGD generalized Josephson radiation consisting of dark photons makes communication of sensory input to MB possible. The signal is coded to the modulation of Josephson frequency depending on the membrane voltage. The cyclotron BEC at MB receives the radiation producing a sequence of resonance peaks.

- ***Negentropy Maximization Principle*** (NMP). NMP [K12] [L39] is the variational principle of consciousness and generalizes SL. NMP states that the negentropy gain in SFR is non-negative and maximal. NMP implies SL for ordinary matter.

- ***Negentropic entanglement*** (NE). NE is possible in adelic physics and NMP does not allow its reduction. NMP implies a connection between NE, the dark matter hierarchy, p-adic physics, and quantum criticality. NE is a prerequisite for an experience defining abstraction as a rule having as instances the state pairs appearing in the entangled state.

- ***Zero energy ontology (ZEO)*** In ZEO physical states are pairs of positive and negative energy parts having opposite net quantum numbers and identifiable as counterparts of initial and final states of a physical event in the ordinary ontology. Positive and negative energy parts of the zero energy state are at the opposite boundaries of a ***causal diamond*** (CD, see **Fig. 12**)) defined as a double-pyramid-like intersection of future and past directed light-cones of Minkowski space.

  CD defines the "spot-light of consciousness": the contents of conscious experience associated with a given CD is determined by the space-time sheets in the embedding space region spanned by CD.

- ***SFR*** is an acronym for state function reduction. The measurement interaction is universal and defined by the entanglement of the subsystem considered with the external world [L19] [K23]. What is measured is the density matrix characterizing entanglement and the outcome is an eigenstate of the density matrix with eigenvalue giving the probability of this particular outcome. SFR can in principle occur for any pair of systems.

  SFR in ZEO solves the basic problem of quantum measurement theory since the zero energy state as a superposition of classical deterministic time evolutions (preferred extremals) is replaced with a new one. Individual time evolutions are not made non-deterministic.

  One must however notice that the reduction of entanglement between fermions (quarks in TGD) is not possible since Fermi- and als Bose statistics predicts a maximal entanglement. Entanglement reduction must occur in WCW degrees of freedom and they are present because point-like particles are replaced with 3-surfaces. They can correspond to the number theoretical degrees of freedom assignable to the Galois group - actually its decomposition in terms of its normal subgroups - and to topological degrees of freedom.

- ***SSFR*** is an acronym for "small" SFR as the TGD counterpart of weak measurement of quantum optics and resembles classical measurement since the change of the state is small [L19] [K23]. SSFR is preceded by the TGD counterpart of unitary time evolution replacing the state associated with CD with a quantum superposition of CDs and zero energy states associated with them. SSFR performs a localization of CD and corresponds to time measurement with time identifiable as the temporal distance between the tips of CD. CD is scaled up in size - at least in statistical sense and this gives rise to the arrow of time.

  The unitary process and SSFR represent also the counterpart for Zeno effect in the sense that the passive boundary of CD as also CD is only scaled up but is not shifted. The states remain unchanged apart from the addition of new fermions contained by the added part of the passive boundary. One can say that the size of the CD as analogous to the perceptive field means that more and more of the zero energy state at the passive boundary becomes visible. The active boundary is however both scaled and shifted in SSFR and states at it change. This gives rise to the experience of time flow and SSFRs as moments of subjective time correspond to geometric time as a distance between the tips of CD. The analog of unitary time evolution corresponds to "time" evolution induced by the exponential of the scaling generator L_0. Time translation is thus replaced by scaling. This is the case also in p-adic thermodynamics. The idea of time evolution by scalings has emerged also in condensed matter physics.

- ***BSFR*** is an acronym for "big" SFR, which is the TGD counterpart of ordinary state function reduction with the standard probabilistic rules [L19] [K23]. What is new is that the arrow of time changes since the roles of passive and active boundaries change and CD starts to increase in an opposite time direction.

  This has profound thermodynamic implications. Second law must be generalized and the time corresponds to dissipation with a reversed arrow of time looking like self-organization for an observed with opposite arrow of time [L17]. The interpretation of BSFR is as analog of biological death and the time reversed period is analogous to re-incarnation but with non-standard arrow of time. The findings of Minev *et al* [L14] give support for BSFR at atomic level. Together with h_{eff} hierarchy BSFR predicts that the world looks classical in all scales for an observer with the opposite arrow of time.

6.3 Figures

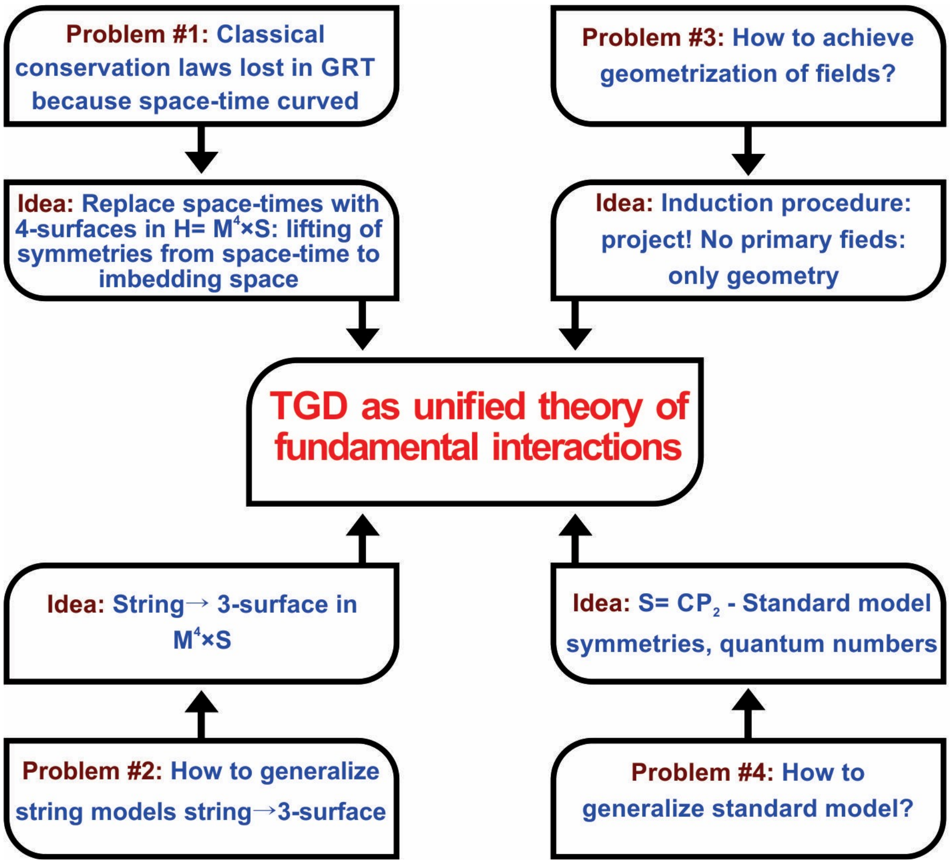

Figure 1: The problems leading to TGD as their solution.

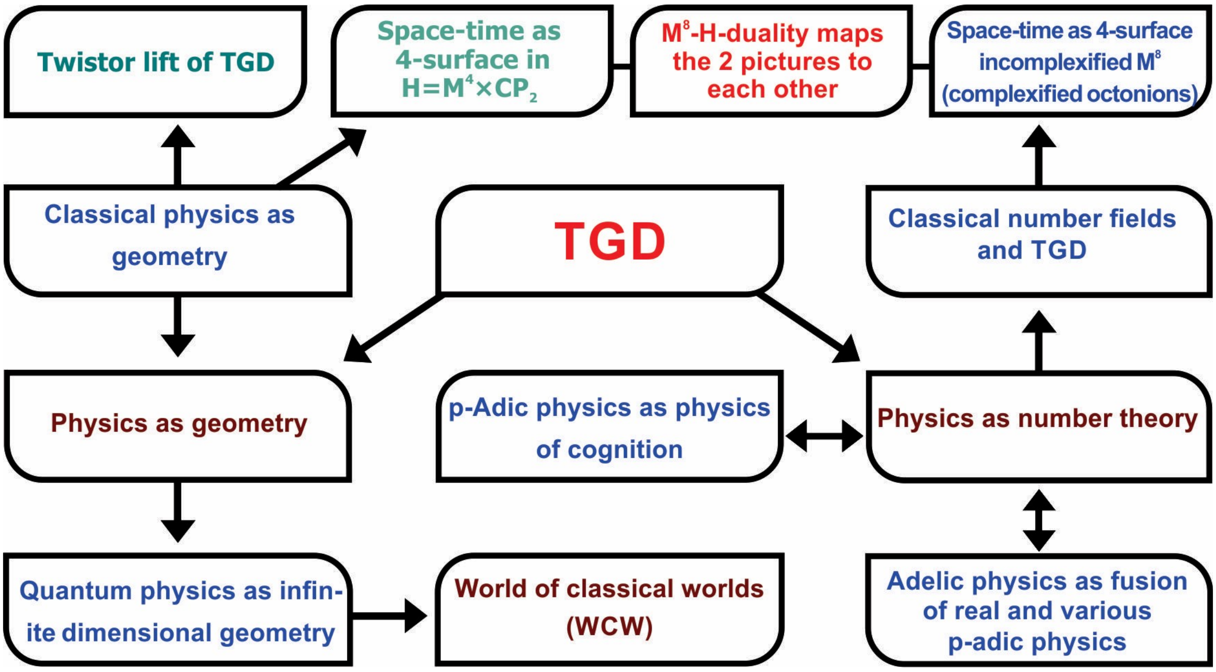

Figure 2: TGD is based on two complementary visions: physics as geometry and physics as number theory.

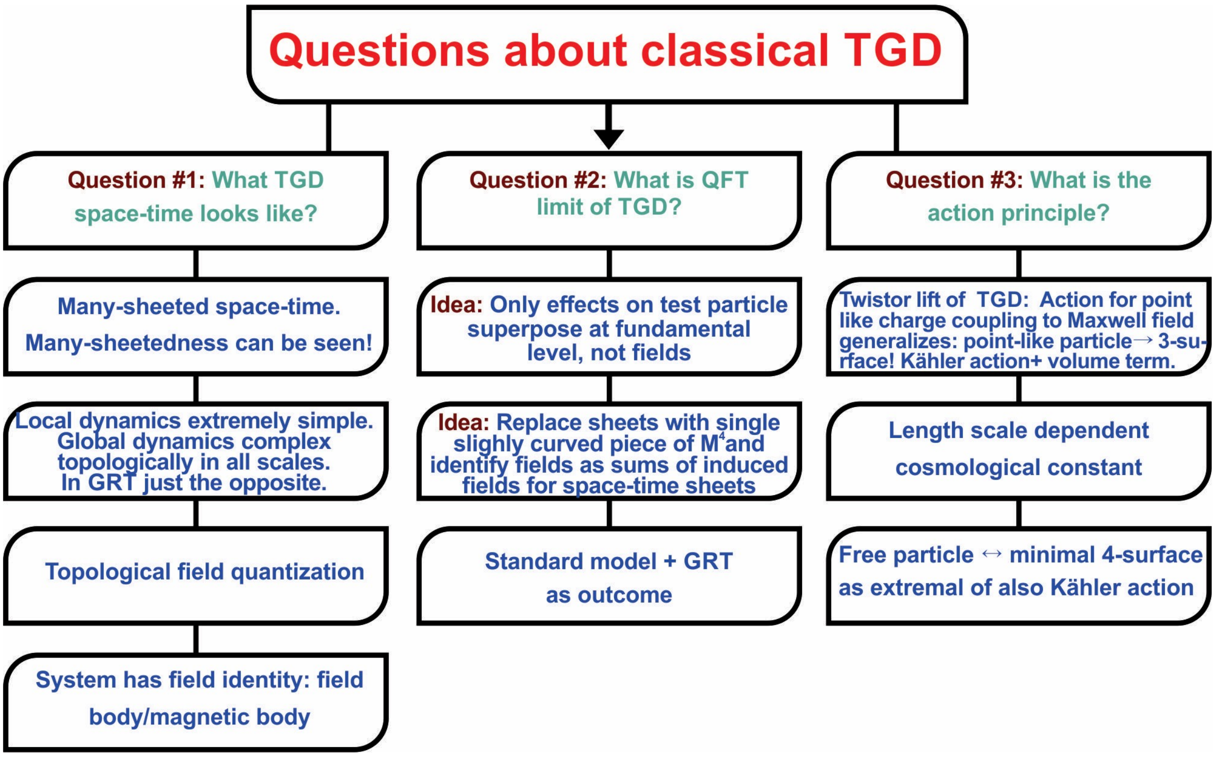

Figure 3: Questions about classical TGD.

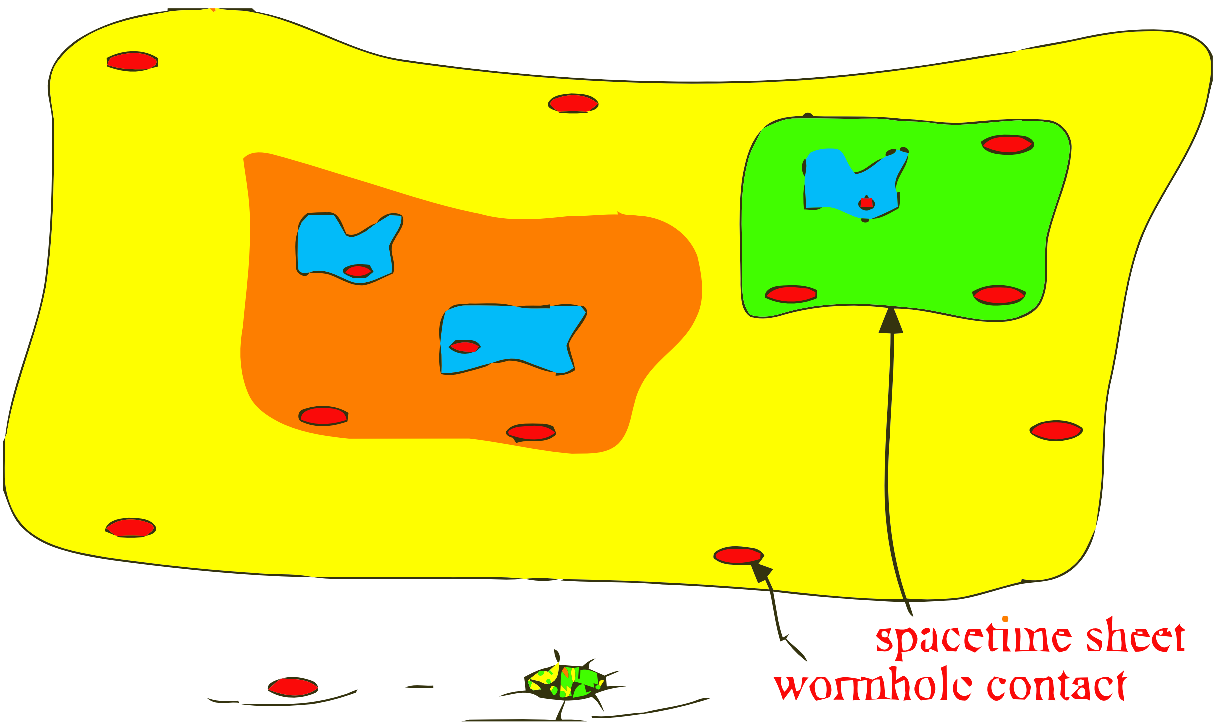

Figure 4: Many-sheeted space-time.

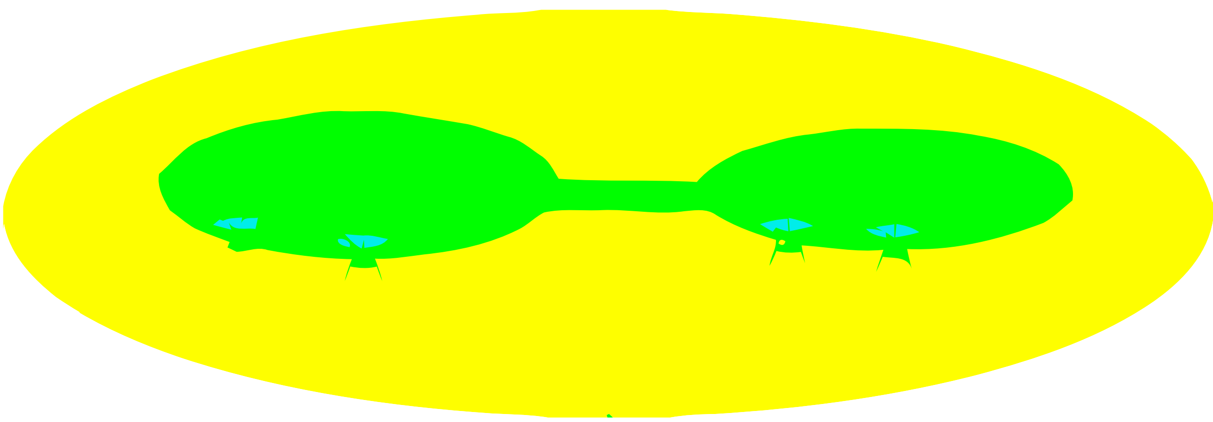

Figure 5: Wormhole contacts.

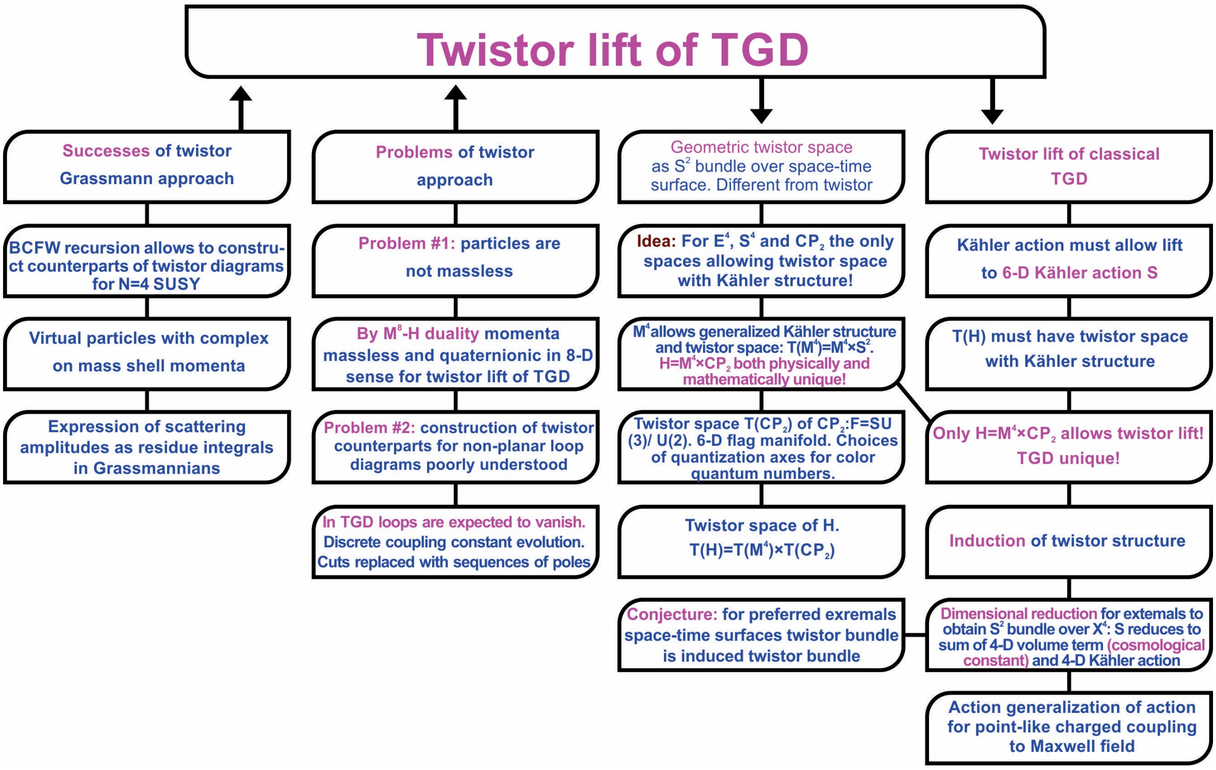

Figure 6: Twistor lift

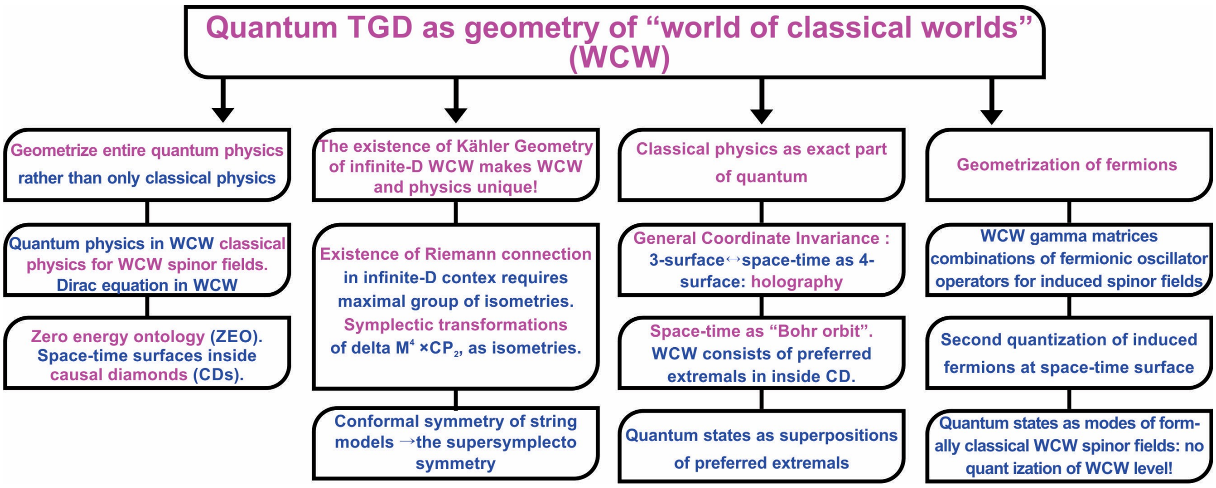

Figure 7: Geometrization of quantum physics in terms of WCW

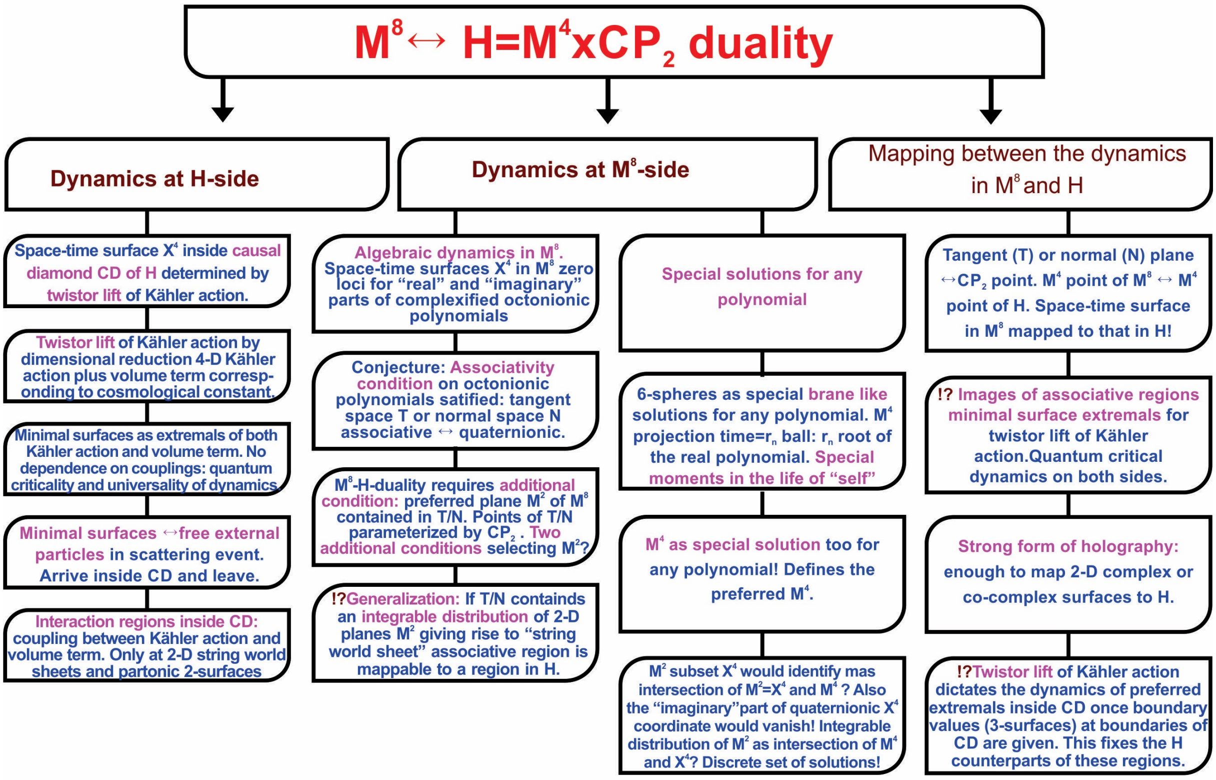

Figure 8: $M^8 - H$ duality

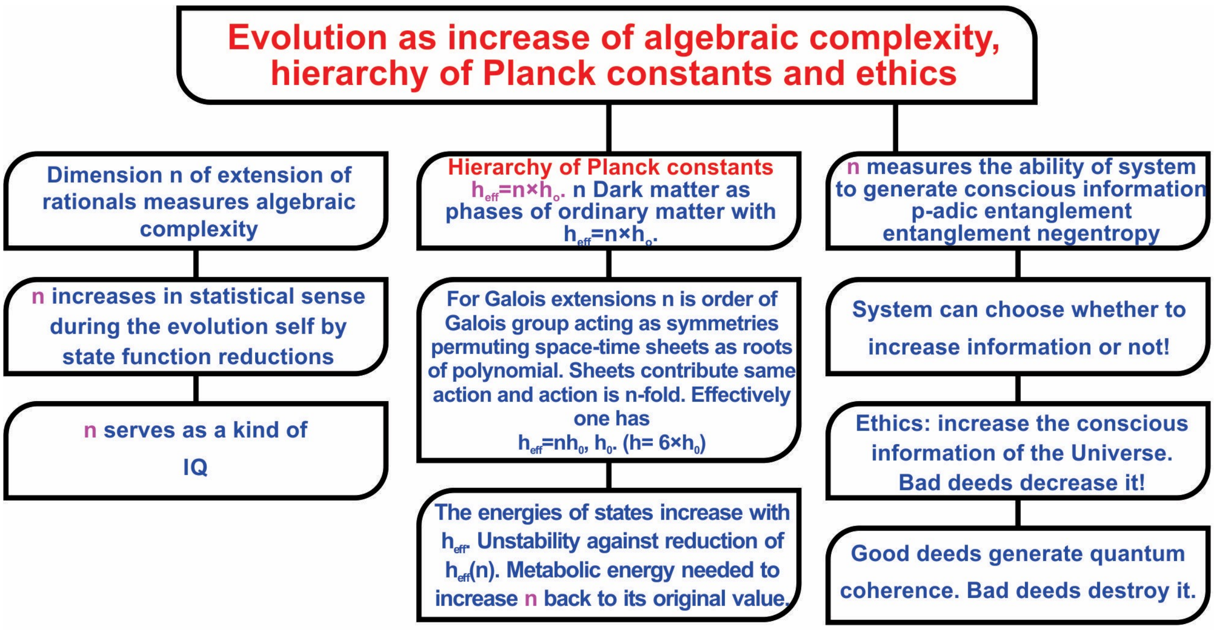

Figure 9: Number theoretic view of evolution

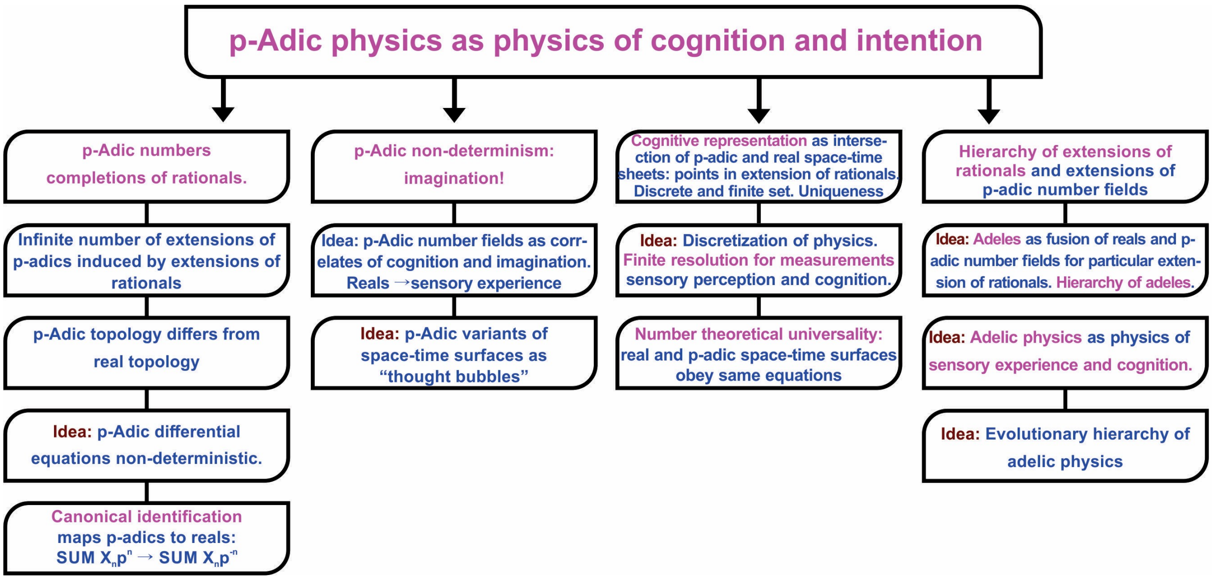

Figure 10: p-Adic physics as physics of cognition and imagination.

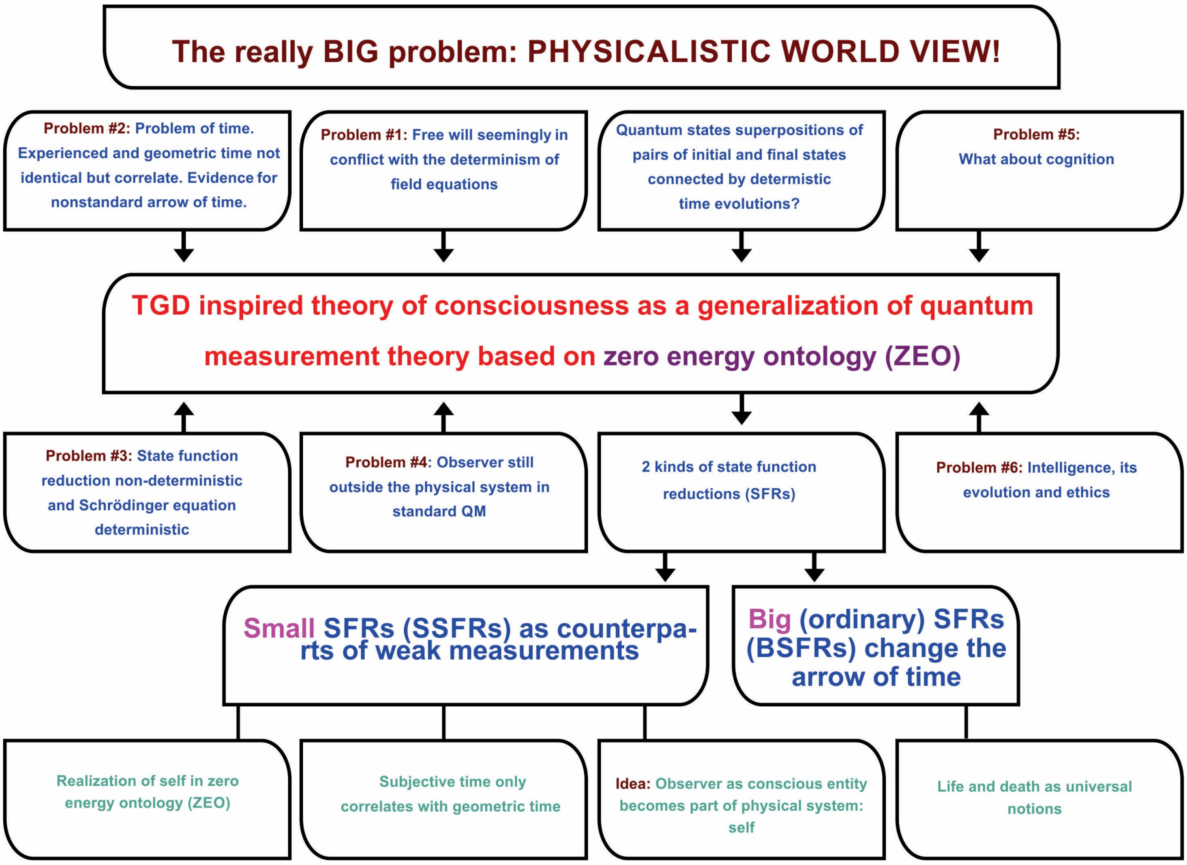

Figure 11: Consciousness theory from quantum measurement theory

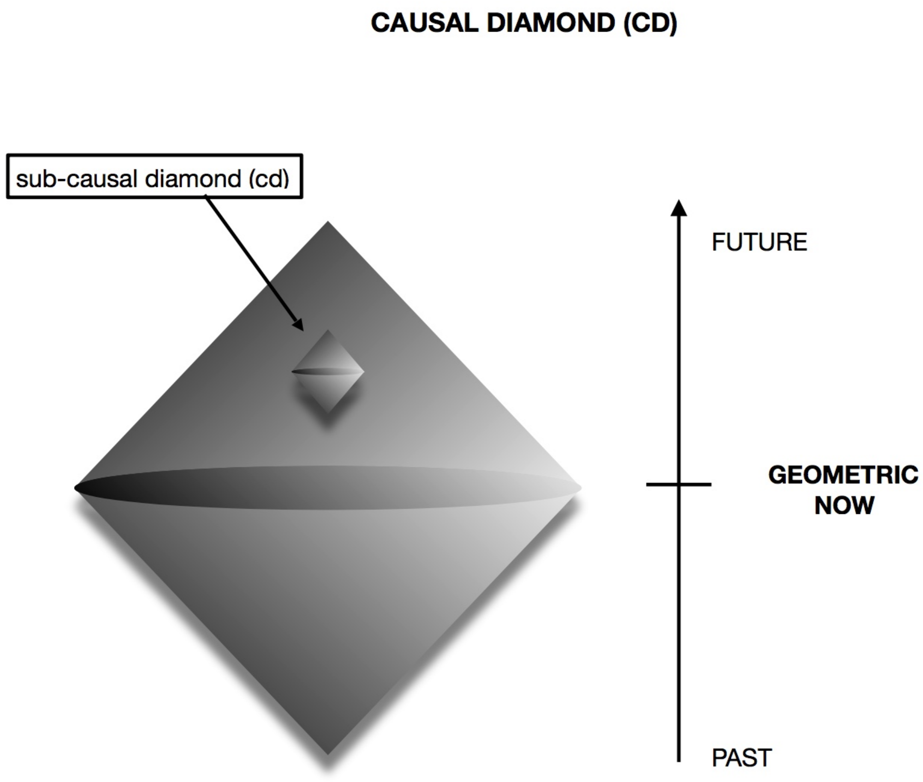

Figure 12: Causal diamond

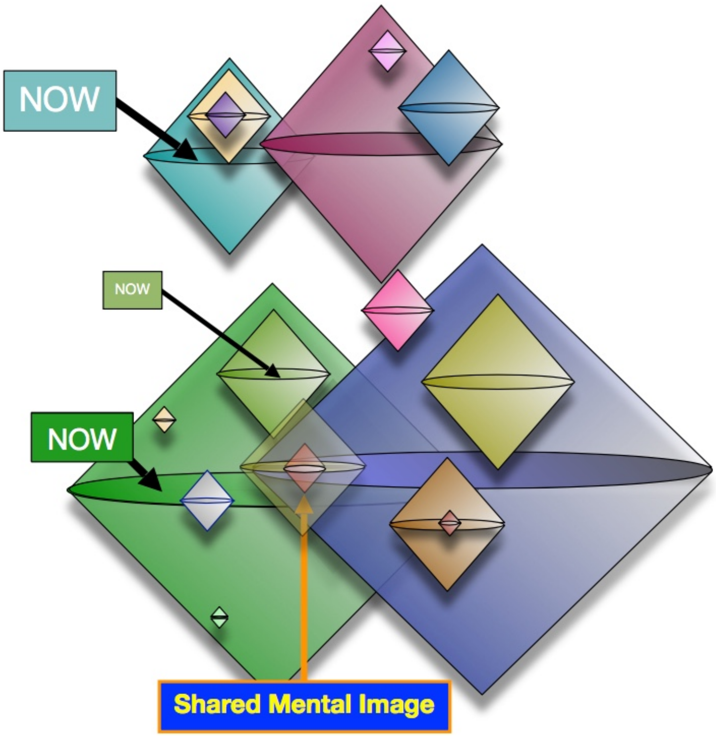

Figure 13: CDs define a fractal "conscious atlas"

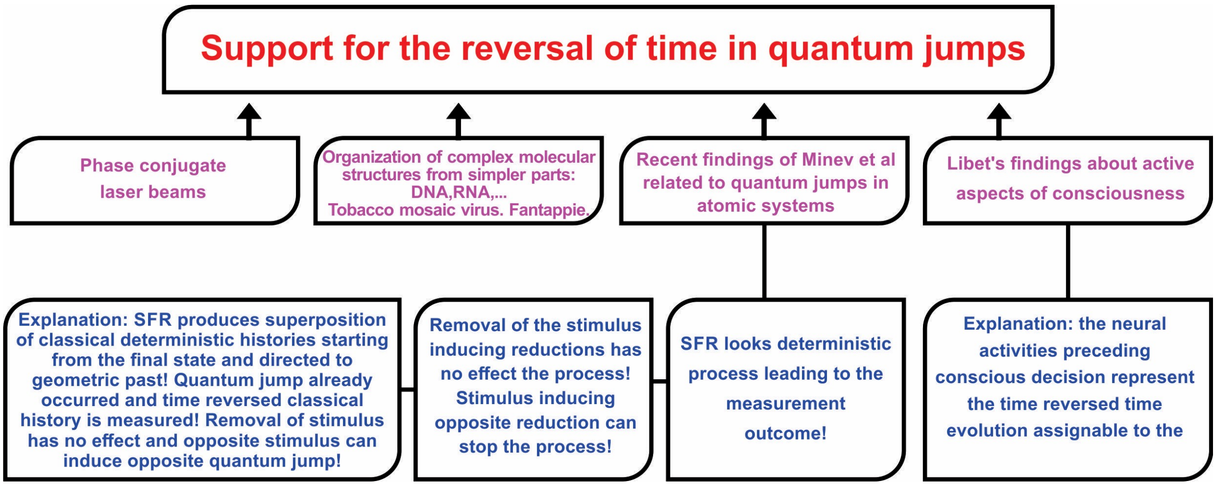

Figure 14: Time reversal occurs in BSFR

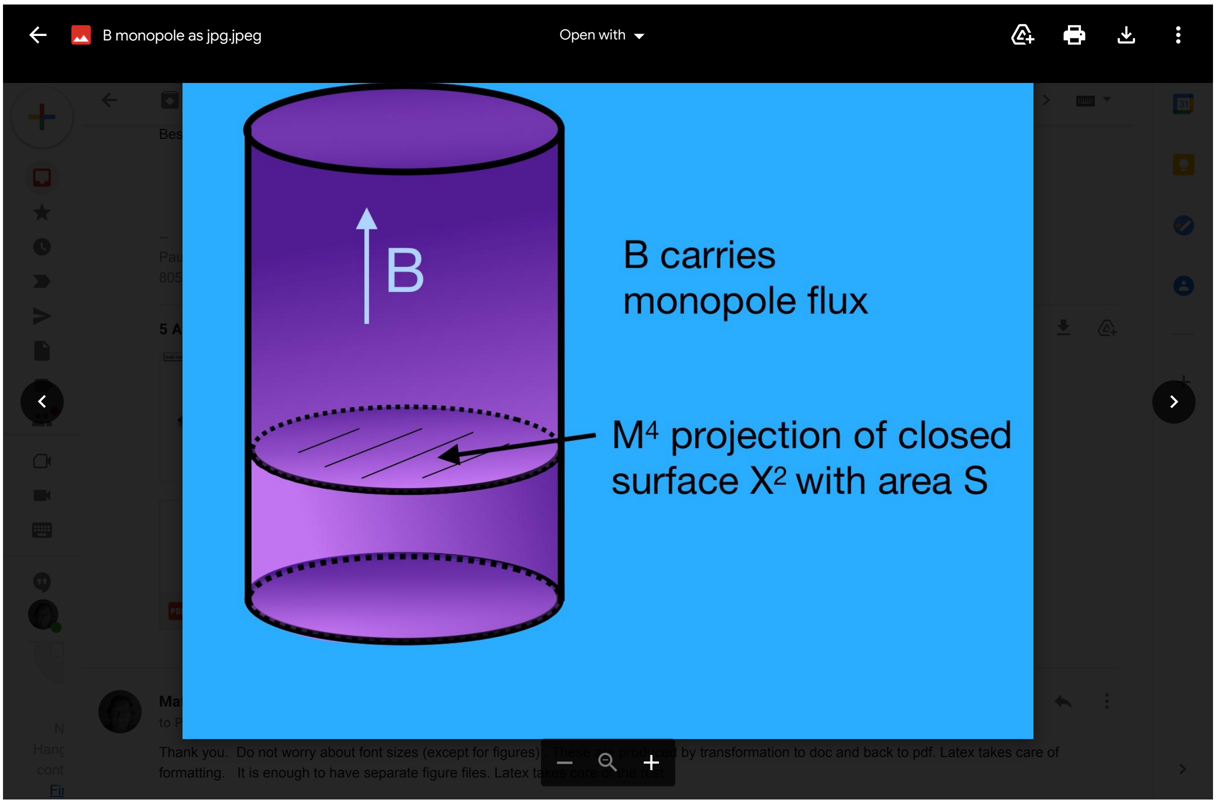

Figure 15: The M^4 projection of a closed surface X^2 with area S defining the cross section for monopole flux tube. Flux quantization $e\oint B\cdot dS = eBS = kh$ at single sheet of n-sheeted flux tube gives for cyclotron frequency $f_c = ZeB/2\pi m = khZ/2\pi mS$. The variation of S implies frequency modulation.

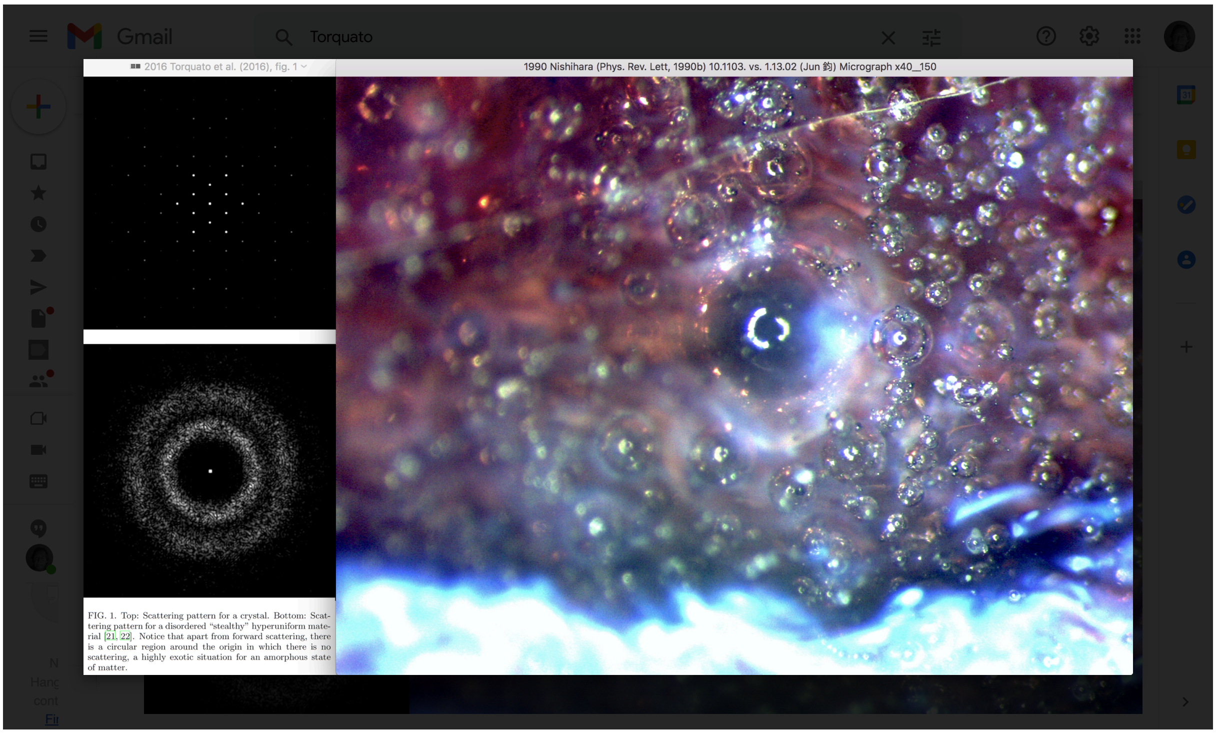

Figure 16: The scattering from a hyperuniform amorphous material shows no scattering in small angles apart from the forward peak (`https://cutt.ly/ZWyLgjk`). This is very untypical in amorphous matter and might reflect the diffraction pattern of dark photons at the magnetic body of the system.

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