Mathematical Speculations Inspired by the Hierarchy of Planck Constants

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Abstract

This chapter contains the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) as separate from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first speculative question is about possible relationship between Jones inclusions of hyperfinite factors of type II_1 (hyper-finite factors are von Neuman algebras emerging naturally in TGD framework). The basic idea is that the discrete groups assignable to inclusions could correspond to discrete groups acting in the effective covering spaces of imbedding space assignable to the hierarchy of Planck constants.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

1 Introduction

I decided to separate the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first highly speculative topic discussed in this chapter is about possible connection of the hierarchy of Planck constants with Jones inclusions.

1. The connection with Jones inclusions was originally a purely heuristic guess based on the observation that the finite groups characterizing Jones inclusion characterize also pages of the Big Book. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and that this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions. The elements of the included algebra $\mathcal{M}$ are invariant under the action of G and $\mathcal{M}$ takes the role of complex numbers in the resulting non-commutative quantum theory.

2. The understanding of quantum TGD at parton level led to the realization that the dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field. This automatically implies cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups closely associated with Jones inclusions [K1]. The Clifford algebra spanned by the fermionic oscillator operators would provide a realization for the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that its unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

3. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 or CP_2 in some sense. The alternative interpretation in terms of two geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom.

4. Number theoretic Universality suggests an answer why the hierarchy of Planck constants is necessary. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in p-adic context. All that one can achieve naturally is the notion of phase defined as root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase if needed. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases geometrically and quantum phases q naturally assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [?].

2 Jones Inclusions And Generalization Of The Imbedding Space

The original motivation for the generalization of the imbedding space was the idea that the pages of the Big Book would provide correlates for Jones inclusions. In the following an attempt to formulate this vision more precisely is carried out.

2.1 Basic Facts About Jones Inclusions

Here only basic facts about Jones inclusions are discussed. Appendix contains a more detailed discussion of inclusions of HFFs.

2.1.1 Jones inclusions defined by subgroups of $SL(2,C)\times SU(2)$

Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ have representation as $R_0^G\subset R^G$ with G a discrete subgroup of $SU(2)$. $SO(3)$ or $SU(2)$ can be interpreted as acting in CP_2 as rotations. On quantum spinors the action corresponds to double cover of G.

A more general choice for G would be as a discrete subgroup $G_a\times G_b\subset SL(2,C)\times SU(2)\times SU(2)$. Poincare invariance suggests that the subgroup of $SL(2,C)$ reduces either to a discrete subgroup of $SU(2)$ and in the case that the rotation are genuinely 3-dimensional (E^6,E^8), the only possible interpretation would be as isotropy group of a particle at rest. When the group acts on plane as in case of A_n and D_{2n}, it could be also assigned to a massless particle.

If the group involves boosts it contains an infinite number of elements and it is not clear whether this kind of situation is physically sensible. In this case Jones inclusion could be interpreted as an inclusion for the tensor product of G invariant algebras associated with CD and CP_2 degrees of freedom and one would have $\mathcal{M}:\mathcal{N}=\mathcal{M}:\mathcal{N}(G_a)\times\mathcal{M}:\mathcal{N}(G_b)$. Since the index increases as the order of G increases one has reasons to expect that in the case of $G_a=SL(2,C)$ $N_a=\infty$ implies larger $\mathcal{M}:\mathcal{N}(G_a)>4$.

A possible interpretation is that the values $\mathcal{M}:\mathcal{N}\leq 4$ are analogous to bound state energies so that a discrete rotation group acting in the relative rotational degrees of freedom can act as a symmetry group whereas the values $\mathcal{M}:\mathcal{N}>4$ are analogous to ionized states for which particles are almost freely moving with respect to each other with a constant velocity.

When one restricts the coefficients to G-invariant elements of Clifford algebra the Clifford field is G-invariant under the natural action of G. This allows two interpretations. Either the Clifford field is G invariant or that the Clifford field is defined in orbifold $CD/G_a \times CP_2/G_b$. CD/G_a is obtained by replacing hyperboloid H_a ($t^2 - x^2 - y^2 - z^2 = a^2$) with H_a/G_a. These spaces have been considered as cosmological models having 3-space with finite volume [K8] (also a lattice like structure could be in question).

2.1.2 The quantum phases associated with sub-groups of $SU(2)$

It is natural to identify quantum phase as that defined by the maximal cyclic subgroup for finite subgroups of $SU(2)$ and infinite subgroups of $SL(2,C)$. Before continuing a brief summary about quantum phases associated with finite subgroups of $SU(2)$ is in order. E_6 corresponds to $N = 24$ and $n = 3$ and E_8 to icosahedron with $N = 120$, $n = 5$ and Golden mean and the minimal value of n making possible universal topological quantum computer [K10].

D_n and A_n have orders $2n$ and $n + 1$ and act as symmetry groups of n-polygon and have n-element cyclic group as a maximal cyclic subgroup. For double covers the orders are twice this. Thus A_n *resp.* D_{2n} correspond to $q = exp(i\pi/n)$ *resp.* $q = exp(i\pi/2n)$. Note that the restriction $n \geq 3$ means geometrically that only non-trivial polygons are allowed.

2.2 Jones Inclusions And The Hierarchy Of Planck Constants

The anyonic arguments for the quantization of Planck constant suggest that one can assign separate scalings of Planck constant to CD and CP_2 degrees of freedom and that these scalings in turn reflect as scalings of $M^4\pm$ and CP_2 metrics. This is definitely not in accordance with the original TGD vision based on uniqueness of imbedding space but makes sense if space-time and imbedding space are emergent concepts as the hierarchy of number theoretical von Neumann algebra inclusions indeed suggests. Indeed, the scaling factors of CD and CP_2 metric remain non-fixed by the general uniqueness arguments since Cartesian product is in question.

2.2.1 Hierarchy of Planck constants and choice of quantization axis

Jones inclusions seem to relate in a natural manner to the selection of quantization axis.

1. In the case of CD the orbifold singularity is for all groups G_a except E_6 and E_8 the time-like plane M^2 corresponding to a radial ray through origin defining the quantization axis of angular momentum and intersecting light-cone boundary along a preferred light-like ray. For E_6 and E_8 (tetrahedral and icosahedral symmetries) the singularity consists of planes M^2 related by symmetries of G sharing time-like line M^1 and in this case there are several alternative identifications of the quantization axes as axis around which the maximal cyclic subgroup acts as rotations.

2. From this it should be obvious that Jones inclusions represented in this manner would relate very closely to the selection of quantization axes and provide a geometric representation for this selection at the level of space-time and WCW . The existence of the preferred direction of quantization at a given level of dark matter level should have observable consequences. For instance, in cosmology this could mean a breaking of perfect rotational symmetry at dark matter space-time sheets. The interpretation would be as a quantum effect in cosmological length scales. An interesting question is whether the observed asymmetry of cosmic microwave background could have interpretation as a quantum effect in cosmological length and time scales.

2.2.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of the singular coverings and and factor spaces? If both geodesic spheres of CP_2 are allowed $\mathcal{M} : \mathcal{N} = 4$ could correspond to the allowance of cosmic strings and other analogous objects. This option is however asymmetric with respect to CD and CP_2 and the more plausible option is that the two kinds of Jones inclusions correspond to singular factor spaces and coverings.

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$. For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane. This picture is also consistent with the G singlets of the quantum states despite the fact that fermionic oscillator operators belong to non-trivial irreps of G.

2.2.3 Coverings and factors spaces form an algebra like structure

It is easy to see that coverings and factor spaces defining the pages of the Big Book form an algebra like structure.

1. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labeled by a subset of discrete subgroups of $SU(2)$.

2. The discrete subgroups of $SU(2)$ with fixed quantization axis possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tetrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2 generated by reflection, and tetrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in WCW labeled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

2.2.4 Connection between Jones inclusions, hierarchy of Planck constants, and finite number of spinor modes

The original generalization of the imbedding space to accommodate the hierarchy of Planck constants was based on the idea that the singular coverings and factor spaces associated with the causal diamond CD and CP_2, which appears as factors of $CD \times CP_2$ correspond somehow to Jones inclusions, and that the integers n_a and n_b characterizing the orders of maximal cyclic groups of groups G_a and G_b associated with the two Cartesian factors correspond to quantum phases

$q = exp(i2\pi/n_i)$ in such a manner that singular factor spaces correspond to Jones inclusions with index $\mathcal{M} : \mathcal{N} < 4$ and coverings to those with index $\mathcal{M} : \mathcal{N} = 4$.

Since Jones inclusions are interpreted in terms of finite measurement resolution, the mathematical realization of this heuristic picture should rely on the same concept realized also by the fact that the number of non-zero modes for induced spinor fields is finite. This allows to consider two possible interpretations.

1. The finite number of modes defines an approximation to the hyper-finite factor of type II_1 defined by WCW Clifford algebra.

2. The Clifford algebra spanned by fermionic oscillator operators is quantum Clifford algebra and corresponds to the somewhat nebulous object $\mathcal{N}/\mathcal{M}$ associated with the inclusion $\mathcal{M} \subset \mathcal{N}$ and coding the finite measurement resolution to a finite quantum dimension of the Clifford algebra. The fact that quantum dimension is smaller than the actual dimension would reflect correlations between spinor components so that they are not completely independent.

If the latter interpretation is correct then second quantized induced spinor fields should obey quantum variant of anti-commutation relations reducing to ordinary anti-commutation relations only for $n_a = n_b = 0$ (no singular coverings nor factor spaces). This would give the desired connection between inclusions and hierarchy of Planck constants. It is possible to have infinite number of quantum group like structure for $\hbar = \hbar_0$.

There are two quantum phases q and one should understand what is the phase that appears in the quantum variant of anti-commutation relations. A possible resolution of the problem relies on the observation that there are two kinds of number theoretic braids. The first kind of number theoretic braid is defined as the intersection of M_+ (or light-like curve of δM^4_+ in more general case) and of δM^4_+ projection of X^2. Second end of braid is defined as the intersection of CP_2 projection of X^2 of homologically non-trivial sphere S^2_{II} of CP_2. The intuitive expectation is that these dual descriptions apply for light-like 3-surfaces associated *resp.* co-associative regions of space-time surface and that both descriptions apply at wormhole throats. The duality of these descriptions is guaranteed also at wormhole throats if physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations. This would suggest that it is $q = exp(i2\pi/r)$, which appears in quantum variant of anti-commutation relations of the induced spinor fields.

2.2.5 The action of $G_a \times G_b$ on WCW spinors and spinor fields

The first question is what kind of measurement resolution is in question. In zero energy ontology the included states would typically correspond to insertion of zero energy states to the positive or negative part of the physical state in time scale below the time resolution defined by the time scale assignable to the smallest CD present in the zero energy state. Does the description in terms of G invariance apply in this case or does it relate only to time and length scale resolution whereas hierarchy of Planck constants would relate to angle resolution? Assume that this is the case.

The second question is how the idea about $\mathcal{M}$ as an included algebra defining finite measurement resolution and G invariance as a symmetry defining $\mathcal{M}$ as the included algebra relate to each other.

1. One cannot say that G creates states, which cannot be distinguished from each other. Rather G-invariant elements of $\mathcal{M}$ create states whose presence in the state cannot be detected.

2. For covering space option $\mathcal{M}$ represents states which are invariant under discrete subgroup of $SU(2)$ acting in the covering. States with integer spin would be below measurement resolution and only factional spins of form j/n would be observable. For factor space option $\mathcal{M}$ would represents states which are invariant under discrete subgroup of $SU(2)$ acting in H-say states with spin. States with spin which is multiple of n would be below measurement resolution. The situation would be very similar to each other. Number theoretic considerations and the fact that the number of fermionic oscillator operators is finite suggest that that for coverings the condition $L_z < 1$ and for factor spaces the condition $L_z < n$ is satisfied by the generators of Clifford algebra regarded as irreducible representation of G. For factor spaces the interpretation could be in terms of finite angular resolution $\Delta\phi \leq 2\pi/n$ excluding

angular momenta $L_z \geq n$. For coverings the resolution would be related to rotations (or rather, braidings) as multiples of 2π: multiples $m2\pi$ $m \geq n$ cannot be distinguished from $m \ mod \ n$ multiples.

3. The minimal assumption is that integer orbital angular momenta are excluded for coverings and n-multiples are excluded for factors spaces. The stronger assumption would be that there is angular momentum cutoff. This point is however very delicate. The states with $j > n$ can be obtained as tensor products of representations with $j = m$. If entanglement is present one cannot anymore express the state as a product of $\mathcal{M}$ element and $\mathcal{N}$ element so that the states $j > n$ created in this manner would not be equivalent with those with $j \ mod \ n$. The replacement of the ordinary tensor product with Connes tensor product would indeed generate automatically entangled states and one could interpret Connes tensor product as a manner to create only the allowed states.

4. For quantum groups allow only finite number of representations up to some maximum spin determined by the integer n characterizing quantum phase q. This would mean angular momentum cutoff leaving only a finite number of representations of quantum group [K1]. This fits nicely with what one obtains in the case of factor spaces. For coverings the new element is that the unit of spin becomes $1/n$: otherwise the situation seems to be similar. Quantum group like structure is obtained if the fermionic oscillator operators satisfy the quantum version of anti-commutation relations. The algebra would be very similar except that the orbital angular momentum labeling oscillator operators has different unit. Oscillator operators are naturally in irreducible representations of G and only the non-trivial representations of G are allowed.

5. Besides Jones inclusions corresponding to $\mathcal{M} : \mathcal{N} < 4$ there are inclusions with $\mathcal{M} : \mathcal{N} = 4$ to which one can also assign quantum phases. It would be natural to assign covering spaces and factor spaces to these two kinds of inclusions. For the minimal option excluding only the orbital angular momentum which are integers or multiples of n the fraction of excluded states is very small for coverings so that $\mathcal{M} : \mathcal{N} = 4$ is natural for this option. $\mathcal{M} : \mathcal{N} < 4$ would in turn correspond naturally to factor spaces.

6. Since the two kinds of number theoretic braids correspond to points which belong to M^2 or S^2, one might argue that several quantum anti-commutation relations must be satisfied simultaneously. This is not the case since the eigen modes of D_{C-S} and hence also oscillator operators code information about partonic surface X^2 itself and also about $X^4(X_l^3)$ rather than being purely local objects. In the case of covering space the oscillator operators can be arranged to irreducible representations of G and in the case of factor space the oscillator operators are G-invariant.

One must distinguish between G invariance for WCW spinors and spinor fields.

1. In the case of factor spaces 3-surface are G invariant so that there is no difference between spinors and spinor fields as far as G is considered. Irreducible representations of G would correspond to the superpositions of G-transforms of oscillator operators for a fixed G-invariant X_l^3.

2. For covering space option G-invariance would mean that 3-surface is a mere G-fold copy of single 3-surface. There is no obvious reason to assume this. Hence one cannot separate spinorial degrees of freedom from WCW degrees of freedom since G affects both the spin degrees of freedom and the 3-surface. Irreducible representations of G would correspond to genuine WCW spinor fields involving a superposition of G-transforms of also X_l^3. The presence of both orbital and spin degrees of freedom could provide alternative explanation for why $\mathcal{M} : \mathcal{N} = 4$ holds true for covering space option.

If the fermionic oscillator algebra is interpreted as a representation for $\mathcal{N}/\mathcal{M}$, allowed fermionic oscillator operators belong to non-trivial irreps of G. One can however ask whether the many-fermion states created by these operators are G-invariant for some physical reason so that one would have kind of G-confinement forcing the states to be many-fermion states with standard unit of quantum numbers for coverings and integer multiples of n for factor spaces. This would conform with the ideas that anyonicity is a microscopic property not visible at the level of entire state and that many-fermion systems in the anyonic state resulting in strong coupling limit for ordinary value of $\hbar$ are in question. The processes changing the value of Planck constant would be phase transitions involving all fermions of the G-invariant state and would be slow for this reason. This would also contribute to the invisibility of dark matter.

2.3 Questions

2.3.1 What is the role of dimensions?

Could the dimensions of CD and CP_2 and the dimensions of spaces defined by the choice of the quantization axes play a fundamental role in the construction from the constraint that the fundamental group is non-trivial?

1. Suppose that the sub-manifold in question is geodesic sub-manifold containing the orbits of its points under Cartan subgroup defining quantization axes. A stronger assumption would be that the orbit of maximal compact subgroup is in question.

2. For M^{2n} Cartan group contains translations in time direction with orbit M^1 and Cartan subgroup of $SO(2n-1)$ and would be M^n so that $\hat{M^{2n}}$ would have a trivial fundamental group for $n > 2$. Same result applies in massless case for which one has $SO(1,1) \times SO(2n-2)$ acts as Cartan subgroup. The orbit under maximal compact subgroup would not be in question.

3. For CP_2 homologically non-trivial geodesic sphere CP_1 contains orbits of the Cartan subgroup. For $CP_n = SU(n+1)/SU(n) \times U(1)$ having real dimension $2n$ the sub-manifold CP_{n-1} contains orbits of the Cartan subgroup and defines a sub-manifold with codimension 2 so that the dimensional restriction does not appear.

4. For spheres $S^{n-1} = SO(n)/SO(n-1)$ the dimension is $n-1$ and orbit of $SO(n-1)$ of point left fixed by Cartan subgroup $SO(2) \times ..$ would for $n = 2$ consist of two points and S_{n-2} in more general case. Again co-dimension 2 condition would be satisfied.

2.3.2 What about holes of WCW ?

One can raise analogous questions at the level of WCW geometry. Vacuum extremals correspond to Lagrangian sub-manifolds $Y^2 \subset CP_2$ with vanishing induced Kähler form. They correspond to singularities of WCW ("world of classical worlds") and WCW spinor fields should vanish for the vacuum extremals. Effectively this would mean a hole in configuration space, and the question is whether this hole could also naturally lead to the introduction of covering spaces and factor spaces of the WCW s. How much information about the general structure of the theory just this kind of decomposition might allow to deduce? This kind of singularities are infinite-dimensional variants of those discussed in catastrophe theory and this suggests that their understanding might be crucial.

2.3.3 Are more general inclusions of HFFs possible?

The proposed scenario could be criticized because discrete subgroups of $SU(2)$ are in a preferred position. The Jones inclusions considered correspond to quantum spinor representations of various quantum groups $SU(2)_q$, $q = exp(i2\pi/n)$. This explains the result $\mathcal{M} : \mathcal{N} \leq 4$. These representations are certainly in preferred role as far as WCW spinor fields are considered but it is possible to assign a hierarchy of inclusions of HFFs labeled by quantum phase q with arbitrary representation of an arbitrary compact Lie group. These inclusions would be analogous to discrete states in the continuum $\mathcal{M} : \mathcal{N} > 4$.

Since the inclusions are characterized by single quantum phase $q = exp(i2\pi/n)$ in the case of compact Lie groups (Appendix), one can ask whether more general discrete groups than subgroups of $SU(2)$ should be allowed. The inclusions of HFFs associated with higher dimensional Lie groups have $\mathcal{M} : \mathcal{N} > 4$ and are analogous to bound states in continuum (Appendix). In the case of CP_2 this would allow to consider much more general sub-groups.

The question is therefore whether some principle selects subgroups of $SU(2)$. There are indeed good arguments supporting the hypothesis that only discrete Abelian subgroups of $SU(2)$ are possible.

1. The notion of number theoretic braid allows only the only subgroups of rotation group leaving M^2 invariant and sub-groups of $SU(3)$ leaving geodesic sphere S^2_i invariant. This would drop groups having genuinely 3-D action. In the case of $SU(3)$ discrete subgroups of $SO(3)$ or $U(2)$ remain under consideration. The geodesic sphere of type II is however analogous to North/South pole of S^2 and second phase factor associated with the coordinates (ξ^1, ξ^2) becomes redundant since $(|\xi^1|^2 + |\xi^2|^2)^{1/2}$ becomes infinite at S^2_{II} so that ξ^1/ξ^2 becomes appropriate coordinate. Hence action of $U(2)$ reduces to that of $SU(2)$ since ξ^1 and ξ^2 correspond to same value of color hyper charge associated with $U(1)$.

2. A physically attractive possibility is that $G_a \times G_b$ leaves the choice of quantization axes invariant. This condition makes sense also for coverings. This would leave only Abelian groups into consideration and drop D_{2n}, E_6, and E_8. It is quite possible that only these groups define sectors of the generalized imbedding space. This means that $G_b = Z_{n_1} \times Z_{n_2} \subset U(1)_I \times U(1)_Y \subset SU(2) \times U(1)_Y$ and even more general subgroups of $SU(3)$ (if non-commutativity is allowed) are a priori possible. Again the first argument reduces the list to cyclic subgroups of $SU(2)$.

3. The products of groups Z_n are also number theoretically in a very special position since they relate naturally to the finite cyclic extensions and also to the maximal Abelian extension of rationals. With this restriction on $G_a \times G_b$ one can consider the hypothesis that elementary particles correspond are maximally quantum critical systems left invariant by all groups $G_a \times G_b$ respecting a given choice of quantization axis and implying that darkness is associated only to field bodies and Planck constant becomes characterizer of interactions rather than elementary particles themselves.

3 Some Mathematical Speculations

3.1 The Content Of Mckay Correspondence In TGD Framework

The possibility to assign Dynkin diagrams with the inclusions of II_1 algebras is highly suggestive concerning possible physical interpretations. The basic findings are following.

1. For $\beta = \mathcal{M} : \mathcal{N} < 4$ Dynkin diagrams code for the inclusions and correspond to simply laced Lie algebras. $SU(2)$, D_{2n+1}, and E_7 are excluded.

2. Extended ADE Dynkin diagrams coding for simply laced ADE Kac Moody algebras appear at $\beta = 4$. Also $SU(2)$ Kac Moody algebra appears.

3.1.1 Does TGD give rise to ADE hierarchy of gauge theories

The first question is whether any finite subgroup $G \subset SU(2)$ acting in CP_2 degrees of freedom could somehow give rise to multiplets of the corresponding gauge group having interactions described by a gauge theory. Orbifold picture suggests that might be the case.

1. The "sheets" for the space-time sheet forming an $N(G)$-fold cover of CD are in one-one correspondence with group G. This degeneracy gives rise to additional states and these states correspond to the group algebra having basis given by group characters $\chi(g)$. One obtains irreducible representations of G with degeneracies given by their dimensions. Altogether one obtains $N(G)$ states in this manner. In the case of $A(n)$ the number of these states is $n + 1$, the number of the states of the fundamental representation of $SU(n + 1)$. In the same manner, for D_{2n} the number of these states equals to the number of states in the fundamental representation of D_{2n}. It seems that the rule is quite general. Thus these representations would in the case of fermions give the states of the fundamental representation of the corresponding gauge group.

2. From fermion and anti-fermion states one can construct in a similar manner pairs giving $N(G)^2$ states defining in the case of $A(n)$ n^2-1-dimensional gauge boson multiplet plus singlet. Also other groups must give boson multiplet plus possible other multiplets. For instance, for $D(4)$ the number of states is 64 and boson multiplet is 8-dimensional so that many other spin 1 states result.

3. These findings give hopes that the orbifold multiplets could be modelled by a gauge theory based on corresponding gauge group. What is nice that this huge hierarchy of gauge theories is associated with dark matter so that the predictivity and falsifiability are not lost unlike in M-theory.

3.1.2 Does one obtain also a hierarchy of conformal theories with ADE Kac Moody symmetry?

Consider next the question Kac Moody interactions correspond to extended ADE diagrams are possible.

1. In this case the notion of orbifold seems to break down since the symmetry related points form a continuum $SU(2)$ and space-time surface would become 6-dimensional if the CD projection is 4-dimensional. If one takes space-time as something which emerges, one could take this possibility half seriously. A more natural natural possibility is that CD projection is 2-dimensional geodesic sphere in which case one would have string like objects so that conformal field theory with Kac-Moody algebra would emerge naturally.

2. The new degrees of freedom would define 2-dimensional continuum and it would not be completely surprising if conformal field theory based on ADE Kac Moody algebra could describe the situation. One possibility is that these continua for different inclusions correspond to $SU(2)$ decompose to an $N(G)$-fold covers of S^2/G orbifold so that also now groups G would be involved with the Jones inclusions, which might provide a hint about how to construct them. S^2/G would play the role of stringy world sheet for the conformal field theory in question. This effective re-arrangement of the topology S^2 might be due to the fact that conformal fields possess G symmetry which effectively groups points of S^2 to $n(G)$-multiplets. The localized representations of the Lie group corresponding to G would correspond to the multiplets obtained from the representations of group algebra of G as in previous case.

3. The formula for the scaling factor of CD metric would give infinite scaling factor if one identifies the scaling factor as maximal order of cyclic subgroup of $SU(2)$. As a matter fact there is no finite cyclic subgroup of this kind. The solution to the problem would be identification of the scaling factor as the order of the maximal cyclic subgroup of G so that the scaling factors would be same for the two situations related by McKay correspondence.

3.1.3 Generalization to CD degrees of freedom

One can ask whether the proposed picture generalizes formally also the case of CD.

1. In this case quantum groups would correspond to discrete subgroups $G \subset SL(2,C)$. Kac Moody group would correspond to G-Kac Moody algebra made local with respect to $SL(2,C)$ orbit in CD divided by G. These orbits are 3-dimensional hyperboloids H_a with a constant value of light cone proper time a so that the division by G gives fundamental domain H_a/G with a finite 3-volume.

2. The 4-dimensionality of space-time would require 1-dimensional CP_2 projection. Vacuum extremals of Kähler action would be in question. Robertson-Walker metric have 1-dimensional CP_2 projection and carry non-vanishing density of gravitational mass so that in this sense the theory would be non-trivial. G would label different lattice like cosmologies defined by tesselations with fundamental domain H_a/G.

3. The multiplets of G would correspond to collections of points, one from each cells of the lattice like structure. Macroscopic quantum coherence would be realized in cosmological scales. If one takes seriously the vision about the role of short distance p-adic physics as a

generator of long range correlations of the real physics reflected as p-adic fractality, this idea does not look so weird anymore.

Complexified modular group $SL(2, Z+iZ)$ and its subgroups are interesting as far as p-adicization is considered. The principal congruence subgroups $\Gamma(N)$ of $SL(2, Z+iZ)$ which are unit matrices modulo N define normal subgroups of the complex modular group and are especially interesting candidates for groups $G \subset SL(2, C)$. The group $\Gamma(N = p^k)$ labeling fundamental domains of the tesselation $H_a/\Gamma(N = p^k)$ defines a mathematically attractive candidate for a point set associated with the intersections of p-adic space-time sheets with real space-time sheets. Also analogous groups for algebraic extensions of Z are interesting.

The simplest discrete subgroup of SL(2, C) with infinite number of elements would corresponds to powers of boost to single direction and correspond at the non-relativistic limit to multiples of basic velocity. This could also give rise to quantization of cosmic recession velocities. There is evidence for the quantization of cosmic recession velocities (for a model in which single object produces quantized redshifts see [K4]) and it is interesting to see whether they could be interpreted in terms of the lattice like periodicity in cosmological length scales implied by the effective reduction of physics to M^4_+/G_n. In [E1] the values $z = 2.63, 3.45, 4.47$ of cosmic red shift are listed. These correspond to recession velocities $v = (z^2-1)/(z^2+1)$ are (0.75, 0.85, 0.90). The corresponding hyperbolic angles are given by $\eta = acosh(1/(1-v^2))$ and the values of η are $(1.46, 1.92, 2.39)$. The differences $\eta(2) - \eta(1) = .466$ and $\eta(3) - \eta(2) = .467$ are same within experimental uncertainties. One has however $\eta(n)/(\eta(2) - \eta(1)) = (3.13, 4.13, 5.13)$ instead of $(3, 4, 5)$. A possible interpretation is in terms of the velocity of the observer with respect to the frame in which quantization of η happens.

3.1.4 Quantitative support for the interpretation

A more detailed analysis of the situation gives support for the proposed vision.

1. A given value of quantum group deformation parameter $q = exp(i\pi/n)$ makes sense for any Lie algebra but now a preferred Lie-algebra is assigned to a given value of quantum deformation parameter. At the limit $\beta = 4$ when quantum deformation parameter becomes trivial, the gauge symmetry is replaced by Kac Moody symmetry.

2. The prediction is that Kac-Moody central extension parameter should vanish for $\beta < 4$. There is an intriguing relationship to formula for the quantum phase q_{KM} associated with (possibly trivial) Kac-Moody central extension and the phase defined by ADE diagram

$$\begin{array}{ll} q_{KM} = exp(i\phi) \ , & \phi_1 = \frac{\pi}{k+h^v} \ , \\ q_{Jones} = exp(i\phi) \ , & \phi = \frac{\pi}{h} \end{array}$$

In the first formula sum of Kac-Moody central extension parameter k and dual Coxeter number h^v appears whereas Coxeter number h appears in the second formula. Internal consistency requires

$$k + h^v = h \ . \qquad (3.1)$$

It is easy see that the dual Coxeter number h^v and Coxeter number h given by $h = (dim(g) - r)/r$, where r is the dimension of Cartan algebra of g, are identical for ADE algebras so that the Kac-Moody central extension parameter k must indeed vanish. For $SO(2n+1)$, $Sp(n)$, G_2, and F_4 the condition $h = h^v$ does not hold true but one has $h(n) = 2n = h^v + 1$ for $SO(2n+1)$, $h(n) = 2n = 2(h^v - 1)$ for $Sp(n)$, $h = 6 = h^v + 2$ for G_2, and $h = 12 = h^v + 3$ for F_4.

What is intriguing that G_2, which seems to play a fundamental role in the dual formulation of quantum TGD based on the identification of space-times as surfaces in hyper-octonionic space M^8 [K9] is not allowed. As a matter fact, $G_2 \rightarrow SU(3)$ reduction occurs also in the dual formulation based on $G_2/SU(3)$ coset model and is required by the separate conservation of quark and lepton numbers predicted by TGD. ADE groups would be associated with the interaction between space-time sheets rather than entire dynamics and need not have anything to do with the Kac-Moody algebra associated with color and electro-weak interactions appearing in the construction of physical states [K6].

3. There seems to be a concrete connection with conformal field theories. This connection would allow to understand the emergence of quantum groups appearing naturally in these theories. Quite generally, the conformal central extension parameter for unitary Virasoro representations resulting by Sugawara construction from Kac Moody representations satisfies either of the conditions

$$\begin{aligned} c &\geq \frac{kdim(g)}{k+h^v}+1 \ , \\ c &= \frac{kdim(g)}{k+h^v}+1-\frac{6}{(h-1)h} \ . \end{aligned} \tag{3.2}$$

For $k=0$, which should be interesting for $\beta<4$, the second formula reduces to

$$c = 1-\frac{6}{(h-1)h} \ . \tag{3.3}$$

The formula gives the values of c for minimal conformal field theories with finite number of conformal fields and real conformal weights. Indeed, h in this formula seems to correspond to the same h as appearing in the expression $\beta \equiv \mathcal{M}:\mathcal{N} = 4cos^2(\pi/h)$.

$\beta=3, h=6$ corresponds to three-state Potts model with $c=4/5$ which should thus have a gauge group for which Coxeter number is 6: the group should be either $SU(6)$ or $SO(8)$. Two-state Potts model, that is Ising model with $\beta=2, h=4$ would correspond to $c=1/2$ and to a gauge group $SU(4)$ or $SO(4)$. For $h=3$ ("one-state Potts model") with group $SU(3)$ one would have $c=0$ and vanishing conformal anomaly so that conformal degrees of freedom would become pure gauge degrees of freedom.

These observations give support for the following picture.

1. Quite generally, the number of states of the generalized β-state Potts model has an interpretation as the dimension $\beta=\mathcal{M}:\mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module. Besides the models with integer number of states there is an infinite number of models for which the number of states is not an integer. The conditions $c \leq 1$ guaranteeing real conformal weights and $\beta \leq 4$ correspond to each other for these models.

2. $\beta>4$ Potts models would be formally obtained by allowing h to be imaginary in the defining formula for $\mathcal{M}:\mathcal{N}$. In this case c would be however complex so that the theory would not be unitary.

3. For minimal models with $(\beta<4, c<1)$ Kac-Moody central extension parameter is vanishing so that Kac Moody algebra indeed acts like gauge symmetries and gauge symmetries would be in question. $(\beta=4, c=1)$ would define a "four-state Potts model" with infinite-dimensional unitary group acting as a gauge group. On the other hand, the appearance of extended ADE Dynkin diagrams suggests strongly that this limit is not realized but that $\beta=\mathcal{M}:\mathcal{N}=4$ corresponds to $k=1$ conformal field theory allowing Kac Moody symmetries for any ADE group, which as simply-laced groups allows vertex operator construction. The appearance of $kdim(g)/(k+g)$ in the more general formula would thus code the Kac Moody group whereas for $\beta<4$ ADE diagram codes for the preferred gauge group characterizing the minimal CFT.

4. The possibility that any ADE gauge group or Kac-Moody group can characterize the interaction between space-time sheets conforms with the idea about Universe as a Topological Quantum Computer able to simulate any conceivable quantum dynamics. Of course, one cannot exclude the possibility that only electro-weak and color symmetries are realized in this manner.

3.1.5 G_a as a symmetry group of magnetic body and McKay correspondence

The group $G_a \subset SU(2) \subset SL(2,C)$ means exact rotational symmetry realized in terms of CD coverings of CP_2. The 5 and 6-cycles in biochemistry (sugars, DNA, ...) are excellent candidates for these symmetries. For very large values of Planck constant, say for the values $\hbar(CD)/\hbar(CP_2) = GMm/v_0 = (n_a/n_b)\hbar_0$, $v_0 = 2^{-11}$, required by the model for planetary orbits as Bohr orbits [K7], G_a is huge and corresponds to either Z_{n_a} or in the case of even value of n_a to the group generated by Z_n and reflection acting on plane and containing $2n_a$ elements.

The notion of magnetic body seems to provide the only conceivable candidate for a geometric object possessing G_a as symmetries. In the first approximation the magnetic field associated with a dark matter system is expected to be modellable as a dipole field having rotational symmetry around the dipole axis. Topological quantization means that this field decomposes into flux tube like structures related by the rotations of Z_n or D_{2n}. Dark particles would have wave functions de-localized to this set of these flux quanta and span group algebra of G_a. Magnetic flux quanta are indeed assumed to mediate gravitational interactions in the TGD based model for the quantization of radii of planetary orbits and this explains the dependence of $\hbar_{gr}$ on the masses of planet and central object [K7].

For the model of dark matter hierarchy appearing in the model of living matter one has $n_a = 2^{11k}$, $k = 1,2,3,..,7$ for cyclotron time scales below life cycle for a magnetic field $B_d = .2$ Gauss at $k = 4$ level of hierarchy (the field strength is fixed by the model for the effects of ELF em fields on vertebrate brain at harmonics of cyclotron frequencies of biologically important ions [K5]). Note that B_d scales as 2^{-11k} from the requirement that cyclotron energy is constant.

ADE correspondence between subgroups of $SU(2)$ and Lie groups in ADE hierarchy encourages to consider the possibility that TGD could mimic ADE hierarchy of gauge theories. In the case of G_a this would mean that many fermion states constructed from single fermion states, which are in one-one correspondence with the elements of G_a group algebra, would define multiplets of the gauge group corresponding to the Dynkin diagram characterizing G_a: for instance, $SU(n_a)$ in the case of Z_{n_a}. Fermion multiplet would contain n_a states and gauge boson multiplet $n_a^2 - 1$ states. This would provide enormous information processing capacity since for $n_a = 2^{11k}$ fermion multiplet would code exactly $11k$ bits of information. Magnetic body could represent binary information using the many-particle states belonging to the representations of say $SU(n_a)$ at its flux tubes.

3.2 Jones Inclusions, The Large N Limit Of $SU(N)$ GaugeTheories and AdS/CFT Correspondence

The framework based on Jones inclusions has an obvious resemblance with larger N limit of $SU(N)$ gauge theories and also with the celebrated AdS/CFT correspondence [B1] so that a more detailed comparison is in order.

3.2.1 Large N limit of gauge theories and series of Jones inclusions

The large N limit of $SU(N)$ gauge field theories has as definite resemblance with the series of Jones inclusions with the integer $n \geq 3$ characterizing the quantum phase $q = exp(i\pi/n)$ and the order of the maximal cyclic subgroup of the subgroup of $SU(2)$ defining the inclusion. Recall that all ADE groups except D_{2n+1} and E_7 are allowed (SU(2) is excluded since it would correspond to $n = 2$).

The limiting procedure keeps the value of g^2N fixed. Rather remarkably, this is equivalent with keeping αN constant but assuming $\hbar$ to scale as $n = N$. Thus the quantization of Planck constants would provide a physical laboratory for the testing of large N limit.

The observation suggesting a description of YM theories in terms of closed strings is that Feynman diagrams can be interpreted as being imbedded at closed 2-surfaces of minimal genus guaranteeing that the internal lines meet except in vertices. The contribution of genus g diagrams is proportional to N^{g-1} at the large N limit. The interpretation in terms of closed partonic 2-surfaces is highly suggestive and the N^{g-1} should come from the multiple covering property of CP_2 by N CD-points (or vice versa) with the finite subgroup of $G \subset SU(2)$ defining the Jones inclusion and acting as symmetries of the surface.

3.2.2 Analogy between stacks of branes and multiple coverings of CD and CP_2

An important aspect of AdS/CFT dualities is a prediction of an infinite hierarchy of gauge groups, which as such is as interesting as the claimed dualities. The prediction relies on the notion Dp-branes. Dp-branes are $p+1$-dimensional surfaces of the target space at which the ends of open strings can end. In the simplest situation one considers N parallel p-branes at the limit when the distances between branes characterized by an expectation value of Higgs fields approach zero to obtain what is called N-stack of branes. There are N^2 different strings connecting the branes and the heuristic idea is that they correspond to gauge bosons of $U(N)$ gauge theory. Note that the requirement that AdS/CFT dualities exist forces the introduction of branes and the optimistic interpretation is that a non-perturbative effect of still unknown M-theory is in question. In the limit of an ideal stack one assumes that $U(N)$ gauge theory at the brane representing the stack is obtained. The branes must also carry a p-form defining gauge potential for a closed $p+1$-form. This Ramond charge is quantized and its value equals to N.

Consider now the group $G_a \times G_b \subset SL(2,C) \times SU(2) \subset SU(3)$ defining double Jones inclusion and implying the scalings $\hbar(M^4) \rightarrow n(G_b)\hbar(M^4)$ and $\hbar(CP_2) \rightarrow n(G_a)\hbar(CP_2)$. These space-time surfaces define $n(G_a)$-fold multiple coverings of CP_2 and $n(G_b)$-fold multiple coverings of CD. In CP_2 degrees of freedom the collection of G_b-related partonic 2-surfaces (/3-surfaces/4-surfaces) is highly analogous to the stack of branes. In CD degrees of freedom the stack of copies of surface typically correspond to along a circle (A_n, D_{2n} or at vertices of tetrahedron or isosahedron.

In TGD framework the interpretation strings are not needed to define gauge fields. The group algebra of G realized as discrete plane waves at G-orbit gives rise to representations of G. The hypothesis supported by few examples is that these additional degrees of freedom allow to construct multiplets of the gauge group assignable to the ADE diagram characterizing the inclusion.

3.2.3 AdS/CFT duality

AdS/CFT duality is a further aspect of the brane construction. The dual description of the situation is in terms of a string theory in a background in which N-brane acts as a macroscopic object giving rise to a black-hole like object in (say) 10-dimensional target space. This background has the form $AdS_5 \times X_5$, where AdS_5 is 5-dimensional hyperboloid of M^6 and thus allows $SO(4,2)$ as isometries. X_5 is compact constant curvature space. S_5 gives rise to $N=4$ SUSY in M^4 with M^4 interpreted as a brane. The first support for the dualities comes from the symmetries: for instance, the $N=4$ super-symmetrized isometries of $AdS_5 \times S^5$ are same as the symmetries of 4-dimensional $N=4$ SUSY for $p=3$ branes. N-branes can be used as models for black holes in target space and black-hole entropy can be calculated using either target space picture or conformal field theory at brane and the results turn out be the same.

Does the TGD equivalent of this duality exists in some sense?

1. As far as partonic 2-surfaces identified as 1-branes are considered, conformal field theory description is trivially true. In TGD framework the analog of Ramond charges are the integers n_a and n_b characterizing the multiplitICIES of the maximal Abelian subgroups having clear topological meaning. This conforms with the observation that large N limit of the gauge field theories can be formulated in terms of closed surfaces at which the Feynman diagrams are imbedded without self crossings. It seems that the integers n_a and n_b characterizing the Jones inclusion naturally take the role of Ramond charge: this does not of course exclude the possibility they can be expressed as fluxes at space-time level as will be indeed found.

2. Conformal field theory description can be generalized in the sense that one replaces the $n(G_a) \times n(G_b)$ partonic surfaces with single one and describes the new states as primary fields arranged into representations of the ADE group in question. This would mean that

the standard model gauge group extends by additional factor which is however non-trivially related to it.

3. If one can accept the idea that the conformal field theory description for partons gives rise to M^4 gauge theory as an approximate description, it is not too difficult to imagine that also ADE hierarchy of gauge theories results as a description of the exotic states. One can say that CFT in p-brane is replaced now with CFT on partonic 2-surface (1-brane) analogous to a closed string.

4. In the minimal interpretation there is no need to add strings connecting the branches of the double covering of the partonic 2-surface whose function is essentially that of making possible gauge bosons as fermion anti-fermion pairs. One could of course imagine gauge fluxes as counterparts of strings but just the fact that G-invariance dictates the configurations completely forces to question this kind of dynamics.

5. There is no reason to expect the emergence of $N = 4$ super-symmetric field theory in M^4 as in the case of super-string models. The reasons should be already obvious: super-conformal generators G anti-commute to L_0 proportional to mass squared rather than four-momentum and the spectrum extended by $G_a \times G_b$ degeneracy contains more states.

One can of course ask whether higher values of p could make sense in TGD framework.

1. It seems that the light-like orbits of the partonic 2-surfaces defining 2-branes do not bring in anything new since the generalized conformal invariance makes it possible the restriction to a 2-dimensional cross section of the light like causal determinant.

2. The idea of regarding space-time surface X^4 as a 3-brane in H in which some kind of conformal field theory is defined is in conflict with the basis ideas of TGD. The role of X^4 interior is to provide classical correlates for quantum dynamics to make possible quantum measurement theory and also introduce correlations between partonic 2-surfaces even in the case that partonic conformal dynamics reduces to a topological string theory. It is quantum classical correspondence which corresponds to this duality.

3.2.4 What is the counterpart of the Ramond charge in TGD?

The condition that there exist a p-form defining $p + 1$-gauge field with p-charge equal to n_a or n_b is a rather stringent additional condition also in TGD framework. For $n < \infty$ this kind of charge is defined by Jones inclusion and represented topologically so that Ramond charge is not needed in $n < \infty$ case. By the earlier arguments one must however be able to assign integers n_a and n_b also to $G = SU(2)$ inclusions with Kac-Moody algebra characterized by an extended ADE diagram with the phases $q_i = exp(i\pi/n_i)$ relating to the monodromy of the theory. Since Jones inclusion does not define in this case the value of $n < \infty$ in any obvious manner, the counterpart of the Ramond charge is needed.

1. For partonic 2-surfaces ordinary gauge potential would define this form and the condition would state that magnetic flux equals to n so that the anyonic partonic two-surfaces would be homologically non-trivial in CP_2 degrees of freedom. String ends would define basic example of this situation. This would be the case also in M^4_+ degrees of freedom: the partonic 2-surface would essentially wind n_a times around the tip of δCD and the gauge field in question would be monopole magnetic field in δCD. This kind of situation need not correspond to anything cosmological since future and past light-cones appear in the basic definition of the scattering amplitudes.

2. For $p = 3$ Chern-Simons action for the induced CP_2 Kähler form associated with the partonic 2-surface indeed defines this kind of charge. Ramond charge should be simply N. CP_2 type extremals or their small deformations satisfy this constraint and are indeed very natural in elementary particle physics context but too restrictive in a more general context.

Note that the light-like orbits of non-deformed CP_2 extremals have light-like random curve as an M^4 projection and the conformal symmetries of M^4 obviously respect light-likeness property. Hence $SO(4,2)$ symmetry characterizing AdS$_5$/CFT is not excluded but would be broken by p-adic thermodynamics and by TGD based Higgs mechanism involving the identification of inertial momentum as average value of non-conserved gravitational momentum parallel to the light-like zitterbewegung orbit.

3.2.5 Can one speak about black hole like structures in TGD framework?

For AdS/CFT correspondence there is also a dynamical coupling to the target space metric. The coupling to H-metric is present also now since the overall scalings of the CD *resp.* CP_2 metrics by n_b *resp.* by n_a are involved. This applies to when multiple covering is used explicitly. In the description in which one replaces the multiple covering by ordinary $M^4 \times CP_2$, the metric suffers a genuine change and something analogous to the black-hole type metrics encountered in AsS/CFT correspondence might be encountered.

Consider as an example an n_a-fold covering of CP_2 points by M^4 points (ADE diagram A_{n_a-1}). The n-fold covering means only $n2\pi$ rotation for the phase angle ψ of CP_2 complex coordinate leads to the original point. The replacement $\psi \to \psi/n_a$ gives rise to what would look like ordinary $M^4 \times CP_2$ but with a modified CP_2 metric. The metric components containing ψ as index are scaled down by $1/n_a$ or $1/n_a^2$. Notice that Ψ effectively disappears from the dynamics at the large n_a limit.

If one uses an effective description in which covering is eliminated the metric is indeed affected at the level of imbedding space black hole like structures at the level of dynamic space might make emerge also in TGD framework at large N limit since the masses of the objects in question become large and CP_2 metric is scaled by N so that CP_2 has very large size at this limit. This need not lead to any inconsistencies if these phases are interpreted as dark matter. At the elementary particle level p-adic thermodynamics predicts that p-adic entropy is proportional to thermal mass squared which implies elementary particle black-hole analogy.

3.2.6 Other dualities

Also quantum classical correspondence defines in a loose sense a duality justifying the basic assumptions of quantum measurement theory. The light-like orbits of 2-D partons are characterized by a generalization of ordinary 2-D conformal invariance so that CFT part of the duality would be very natural. The dynamical target space would be replaced with the space-time surface X^4 with a dynamical metric providing classical correlates for the quantum dynamics at partonic 2-surfaces. The duality in this sense cannot be however exact since classical dynamics cannot fully represent quantum dynamics.

Classical description is not expected to be unique. The basic condition on space-time surfaces assignable to a given configuration of partonic 2-surfaces associated with the surface X^3_V defining S-matrix element are posed by quantum classical correspondence. Both hyper-quaternionic and co-hyper-quaternionic space-time surfaces are acceptable and this would define a fundamental duality.

A concrete example about this HQ-coHQ duality would be the equivalence of space-time descriptions using 4-D CP_2 type extremals and 4-D string like objects connecting them. If one restricts to CP_2 type extremals and string like objects of from $X^2 \times Y^2$, the target space reduces effectively to M^4 and the dynamical degrees of freedom correspond in both cases to transversal M^4 degrees of freedom. Note that for CP_2 type extremals the conditions stating that random light-likeness of the M^4 projection of the CP_2 type extremal are equivalent to Virasoro conditions. CP_2 type extremals could be identified as co-HQ surfaces whereas stringlike objects would correspond to HQ aspect of the duality.

HQ-coHQ provides dual classical descriptions of same phenomena. Particle massivation would be a basic example. Higgs mechanism in a gauge theory description based on CP_2 type extremals would rely on zitterbewegung implying that the average value of gravitational mass identified as inertial mass is non-vanishing and is discussed already. Higgs field would be assigned to the wormhole contacts. The dual description for the massivation would be in terms of string tension and mass squared would be proportional to the distance between G-related points of CP_2.

These observations would suggest that also a super-conformal algebra containing $SL(2,R) \times SU(2)_L \times U(1)$ or its compact version exists and corresponds to a trivial inclusion. This is indeed the case [A3]. The so called large $N = 4$ super-conformal algebra contains energy momentum current, 2+2 super generators G, $SU(2) \times SU(2) \times U(1)$ Kac-Moody algebra (both $SU(2)$ and SL(2, R) could be interpreted as acting on M^4 spin degrees of freedom, and 2 spin 1/2 fermionic currents having interpretation in terms of right handed neutrinos corresponding to two H-chiralities. Interestingly, the scalar generator is now missing.

3.3 Could Mckay Correspondence And Jones Inclusions Relate To Each Other?

The understanding of Langlands correspondence for general reductive Lie groups in TGD framework seems to require some physical mechanism allowing the emergence of these groups in TGD based physics. The physical idea would be that quantum dynamics of TGD is able to emulate the dynamics of any gauge theory or even stringy dynamics of conformal field theory having Kac-Moody type symmetry and that this emulation relies on quantum deformations induced by finite measurement resolution described in terms of Jones inclusions of sub-factors characterized by group G leaving elements of sub-factor invariant. Finite measurement resolution would would result simply from the fact that only quantum numbers defined by the Cartan algebra of G are measured.

There are good reasons to expect that infinite Clifford algebra has the capacity needed to realize representations of an arbitrary Lie group. It is indeed known that that any quantum group characterized by quantum parameter which is root of unity or positive real number can be assigned to Jones inclusion [A5]. For $q = 1$ this would gives ordinary Lie groups. In fact, all amenable groups define unique sub-factor and compact Lie groups are amenable ones.

It was so called McKay correspondence [A8] which originally stimulated the idea about TGD as an analog of Universal Turing machine able to mimic both ADE type gauge theories and theories with ADE type Kac-Moody symmetry algebra. This correspondence and its generalization might also provide understanding about how general reductive groups emerge. In the following I try to cheat the reader to believe that the tensor product of representations of SU(2) Lie algebras for Connes tensor powers of $\mathcal{M}$ could induce ADE type Lie algebras as quantum deformations for the direct sum of n copies of $SU(2)$ algebras This argument generalizes also to the case of other compact Lie groups.

3.3.1 About McKay correspondence

McKay correspondence [A8] relates discrete finite subgroups of $SU(2)$ ADE groups. A simple description of the correspondences is as follows [A8].

1. Consider the irreps of a discrete subgroup $G \subset SU(2)$ which correspond to irreps of G and can be obtained by restricting irreducible representations of $SU(2)$ to those of G. The irreducible representations of $SU(2)$ define the nodes of the graph.

2. Define the lines of graph by forming a tensor product of any of the representations appearing in the diagram with a doublet representation which is always present unless the subgroup is 2-element group. The tensor product regarded as that for $SU(2)$ representations gives representations $j - 1/2$, and $j + 1/2$ which one can decompose to irreducibles of G so that a branching of the graph can occur. Only branching to two branches occurs for subgroups yielding extended ADE diagrams. For the linear portions of the diagram the spins of corresponding SU(2) representations increase linearly as $.., j, j + 1/2, j + 1, ...$

One obtains extended Dynkin diagrams of ADE series representing also Kac-Moody algebras giving A_n, D_n, E_6, E_7, E_8. Also A_∞ and $A_{-\infty,\infty}$ are obtained in case that subgroups are infinite. The Dynkin diagrams of non-simply laced groups B_n ($SO(2n + 1)$), C_n (symplectic group $Sp(2n)$ and quaternionic group $Sp(n)$), and exceptional groups G_2 and F_4 are not obtained.

ADE Dynkin diagrams labeling Lie groups instead of Kac-Moody algebras and having one node less, do not appear in this context but appear in the classification of Jones inclusions for

$\mathcal{M} : \mathcal{N} < 4$. As a matter fact, ADE type Dynkin diagrams appear in very many contexts as one can learn from John Baez's This Week's Finds [A2] .

1. The classification of integral lattices in Rn having a basis of vectors whose length squared equals 2
2. The classification of simply laced semisimple Lie groups.
3. The classification of finite sub-groups of the 3-dimensional rotation group.
4. The classification of simple singularities. In TGD framework these singularities could be assigned to origin for orbifold CP_2/G, $G \subset SU(2)$.
5. The classification of tame quivers.

3.3.2 Principal graphs for Connes tensor powers $\mathcal{M}$

The thought provoking findings are following.

1. The so called principal graphs characterizing $\mathcal{M} : \mathcal{N} = 4$ Jones inclusions for $G = SU(2)$ are extended Dynkin diagrams characterizing ADE type affine (Kac-Moody) algebras. D_n is possible only for $n \geq 4$.
2. $\mathcal{M} : \mathcal{N} < 4$ Jones inclusions correspond to ordinary ADE type diagrams for a subset of simply laced Lie groups (all roots have same length) A_n $(SU(n))$, D_{2n} $(SO(2n))$, and E_6 and E_8. Thus D_{2n+1} $(SO(2n+2))$ and E_7 are not allowed. For instance, for $G = S_3$ the principal graph is not D_3 Dynkin diagram.

The conceptual background behind principal diagram is necessary if one wants to understand the relationship with McKay correspondence.

1. The hierarchy of higher commutations defines an invariant of Jones inclusion $\mathcal{N} \subset \mathcal{M}$. Denoting by $\mathcal{N}'$ the commutant of $\mathcal{N}$ one has sequences of horizontal inclusions defined as $C = \mathcal{N}' \cap \mathcal{N} \subset \mathcal{N}' \cap \mathcal{M} \subset \mathcal{N}' \cap \mathcal{M}^1 \subset ...$ and $C = \mathcal{M}' \cap \mathcal{M} \subset \mathcal{M}' \cap \mathcal{M}^1 \subset ....$ There is also a sequence of vertical inclusions $\mathcal{M}' \cap \mathcal{M}^k \subset \mathcal{N}' \cap \mathcal{M}^k$. This hierarchy defines a hierarchy of Temperley-Lieb algebras [A7] assignable to a finite hierarchy of braids. The commutants in the hierarchy are direct sums of finite-dimensional matrix algebras (irreducible representations) and the inclusion hierarchy can be described in terms of decomposition of irreps of k^{th} level to irreps of $(k-1)^{th}$ level irreps. These decomposition can be described in terms of Bratteli diagrams [A4].
2. The information provided by infinite Bratteli diagram can be coded by a much simpler bi-partite diagram having a preferred vertex. For instance, the number of $2k$-loops starting from it tells the dimension of k^{th} level algebra. This diagram is known as principal graph.

Principal graph emerges also as a concise description of the fusion rules for Connes tensor powers of $\mathcal{M}$.

1. It is natural to decompose the Connes tensor powers [A8] $\mathcal{M}_k = \mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$ to irreducible $\mathcal{M} - \mathcal{M}$, $\mathcal{N} - \mathcal{M}$, $\mathcal{M} - \mathcal{N}$, or $\mathcal{N} - \mathcal{N}$ bi-modules. If $\mathcal{M} : \mathcal{N}$ is finite this decomposition involves only finite number of terms. The graphical representation of these decompositions gives rise to Bratteli diagram.
2. If $\mathcal{N}$ has finite depth the information provided by Bratteli diagram can be represented in nutshell using principal graph. The edges of this bipartite graph connect $\mathcal{M} - \mathcal{N}$ vertices to vertices describing irreducible $\mathcal{N} - \mathcal{N}$ representations resulting in the decomposition of $\mathcal{M} - \mathcal{N}$ irreducibles. If this graph is finite, $\mathcal{N}$ is said to have finite depth.

3.3.3 A mechanism assigning to tensor powers Jones inclusions ADE type gauge groups and Kac-Moody algebras

The earliest proposals inspired by the hierarchy of Jones inclusions is that in $\mathcal{M}:\mathcal{N}<4$ case it might be possible to construct ADE representations of gauge groups or quantum groups and in $\mathcal{M}:\mathcal{N}=4$ using the additional degeneracy of states implied by the multiple-sheeted cover $H\to H/G_a\times G_b$ associated with space-time correlates of Jones inclusions. Either G_a or G_b would correspond to G. In the following this mechanism is articulated in a more refined manner by utilizing the general properties of generators of Lie-algebras understood now as a minimal set of elements of algebra from which the entire algebra can be obtained by repeated commutation operator (I have often used " Lie algebra generator" as an synonym for "Lie algebra element"). This set is finite also for Kac-Moody algebras.

1. Two observations

The explanation to be discussed relies on two observations.

1. McKay correspondence for subgroups of G ($\mathcal{M}:\mathcal{N}=4$) *resp.* its variants ($\mathcal{M}:\mathcal{N}<4$) and its counterpart for Jones inclusions means that finite-dimensional irreducible representations of allowed $G\subset SU(2)$ label both the Cartan algebra generators and the Lie (Kac-Moody) algebra generators of t_+ and t_- in the decomposition $g=h\oplus t_+\oplus t_-$, where h is the Lie algebra of maximal compact subgroup.

2. Second observation is related to the generators of Lie-algebras and their quantum counterparts (see Appendix for the explicit formulas for the generators of various algebras considered). The observation is that each Cartan algebra generator of Lie- and quantum group algebras, corresponds to a triplet of generators defining an SU(2) sub-algebra. The Cartan algebra of affine algebra contains besides Lie group Cartan algebra also a derivation d identifiable as an infinitesimal scaling operator L_0 measuring the conformal weight of the Kac-Moody generators. d is exceptional in that it does not give rise to a triplet. It corresponds to the preferred node added to the Dynkin diagram to get the extended Dynkin diagram.

2. Is ADE algebra generated as a quantum deformation of tensor powers of $SU(2)$ Lie algebras representations?

The ADE type symmetry groups could result as an effect of finite quantum resolution described by inclusions of HFFs in TGD inspired quantum measurement theory.

1. The description of finite resolution typically leads to quantization since complex rays of state space are replaced as $\mathcal{N}$ rays. Hence operators, which would commute for an ideal resolution cease to do so. Therefore the algebra $SU(2)\otimes ...\otimes SU(2)$ characterized by n mutually commuting triplets, where n is the number of copies of $SU(2)$ algebra in the original situation and identifiable as quantum algebra appearing in $\mathcal{M}$ tensor powers with $\mathcal{M}$ interpreted as $\mathcal{N}$ module, could suffer quantum deformation to a simple Lie algebra with $3n$ Cartan algebra generators. Also a deformation to a quantum group could occur as a consequence.

2. This argument makes sense also for discrete groups $G\subset SU(2)$ since the representations of G realized in terms of WCW spinor s extend to the representations of SU(2) naturally.

3. Arbitrarily high tensor powers of $\mathcal{M}$ are possible and one can wonder why only finite-dimensional Lie algebra results. The fact that $\mathcal{N}$ has finite depth as a sub-factor means that the tensor products in tensor powers of $\mathcal{N}$ are representable by a finite Dynkin diagram. Finite depth could thus mean that there is a periodicity involved the kn tensor powers decomposes to representations of a Lie algebra with $3n$ Cartan algebra generators. Thus the additional requirement would be that the number of tensor powers of $\mathcal{M}$ is multiple of n.

3. Space-time correlate for the tensor powers $\mathcal{M}\otimes_{\mathcal{N}}...\otimes_{\mathcal{N}}\mathcal{M}$

By quantum classical correspondence there should exist space-time correlate for the formation of tensor powers of $\mathcal{M}$ regarded as $\mathcal{N}$ module. A concrete space-time realization for this kind of situation in TGD would be based on n-fold cyclic covering of H implied by the $H \to H/G_a \times G_b$ bundle structure in the case of say G_b. The sheets of the cyclic covering would correspond to various factors in the n-fold tensor power of $SU(2)$ and one would obtain a Lie algebra, affine algebra or its quantum counterpart with n Cartan algebra generators in the process naturally. The number n for space-time sheets would be also a space-time correlate for the finite depth of $\mathcal{N}$ as a factor.

WCW spinors could provide fermionic representations of $G \subset SU(2)$. The Dynkin diagram characterizing tensor products of representations of $G \subset SU(2)$ with doublet representation suggests that tensor products of doublet representations associated with n sheets of the covering could realize the Dynkin diagram.

Singlet representation in the Dynkin diagram associated with irreps of G would not give rise to an SU(2) sub-algebra in ADE Lie algebra and would correspond to the scaling generator. For ordinary Dynkin diagram representing gauge group algebra scaling operator would be absent and therefore also the exceptional node. Thus the difference between ($\mathcal{M} : \mathcal{N} = 4$) and ($\mathcal{M} : \mathcal{N} < 4$) cases would be that in the Kac-Moody group would reduce to gauge group $\mathcal{M} : \mathcal{N} < 4$ because Kac-Moody central charge k and therefore also Virasoro central charge resulting in Sugawara construction would vanish.

4. Do finite subgroups of $SU(2)$ play some role also in $\mathcal{M} : \mathcal{N} = 4$ case?

One can ask wonder the possible interpretation for the appearance of extended Dynkin diagrams in ($\mathcal{M} : \mathcal{N} = 4$) case. Do finite subgroups $G \subset SU(2)$ associated with extended Dynkin diagrams appear also in this case. The formal analog for $H \to G_a \times G_b$ bundle structure would be $H \to H/G_a \times SU(2)$. This would mean that the geodesic sphere of CP_2 would define the fiber. The notion of number theoretic braid meaning a selection of a discrete subset of algebraic points of the geodesic sphere of CP_2 suggests that $SU(2)$ actually reduces to its subgroup G also in this case.

5. Why Kac-Moody central charge can be non-vanishing only for $\mathcal{M} : \mathcal{N} = 4$?

From the physical point of view the vanishing of Kac-Moody central charge for $\mathcal{M} : \mathcal{N} < 4$ is easy to understand. If parton corresponds to a homologically non-trivial geodesic sphere, space-time surface typically represents a string like object so that the generation of Kac-Moody central extension would relate directly to the homological non-triviality of partons. For instance, cosmic strings are string like objects of form $X^2 \times Y^2$, where X^2 is minimal surface of M^2 and Y^2 is a holomorphic sub-manifold of CP_2 reducing to a homologically non-trivial geodesic sphere in the simplest situation. A conjecture that deserves to be shown wrong is that central charge k is proportional/equal to the absolute value of the homology (Kähler magnetic) charge h.

6. More general situation

McKay correspondence generalizes also to the case of subgroups of higher-dimensional Lie groups [A8]. The argument above makes sense also for discrete subgroups of more general compact Lie groups H since also they define unique sub-factors. In this case, algebras having Cartan algebra with nk generators, where n is the dimension of Cartan algebra of H, would emerge in the process. Thus there are reasons to believe that TGD could emulate practically any dynamics having gauge group or Kac-Moody type symmetry. An interesting question concerns the interpretation of non-ADE type principal graphs associated with subgroups of SU(2).

7. Flavor groups of hadron physics as a support for HFF?

The deformation assigning to an n-fold tensor power of representations of Lie group G with k-dimensional Cartan algebra a representation of a Lie group with nk-dimensional Cartan algebra could be also seen as a dynamically generated symmetry. If quantum measurement is characterized by the choice of Lie group G defining measured quantum numbers and defining Jones inclusion characterizing the measurement resolution, the measurement process itself would generate these dynamical symmetries. Interestingly, the flavor symmetry groups of hadron physics cannot be justified from the structure of the standard model having only electro-weak and color group as fundamental symmetries. In TGD framework flavor group $SU(n)$ could emerge naturally as a fusion of n quark doublets to form a representation of $SU(n)$.

3.4 Farey Sequences, Riemann Hypothesis, Tangles, And TGD

Farey sequences allow an alternative formulation of Riemann Hypothesis and subsequent pairs in Farey sequence characterize so called rational 2-tangles. In TGD framework Farey sequences relate very closely to dark matter hierarchy, which inspires *"Platonia as the best possible world in the sense that cognitive representations are optimal"* as the basic variational principle of mathematics. This variational principle supports RH.

Possible TGD realizations of tangles, which are considerably more general objects than braids, are considered. One can assign to a given rational tangle a rational number a/b and the tangles labeled by a/b and c/d are equivalent if $ad - bc = \pm 1$ holds true. This means that the rationals in question are neighboring members of Farey sequence. Very light-hearted guesses about possible generalization of these invariants to the case of general N-tangles are made.

3.4.1 Farey sequences

Some basic facts about Farey sequences [A1] demonstrate that they are very interesting also from TGD point of view.

1. Farey sequence F_N is defined as the set of rationals $0 \leq q = m/n \leq 1$ satisfying the conditions $n \leq N$ ordered in an increasing sequence.
2. Two subsequent terms a/b and c/d in F_N satisfy the condition $ad - bc = 1$ and thus define and element of the modular group $SL(2, Z)$.
3. The number $|F(N)|$ of terms in Farey sequence is given by

$$|F(N)| = |F(N-1)| + \phi(N-1) \ . \tag{3.4}$$

   Here $\phi(n)$ is Euler's totient function giving the number of divisors of n. For primes one has $\phi(p) = 1$ so that in the transition from p to $p+1$ the length of Farey sequence increases by one unit by the addition of $q = 1/(p+1)$ to the sequence.

The members of Farey sequence F_N are in one-one correspondence with the set of quantum phases $q_n = exp(i2\pi/n)$, $0 \leq n \leq N$. This suggests a close connection with the hierarchy of Jones inclusions, quantum groups, and in TGD context with quantum measurement theory with finite measurement resolution and the hierarchy of Planck constants involving the generalization of the imbedding space. Also the recent TGD inspired ideas about the hierarchy of subgroups of the rational modular group with subgroups labeled by integers N and in direct correspondence with the hierarchy of quantum critical phases [K3] would naturally relate to the Farey sequence.

3.4.2 Riemann Hypothesis and Farey sequences

Farey sequences are used in two equivalent formulations of the Riemann hypothesis. Suppose the terms of F_N are $a_{n,N}$, $0 < n \leq |F_N|$. Define

$$d_{n,N} = a_{n,N} - \frac{n}{|F_N|} \ .$$

In other words, $d_{n,N}$ is the difference between the n: th term of the N: th Farey sequence, and the n: th member of a set of the same number of points, distributed evenly on the unit interval. Franel and Landau proved that both of the following statements

$$\sum_{n=1,\ldots,|F_N|} |d_{n,N}| = O(N^r) \text{ for any } r > 1/2 \ ,$$
$$\sum_{n=1,\ldots,|F_N|} d_{n,N}^2 = O(N^r) \text{ for any } r > 1 \ . \tag{3.5}$$

are equivalent with Riemann hypothesis.

One could say that RH would guarantee that the numbers of Farey sequence provide the best possible approximate representation for the evenly distributed rational numbers $n/|F_N|$.

3.4.3 Farey sequences and TGD

Farey sequences seem to relate very closely to TGD.

1. The rationals in the Farey sequence can be mapped to the roots of unity by the map $q \rightarrow exp(i2\pi q)$. The numbers $1/|F_N|$ are in turn mapped to the numbers $exp(i2\pi/|F_N|)$, which are also roots of unity. The statement would be that the algebraic phases defined by Farey sequence give the best possible approximate representation for the phases $exp(in2\pi/|F_N|)$ with evenly distributed phase angle.

2. In TGD framework the phase factors defined by F_N corresponds to the set of quantum phases corresponding to Jones inclusions labeled by $q = exp(i2\pi/n)$, $n \leq N$, and thus to the N lowest levels of dark matter hierarchy. There are actually two hierarchies corresponding to M^4 and CP_2 degrees of freedom and the Planck constant appearing in Schrödinger equation corresponds to the ratio n_a/n_b defining quantum phases in these degrees of freedom. $Z_{n_a \times n_b}$ appears as a conformal symmetry of "dark" partonic 2-surfaces and with very general assumptions this implies that there are only in TGD Universe [K3, K2].

3. The fusion of physics associated with various number fields to single coherent whole requires algebraic universality. In particular, the roots of unity, which are complex algebraic numbers, should define approximations to continuum of phase factors (angle is not well-defined notion p-adically but trigonometric functions are if algebraic extensions involving roots of unity are allowed). The S-matrix associated with p-adic-to-padic transitions can involve only this kind of algebraic phases. One can also say that cognitive representations can involve only algebraic phases and algebraic numbers in general. For completions of algebraic extensions of rationals U-matrix, M-matrix and S-matrix would be obtained by algebraic continuation from from that in the extension of rationals. One can also say that in the intersection all parameters belong to an extension of rationals and various transition amplitudes have parameters in this intersection. The core of physics (its "genes") would be number theoretically universal [**?**]

4. The subgroups of the hierarchy of subgroups of the modular group with rational matrix elements are labeled by integer N and relate naturally to the hierarchy of Farey sequences. The hierarchy of quantum critical phases is labeled by integers N with quantum phase transitions occurring only between phases for which the smaller integer divides the larger one [K3].

3.4.4 Interpretation of RH in TGD framework

Number theoretic universality of physics suggests an interpretation for the Riemann hypothesis in TGD framework. RH would be equivalent to the statement that the Farey numbers provide best possible approximation to the set of rationals $k/|F_N|$ or to the statement that the roots of unity contained by F_N define the best possible approximation for the roots of unity defined as $exp(ik2\pi/|F_N|)$ with evenly spaced phase angles. The roots of unity allowed by the lowest N levels of the dark matter hierarchy allows the best possible approximate representation for algebraic phases represented exactly at $|F_N|$: th level of hierarchy.

A stronger statement would be that the Platonia, where RH holds true would be the best possible world in the sense that algebraic physics behind the cognitive representations would allow the best possible approximation hierarchy for the continuum physics (both for numbers in unit interval and for phases on unit circle). Platonia with RH would be cognitive paradise.

One could see this also from different view point. "Platonia as the cognitively best possible world" could be taken as the "axiom of all axioms": a kind of fundamental variational principle of mathematics. Among other things it would allow to conclude that RH is true: RH must hold true either as a theorem following from some axiomatics or as an axiom in itself.

3.4.5 Could rational N-tangles exist in some sense?

The article of Kauffman and Lambropoulou [A6] about rational 2-tangles having commutative sum and product allowing to map them to rationals is very interesting from TGD point of view. The illustrations of the article are beautiful and make it easy to get the gist of various ideas. The theorem of the article states that equivalent rational tangles giving trivial tangle in the product correspond to subsequent Farey numbers a/b and c/d satisfying $ad - bc = \pm 1$ so that the pair defines element of the modular group SL(2, Z).

1. Rational 2-tangles

1. The basic observation is that 2-tangles are 2-tangles in both "s- and t-channels". Product and sum can be defined for all tangles but only in the case of 2-tangles the sum, which in this case reduces to product in t-channel obtained by putting tangles in series, gives 2-tangle. The so called rational tangles are 2-tangles constructible by using addition of $\pm[1]$ on left or right of tangle and multiplication by $\pm[1]$ on top or bottom. Product and sum are commutative for rational 2-tangles but the outcome is not a rational 2-tangle in the general case. One can also assign to rational 2-tangle its negative and inverse. One can map 2-tangle to a number which is rational for rational tangles. The tangles $[0]$, $[\infty]$, $\pm[1]$, $\pm 1/[1]$, $\pm[2]$, $\pm[1/2]$ define so called elementary rational 2-tangles.

2. In the general case the sum of $M-$ and $N-$tangles is $M+N-2$-tangle and combines various $N-$tangles to a monoidal structure. Tensor product like operation giving $M+N$-tangle looks to me physically more natural than the sum.

3. The reason why general 2-tangles are non-commutative although 2-braids obviously commute is that 2-tangles can be regarded as sequences of $N-$tangles with 2-tangles appearing only as the initial and final state: N is actually even for intermediate states. Since $N > 2$-braid groups are non-commutative, non-commutativity results. It would be interesting to know whether braid group representations have been used to construct representations of $N-$tangles.

2. Does generalization to $N >> 2$ case exist?

One can wonder whether the notion of rational tangle and the basic result of the article about equivalence of tangles might somehow generalize to the $N > 2$ case.

1. Could the commutativity of tangle product allow to characterize the $N > 2$ generalizations of rational 2-tangles. The commutativity of product would be a space-time correlate for the commutativity of the S-matrices defining time like entanglement between the initial and final quantum states assignable to the N-tangle. For 2-tangles commutativity of the sum would have an analogous interpretation. Sum is not a very natural operation for N-tangles for $N > 2$. Commutativity means that the representation matrices defined as products of braid group actions associated with the various intermediate states and acting in the same representation space commute. Only in very special cases one can expect commutativity for tangles since commutativity is lost already for braids.

2. The representations of 2-tangles should involve the subgroups of N-braid groups of intermediate braids identifiable as Galois groups of N: th order polynomials in the realization as number theoretic tangles. Could non-commutative 2-tangles be characterized by algebraic numbers in the extensions to which the Galois groups are associated? Could the non-commutativity reflect directly the non-commutativity of Galois groups involved? Quite generally one can ask whether the invariants should be expressible using algebraic numbers in the extensions of rationals associated with the intermediate braids.

3. Rational 2-tangles can be characterized by a rational number obtained by a projective identification $[a,b]^T \rightarrow a/b$ from a rational 2-spinor $[a,b]^T$ to which SL(2(N-1), Z) acts. Equivalence means that the columns $[a,b]^T$ and $[c,d]^T$ combine to form element of SL(2, Z) and thus defining a modular transformation. Could more general 2-tangles have a similar representation but in terms of algebraic integers?

4. Could N-tangles be characterized by $N-1$ $2(N-1)$-component projective column-spinors $[a_i^1, a_i^2, .., a_i^{2(N-1)}]^T$, $i = 1, ...N-1$ so that only the ratios $a_i^k/a_i^{2(N-1)} \leq 1$ matter? Could equivalence for them mean that the $N-1$ spinors combine to form $N-1+N-1$ columns of $SL(2(N-1), Z)$ matrix. Could N-tangles quite generally correspond to collections of

projective $N-1$ spinors having as components algebraic integers and could $ad-bc=\pm 1$ criterion generalize? Note that the modular group for surfaces of genus g is SL(2g, Z) so that $N-1$ would be analogous to g and $1 \leq N \geq 3$- braids would correspond to $g \leq 2$ Riemann surfaces.

5. Dark matter hierarchy leads naturally to a hierarchy of modular sub-groups of $SL(2,Q)$ labeled by N (the generator $\tau \to \tau+2$ of modular group is replaced with $\tau \to \tau+2/N$). What might be the role of these subgroups and corresponding subgroups of $SL(2(N-1),Q)$. Could they arise in "anyonization" when one considers quantum group representations of 2-tangles with twist operation represented by an N: th root of unity instead of phase U satisfying $U^2=1$?

3.4.6 How tangles could be realized in TGD Universe?

The article of Kauffman and Lambropoulou stimulated the question in what senses N-tangles could be be realized in TGD Universe as fundamental structures.

1. Tangles as number theoretic braids?

The strands of number theoretical $N-$braids correspond to roots of N: th order polynomial and if one allows time evolutions of partonic 2-surface leading to the disappearance or appearance of real roots $N-$tangles become possible. This however means continuous evolution of roots so that the coefficients of polynomials defining the partonic 2-surface can be rational only in initial and final state but not in all intermediate "virtual" states.

2. Tangles as tangled partonic 2-surfaces?

Tangles could appear in TGD also in second manner.

1. Partonic 2-surfaces are sub-manifolds of a 3-D section of space-time surface. If partonic 2-surfaces have genus $g>0$ the handles can become knotted and linked and one obtains besides ordinary knots and links more general knots and links in which circle is replaced by figure eight and its generalizations obtained by adding more circles (eyeglasses for $N-$eyed creatures).

2. Since these 2-surfaces are space-like, the resulting structures are indeed tangles rather than only braids. Tangles made of strands with fixed ends would result by allowing spherical partons elongate to long strands with fixed ends. DNA tangles would the basic example, and are discussed also in the article. DNA sequences to which I have speculatively assigned invisible (dark) braid structures might be seen in this context as space-like "written language representations" of genetic programs represented as number theoretic braids.

3.5 Only The Quantum Variants Of M^4 And M^8 Emerge FromLocal Hyper-Finite II_1 Factors

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2,F)$ matrix, $F=C,H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \quad , \tag{3.6}$$

read as

$$\begin{array}{ll} ab = qba \ , & ac = qac \ , \quad bd = qdb \ , cd = qdc \ , \\ [ad,da] = (q-q^{-1})bc \ , & bc = cb \ . \end{array} \tag{3.7}$$

These relations can be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a,a^\dagger] = [b,b^\dagger] = [c,c^\dagger] = [d,d^\dagger] = 1 \ . \tag{3.8}$$

The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a-a^\dagger, d-d^\dagger, b-c^\dagger, c-b^\dagger\} \ . \end{array} \tag{3.9}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. This is the case and means also that the operators representing M^4 coordinates commute and it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

$M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be the case. For instance, the replacement $a - \overline{a} \rightarrow a + \overline{a}$ making also time variable Euclidian is impossible since $[a + \overline{a}, d - \overline{d}] = 2(q - q^{-1})bc$ does not vanish. The observation that M^4 coordinates can be regarded as eigenvalues of commuting observables proves that quantum CD and its orbifold description are equivalent.

What about M^8: does it have analogous description? The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anti-commutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

Introduce the coefficients of E^4 gamma matrices having interpretation as quaterionic units as

$$\begin{array}{ll} a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\ a_1 = x(b+c) \ , & a_2 = x(ib-c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

The selections of commutative and quaternionic sub-algebras of octonion space are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of octonions the selection of quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Quaternionic sub-algebra obeys the commutations of $M_q(2, C)$ whereas the coefficients in in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned}
[a_0, a_3] &= -i(q - q^{-1})(a_1^2 + a_2^2) \ , \\
[a_i, a_j] &= 0 \ , \ i, j \neq 0, 3 \ , \\
a_0 a_i &= q a_i a_0 \ , \ i \neq 0, 3 \ , \\
a_3 a_i &= q a_i a_3 \ , \ i \neq 0, 3 \ .
\end{aligned} \tag{3.10}$$

Checking that M^8 indeed corresponds to commutative subspace defined by the eigenvalues of operators is straightforward.

The argument generalizes easily to other dimensions $D \geq 4$ but now quaternionic and octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

Thus the special role of classical number fields and uniqueness of space-time and imbedding space dimensions becomes really manifest only when a quantal deformation of the quaternionic and octonionic matrix algebras is performed. It is possible to construct the quantal variants of the coset spaces $M^4 \times E^4/G_a \times G_b$ by simply posing restrictions on the of eigen states of the commuting coordinate operators. Also the quantum variants of the space-time surface and quite generally, manifolds obtained from linear spaces by geometric constructions become possible.

REFERENCES

Mathematics

[A1] Farey sequence. Available at: http://en.wikipedia.org/wiki/Farey_sequence.

[A2] This Week's Finds in Mathematical Physics: Week 230. Available at: http://math.ucr.edu/home/baez/week230.html.

[A3] Ali A. Types of 2-dimensional $N = 4$ superconformal field theories. *Pramana*, 61(6):1065–1078, 2003.

[A4] Robinson DW Bratteli O. *Operator Algebras and Quantum Statistical Mechanics*. Springer Verlag, New York, 1979.

[A5] Kawahigashi Y Evans D. *Quantum symmetries on operator algebras*. Oxford University Press, New York, 1998.

[A6] Lambropoulou S Kauffman LH. Hard Unknots and Collapsing Tangles. Available at: http://arxiv.org/abs/math/0601525, 2006.

[A7] Lieb EH Temperley NH. Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices:some exact results for the percolation problem. *Proc R Soc London*, 322(1971), 1971.

[A8] Jones V. In and around the origin of quantum groups. Available at: http://arxiv.org/abs/math/0309199, 2003.

Theoretical Physics

[B1] Klebanov IR. TASI Lectures: Introduction to the AdS/CFT Correspondence. Available at: http://arxiv.org/abs/hep-th/0009139, 2000.

Cosmology and Astro-Physics

[E1] Helmi A. Halo streams as relicts from the formation of the Milky Way. Available at: http://arxiv.org/abs/astro-ph/008086, 2000.

Books related to TGD

[K1] Pitkänen M. Appendix A: Quantum Groups and Related Structures. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#bialgebra, 2006.

[K2] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#elvafu, 2006.

[K3] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#quthe, 2006.

[K4] Pitkänen M. Cosmic Strings. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#cstrings, 2006.

[K5] Pitkänen M. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#eegdark, 2006.

[K6] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#mless, 2006.

[K7] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#astro, 2006.

[K8] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#cosmo, 2006.

[K9] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#visionb, 2006.

[K10] Pitkänen M. Topological Quantum Computation in TGD Universe. In *Genes and Memes*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html#tqc, 2006.