

TGD involves geometric and number theoretic physics as complementary views of physics. Almost all basic number fields: rationals and their algebraic extensions, p-adic number fields and their extensions, reals, complex number fields, quaternions, and octonions play a fundamental role in the number theoretical vision of TGD.

Even a hierarchy of infinite primes and corresponding number fields appears. At the first level of the hierarchy of infinite primes, the integer coefficients of a polynomial Q defining infinite prime have no common prime factors. $P = Q$ hypothesis states that the polynomial P defining space-time surface is identical with a polynomial Q defining infinite prime at the first level of hierarchy.

However, finite fields, which appear naturally as approximations of p-adic number fields, have not yet gained the expected preferred status as atoms of the number theoretic Universe. Also additional constraints on polynomials P are suggested by physical intuition.

Here the notions of prime polynomial and concept of infinite prime come to rescue. Prime polynomial P with prime order $n = p$ and integer coefficients smaller than p can be regarded as a polynomial in a finite field. The proposal is that all physically allowed polynomials are constructible as functional composites of prime polynomials satisfying $P = Q$ condition.

One of the long standing mysteries of TGD is why preferred p-adic primes, characterizing elementary particles and even more general systems, satisfy the p-adic length scale hypothesis. The proposal is that p-adic primes correspond to ramified primes as factors of discriminant D of polynomial $P(x)$. $D = P$ condition reducing discriminant to a single prime is an attractive hypothesis for preferred ramified primes. $M^8 - H$ duality suggests that the exponent $\exp(K)$ of Kähler function corresponds to a negative power D^{-k} . Spin glass character of WCW suggests that the preferred ramified primes for, say prime polynomials of a given degree, and satisfying $D = P$, have an especially large degeneracy for certain ramified primes P , which are therefore of a special physical importance.