Langlands' program seeks to relate Galois groups in algebraic number theory to automorphic forms and representation theory of algebraic groups over local fields and adeles. Langlands program is described by Edward Frenkel as a kind of grand unified theory of mathematics.

In the TGD framework, $M^8 - M^4 \times CP_2$ duality assigns to a rational polynomial a set of mass shells H^3 in $M^4 \subset M^8$ and by associativity condition a 4-D surface in M^8, and its it to $H = M^4 \times CP_2$. $M^8 - M^4 \times CP_2$ means that number theoretic vision and geometric vision of physics are dual or at least complementary. This vision could extend to a trinity of number theoretic, geometric and topological views since geometric invariants defined by the space-time surfaces as Bohr orbit-like preferred extremals could serve as topological invariants.

Concerning the concretization of the basic ideas of Langlands program in TGD, the basic principle would be quantum classical correspondence (QCC), which is formulated as a correspondence between the quantum states in the "world of classical worlds" (WCW) characterized by analogs of partition functions as modular forms and classical representations realized as space-time surfaces. L-function as a counter part of the partition function would define as its roots space-time surfaces and these in turn would define via Galois group representation partition function. QCC would define a kind of closed loop giving rise to a hierarchy.

If Riemann hypothesis (RH) is true and the roots of L-functions are algebraic numbers, L-functions are in many aspects like rational polynomials and motivate the idea that, besides rationals polynomials, also L-functions could define space-time surfaces as kinds of higher level classical representations of physics.

One concretization of Langlands program would be the extension of the representations of the Galois group to the polynomials P to the representations of reductive groups appearing naturally in the TGD framework. Elementary particle vacuum functionals are defined as modular invariant forms of Teichmüller parameters. Multiple residue integral is proposed as a manner to obtain L-functions defining space-time surfaces.

One challenge is to construct Riemann zeta and the associated ξ function and the Hadamard product leads to a proposal for the Taylor coefficients c_k of $\xi(s)$ as a function of $s(s-1)$. One would have $c_k = \sum_{i,j} c_{k,ij} e^{i/k} e^{\sqrt{-1}2\pi j/n}$, $c_{k,ij} \in \{0, \pm 1\}$. $e^{1/k}$ is the hyperbolic analogy for a root of unity and defines a finite-D transcendental extension of p-adic numbers and together with n :th roots of unity powers of $e^{1/k}$ define a discrete tessellation of the hyperbolic space H^2.

This construction leads to the question whether also finite fields could play a fundamental role in the number theoretic vision. Prime polynomial with prime order $n = p$ and integer coefficients smaller than $n = p$ can be regarded as a polynomial in a finite field. If it satisfies the condition that the integer coefficients have no common prime factors, it defines an infinite prime. The proposal is that all physically allowed polynomials are constructible as functional composites of these.