

TGD as a Generalized Number Theory

The focus of this book is the number theoretical vision about physics. This vision involves three loosely related parts.

1. The fusion of real physics and various p-adic physics to a single coherent whole by generalizing the number concept by fusing real numbers and various p-adic number fields along common rationals. Extensions of p-adic number fields can be introduced by gluing them along common algebraic numbers to reals. Algebraic continuation of the physics from rationals and their extensions to various number fields (generalization of completion process for rationals) is the key idea, and the challenge is to understand whether how one could achieve this dream. A profound implication is that purely local p-adic physics would code for the p-adic fractality of long length length scale real physics and vice versa, and one could understand the origins of p-adic length scale hypothesis.
2. Second part of the vision involves hyper counterparts of the classical number fields defined as subspaces of their complexifications with Minkowskian signature of metric. Allowed space-time surfaces would correspond to what might be called hyper-quaternionic sub-manifolds of a hyper-octonionic space and mappable to $M^4 \times CP_2$ in natural manner. One could assign to each point of space-time surface a hyper-quaternionic 4-plane which is the plane defined by the modified gamma matrices but not tangent plane in general. Hence the basic variational principle of TGD would have deep number theoretic content.
3. The third part of the vision involves infinite primes identifiable in terms of an infinite hierarchy of second quantized arithmetic quantum fields theories on one hand, and as having representations as space-time surfaces analogous to zero loci of polynomials on the other hand. Single space-time point would have an infinitely complex structure since real unity can be represented as a ratio of infinite numbers in infinitely many manners each having its own number theoretic anatomy. Single space-time point would be in principle able to represent in its structure the quantum state of the entire universe. This number theoretic variant of Brahman=Atman identity would make Universe an algebraic hologram.

Number theoretical vision suggests that infinite hyper-octonionic or -quaternionic primes could correspond directly to the quantum numbers of elementary particles and a detailed proposal for this correspondence is made. Furthermore, the generalized eigenvalue spectrum of the Chern-Simons Dirac operator could be expressed in terms of hyper-complex primes in turn defining basic building bricks of infinite hyper-complex primes from which hyper-octonionic primes are obtained by discrete $SU(3)$ rotations performed for finite hyper-complex primes.

Besides this holy trinity I will discuss loosely related topics. Included are possible applications of category theory in TGD framework; TGD inspired considerations related to Riemann hypothesis; topological quantum computation in TGD Universe; and TGD inspired approach to Langlands program.