

The formula for inversion of the harmonic keeping note X as fixed can be represented as a product of translation taking X to C , inversion keeping C fixed, and translation taking C back to X . The inversion maps the chord having C as basic note to its mirror image so that the order of notes can change and basic note can change. For instance, the major chord $CM = CEG$ goes to minor chord $CG\sharp F = Fm$ so that $k = 0$ goes to $k \equiv \Delta k_{inv} = 11$. This delicacy must be taken into account. If X remains fixed inversion is just the transformation

$$k \rightarrow k_{inv} = (2 \times k(X) - \Delta k_{inv}) \text{ mod } 12 . \quad (0.1)$$

The following table gives the inversion of the scale leaving C (and also $F\sharp$) invariant:

C	G	D	A	E	H	F+	C+	G+	D+	B-	F
C	F	B \flat	D+	G+	C+	F+	H	E	A	D	G

The inversion for the types of the chords does not depend on the basic note as is clear from the distance preserving character of the inversion. The following table gives the inversion of for the types of the chords leaving C fixed. The elements of the rows give the type of the chord and the number of quints k corresponding to it. For chords having C as basic note one has $k = 0$. It is easy to deduce the transformation formula in more general case from the table.

M,0	m,0	sus4,0	aug,0	4,0	9,0	4+,0	9-,0	6-,0	maj7,0
m,11	M,11	sus,0	aug,0	4,0	9,10	9-,11	4+,11	maj7,11	6-,11
6,0	7,0	ex1,0	ex2,0	ex3,0	ex4,0	ex5,0	ex6,0	ex7,0	ex8,0
7,11	6,11	ex1,10	ex3,3	ex2,3	ex4,8	ex6,8	ex5,80	ex8,6	ex7,6

Table 5: Table gives the transformation of inversion leaving C invariant on the basic chords having C as basic note.

The following tables give the chords and corresponding inverse chords for the 11 icosahedral harmonies.

ro6	iro6	re41	ire41	re42	ire42	ro21	iro21
F.aug	F.aug	D.7	A.6	C.ex3	A.ex2	E.m	F.M
G.aug	D+.aug	D.6	A.7	E.ex2	F.ex3	B-.m	B.M
C.m	F.M	G+.7	D+.6	F+.ex3	D+.ex2	C.m	A.M
D.m	D+.M	G+.6	D+.7	B-.ex2	B.ex3	F+.m	D+.M
E.m	C+.M	G.4+	E.9-	D.maj7	B.6-	G.6	D.7
F+.m	B.M	A.9-	D.4+	E.9-	A.4+	C+.6	G+.7
G+.m	A.M	C+.4+	B-.9-	A.7	E.6	A.6	C.7
B-.m	G.M	D+.9-	G+.4+	A.6	E.7	D+.6	F+.7
F.6	C.7	E.maj7	G.6-	G+.maj7	F.6-	D.4+	G.9-
G.6	B-.7	G.maj7	E.6-	B-.9-	D+.4+	G+.4+	C+.9-
A.6	G+.7	B-.maj7	C+.6-	D+.7	B-.6	B.4+	B-.9-
B.6	F+.7	C+.maj7	B-.6-	D+.6	B-.7	F.4+	E.9-
C+.6	E.7	C.9-	B.4+	F.9	D+.9	C.maj7	A.6-
D+.6	D.7	A.9-	D.4+	C.9	G+.9	F+.maj7	D+.6-
C.9	C.9	F+.9-	F.4+	G.9	C+.9	G.6-	D.maj7
D.9	B-.9	D+.9-	G+.4+	E.9	E.9	C+.6-	G+.maj7
E.9	G+.9	B.9	G.9	B.9	A.9	D.9	D.9
F+.9	F+.9	E.9	D.9	F+.9	D.9	G+.9	G+.9
G+.9	E.9	F.9	C+.9	C+.9	G.9	E.9	C.9
B-.9	D.9	B-.9	G+.9	B-.9	B-.9	B-.9	F+.9

Table 6. Pairs“X” and“iX” of columns give the chords of the bio-harmonies and their inversions depicted in figures ??,??, ??,??.

ro22	iro22	ro23	iro23	re21	ir21	re22	ir22
A.ex4	G.ex4	A.ex2	B-.ex3	F+.ex3	D+.ex2	D.ex4	E.ex4
D+.ex2	C.ex3	H.ex8	B-.ex7	H.ex4	B-.ex4	H.ex4	F+.ex4
A.m	B-.M	D+.ex2	E.ex3	A.m	E.M	F.M	E.m
D+.m	E.M	F.ex8	F.ex7	D+.M	B-.m	F.m	E.M
G.9-	C.4+	D.7	A.6	A.6	E.7	C.6-	A.maj7
C+.9-	F+.4+	G+.7	D+.6	D+.7	B-.6	B-.maj7	B.6-
C.4	C.4	A.maj7	D.6-	D.7	B.6	C.9-	A.4+
F+.4	F+.4	D+.maj7	G+.6-	B-.6	D+.7	D.7	G.6
E.4+	D+.9-	A.4+	D.9-	G.6-	F+.maj7	G+.6	C+.7
B-.4+	A.9-	D+.4+	G+.9-	F.maj7	G+.6-	G.maj7	D.6-
D.maj7	F.6-	E.7	G.6	D.4+	B.9-	D+.6-	F+.maj7
G+.maj7	B.6-	B-.7	C+.6	B-.9-	D+.4+	C+.4	C+.4
B.maj7	G+.6-	B-.9	G+.9	G+.4+	F.9-	A.4+	C.9-
F.maj7	D.6-	G.9	B.9	E.9-	A.4+	E.4+	F.9-
C.9	D.9	C+.9	F.9	C.9	G+.9	F+.6	D+.7
F+.9	G+.9	A.9	A.9	F.9	D+.9	D+.9	C+.9
A.9	F.9	B.9	G.9	B.9	A.9	C+.9	D+.9
D+.9	B.9	F.9	C+.9	F+.9	D.9	E.9	C.9
D.9	C.9	E.9	D.9	E.9	E.9	B.9	F.9
G+.9	F+.9	D+.9	D+.9	C+.9	G.9	D+.9	C+.9

Table 7. Pairs“X” and“iX” of columns give the chords of the bio-harmonies and their inversions depicted in figures ??,??, ??,??.

re23	ire23	re24	ire24	re25	ire25		
F.ex1	F.ex1	H.ex3	G.ex2	F+.ex2	F.ex3		
D+.ex3	G+.ex2	E.ex7	F+.ex8	F.ex3	F+.ex2		
G+.ex1	D.ex1	D.7	A.6	F.M	B-.m		
A.ex2	D.ex3	G+.6	D+.7	B-.m	F.M		
E.7	B.6	G-.M	B.m	C.7	D+.6		
E.6	B.7	D+.m	G+.M	G+.6	G.7		
A.maj7	F+.6-	F.M	F+.m	A.maj7	F+.6-		
B.9-	E.4+	F.m	F+.M	B.9-	E.4+		
G.M	G+.m	C.6-	B.maj7	E.6	B.7		
C+.m	D.M	B-.maj7	C+.6-	E.7	B.6		
D.7	C+.6	A.9-	D.4+	G.M	G+.m		
F+.6	A.7	C+.4+	B-.9-	C+.m	D.M		
B-.9	C.9	E.7	G.6	D.7	C+.6		
D.9	G+.9	F+.6	F.7	B.6	E.7		
B.9	B.9	C.9	F+.9	D+.9	G.9		
C.9	B-.9	D+.9	D+.9	C.9	B-.9		
F.9	F.9	D.9	E.9	C+.9	A.9		
G+.9	D.9	C+.9	F.9	B-.9	C.9		
D+.9	G.9	E.9	D.9	D.9	G+.9		
C+.9	A.9	B.9	G.9	H.9	B-.9		

Table 8. Pairs“X” and“iX” of columns give the chords of the bio-harmonies and their inversions depicted in figures ??,??, ??.